Is Labor Supply Important for Business Cycles? *

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October 2011

Abstract

We build a general equilibrium model that features idiosyncratic shocks, search frictions and an operative labor supply choice along the extensive margin. We use this model to study the implications of several aggregate shocks for the behavior of labor market aggregates and flows, and in particular the role of labor supply. While shocks to only job finding and job loss rates can account for unemployment fluctuations, the presence of labor supply responses implies that they account for only a small fraction of employment fluctuations and have counterfactual predictions for participation. A model that features shocks to the return to market activity in addition to the job finding and job loss rates accounts for fluctuations in employment and unemployment as well as the main patterns found in the labor market flows. Employment fluctuations in this model are driven by labor supply responses.

Keywords: Labor Supply, Labor Market Frictions, Business Cycles

JEL Classifications: E24, J22, J64

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*We thank seminar participants as well as conference participants at the Asian Meeting of the Econometric Society (2011), Chicago Fed, Hitotsubashi Macro Econometrics Conference, National University of Singapore, Search and Matching Network Conference (2011), NBER Summer Institute (2011), NBER Conference on Macroeconomics Across Time and Space (2011), SED (2011), Oslo University, Norges Bank, European University Institute, Bank of Korea, and St. Louis Fed. We thank Joe Song for excellent research assistance. Krusell thanks the NSF for financial support, Mukoyama thanks the Bankard Fund for Political Economy for financial support, and Rogerson thanks the NSF and the Korean Science Foundation (WCU-R33-10005) for financial support. The views expressed in this article are those of the authors and do not necessarily reflect those of the Federal Reserve Bank of New York or the Federal Reserve System. Corresponding author: Richard Rogerson. Address for correspondence: Woodrow Wilson School, Princeton University, Princeton, NJ 08544. Phone: 609-258-4839 Fax: 609-258-5349. Email: rdr@princeton.edu

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1 Introduction

The idea that labor supply plays a key role in accounting for business cycle fluctuations has been challenged ever since it was first introduced by Lucas and Rapping (1969). An early issue was the apparent inconsistency between the relatively low Frisch labor supply elasticity estimated from micro data and the relatively high value required to generate substantial labor supply effects in a representative agent model. It is now known that explicit modelling of adjustment along the intensive and extensive margins can account for much of this discrepancy.

A second challenge concerns the explicit consideration of unemployment. While this critique has been around for a long time, it was recently revived in a more formal manner in the context of modern unemployment theory by Hall (2009). Hall (2009) argues that although aggregate models featuring labor supply choices along the extensive margin can generate large fluctuations in employment via labor supply responses, they are inconsistent with the US evidence that roughly 80% of business cycle fluctuations in employment show up as offsetting changes in unemployment rather than changes in participation.

This leads Hall to consider a model which exogenously eliminates labor supply choice along the extensive margin, assuming instead that the labor force is fixed and that employment is completely determined by the level of search frictions, as captured by the job offer arrival rate facing unemployed workers. In this paper we challenge the conclusion that explicit consideration of empirical evidence on unemployment and participation implies a
minimal role for labor supply. In particular, our model featuring labor supply and frictions is consistent with business cycle fluctuations in employment, unemployment and participation at the same time that employment fluctuations are driven by labor supply responses.

To make this argument we build a new model of business cycle fluctuations in the labor market. There is widespread agreement that a good theory of labor market fluctuations should have solid microfoundations. Following more than forty years of applied research on labor market dynamics, we believe that the appropriate microfoundations in this context should include a theory of gross worker flows among the three labor market states of employment, unemployment and non-participation. In effect, we go one step further than Hall, in that we require our model to match the gross worker flows as opposed to simply the net flows.

A key building block of our framework is a model of individual labor supply in the presence of uncertainty, frictions and incomplete markets. When embedded into an otherwise standard aggregate model, it yields a theory of gross labor market flows. Our model relaxes two problematic features of the early indivisible labor models of Hansen (1985) and Rogerson (1988); it rules out trade in employment lotteries, thereby forcing individuals to self insure by accumulating assets, and it assumes that individuals are heterogeneous. While this sort of market incompleteness has been found to not matter much for some business cycle properties, we note that Galí et al (2011) find it to be very important for the cyclical properties of participation. It also extends these models to allow for search frictions, thereby allowing us
to separate the unemployed from the nonparticipants.

A prominent feature of our model, in contrast to Hall (2009) and many other business cycle analyses of unemployment, is that it does not exogenously assume that there are no fluctuations in participation. Even though fluctuations in participation in the US are a relatively small component of fluctuations in employment, there are several reasons to think that shutting them down exogenously is problematic in the context of the effort to understand the underlying forces that shape labor market fluctuations.

First, to the extent that fluctuations in participation are a part of fluctuations in employment, it is counterfactual. But more importantly, even if participation did not change at all over the business cycle, it is surely important to understand why. The fact that many individuals move between participation and non-participation each month suggests that this is surely an operative margin for many individuals. Moreover, there are many examples of large responses in participation over time at the aggregate level.\footnote{An obvious example is the large increase in female participation rates over time. But other examples include a large decline in participation rates for low-skilled males, as well as a decrease for all older males prior to 1995 followed by a subsequent increase.} Second, as we detail later in the paper, the relative constancy of the aggregate participation rate hides the fact there are large changes in the flows between participation and non-participation over the business cycle that turn out to mostly offset each other. Third, although participation plays a relatively small role in the US in terms of statistically accounting for employment fluctuations, this is much less true for some other OECD economies.\footnote{See for example Rogerson and Shimer (2010).} Additionally, the statistical importance
of participation in accounting for employment fluctuations varies across demographic groups within the US. One cannot understand the source of these differences if one fixes participation exogenously.

As is standard in the modern business cycle literature, we calibrate our model so as to be consistent with the average behavior of the US economy, including the average gross worker flows between the three labor market states. We then use our calibrated model to study the effects of different aggregate shocks on standard labor market aggregates and gross worker flows. Two key findings emerge.

First, we find that abstracting from labor supply along the extensive margin (i.e., participation) leads one to dramatically overstate the extent to which fluctuations in job finding and job loss rates can account for fluctuations in both employment and unemployment. Specifically, even if fluctuations in these frictions are large enough to account for observed fluctuations in unemployment, labor supply responses significantly dampen their effect on employment. Closely related, fluctuations in frictions alone generate fluctuations in participation that are counterfactually large and have the wrong correlation with output. The key message is that even in a context in which labor supply is not the main driving force behind fluctuations, the presence of an empirically plausible labor supply response along the extensive margin has important implications for how labor market variables respond to shocks. Put somewhat differently, exogenously shutting down the participation margin hides the fact that this specification contains quantitatively important forces with counterfactual
Second, we show that a specification with shocks to both the return to market activity and job finding and job loss rates can account for many of the patterns found in the data. Specifically, it not only accounts for fluctuations in the levels of employment, unemployment and out of the labor force, but also for fluctuations in the gross flows of individuals among these three states. In the context of this specification we can decompose the resulting fluctuations into the contribution attributed to each shock. We find that the dominant source of employment fluctuations is the shock to the return to overall market activity, whereas the dominant source of unemployment fluctuations is the shock to the job offer arrival rate. Shocks to the job loss rate play only a minor role overall. Moreover, shocks to the job finding rate account for much of the fluctuations in measured flows from employment to unemployment. Importantly, the key channel through which shocks to the return to market activity influence employment is through labor supply. The implication is that our model can match the cyclical properties of employment and unemployment at the same time that labor supply is the dominant source of employment fluctuations. In other words, the relative constancy of the participation rate over the business cycle is not evidence against the importance of labor supply as a driving force behind aggregate fluctuations.

Beyond these substantive findings regarding the role of labor supply for business cycle fluctuations, our paper offers a contribution to the broader literature on building cyclical models of labor market flows. In particular, we present a benchmark model for analysing
flows across all three labor market states and deliver a specification that can match the key features of these flows over the cycle.

Given its focus on business cycle fluctuations in an aggregate three state model of the labor market, our analysis is closely related to recent similar analyses by Tripier (2004), Veracierto (2008), Galí (2011a, 2011b), Galí et al (2011), Christiano et al (2010) and Shimer (2011). Relative to these authors, one key distinction of our analysis is its focus on gross flows. Each of these papers focuses exclusively on the fluctuations in the levels of $E$, $U$ and $N$ and pays no attention to the properties of the underlying flows, either in steady state or over the cycle. Our setting also places stronger emphasis on individual heterogeneity and the uninsurability of idiosyncratic shocks. Relative to Tripier and Veracierto, we allow for shocks to labor market frictions. Relative to Shimer (2011), the job finding frictions are assumed exogenous here; thus, we focus on the economy’s response to cyclical variations in job finding and job loss rates as opposed to their origins.

An outline of the paper follows. In the next section we document the key business cycle facts for gross worker flows among the three labor market states for the US over the period 1968-2009. Section 3 describes our theoretical framework. Section 4 presents our quantitative results on the effects of different aggregate shocks on gross worker flows and Section 5 concludes.
2 Business Cycle Fluctuations in the Labor Market

The early real business cycle literature focused on a small set of second moments that were thought to represent first order properties of business cycle fluctuations. In particular, this literature sought to understand the relative volatility and comovement between aggregate series such as output, consumption \((C)\), investment \((I)\), and employment \((E)\). Table 1 presents these statistics for the US economy over the period 1968-2009. We limit ourselves to this time period since this is the period for which we have consistent measures of the labor market statistics on which we will focus.

<table>
<thead>
<tr>
<th></th>
<th>US Business Cycle Facts for Aggregates</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>(\text{std}(x)/\text{std}(Y))</td>
</tr>
<tr>
<td>(C)</td>
<td>.81</td>
</tr>
<tr>
<td>(I)</td>
<td></td>
</tr>
<tr>
<td>(E)</td>
<td></td>
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As is standard, the raw quarterly data has been logged and HP filtered using a smoothing parameter of 1600 in order to isolate the cyclical component of the time series. Note that we present data for employment rather than total hours for the reason that our subsequent analysis will impose indivisible labor and focus solely on adjustment along the extensive margin. This table confirms the standard set of stylized facts regarding business cycle: fluctuations in output, consumption, investment and employment are persistent and highly positively correlated, with investment fluctuating more than output, whereas consumption and employment fluctuate somewhat less than output.\(^4\)

\(^3\) As is well known, roughly two-thirds of the fluctuations in total hours are accounted for by fluctuations along the extensive margin. See, for example, Hansen (1985).

\(^4\) If we limit attention to nondurable consumption expenditure then the relative volatility of consumption is reduced by almost half.
While focusing on this small set of second moments has proven to be a very useful device for focusing and organizing quantitative research on business cycles, such a small set of moments necessarily captures only a small subset of business cycle properties. In order to better understand and distinguish between the forces that shape business cycle fluctuations in the labor market we expand the set of labor market statistics under consideration along two dimensions. First, as in recent work by Galí (2011a, 2011b), Galí et al (2011), Christiano et al (2010) and Shimer (2011), we include information on how non-employed workers are categorized between the states of unemployment and out of the labor force. Second, and differently than the studies just mentioned, we consider not only the behavior of the stocks of workers in each of the three labor market states but also the gross flows of workers between states. From the perspective of connecting an aggregate model of the labor market to micro data, we believe the behavior of individual labor market flows is of first order importance.

Table 2 presents data for average monthly transition rates among the three labor market states: employment ($E$), unemployment ($U$), and not in the labor force ($N$). Details regarding data sources and construction are provided in the appendix.

<table>
<thead>
<tr>
<th>FROM</th>
<th>TO</th>
<th></th>
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<tbody>
<tr>
<td>$E$</td>
<td>0.954</td>
<td>0.016</td>
</tr>
<tr>
<td>$U$</td>
<td>0.270</td>
<td>0.508</td>
</tr>
<tr>
<td>$N$</td>
<td>0.048</td>
<td>0.027</td>
</tr>
</tbody>
</table>

The key feature of this data that we want to stress is that the flows among all of the states are all very large. While a large literature has previously emphasized the high transition rates
between $E$ and $U$, we specifically want to highlight the large flows into and out of the labor force. In particular, Table 2 shows that in each month, more than 7% of the workers who are out of the labor force will enter the labor force in the subsequent month. As we detail later in the paper, part of the measured flows between $N$ and $U$ may be the result of classification error. But even if we focus only on the flow of workers from $N$ to $E$ we see that this is almost five percent per month. The key point is that the decision to move between participation and non-participation seems an operative margin for a large number of individuals.

Table 3 presents the richer set of labor market business cycle statistics that will be the focus of our study. Note that $u$ denotes the unemployment rate $(U/(E+U))$ and $lfpr$ is the labor force participation rate $((E+U)/(E+U+N))$.

<table>
<thead>
<tr>
<th></th>
<th>$u$</th>
<th>$lfpr$</th>
<th>$f_{EU}$</th>
<th>$f_{EN}$</th>
<th>$f_{UE}$</th>
<th>$f_{UN}$</th>
<th>$f_{NE}$</th>
<th>$f_{NU}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$std(x)/std(Y)$</td>
<td>7.6</td>
<td>.21</td>
<td>5.4</td>
<td>2.0</td>
<td>4.9</td>
<td>3.8</td>
<td>2.7</td>
<td>4.0</td>
</tr>
<tr>
<td>$corrcoef(x,Y)$</td>
<td>-.87</td>
<td>.46</td>
<td>-.82</td>
<td>.33</td>
<td>.78</td>
<td>.78</td>
<td>.64</td>
<td>-.70</td>
</tr>
<tr>
<td>$corrcoef(x,x-1)$</td>
<td>.92</td>
<td>.72</td>
<td>.73</td>
<td>.20</td>
<td>.84</td>
<td>.73</td>
<td>.41</td>
<td>.75</td>
</tr>
</tbody>
</table>

Our notational convention is that $f_{ij}$ denotes the fraction of workers that moved from state $i$ in the previous period to state $j$ in the current period. Note that although labor market data are available at monthly frequency, NIPA data is available only at quarterly frequency. In order to evaluate the correlation of labor market flows with output we have generated labor market flow data at quarterly frequency by setting the quarterly value equal to the average of the three monthly values within the quarter. To produce business cycle statistics we then log and HP filter the data.
Several features are worth noting. Consider first the behavior of the transition rates between $E$ and $U$. Over our sample period, these two rates exhibit roughly equal volatility, are very persistent and are strongly correlated with the cycle, with $f_{EU}$ being strongly countercyclical and $f_{UE}$ being strongly procyclical. Second, fluctuations in the unemployment rate are more than an order of magnitude larger than fluctuations in the participation rate. The former is strongly countercyclical whereas the latter is weakly procyclical. It follows that most of the cyclical fluctuations in employment show up as offsetting changes in unemployment as opposed to changes in participation.

While these patterns are relatively well-known, there are some other patterns that are less well known. For example, although the stock of non-participants does not vary that much over the business cycle, the flows between non-participation and the other states exhibit pronounced movements at business cycle frequencies. For example, whereas the fluctuations in the participation rate are an order of magnitude smaller than the fluctuations in the unemployment rate, the fluctuations in the flows into and out of non-participation are of roughly the same order of magnitude as the fluctuations in the flows between $E$ and $U$ that have received so much attention. There is also an interesting pattern regarding the cyclicality of the flows between $U$ and $N$. Specifically, the flow from $U$ to $N$ has the same cyclical correlation as the flow from $U$ into $E$, and the flow from $N$ into $U$ has the same

\footnote{Note that there are some differences between the statistics that we report here and those that are used in several studies. For example, Shimer (2005) focuses on the flows into and out of unemployment without distinguishing whether the workers that enter unemployment come from employment or out of the labor force, or whether the workers that leave unemployment leave to employment or out of the labor force.}
cyclical pattern as the flow from $E$ into $U$. That is, both rates of flow into $U$ have the same cyclical properties, as do both rates of flow out of $U$.

The flow from $N$ into $E$ is strongly procyclical, just as is the flow from $U$ into $E$, though it is somewhat less volatile. If one were to only look at these two flows, one would not be lead to infer that the participation rate plays only a minor role in accounting for employment fluctuations. The reason that the stock of participation does not move more over the cycle is because of the offsetting effect of an increased $U$ to $N$ flow during good times. Finally, the flow rate from $E$ to $N$ is countercyclical, though only weakly so and exhibits less volatility than the other flows.

3 Theory

As is standard in the business cycle literature, we consider business cycles as resulting from aggregate shocks that cause the economy to fluctuate around the steady-state equilibrium that would result in the absence of aggregate shocks. We first describe our model.

3.1 Economy

At its core, the model captures the problem of optimal labor supply in the presence of frictions. Consumer-workers thus go through life, modeled here as lasting forever, facing a variety of shocks, and they are imperfectly able to insure against these shocks. One risk concerns unemployment, either as a result of failed job search or because of an involuntary layoff without immediate reemployment. Another risk concerns the real wage and its fluc-
tuations. Both of these risks have idiosyncratic and aggregate components: job finding and job loss rates fluctuate over time, and real wages do too, but with significant wage dispersion due to individual wage shocks. In addition to developing a framework appropriate for the issue at hand, our broader aim is to provide a rather comprehensive and realistic description of the main economic events for a household as it goes through time, including both significant discrete shocks such as job loss and gradual changes in wages. We thus formulate a model that arguably captures the key income-relevant events for a household and captures its main vehicles for dealing with these events: it can hold an asset, although restrictedly, thus being able to insure itself but only to some extent. One advantage of this approach is that it allows us to build a solid connection with micro data and the rich set of studies on individuals; we will use this in the calibration of our model. We believe that our model is the first macroeconomic model with these features.

Abstracting from frictions, our setting represents a canonical dynamic model of labor supply in a context that features idiosyncratic shocks and incomplete markets for both credit and risk-sharing.\textsuperscript{6} Krusell et al (2011) showed that idiosyncratic shocks are an essential element in allowing the model to match the key features of average labor market flows. Although that paper also showed that incomplete markets per se is not essential in matching the key features of average flows, the recent work of Gali et al (2011) shows that the assumption of

\textsuperscript{6}Several papers have recently analyzed labor supply in models with idiosyncratic shocks and incomplete markets, including Floden and Linde (2001), Low (2005), Domeij and Floden (2006), Pijoan-Mas (2006), and Chang and Kim (2006, 2007). Relative to these papers the distinguishing feature of our model is the presence of frictions.
complete markets does lead to counterfactual predictions for the behavior of participation rates in the context of business cycle analysis. For this reason we think that it is important to include incomplete markets as a feature of the model. Our model extends this standard model of labor supply to allow for the two key frictional shocks that characterize steady state labor market outcomes in almost any search and matching model: a separation shock for employed individuals, and an offer arrival shock for non-employed individuals. While the model is somewhat minimalist, our earlier work shows that it is capable of accounting for the average behavior of labor market flows, therefore making it a natural benchmark for an initial quantitative assessment of how frictions interact with labor supply in the business cycle context.

The economy is populated by a continuum of workers with total mass equal to one. All workers have identical preferences over streams of consumption and time devoted to work given by:

$$E_t \sum_{t=0}^{\infty} \beta^t [\log(c_t) - \alpha e_t]$$

where $c_t \geq 0$ is consumption in period $t$, $e_t \in \{0, 1\}$ is time devoted to work in period $t$, $0 < \beta < 1$ is the discount factor and $\alpha > 0$ is the disutility of work. Individuals are subject to idiosyncratic shocks that affect the static payoffs of working in the market relative to not working. While many distinct shocks may have this feature, including, for example, shocks to market opportunities, shocks to home production opportunities, health shocks, family shocks, preference shocks etc..., we represent the net effect of all of these shocks as a single
shock, and model it as a shock to the return to market work.\textsuperscript{7} In particular, letting $z_t$ denote the quantity of labor services that an individual contributes if working, we assume an $AR(1)$ stochastic process in logs:

$$\log z_{t+1} = \rho \log z_t + \varepsilon_{t+1}$$

where the innovation $\varepsilon_t$ is a mean zero normally distributed random variable with standard deviation $\sigma_\varepsilon$. This process is the same for all workers, but realizations are $iid$ across workers.

Frictions in the labor market are captured by two exogenous variables: $\lambda_t$ and $\sigma_t$, where $\lambda$ is the employment opportunity arrival rate and $\sigma$ is the employment separation rate. These values are common to all individuals but may fluctuate randomly over time, though we will often suppress the time subscript in describing the model. To understand how the frictions impact the economy it is useful to think that the economy consists of two islands: a production island and a non-production island. An individual begins period $t$ on the production island if $e_{t-1} = 1$, and otherwise begins on the non-production island. Next, each individual will observe the realizations of several shocks. First, each worker receives a new realization for the value of their idiosyncratic productivity shock and the aggregate shock $Z_t$, which will be described below, is realized. Second, each individual on the production island observes the realization of an $iid$ separation shock: with probability $\sigma$ the individual is relocated to the non-production island. Third, each individual who is now on the non-production island, either because they started the period there or were relocated there by the separation shock,

\textsuperscript{7}Alternatively, we could have assumed that the only shock is to the disutility of working. Either way, the key economic mechanism is that the shock serves to change the relative return to working versus not working.
observes the realization of an \( iid \) employment opportunity shock that with probability \( \lambda \) relocates them to the production island.

Note that given our timing assumption, an individual who suffers the \( \sigma \) shock will not necessarily spend a period in nonemployment. Once the shocks have been realized, individuals make their labor supply and consumption decisions, though only individuals on the production island can choose \( e \) equal to 1. An individual on the production island who chooses not to work will then be relocated to the leisure island at the end of period \( t \) and will therefore not have the opportunity to return to the production island until receiving a favorable employment opportunity shock.

The production technology is described by a Cobb-Douglas aggregate production function:

\[
Y_t = Z_t K_t^\theta L_t^{1-\theta}.
\]

where \( K_t = \int k_{it} di \) is aggregate input of capital services and \( L_t = \int e_{it} z_{it} di \) is aggregate input of labor services. \( Z_t \) is a standard technology shock. Output can be used either as consumption or investment, and capital depreciates at rate \( \delta \).

### 3.2 Equilibrium

We formulate equilibrium recursively. In each period there are markets for output, capital services and labor services, but there are no insurance markets, so individuals will (potentially) accumulate assets to self-insure. We normalize the price of output to equal one in all periods, and let \( r_t \) and \( w_t \) denote rental rates for a unit of capital and a unit of labor services, respectively. If a worker with productivity \( z_t \) chooses to work then he or she earns
$w_t z_t$ in labor income. We assume that individual capital holdings must be nonnegative, or equivalently, that individuals are not allowed to borrow. We include a simple and stylized tax and transfer program in the model to capture the fact the presence of such programs in reality. The government taxes labor income at constant rate $\tau$ and uses the proceeds to finance a lump-sum transfer payment $T_t$ subject to a period-by-period balanced budget constraint. The one period budget equation for an individual with $k_t$ units of capital and productivity $s_t$ is given by:

$$c_t + k_{t+1} = r_t k_t + (1 - \tau) w_t z_t e_t + (1 - \delta) k_t + T_t.$$  

An individual’s state consists of his or her location at the time that the labor supply decision needs to be made, the level of asset holdings, and productivity. The aggregate state will include any information that individuals have and is useful in forecasting prices. We denote this information by $\Omega$. In this model, $\Omega$ includes both exogenous aggregate shocks $(Z, \lambda, \text{and } \sigma)$ and the distribution of wealth and labor-market status across all individuals. Since the notion of recursive competitive equilibrium that we employ is standard we do not include a formal definition of the consistency conditions, in particular for how $\Omega$ evolves over time.\(^8\) We use the notation $\Omega'(\Omega)$ to denote this evolution. We do, however, present detailed information on the Bellman equations for individual workers.

Let $W(k, z, \Omega)$ be the maximum value for an individual who works and $N(k, z, \Omega)$ be the

\(^8\)In all essential respects we follow Krusell and Smith (1998). Note that because firms make zero profits here, one does not need to consider firm equity, as in Krusell, Mukoyama, and Şahin (2010), where consumers consequently have a more demanding portfolio problem.
maximum value for an individual who does not work given that he or she has individual values of \((z, k)\) and the aggregate state is \(\Omega\). Define \(V(k, z, \Omega)\) by:

\[
V(k, z, \Omega) = \max\{W(k, z, \Omega), N(k, z, \Omega)\}.
\]

The Bellman equations for \(W\) and \(N\) are given by:

\[
W(k, z, \Omega) = \max_{c, k'} \{\log(c) - \alpha + \beta E_{z', \Omega'}[(1 - \sigma)V(k', z', \Omega') + \sigma(1 - \lambda)N(k', z', \Omega')]\}
\]

\[
s.t. \ c + k' = r(\Omega)k + (1 - \tau)w(\Omega)z + (1 - \delta)k + T(\Omega)
\]

\[
c \geq 0, \ k' \geq 0
\]

and

\[
N(k, z, \Omega) = \max_{c, k'} \{\log(c) + \beta E_{z', \Omega'}[\lambda V(k', z', \Omega') + (1 - \lambda)N(k', z', \Omega')]\}
\]

\[
s.t. \ c + k' = r(\Omega)k + (1 - \delta)k + T(\Omega)
\]

\[
c \geq 0, \ k' \geq 0.
\]

In these expressions, the expectation with respect to \(\Omega'\) refers to the stochastic aggregate components in \(\Omega\) next period: next period’s values for \(Z\), \(\lambda\), and \(\sigma\).
4 Quantitative Analysis

We now begin the quantitative analysis of our model economy, starting with the calibration. The calibration is of two sorts: we need to select the driving processes for our main business cycle shocks, $Z$, $\lambda$, and $\sigma$, and we need to select the rest of the model parameters. We begin with the latter part, which is accomplished by looking at the model’s steady state. Thus, the basic parameters here are selected to match long run aggregate facts and average labor market facts: stocks and flows across the three states.

4.1 Steady State Calibration

In a steady state equilibrium of this economy, $r$ and $w$ will be constant over time, as are job finding and job loss rates. Accordingly, this section describes the steady state version of the economy and its calibration. In particular, we will be interested in the model’s ability to account for the average transition rates documented in the previous section. In the next section we will add various aggregate shocks to the model and assess the ability of different shocks to account for the cyclical patterns in the transition rates. The model in this section draws heavily on the earlier work of Krusell et al (2010, 2011).

A key aspect of the calibration procedure is to choose parameters so that the distribution of workers across the three labor market states and the flows of workers between states in the steady state equilibrium are similar to their average values over time in the US economy. Official statistics divide non-employed workers into the two categories of unemployed and out of the labor force based primarily on how they answer a question regarding active search in
the previous four weeks. Although our model does not feature a search decision, it can be mapped into this definition. Specifically, if active search is a discrete decision and the cost of search is very small, the decision to search amounts to asking an individual if he or she would prefer working to not working.⁹ Among those individuals in our model who are not employed in period \( t \), we will label them as unemployed if they would prefer to be employed and out of the labor force if they would prefer to not work.¹⁰ There is strong evidence in the literature (see, e.g., Poterba and Summers (1981)) that classification errors lead to spurious flows, especially between unemployment and not in the labor force. To capture this we will also add a measurement equation to our model. To facilitate exposition we will calibrate the model assuming no classification error and then show how the model’s implications are affected when we add this type of error.

The model has nine parameters that need to be assigned: preference parameters \( \beta \) and \( \alpha \), production parameters \( \theta \) and \( \delta \), idiosyncratic shock parameters \( \rho_z \) and \( \sigma_{\varepsilon_z} \), frictional parameters \( \sigma \) and \( \lambda \), and the tax rate \( \tau \). Because data on labor market transitions are available monthly, we set the length of a period to be one month. We set \( \tau = .30 \).¹¹ Because our

⁹Given evidence from time use data on the amount of time devoted to search, we think it is reasonable to assume that the cost of active search is relatively small.

¹⁰In our earlier work we argued that a more natural way to connect the model to the data was to adopt a more inclusive definition of unemployment in the data, based on the desire to work rather than active search. Nonetheless, we found that the broader definition was not substantively important either in terms of the features in the data or the ability of the model to account for the data. We revert to the standard definition of unemployment in this paper because of the difficulty in getting a longer time series for flows between the states with the broader measure.

¹¹Following the work of Mendoza et al (1994) there are several papers which produce estimates of the average effective tax rate on labor income across countries. Examples include Prescott (2004) and McDaniel (2006). There are minor variations in methods across these studies, which do produce some small differences in the estimates, and the value .30 is chosen as representative of these estimates.
model is a variation of the standard growth model, we can assign some parameter values following standard procedures used to calibrate versions of the growth model. Because of incomplete markets and idiosyncratic uncertainty, we cannot derive analytic expressions for the steady state, and so cannot isolate the connection between certain parameters and target values. Nonetheless, it is still useful and intuitive to associate particular targets and parameter values. Specifically, given values for \( \lambda, \sigma, \rho_z, \) and \( \sigma_{\varepsilon_z} \), we choose \( \theta = 0.3 \) to target a capital share of 0.3, \( \delta \) to achieve an investment to output ratio equal to 0.2, and the discount factor \( \beta \) to target an annual real rate of return on capital equal to 4%. The other preference parameter \( \alpha \), which captures the disutility of working, is set so that the steady state value of the employment to population ratio is equal to 0.61. This is the value of the employment to population ratio for the population aged 16 and older for the period 1968 – 2009.\(^{12}\)

It remains to choose values for \( \lambda, \sigma, \rho_z \) and \( \sigma_{\varepsilon_z} \). Recall that our idiosyncratic shock process should be viewed as a composite of all idiosyncratic shocks that affect the static return to working versus not working. Shocks to wages are of course only one such component. However, since these are the shocks that we have the best measures of, for our benchmark specification we calibrate the shock process based on estimates of idiosyncratic wage shocks. Specifically, we choose values for \( \rho_z \) and \( \sigma_{\varepsilon} \) based on Floden and Linde (2001), who estimated \( \rho_z = 0.92 \) and \( \sigma_{\varepsilon} = 0.21 \) expressed on an annual basis.\(^{13}\) There is an intimate connection

\(^{12}\)We calibrate to values for the period 1968-2009 because this is the period for which we have consistent measures of labor market flows.

\(^{13}\)Krusell et al (2009) showed that the ability of the model to account for the flows between states remains relatively unchanged over a wide range of values of \( \rho \) and \( \sigma_z \). What mattered most was that \( \rho \) was reasonably persistent (at least 0.5), but not too close to being a unit root (say less than 0.97), and that \( \sigma_z \) was not too
between $\lambda$ and the unemployment rate in the model. If $\lambda = 1$ then unemployment will be zero, since everyone always has the opportunity to work. We therefore choose $\lambda$ so that the steady state unemployment rate matches the average value for the unemployment rate in the US data for the period 1968 – 2009, which is .061. We choose $\sigma$ to target the average flow rate out of employment over our sample period, which is 3.6%. We target this rate based on our belief that the employment state is the one subject to the least amount of measurement error. The calibrated parameter values are displayed in Table 4.

<table>
<thead>
<tr>
<th>Table 4</th>
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<tbody>
<tr>
<td>Benchmark Calibration</td>
</tr>
<tr>
<td>Targets</td>
</tr>
<tr>
<td>$\frac{1}{K} = .20, \frac{E^K}{E} = .3, \frac{E}{K} = .610, \frac{U}{E+U} = .061, 1 + r - \delta = (1.04)^{1/12}, E \rightarrow E = .954$</td>
</tr>
<tr>
<td>Parameter Values</td>
</tr>
<tr>
<td>$\theta$</td>
</tr>
<tr>
<td>.30</td>
</tr>
</tbody>
</table>

The labor market flows in our calibrated model and the data are displayed in Table 5.

<table>
<thead>
<tr>
<th>Table 5</th>
</tr>
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<tbody>
<tr>
<td>Flows in the Model and Data</td>
</tr>
<tr>
<td>US 1968-2009</td>
</tr>
<tr>
<td>FROM</td>
</tr>
<tr>
<td>$E$</td>
</tr>
<tr>
<td>$E$</td>
</tr>
<tr>
<td>$U$</td>
</tr>
<tr>
<td>$N$</td>
</tr>
</tbody>
</table>

Overall the model does a reasonable job of capturing the salient features of the data. Specifically, it does a good job of capturing the degree of persistence in each of the three states. One major discrepancy is that the model does not generate enough flows from $U$ to small. An issue for our quantitative exercises is the extent to which different specifications of the shock process influence our results, despite having little impact on worker flows. We carry out sensitivity analysis to assess this.
Given our strategy of targeting the stock of workers in $U$, this necessarily implies that the other flow out of $U$ (i.e., the flow from $U$ to $E$) must also be off. As previously noted, authors such as Poterba and Summers (1979) have argued that there is substantial measurement error in the flows, especially for flows between $U$ and $N$. This is also quite plausible in the context of our model, since one could imagine that individuals who are not working but for whom $N(k, z)$ and $W(k, z)$ are close to each other may misreport their status.

One strategy to address this would be to try to purge the official data of measurement error. Unfortunately, this is not feasible. The survey that Poterba and Summers used to estimate the extent of classification error on transition rates was discontinued shortly thereafter. Instead, we deal with this issue by adding some measurement error to the data generated by our model. We provide details on this procedure in the appendix and show that with an empirically plausible amount of measurement error the model does a much better job of matching the flows.

While our calibrated model is able to reproduce the key features of both aggregate facts and labor market flows in the US economy, we think it is also relevant to note the minimalist nature of the model. At the aggregate level, the basic structure is that of the standard one-sector neoclassical growth model. We have not included features such as adjustment costs that are common in current medium scale DSGE models. At the micro level we have abstracted from empirically reasonable features such as an interaction between individual market productivity and employment status, or interaction between individual productivity
and the two frictional parameters. We have also abstracted from the choice of search intensity. Note also that we only model frictions as they explicitly influence movements between non-
employment and employment. That is, we do not explicitly model the process of job-to-job transitions. To the extent that these occur, they are subsumed into the evolution of the idiosyncratic shocks to z.

While all of these features could be added to the framework, we have instead opted for parsimony. Our simple model is able to capture the key features of the average flow data, and therefore seems a natural starting point for the analysis of aggregate fluctuations in a model that explicitly models the flows. Incorporating the additional features mentioned above is something that we leave for future work.

4.2 Business Cycle Analysis

In this section we subject the calibrated steady state model of the previous section to aggregate shocks. While we could subject the model to various sorts of aggregate shocks we will limit our attention to two types of aggregate shocks. In our steady state model we found that idiosyncratic shocks to the benefit of working relative to not working and to frictions were sufficient to capture average movements of workers across the three labor market states. Here we will focus on aggregate shocks to these same two factors. We capture aggregate shocks to the relative value of working versus not working by our TFP shock $Z_t$. This is obviously a simple way to generate a common increase in the benefit of working relative to not working. However, to the extent that there are other mechanisms for achieving this same outcome,
for example, informational shocks coupled with some form of increasing returns to scale, we
do not necessarily insist on the interpretation of this driving force as a true technological
shock. We capture aggregate shocks to frictions through stochastic movements in $\lambda$ and $\sigma$.
Note that we take the changes in these frictional parameters to be exogenous. Our goal at
this point is to assess the ability of various types of shocks to generate outcomes that match
those found in the data, rather than the analysis of how these shocks arise.

We proceed by considering a sequence of different assumptions about the aggregate
shocks. We first consider the special case when labor market frictions are random but pro-
ductivity is not. That section makes precise how the labor supply margin plays an important
role in this economy and how the equilibrium outcome involves counterfactual behavior for
participation as well as for flows. We next shut down shocks to frictions and look at TFP
shocks only. Here as well, the model’s predictions are far from the data in a number of re-
spects, particularly for both unemployment and flows. Finally, we present our “benchmark”,
which is a model where there are shocks to frictions and TFP.

The details of how the aggregate shocks are chosen are presented in each section. Prior
to describing our findings, we also briefly summarize how the model is solved. The present
model is similar in spirit to that in Krusell and Smith (1998), where there is no participation
decision. As a result, here the consumer’s problem is considerably more nonlinear, as there are
always consumers (i) deeply in the non-participation region, (ii) deeply in the participation
region, and (iii) near indifference between participating and not participating in the labor
market. For readers not interested in the computational detail it is safe to skip ahead to the results section.

4.2.1 Computation

We follow Krusell and Smith’s (1998) limited information approach—we restrict $\Omega$ to a few variables that can easily be kept track of, let the consumers make decisions based on this limited information and simple forecasting rules that are based on this information, and check whether their forecast is consistent with what actually happens in the economy that consists of these consumers. Specifically, the computation follows the algorithm described below.

1. Replace $\Omega$ by more limited information that can easily be kept track of. Here, we choose the current aggregate capital stock $K$ and the aggregate capital-labor ratio in the previous period, $M_{-1} \equiv K_{-1}/L_{-1}$, as the information that the consumers use when they make decisions.

2. The consumers have to forecast tomorrow’s aggregate capital $K'$ and also need to calculate today’s aggregate capital-labor ratio $M = K/L$ (to know the prices today).

We use the following simple forecasting rules:

$$\log(K') = a_0 + a_1 \log(K) + a_2 \log(z) + a_3 \log(M_{-1})$$

and

$$\log(M) = b_0 + b_1 \log(K) + b_2 \log(z) + b_3 \log(M_{-1}).$$

At the first iteration, make a guess for the values of $a_0$, $a_1$, $a_2$, $b_0$, $b_1$, and $b_2$. 
3. Obtain the prices \( r \) and \( w \) from \( z \) and the forecasted \( M \). Obtain \( T \) from \( w, K, \) and the forecasted \( M \). Solve the optimization problem of the consumers.

4. Simulate the economy using the decision rules of the consumers obtained above. In particular, we can obtain the time series of \( K \) and \( M \). Check whether the law of motion for \( K' \) and the forecasting rule for \( M \) guessed above are consistent with the simulated values. That is, run a regression using the simulated data to see if the coefficients conjectured above are identical to the ones obtained from the regression (also check the fit of the regression). If they are different, modify the coefficients and go back to the previous step. Repeat until the coefficients have converged.

We find that this procedure works well in our model, and the resulting forecasting rules are remarkably accurate. This means that even if we add more information to each consumer’s information set, the consumer cannot forecast much better.

### 4.2.2 Results 1: The Model With Friction Shocks Only

One point that we want to emphasize is that a focus on two state models that completely abstract from labor supply considerations can deliver misleading conclusions regarding the kinds of shocks that account for labor market fluctuations. Consider the simple two state model that Shimer (2005) used in his analysis. By construction, as in any two state model, fluctuations in \( E \) and \( U \) are mirror images of each other. Moreover, since in the data there is relatively little movement in the participation rate, it follows that if one takes the flow
rates between $E$ and $U$ as measured in the data and exogenously feeds them into the model, one will mechanically reproduce the fluctuations in $E$ and $U$ found in the data. Researchers who view the world through the lens of such a two state model will conclude that a sufficient condition for understanding the key features of labor market fluctuations is to understand movements in the two flow rates between $E$ and $U$. In this section we examine this conclusion in the context of our three state model.

To do this we assume that both $\lambda$ and $\sigma$ follow symmetric two state Markov processes with persistence parameters $\rho_{\lambda}$ and $\rho_\sigma$ respectively. Specifically, if we let $\bar{\lambda}$ and $\bar{\sigma}$ denote the calibrated values of the two frictions, we assume that $\lambda$ takes values in the set $\{\lambda_G, \lambda_B\}$ where $\lambda_G = \bar{\lambda} + \varepsilon_\lambda$ and $\lambda_B = \bar{\lambda} - \varepsilon_\lambda$, and similarly that $\sigma$ takes values in the set $\{\sigma_G, \sigma_B\}$ where $\sigma_G = \bar{\sigma} - \varepsilon_\sigma$ and $\sigma_B = \bar{\sigma} + \varepsilon_\sigma$. For simplicity, we assume that the two shocks are perfectly (negatively) correlated, so that the aggregate realizations are either both good or both bad. We therefore assume that $\rho_{\lambda} = \rho_\sigma = \rho$, and consistent with much of the real business cycle literature assume that $\rho = .983$. The values of $\varepsilon_\lambda$ and $\varepsilon_\sigma$ are then set so that in the simulated equilibrium, we match the volatility of $f_{EU}$ and $f_{UE}$ reported in Table 3. The calibrated values are $(\lambda_G, \lambda_B) = (0.51, 0.37), (\sigma_G, \sigma_B) = (0.01254, 0.01346)$.

By construction, the model will do a good job of accounting for the cyclical movements in the two rates $f_{EU}$ and $f_{UE}$. But how will it do in terms of the other dimensions? We start by examining its ability to account for the cyclical properties of the three stocks: $E$, $U$ and $N$ as well as output. Table 6 shows the results.
Several interesting results emerge. First, note that this model accounts for almost all of the fluctuations in unemployment that are found in the data. Moreover, unemployment is strongly negatively correlated with output fluctuations. Based on these two statistics, one would conclude that the model with shocks to frictions does a good job of accounting for the behavior of unemployment found in the data. And if we were in the context of a two-state model, this result would necessarily carry over to the case of employment fluctuations as well, since in such a model the two are mirror images of each other, a property which also holds approximately in the data.

However, looking at the behavior of employment in Table 6, we see that the model accounts for less than 20% of the movements in employment. Why is it that employment fluctuations are not a mirror image of unemployment fluctuations in this model? The reason is that in a model with an operative labor supply margin, individuals have the ability to make choices that offset the direct effect of changes in frictions.\(^{14}\) We will see this mechanism most clearly when we look at the flows, but as a first step it is instructive to note the behavior of the participation rate in Table 6. The magnitude of participation rate fluctuations in the

\(^{14}\)In Krusell et al (2010) we demonstrated that this effect was quantitatively important in terms of steady state outcomes. Here we see that the effect is also quantitatively significant in a business cycle context as well.

<table>
<thead>
<tr>
<th></th>
<th>Volatilities: $\text{std}(x)$</th>
<th>Correlations: $\text{corrcoef}(x,Y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$Y$</td>
<td>$u$</td>
</tr>
<tr>
<td>Data</td>
<td>.016</td>
<td>.12</td>
</tr>
<tr>
<td>Frictions</td>
<td>.001</td>
<td>.11</td>
</tr>
</tbody>
</table>
model are more than two times as large as in the data. Moreover, whereas in the data the participation rate is mildly procyclical, in the model with only friction shocks it is strongly countercyclical.

To examine the economic mechanisms in more detail, Table 7 reports results for the behavior of labor market flows in the model and in the data.

| Table 7 |
|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|
| Cyclical Properties of Transition Rates: Model with Friction Shocks Only |
| Volatilities: std(x) | Correlations: corr(x, Y) |
| f_{EU} | f_{EN} | f_{UE} | f_{UN} | f_{NE} | f_{NU} | f_{EU} | f_{EN} | f_{UE} | f_{UN} | f_{NE} | f_{NU} |
| Data | .085 | .032 | .077 | .060 | .043 | .064 | −.82 | .33 | .78 | .78 | .64 | −.70 |
| Model | .085 | .044 | .077 | .043 | .087 | .066 | −.72 | .43 | .76 | .43 | .84 | −.53 |

By construction, the model matches the volatility of the transition rates $f_{EU}$ and $f_{UE}$. Interestingly, the model also looks to do a reasonable job of capturing the behavior of many of the other flows as well. However, the magnitude of the fluctuations in the two flows between $E$ and $N$ are both roughly twice as large in the model as in the data. The relatively high volatility in the transition rate from $N$ to $E$ is essentially a mechanical feature of our model. In our model, transitions from $N$ to $E$ occur when an individual transitions from not wanting to work in period $t-1$ to wanting to work in period $t$ and at the same time receives an employment opportunity. Holding all else constant, this transition rate will increase proportionately if the probability of receiving an employment opportunity changes.

In contrast, the change in the $E$ to $N$ transition rate reflects the influence of the operative labor supply margin. When frictions are high (i.e., during bad times), individuals adjust behavior in two ways. First, knowing that it is more difficult to receive employment
opportunities, they become less likely to voluntarily leave employment, thereby generating a procyclical flow from $E$ to $N$.

We now return to the strong countercyclical pattern found for the labor force participation rate. The intuition for this result is that when the employment opportunity arrival rate decreases, individuals who are not employed understand that it is now harder to receive employment opportunities and they become less concerned about trying to time employment spells to coincide with periods in which their idiosyncratic value of $z$ is relatively high. In other words, the “reservation wage” (or “reservation productivity”) falls. Hence, when the economy is hit by a negative friction shock, some individuals switch from not wanting to work to wanting to work, leading to an increase in the flow of workers from $N$ into $U$. This effect is the mechanism behind the countercyclical fluctuations in $f_{NU}$, which is the pattern we observe in the data.\footnote{Potentially, there is also a “liquidity effect” at work—in recessions there are more nonemployed workers that are close to the borrowing constraint, thus increasing their desire to work. We suspect that this effect is quantitatively minor.}

In our framework, finding a job does not require any search effort on the worker’s side. Consider an extension of our model in which a worker who is nonemployed has to exert some (indivisible) search effort in order to receive a job offer. If this search effort is costly, only some of the workers that we classify as being in $U$ would choose to exert job search effort.\footnote{Workers that we identify as being in $N$ do not exert any effort since they prefer to stay nonemployed even without any search cost.} In the limit as the search cost approaches zero, all workers that we categorize as being in $U$ would choose to exert job search effort. In the case of a very small but strictly positive search
cost, the discussion in the previous paragraph suggests that aggregate search intensity among nonemployed workers increases during recessions. To some, this may sound counterintuitive—since market work is more attractive during booms one might expect nonemployed workers to search harder.\textsuperscript{17} However, in the presence of a shock that makes it easier for individuals to find work, a nonemployed worker can search less precisely because it is easier to find a job.

Whether nonemployed workers search harder during recessions is ultimately an empirical issue. Recent work suggests that if anything, search intensity is countercyclical. For example, Shimer (2004) examines several indicators of aggregate search intensity over time. One of his indicators, the number of search methods used by attached workers, increased during 2001 (Figure 4). A similar pattern holds even when the sample is limited to active job searchers (see his Figure 7). For earlier periods this measure of search intensity does not exhibit a strong cyclical pattern, but it does always increase during recessionary periods (see his Figure 8).\textsuperscript{18}

In summary, although the model with shocks only to frictions can generate substantial movements in unemployment, and captures many features of the transition rates, it generates

\textsuperscript{17}This is the intuition behind Veracierto’s (2008) “puzzle”. If job search is a costly activity, workers will choose to put more effort in job search when return to job search is higher, i.e. during booms, which in his model correspond to periods of higher wages. In this case, job search effort, unemployment and labor force participation rate all become strongly procyclical. We return to this issue later when we add productivity shocks to the model.

\textsuperscript{18}Shimer (2004) questions the assumption that search effort and the ease of finding a job are complements and he argues that it is plausible to have a modified view of job search effort where some workers respond to deteriorating labor market conditions by increasing their search intensity. He then constructs a model where search intensity can be countercyclical.
changes in employment that are far too small, and changes in participation that are too large and of the wrong cyclical correlation. While the appeal of this type of specification is derived from the fact that researchers have examined it in settings with the participation rate exogenously fixed, we find that this assumption of constant participation is in fact inconsistent with the other features of the model when embedded into a setting that allows for an empirically reasonable labor supply response along the extensive margin.

4.2.3 Results II: The Model With TFP Shocks Only

In this subsection we consider the case in which the only shock is a shock to aggregate TFP. It is well known in the business cycle literature that this type of shock is capable of generating fluctuations in employment and the components of aggregate output that capture the key patterns found in the data. Our main objective here is to assess the ability of this case to match the additional facts that we documented in Section 2. While there is no presumption that this type of shock will work, we carry out this exercise because it is informative to understand the successes and failures of this model vis-a-vis the data.

To that end we consider a symmetric two state Markov process on $Z$ with persistence parameter $\rho_Z$ and $Z$ taking values in the set $\{Z_G, Z_B\}$, where $Z_G = 1 + \varepsilon_Z$ and $Z_B = 1 - \varepsilon_Z$.

To illustrate the effects of this type of shock we choose $\rho_Z = 0.983$ to be consistent with much of the real business cycle literature. We choose $\varepsilon_Z$ to match the standard deviation of employment in the data. The resulting value is $\varepsilon_Z = 0.0290$.

Panel B in Table 8 displays the cyclical properties for model generated data. For com-
pleteness, we include values from the data in Panel A.

Table 8

Cyclical Properties of Labor Market Variables

|                   | A. Data |                |                |                |                |                |                |                |
|-------------------|---------|----------------|----------------|----------------|----------------|----------------|----------------|
|                   | E       | u              | Irfpr          | f_{EU}        | f_{EN}        | f_{UE}        | f_{UN}        | f_{NE}        |
| std(x)            | .011    | .12            | .003           | .085          | .032          | .077          | .060          | .043          | .064          |
| corrcoef(x, Y)    | .84     | -.87           | .46            | -.82          | .33           | .78           | .78           | .64           | -.70          |
| corrcoef(x, x-1)  | .92     | .92            | .72            | .73           | .20           | .84           | .73           | .41           | .75           |

|                   | B. Model |                |                |                |                |                |                |
|                   | E       | u              | Irfpr          | f_{EU}        | f_{EN}        | f_{UE}        | f_{UN}        | f_{NE}        | f_{NU}        |
| std(x)            | .011    | .03            | .010           | .027          | .053          | .012          | .051          | .035          | .034          |
| corrcoef(x, Y)    | .96     | -.57           | .94            | .02           | -.28          | -.01          | .05           | -.16          | -.17          |
| corrcoef(x, x-1)  | .75     | .38            | .72            | -.04          | .07           | -.07          | .01           | -.13          | -.15          |

By construction, the model accounts for all of the fluctuations in employment in the data. However, it fares poorly on many other dimensions. First, although the model does capture the fact that the participation rate is procyclical and the unemployment rate is countercyclical, it misses on the relative volatility of the two by an order of magnitude. In fact, the standard deviation of the employment to population ratio is only slightly larger than that of the labor force participation rate, implying that virtually all of the fluctuations in employment are due to fluctuations in participation. One critique of standard real business cycle models such as Hansen (1985) was that although they could generate substantial fluctuations in employment, they were silent on the issue of unemployment. By including frictions, our model does allow one to connect with data on unemployment, and perhaps not surprisingly, shows that when the only shocks to the model are to aggregate TFP, virtually none of the employment fluctuations show up as offsetting fluctuations in unemployment.

Looking at the properties of the transitions, if one were to only look at the standard
deviations one might conclude that the model is doing a reasonable job in capturing some of the fluctuations. But when one looks at the correlations with output and the autocorrelations it is apparent that the model is capturing virtually none of the dynamics that is present in the flows in the data. The reason for the failure along this dimension is that this model does not contain a mechanism that leads to persistence in flows. Consider the case in which the economy receives a positive shock to $Z$. Working in the market becomes more favorable, and effectively increases the region of $(k, z)$ space in which it is optimal for an individual to work if given the opportunity. This has two direct effects in terms of flows. One is that there is a flow of people from out of the labor force into the labor force. Second, there is a decrease in the flow of people from in the labor force to out of the labor force. However, these effects are responses to the initial change in $Z$. If the value of $Z$ remains high, there will not be similar effects in subsequent periods.

It is of some interest to compare our findings from this exercise with the results in Veracierto (2008). We note that although Veracierto’s model has predictions for the stocks of workers in the three labor market states, the gross flows are not uniquely determined due to the fact that in equilibrium many workers are necessarily indifferent between two transitions. However, we can compare predictions about the number of workers in each of the three states. While there are some differences in details regarding model specification, he also considers aggregate TFP shocks. Similar to us, he finds that although the model can generate substantial fluctuations in employment, it fares very poorly in accounting for the
behavior of unemployment and participation. In particular, he finds that unemployment becomes procyclical. Veracierto ascribes the procyclical unemployment rate to the fact that when a high TFP shock occurs, individuals move from not-participating to participating, thereby increasing unemployment because it takes time to find a job. Figure 1 shows the dynamics in our model following an increase in $Z$ from $Z_B$ to $Z_G$ assuming that $Z$ remains at this level. The figure shows that our model has the same initial response to an increase in TFP as in Veracierto’s model; that is, there is an immediate jump in the size of the labor force and an increase in unemployment. However, over time these individuals will become employed, and in our model the unemployment rate approaches a lower level than attained prior to the shock. This asymptotic response turns out to dominate the immediate effect in terms of its effect on the correlation between the unemployment rate and output. Hence,
while our model does not match the volatility of the unemployment rate, it does produce a countercyclical unemployment rate. Despite this difference with the results in Veracierto, our model displays quantitative responses that are quite similar to his.

4.3 Results III: Shocks to TFP and Frictions—The Benchmark

A very simple characterization of the findings from our first two exercises is that a model with shocks to frictions can do a reasonable job of accounting for the behavior of unemployment and many of the transition rates, whereas a model with shocks to TFP can do a reasonable of accounting for the behavior of employment. Neither model seems capable of accounting for the behavior of participation, in that in both cases participation seems to fluctuate too much. However, it is interesting to note that although in both cases the participation rate fluctuates too much, in the case of TFP shocks it is procyclical whereas in the case of friction shocks it is countercyclical. Noting these results, it seems possible that a model that features shocks to both frictions and TFP could do a reasonable job of accounting for a large set of the labor market facts. In this section we show this to be the case.

As before, we assume that $Z$, $\lambda$, and $\sigma$ all follow symmetric two state Markov processes. We adopt the same notation as before in terms of denoting good and bad states. Once again, for ease of exposition, we focus on the case in which the shocks are all perfectly correlated, so that there are only two realization that can occur: either all values are good or all values are bad. Our main goal is to show that this type of specification can capture many of the facts from Section 2. We set $\rho_Z = 0.983$ similar to above. $\varepsilon_Z$ is calibrated to 0.0287 to match the
standard deviation of employment in the data. We choose the volatility of the two transition rates between employment and unemployment so that their standard deviations are the same as in the data. The calibrated values for the shock processes are $(\lambda_G, \lambda_B) = (0.509, 0.371)$ and $(\sigma_G, \sigma_B) = (0.01249, 0.01351)$. Table 9 shows the results for the cyclical behavior of employment, the unemployment rate and the participation rate.

<table>
<thead>
<tr>
<th>Table 9</th>
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<tr>
<td>Behavior of Stocks with TFP and Friction Shocks</td>
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<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Data</td>
</tr>
<tr>
<td>Model</td>
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</tbody>
</table>

While the volatility of employment in the model is the same as that in the data by construction, note that the model also does a good job of accounting for the size of fluctuations in both unemployment and participation. Additionally, it also correctly accounts for the fact that employment is strongly procyclical, the unemployment rate is strongly countercyclical, and the labor force participation rate is weakly procyclical. We noted previously that in each of the first two exercises, the participation rate fluctuated too much. However, because the two shocks move participation in opposite directions, when both shocks are present the overall effect on participation is much smaller than the individual effects. The cyclicity of the participation rate is dependent on the relative magnitudes of the shocks. But a robust feature of this specification is that the participation rate will exhibit much less cyclicity than either employment or the unemployment rate.

Table 10 shows the results for the transition probabilities. This table shows that the model
does a good job of accounting for the key patterns. The model captures the countercyclicality of unemployment inflows (EU and NU flow rates), procyclicality of unemployment outflows (UE and NU flow rates) and mild procyclicality of the EN flow rate. The model delivers a volatility and correlation (with output) for $f_{NE}$ that are somewhat too high.

**Table 10**

<table>
<thead>
<tr>
<th>Flows in the Model with TFP Shocks and Friction Shocks</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A. Data</strong></td>
</tr>
<tr>
<td>$std(x)$</td>
</tr>
<tr>
<td>0.085</td>
</tr>
<tr>
<td>corrcoef$(x, Y)$</td>
</tr>
<tr>
<td>corrcoef$(x, x_{-1})$</td>
</tr>
<tr>
<td><strong>B. Model</strong></td>
</tr>
<tr>
<td>$std(x)$</td>
</tr>
<tr>
<td>0.085</td>
</tr>
<tr>
<td>corr$(x, Y)$</td>
</tr>
<tr>
<td>corr$(x, x_{-1})$</td>
</tr>
</tbody>
</table>

Figure 2 shows the dynamics of labor market flow transition rates in the benchmark model following an increase in $Z$ from $Z_B$ to $Z_G$ assuming that $Z$ remains at this level. When aggregate productivity switches from $B$ to $G$, the employment opportunity arrival rate $\lambda$ goes up and the separation rate $\sigma$ goes down. These two shocks account for the behavior of $EU$ and $UE$ transition rates following a transition from $Z_B$ to $Z_G$. The behavior of $NU$ and $NE$ flows can be explained by the evolution $\lambda$. Recall that in our framework when a worker decides that she wants a job, she can receive an employment opportunity in the same period. If $\lambda$ is high, a worker is more likely to receive an employment opportunity within the first period and less likely to flow into unemployment. Therefore, $NE$ flow rate increases and $NU$ flow rate decreases when $Z$ increases from $Z_B$ to $Z_G$. Both $UN$ and $EN$ flow rates show an
initial decline right after the aggregate shock. This is the effect of the increase in the return to work. Since the aggregate state is good, working becomes more attractive for consumers and they react by lowering their transition rates into nonparticipation from employment and unemployment. As the good state continues, this initial drop is reversed and inflows into participation (UN and EN flow rates) increase. The behavior of EN flows is due to what we call “the attenuation effect”. When job-finding prospects are better, consumers raise their reservation productivity levels and hence are less willing to work. The change in the average productivity of unemployed workers explains the increase in UN flows in the long-run. During booms, the average productivity of unemployed workers is lower since more productive workers can move to E quickly. As a result, there are many workers in U who are close to the threshold of moving from U to N, causing an increase in the UN transition rate.

Figure 2: Response of transition probabilities to a positive Z shock in the benchmark model.

---

19This effect is very similar to the effect that we have seen in the model with shocks to market activity.
Given that the above specification does a reasonable job of accounting for the cyclical fluctuations of the labor market variables, it is of interest to decompose the fluctuations in various series into the parts attributable to each of the three shocks: Z, λ, and σ. To do this we simply recompute the model assuming that two of the three shocks are shut down. Table 11 does this for the three aggregates. It also reports the correlations of the variables with output for each specification as well.

<table>
<thead>
<tr>
<th></th>
<th>Contribution of the Shocks: Stocks</th>
<th>Volatilities: std(x)</th>
<th>Correlations: corr(x, Y)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Y</td>
<td>u</td>
<td>lfpr</td>
</tr>
<tr>
<td>Data</td>
<td>.016</td>
<td>.12</td>
<td>.003</td>
</tr>
<tr>
<td>All</td>
<td>.020</td>
<td>.13</td>
<td>.004</td>
</tr>
<tr>
<td>Z</td>
<td>.018</td>
<td>.03</td>
<td>.010</td>
</tr>
<tr>
<td>λ</td>
<td>.001</td>
<td>.10</td>
<td>.007</td>
</tr>
<tr>
<td>σ</td>
<td>.000</td>
<td>.01</td>
<td>.001</td>
</tr>
</tbody>
</table>

Several results are notable. First, consistent with our earlier results, shocks to λ are most responsible for fluctuations in the unemployment rate whereas fluctuations in Z are most responsible for fluctuations in employment. Note that allowing for fluctuations in frictions does not diminish the role that TFP shocks in accounting for fluctuations in employment. From the perspective of employment fluctuations the model looks very much like the model that has only TFP shocks. The key effect of frictions vis-a-vis employment fluctuations is that they change the participation dynamics so that unemployment fluctuations are now much closer to being the mirror image of employment fluctuations. Fluctuations in σ are relatively unimportant for all three stocks. It may seem somewhat surprising that shocks to σ are so unimportant given that the transition rates between E and U fluctuate by roughly
the same amount. To obtain some insight into this Table 12 carries out the same exercise for the two transition rates between $E$ and $U$.

![Table 12](https://example.com/table12.png)

<table>
<thead>
<tr>
<th>Contribution of the Shocks: Flows</th>
<th>std($x$)</th>
<th>corrcoef($x, Y$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{EU}$</td>
<td>Data</td>
<td>.085</td>
</tr>
<tr>
<td></td>
<td>All</td>
<td>.085</td>
</tr>
<tr>
<td></td>
<td>$z$</td>
<td>.026</td>
</tr>
<tr>
<td></td>
<td>$\lambda$</td>
<td>.068</td>
</tr>
<tr>
<td></td>
<td>$\sigma$</td>
<td>.033</td>
</tr>
<tr>
<td>$f_{UE}$</td>
<td>.077</td>
<td>-.82</td>
</tr>
<tr>
<td>$f_{EU}$</td>
<td>-.90</td>
<td>.78</td>
</tr>
<tr>
<td>$f_{UE}$</td>
<td>.03</td>
<td>.00</td>
</tr>
<tr>
<td>$f_{EU}$</td>
<td>-.67</td>
<td>.74</td>
</tr>
<tr>
<td>$f_{UE}$</td>
<td>-.48</td>
<td>-.02</td>
</tr>
</tbody>
</table>

An interesting result here is that even when there are only shocks to $\lambda$ so that $\sigma$ is constant, the model accounts for roughly 80% of the volatility in $f_{EU}$. This is because we allow individuals to receive an employment opportunity in the same period that they experience an employment separation shock. It follows that the likelihood of an $E$ to $U$ flow is affected by the value of $\lambda$. To see why this is, note that when $\lambda$ is low it is more likely that a worker who experiences an employment separation will remain in the unemployment state. This phenomenon in the model captures an important feature that is present in the actual data. That is, in the household survey, an individual who transits from employment to unemployment after an interview date, but who receives a job offer and starts to work prior to the next interview date will be recorded as an $E$ to $E$ flow. The greater is the job offer arrival rate, the greater is the chance that this worker will be recorded as an $E$ to $E$ transition.\(^{20}\)

Note that in the model with only shocks to $\sigma$ there is also an effect on the transition

\(^{20}\)As an alternative exercise we could ask what types of fluctuations result if we assumed that there were only shocks to $\sigma$ but that the size of these shocks were sufficient to match the volatility in $f_{EU}$.
rate from $U$ to $E$. This may at first seem somewhat surprising. However, this effect is due to a one-time effect associated with changes in the value of $\sigma$. The nature of the effect is that during good times (i.e., when $\sigma$ decreases), some people move from participation to non-participation due to the fact that as it becomes easier to work when one desires, one becomes more particular about timing periods of work with high values of the idiosyncratic shocks. As a result, during the period in which $\sigma$ changes from high to low, some of the previously unemployed move to non-participation and hence even though they receive employment opportunities they choose not to work. This causes a one-time reduction in $f_{UE}$. The fact that it is a one-time reduction is evident in the very low correlation between $f_{UE}$ and output in the case of shocks to only $\sigma$.

5 Conclusion

We have developed a general equilibrium model of gross worker flows and used it to study the role of various shocks in accounting for business cycle fluctuations in the labor market. A key message from our analysis is that our model offers important insights into the forces that drive business cycle fluctuations in the labor market. In contrast to the implications of standard two-state models that stress matching frictions, we find that fluctuations in employment loss and employment offer rates are unable to account for the key patterns in labor market variables over the business cycle. However, we find that a model that features both fluctuations in these rates in addition to fluctuations in the aggregate return to market activity is able to account for the main features not only of labor market aggregates but also
of gross worker flows.

Our model offers a rich description of individual labor supply in a setting with heterogeneity, search frictions and an empirically reasonable market structure. It is the first paper to consider the effects of aggregate shocks on individual labor market transitions in this setting. It is therefore a useful framework for assessing a variety of further issues, such as the heterogeneous effects of business cycles on various subgroups of the population. While we have focused on aggregate shocks to frictions and the return to market activity, we can also study other aggregate shocks, including various candidates for demand shocks.
References


