Competitive Poaching in Unsecured Lending*

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ABSTRACT

The paper studies the effects of non-exclusivity of credit card contracts on the provision of insurance through the institution of personal bankruptcy. In our model, lenders can continually observe borrower's time-varying creditworthiness and provide credit to them by undercutting (poaching) the existing lender(s). Contracts are non-exclusive and, to rollover their debt, borrowers may accept multiple credit agreements to economize on the cost of credit. The main result of the paper, which holds for a broad range of parameter values, is that the level of insurance provided under bankruptcy is largely independent from borrowers' preferences and features a bang-bang property: Either too little insurance is provided or, generically, there is overinsurance (potentially severe). Comparing to the exclusivity regime, our results suggest that non-exclusivity regime is inferior in terms of welfare. The key novel mechanism of the model is a strategic entry deterrence motive of lenders.

JEL: D1,D8,G2
Keywords: personal bankruptcy, credit cards, credit lines, unsecured credit

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I. Introduction

In the 80s, credit cards, and the associated with them credit lines, accounted for a scant portion of consumer borrowing. Over the 90s, however, the credit card market has been expanding rapidly. As a result of this growth, as much as 78% of US households now report having access to this form of borrowing[1], credit card debt accounts for more than a third of total consumer borrowing[2], and a statistical US household holds an option to draw as much as $40k[3] in credit card funds. Since credit card debt is unsecured and in the US can be defaulted on[4], credit cards not only facilitate intertemporal smoothing of consumption, but also provide an unprecedented by historical standards source of insurance against severe negative income (expense) shocks. This insurance role of credit card funds is the subject of our study.

By design, the institution of personal bankruptcy in the unsecured credit market is meant to provide state contingency when private contracts cannot be made state contingent. In the case of borrowing contracts with non-contingent debt, according to theory, such an arrangement should yield a constrained efficient allocation when two-sided commitment contracts are available, and default for borrowers is sufficiently costly 'in normal times' to guarantee unobstructed intertemporal smoothing in non-default states. If, however, as is the case in the credit card market, the borrower cannot commit to use credit from a particular lender, the theory predicts that the amount of insurance provided may be distorted by the (voluntary) participation constraint implied by non-commitment. Thus, in light of theory, the current institutional regime raises an important question how outcomes in this market are affected

[1]Source: Table 4 in "The Survey of Consumer Payment Choice," Federal Reserve Bank of Boston, January 2010, available at http://www.bos.frb.org/economic/ppdp/2009/ppdp0910.pdf.

[2]As of March 2010, total revolving debt outstanding was $835 billion (Federal Reserve's G.19 report on consumer credit, March 2010). It is estimated that about 98% of this figure is accounted for by the credit card debt. Total consumer borrowing, both secured and unsecured, has been about 2.5 trillion during the same time period.

[3]Data for 2007. Source: Page 226 of H.R. 5244, the Credit Cardholder's Bills of Rights: Providing New Protections for Consumers, Hearing Before the Subcommittee on Financial Institutions and Consumers Credit, House Committee on Financial Services, 110 Cong. 109, 226 (April 17, 2008) (testimony of Travis Plunkett, Consumer Federation of America), available at http://frwebgate.access.gpo.gov/cgi-bin/getdoc.cgi?dbname=110_house_hearings&docid=f:42721.pdf.

[4]In the US, a complete discharge of all unsecured debts is possible under Chapter 7 of the Bankruptcy Code, with a fairly generous asset exemption (regulated by state law). In addition, a partial discharge (5 years repayment plan) of debts is allowed under Chapter 13.

by non-commitment of borrowers.

In this paper, we are motivated by this question, and study how non-exclusivity of contracts in the credit card market impacts the amount of insurance provided through personal bankruptcy. To this end, we propose a positive equilibrium theory consistent with three salient features of the credit card market in the US: (i) credit line contracts that allow lenders to commit to terms, (ii) non-exclusive contracts that allow the borrowers to transfer their outstanding balances to optimize on interest rate charges paid to the existing lenders, and (iii) a possibility of entry by the competing lenders in response to new information regarding the borrower's changing credit-worthiness.

In this environment, we show that non-exclusivity of contracts, due to the presence of a voluntary participation constraint, creates a strategic entrant-incumbent interdependency between lenders, resulting in a preemptive entry deterrence behavior of lenders. As a result, for most parameter values, the level of insurance implied by bankruptcy in our model is largely independent from borrower's preferences and features a bang-bang property: Either no insurance at all is provided or, generically, there is overinsurance (potentially severe).

In particular, our numerical experiments suggest that, under some parameter values, the threat of undercutting entry (a balance transfer) can make credit limits excessive by as much as 20-30% with respect to the underlying constrained efficient allocation. In the presence of other social insurance programs, we think this result is particularly worrying, as it may unnecessarily hurt intertemporal smoothing and contribute to the fragility of the banking system.

In terms of policy prescriptions, our results imply that some form of exclusivity imposed by law (commitment of borrowers) would be welfare improving. In the context of the current regulations, our model suggests that an amendment of the bankruptcy law in the spirit of the means testing introduced by the 2005 Act may exacerbate the overinsurance result, and thus in some cases worsen outcomes in terms of bankruptcy statistics[5]. All of these results

[5]The Bankruptcy Abuse Prevention and Consumer Protection Act of 2005 (the Act) was signed into law on April 20, 2005, with the explicit intent of discouraging filings under the Bankruptcy Code. The most controversial change was the creation of a means test for eligibility to file under liquidation Chapter 7 (full discharge of debt) versus restructuring under Chapter 13 (partial discharge). The Act requires a comparison of the debtor's income to the median income in the individual's domiciled state. If the debtor's income is above the median, and he is able to pay at least a minimal amount per month to creditors, he is barred from

are in stark contrast with the standard models of consumer bankruptcy which effectively assume exclusivity of contracts for the admissible duration of the borrowing relationship (e.g. Livshits, McGee & Tertilt (2006), Chatterjee, Corbae, Nakajima & Rios-Rull (2007) or Athreya (2002)).

The intuition behind our results is related to the classical effect of entry deterrence through capacity. In our model, borrowers, even in normal times, may choose to default strategically. As a result, there is a well-defined upper bound credit limit (notion of capacity) above which strategic default is triggered. Lenders, exploit this possibility of a 'strategic default' to pre-empt entry by other lenders—especially, when the latter can undercut them by conditioning entry on a favorable resolution of uncertainty regarding borrower's future creditworthiness.

More specifically, the above results stem from the fact that, due to possibility of strategic default, higher credit limits of the initial lenders not only provide more insurance to the borrowers, but also, from initial lender's perspective (insurers), have an additional benefit of crowding out future lenders (poachers). As a result, for the range of interest rates that are consistent with initial lender's zero profit condition, a deviation that raises credit limits may actually yield higher profits, despite the increased by that losses in the default states. Higher profits, in turn, imply that initial lenders can at the same time lower the interest rate. Since lower interest rate reduces intertemporal distortion of consumption in non-default states, it implies that initial lender can this way transfer resources (consumption) from high state (non-default states) to low state (default states) at a rate that is more favorable than the actuarially fair one (ratio of probabilities). This is important because starting from a realistic situation in which the consumers are always incompletely insured in equilibrium against shocks that prompt default, equilibrium allocation will necessarily feature a strong entry deterrence motive leading to a corner solution (note: preferences are flatter than the actuarially fair rate in this case). Moreover, the outcome will potentially involve severe inefficiency, as the corresponding constrained efficient allocation will always features a trade-off strictly below the actuarially fair rate. This separation non-exclusivity and constrained

Chapter 7 filing and can only default under Chapter 13. If the debtor fails the test for filing under Chapter 7, there may now also arise a presumption of bad faith. These changes were openly intended to reduce the national default rate under Chapter 7, and limit any potentially fraudulent behavior.

efficient allocation by the actuarially fair hyperplane is the key result of our paper.

Since our theory is positive in nature, several additional comments are in order to justify our key modeling assumptions.

Our focus on credit lines exclusively characterized by a credit limit and an interest rate, while not exhausting all the possibilities seen in the market (e.g. annual fees, transaction fees or cash back), is motivated by the revenue structure of the credit card companies in the US. In an excellent monograph on the credit card market, Evans & Schmalensee (2005) (page 223) document that for an average credit card issuer (with the exception of American Express) as much 70% of total revenue of credit card issuers is accounted for by financing charges (interest, service fees, late fees and balance transfer fees charged for borrowing). Penalty and cash-advance fees account for about 12%, annual fees for as little as 3% (up to about 5% in 2007), and interchange (transaction) fees for the remaining 15% or so (as of 2001-2). Our reading of this evidence is that, except for the transactional role of credit cards, credit limits and interest rates are the main characteristics of this type of contract, and borrowing is an important aspect of this product.

The assumption of an option to commit on the lender's side is now largely required by law[6]. In addition, in our view, reputational considerations of banks are likely to further contribute to stability of terms in times when reputation is not valued[7].

Finally, we should stress that our results are fairly limited to the credit card market, and

[6] As a result of the Credit Card Act of 2009, consumers now have the right to opt out of—or reject—certain significant changes in terms on their accounts. Opting-out means cardholders agree to close their accounts and pay off the balance under the old terms. Moreover, interest rate hikes on existing balances are allowed only under limited conditions, such as when a promotional rate ends, there is a variable rate or the cardholder makes a late payment. Interest rates on new transactions can increase only after the first year. Significant changes in terms cannot occur without a 45 days' advance notice of the change.

[7] While lenders did change terms in the past, such a practice has never been widespread (with the exception of the recent crisis). Moreover, In many cases, if terms were changed, borrowers were given an option to opt-out of these changes—sometimes without any consequences. For example, the exact wording of the opt-out option offered by the Citibank, during its famous 2008 interest rate hikes, was (bottom of the page reads): "Right to Opt Out. To opt out of these changes, you must call or write us by Jan 31 2009. When you do, you must tell us that you are opting out. Call us toll-free at 1-866-565-7030. (Please have your account number available.) (...), "**If you opt out of these changes, you may use your account under the current terms until the end of your current membership year or the expiration date on your card, whichever is later.** (...)". According to industry analysts, with the exception of a few credit card issuers, this was a standard practice: suggesting that, due to reputational considerations, lenders were rather careful breaking their implicit commitment expected from them by borrowers. A copy of the actual notice is available at the end.

do not necessarily extend to other defaultable credit relations. The main effects we identify in this paper crucially depend on the contracts being restricted to non-exclusive credit lines, featuring no practical possibility of charging any fixed fees[8] or prepayment penalties[9]. It is also important that the borrowers cannot commit to close an existing account only upon receiving a contract from another lender. In the credit card market, pre-existing debt and the lack of commitment on the borrower's side to close accounts are likely to make this feature more realistic than in the context of other credit markets.

Related literature There are three papers that are most closely related to ours: DeMarzo & Bizer (1992), Parlour & Rajan (2001), and Petersen & Rajan (1995). These papers, with the exception of Parlour & Rajan (2001), predomantely focus on the corporate finance aspects of the bankruptcy protection. [10] While exploring effects related to ours, the main difference between our paper and this literature is the type of contractual restrictions that are considered giving rise to a novel effect of entry deterrence through capacity. More specifically, this literature considers loan contracts, which in contrast to credit lines, can affect lenders only through moral hazard considerations, while credit lines introduce a new effect related to business stealing of non-committed borrowers.

Our paper is also related to and complementary to the quantitative modeling of the credit card market, which at this point is rather scant. Notable exceptions include: Rios-Rull & Mateos-Planas (2007), Drozd & Nosal (2007) and Narajabad (2007). In contrast to this growing literature, here we allow for multiple credit relations to coexist at the same time.

The link between information and provision of credit considered by us in the numerical

[8]One way of resolving the contracting problem we consider is to charge a large fixed fee upon contract initiation. However, such practices do not take place in the credit card market. Annual fees contribute to as little as 3 percent of revenue of credit card companies (see source in text). Our conjecture is that the reason is an adverse selection problem that such a contract would implicate. In the model, instead of modeling such problems explicitly, we assume it is not possible to impose fixed fees.

[9]Unlike in some other markets, in the credit card market, lenders do not seem to have the ability of detecting a hidden prepayment, in which the borrower finances all current consumption using a new credit card, while repaying quickly the old one. Because of this problem, implementation of such fees is highly problematic.

[10]Parlour & Rajan (2001), study the sources of imperfect competition in the presence of a seemingly competitive environment and DeMarzo & Bizer (1992) the effects of sequential competition for loan contracts in presence of moral hazard involved in repayment. In a similar context, Petersen & Rajan (1995) emphasize the benefits of long-term relationship borrowing in the context of small business borrowing.

section is related to the recent studies exploring such effects in the context of the recent bankruptcy trends. For example, see Livshits, MacGee & Tertilt (2007), Athreya, Tam & Young (2008) and Sanches (2008).

More broadly, our paper is also related to the literature of implicit contracts, that in other contexts, studies the effects of voluntary participation constraints (e.g. Harris & Holmstrom (1982)) on the broadly defined provision of insurance.

The structure of the rest of the paper is as follows. In section 2, we present the model. In section 3, we prove our main analytic results. In section 4-5, we extend the model and provide a series of numerical examples to illustrate our results and verify their robustness.

II. Model

The economy is populated by two types of agents: consumers (households) and lenders. Time is discrete and there are two periods, referred to as the current period and the future period.

Consumers optimize to smooth consumption across dates and states by saving or borrowing from the lenders. In consistency with the US Bankruptcy Code, they can default on their debts. Their current income is deterministic and is normalized to 1. Future income is random $y \in Y = \{y_1, ..., y_n\}$, and consumers observe its realization before first period consumption takes place.

Lenders have deep pockets, and the cost of funds is normalized to one, and they complete in a Bertrand fashion in two rounds of competition. Markets are incomplete and unsecured credit is restricted to take the form of non-exclusive unsecured credit lines. Lenders can commit to terms, but they do not have to.

A. Timing, information and contractual restrictions

Access to credit is acquired by consumers during two rounds of Bertrand competition between lenders. Both rounds are assumed to take place in the first period before consumers make any consumption decision, but their timing differs with respect to the information revelation. Specifically, while no new information is revealed before the first round takes place, the second round takes place after a public signal $s \in Y$ of the future income realization $y \in Y$ of the borrower is revealed to all lenders (and the borrower). The signal can is assumed

to be revealing with an exogenous probability $0 \leq \pi \leq 1$.

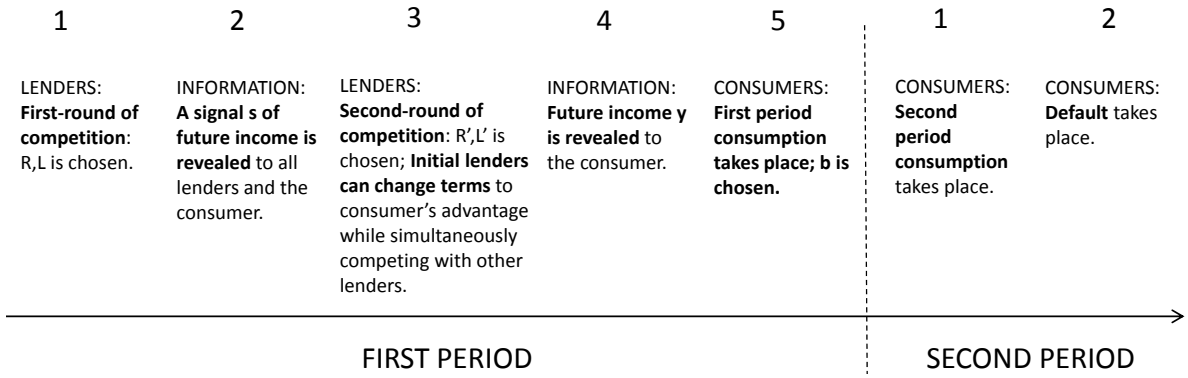

Figure 1: Timing of events.

In the baseline setup, we assume that only one contract per round can be accepted. However, to the end of our analytical section, we relax this assumption and show that for the case of fully revealing signal and two income states it is largely without loss of generality.

Contracts between lenders and consumers take the form of unsecured credit lines $\mathcal{C} = (L, R) \in \mathcal{R}_+^2$, where L denotes a credit limit and $R \geq 1$ includes the principal and the interest rate. A few words are in order regarding commitment to terms by lenders. As already mentioned, initial lenders, while specifying the initial terms, can, but do not have to commit. Nevertheless, in what follows, by $\mathcal{C}$ we mean a commitment contract from the first round. This is because the timing of events in our model (Figure 1), and the details of our setup, imply that the choice of no-commitment contract, or alternatively, sweetening of terms under commitment is actually equivalent to initial lenders choosing not to enter ($L = 0$).

B. Consumers

Consumers are born in the current period and live for two periods. They choose borrowing (saving) level for the future period and decide whether to default on their debts in the second period[11]. In their choice, they are constrained by the credit lines made available to them by lenders in the aforementioned two rounds of competition.

As implied by the timing of events illustrated in Figure 1, at the beginning of the first

[11] Allowing default in first period would not change any of the results. Such extended setup is considered by us in the technical appendix.

period, the consumers acquire a contract from the initial lenders[12]. This happens before any information regarding the realization of consumer's future income y is revealed. After the first contract is accepted (can be null), a signal $s \in S$ of the future income realization is observed by all parties, and the second round of competition takes place.

During the second round, the consumers are allowed to acquire another contract, referred to as the second credit line[13]. Having the two contracts, the consumers rank them in terms of the the interest rate charged on them, and the lowest interest rate credit line is labeled as $\mathcal{C}'$. The consumers then learn their future period income realization $y \in Y$, choose consumption in the first period c_1, borrowing/saving decision b, consumption in the second period c_2, and binary default decision $D \in \{0, 1\}$, to maximize their lifetime utility

$$u(c), \tag{1}$$

subject to intertemporal consumption aggregation,

$$c = G(c_1, c_2), \tag{2}$$

the first period budget constraint (first period income normalized to 1),

$$c_1 = 1 + b, \tag{3}$$

a borrowing constraint implied by the two credit lines[14],

$$b \leq L + L', \tag{4}$$

[12]Initial lenders can commit to L,R, if it improves borrowers' welfare. See Figure 1.

[13]It is assumed that during the second-round consumers can not cancel the first-round credit line. There are two ways of rationalizing that. The first is that the borrower enters the model with some debt. Thus, the lack of commitment to close an account only upon receiving a new contract makes it costly for the borrower to close the account. In such case, the math of our model is identical if we set income in first period equal to Y minus initial debt, and normalize Y so that it is Y minus initial debt=1. Another way, is to assume that credit line of the second round lender can not be used to finance consumption, and only gives the option of balance transfer. This second formulation changes our results by strengthening them.

[14]Our formal setup, for simplicity allows the borrower to use second credit line, but results we have would actually be stronger if we allowed only for balance transfers. We have chosen this setup as the characterization becomes tedious, and we do not want our results to come from the fact that first round lender has a built in advantage (even though such possibility may actually be realistic).

and the second period budget constraint,

$$c_2 = \begin{cases} \theta_y y - b + D(L + L') \text{ for } b \leq 0, \\ \theta_y y - R'b + D(L + L') \text{ for } 0 < b \leq L', \\ \theta_y y - R(b - L') - R'L' + D(L + L') \text{ for } b \geq L'. \end{cases} \tag{5}$$

This constraint depends on the contract offered during the second round of bank competition $\mathcal{C}'$, and intended to capture the idea that the consumers can use the second credit line to either directly finance their first period consumption or alternatively to transfer the outstanding balances to the less expensive account to lower interest rate cost of debt. Both, the utility function u, as well as the intertemporal aggregator G are assumed to be symmetric ($G(x, y) = G(y, x)$), continuously differentiable and strictly concave. In addition, G is homogenous of degree one.

Default (bankruptcy) option in the model is meant to be consistent with the US Bankruptcy Code (Chapter 7, and partly also Chapter 13) that governs personal default. As is clear from the equations, the consumer is allowed to max out on all available credit cards to obtain an effective transfer *equal* to the total available credit limit $L + L'$ *less* the interest rate charged in the process of rolling over the debt from the first to the second period. The pecuniary punishment for defaulting is $(1 - \theta_y)y$, and it is assumed increasing with income.

As we show in the lemma below, our assumptions make the default decision rule particularly tractable, and rise to the standard property shared by many models of default: The default set (the range of debt that triggers default) shrinks with the level of income y. This property creates a potential insurance role of the bankruptcy protection, as the incentive to repay debts is positively correlated with the ex-post realization of borrower's income.

LEMMA 1. *The consumer defaults iff $L + L'$ exceeds $\mathcal{L}_{\max}(y) = y - \theta_y y$.*

Proof. Since utility function is increasing in consumption, the sufficient condition for default on a given income path y is that the budget set expands due to default for all values of b, i.e. for all b

$$\theta_y y - \rho(b, \mathcal{C}, \mathcal{C}') + \mathcal{L}_{\max}(y) > y - \rho(b, \mathcal{C}, \mathcal{C}'),$$

where ρ stands for a repayment function of debt b under contracts $\mathcal{C}, \mathcal{C}'$. Since repayment

function cancels out in the expression above, the condition does not depend on b, and thus is also necessary. After simplifications, we obtain the formula characterizing the default set in our model:

$$\mathcal{L}_{\max}(y) \leq y - \theta_y y. \tag{6}$$

□

DEFINITION 1. $\mathcal{L}_{\max}(y)$ *is referred to as the* ***maximum repayment capacity*** *associated with income path* y.

Finally, to formally define the default decision problem faced by the consumer, let $\mathcal{W}(D; y, \mathcal{C}, \mathcal{C}')$ be the indirect utility of the consumer associated with a fixed default decision D. The choice of D then satisfies the following

$$\mathcal{V}(y, \mathcal{C}, \mathcal{C}') = \max_{D=0,1} \left[D\mathcal{W}(1; y, \mathcal{C}, \mathcal{C}') + (1-D)\mathcal{W}(0; y, \mathcal{C}, \mathcal{C}') \right]. \tag{7}$$

The indirect utility function $\mathcal{V}$ is the consumers welfare that determines consumer's preference relation defined on the space of contracts.

C. Lenders

Lenders compete in Bertrand fashion in two rounds of competition. As already explained, both rounds take place in the first period, but their timing differs with respect to the information revelation implied by the signal (see Figure 1). The first round takes place in the very beginning, before any information is known regarding consumer's future income realization y. The second round occurs after lenders and the consumer receives a noisy public signal of this realization (consumers do not know income yet).

The contracts offered by lenders are assumed to constitute a subgame perfect Nash equilibrium. By backward induction, it is convenient to start defining equilibrium in the credit markets from the second round. We do so in what follows next.

Second round of Bertrand competition (problem of the poacher) In the second round, lenders observe a noisy signal of the borrower's income realization $s \in S$ and are assumed to know the contract that the borrower has already received from the initial lenders.

Given this information, the second-round lenders choose their contract $\mathcal{C}' = (L', R')$ to best respond to the contract given of initial lenders $\mathcal{C}$ and the contracts expected to be offered by the lateral competitors from the same round.

Formally, the second-round contract solves

$$\mathcal{C}'(\mathcal{C}, s) = \arg\max_{\mathcal{C}'(s)} E_s \mathcal{V}(y, \mathcal{C}, \mathcal{C}'), \tag{8}$$

subject to the expected zero profit condition given the realized signal s,

$$E_s \pi'(y, \mathcal{C}, \mathcal{C}'(s)) = 0.$$

The profit function of second round lenders π' depends on the policy function of the agent (D, b), and on the relevant part where the second round lenders undercut the initial lender by offering a lower interest rate ($R' \leq R$), it is defined as follows:

$$\pi'(y, \mathcal{C}, \mathcal{C}') = \begin{cases} -DL' \text{ for } b \leq 0, \\ (1-D)(R'-1)b - D(L' - (R'-1)b) \text{ for } 0 < b \leq L', \\ (1-D)(R'-1)L' - D(L' - (R'-1)L') \text{ for } b > L'. \end{cases} \tag{9}$$

In words, in the case of no default (D=0), second-round lenders profit on the interest rate $R' - 1$ charged on borrowing up to the specified limit L', and incur a loss in case of default ($D = 1$) *equal* to the credit limit granted by them *less* any interest paid on the rolled-over debt.

First round of Bertrand competition (problem of the insurer) In the first round, lenders choose a contract $\mathcal{C}$ that is the best response to the expected offering of lateral competitors, as well as the strategy of the second-round lenders $\mathcal{C}'(\mathcal{C}, s)$.

It is straightforward to show that given the expected indirect utility of the consumer, $E\mathcal{V}(y, \mathcal{C}, \mathcal{C}'(\mathcal{C}, s))$, in the Nash equilibrium, first-round contract $\mathcal{C} = (L, R)$ solves:

$$\mathcal{C} = \arg\max_{\mathcal{C}} E\mathcal{V}(y, \mathcal{C}, \mathcal{C}'(\mathcal{C}, s)) \tag{10}$$

subject to a zero profit condition

$$E\pi(y, \mathcal{C}, \mathcal{C}'(\mathcal{C}, s)) = 0.$$

In this case, due to informational advantage of second round lenders, the relevant part of the profit function is the case of $R \geq R'$ is

$$\pi(y, \mathcal{C}, \mathcal{C}'(\mathcal{C}, s)) = \begin{cases} -DL \text{ for } b \leq L', \\ (1-D)(R-1)(b-L') - D(L-(R-1)(b-L')) \text{ for } b > L'. \end{cases} \tag{11}$$

Here, the initial lenders profit in the case of no default if and only if borrowing b exceeds the second-round lender's credit limit L'. In case of default, the loss is equal to their credit limit *less* any interest paid on the rolled-over debt.

In our model, the initial lenders choose their contract while taking into account the best response of the second-round lenders. As a result, the initial lenders solve a problem that not necessarily yields an efficient solution.

Equilibrium By equilibrium in this economy, we mean: policy functions

$$D(y, \mathcal{C}, \mathcal{C}'), b\left(y, \mathcal{C}, \mathcal{C}'\right), \mathcal{C}, \mathcal{C}'(\mathcal{C}, s),$$

value functions

$$\mathcal{W}(D, y, \mathcal{C}, \mathcal{C}'), \mathcal{V}(y, \mathcal{C}, \mathcal{C}'), \pi\left(y, \mathcal{C}, \mathcal{C}'\right), \pi'\left(y, \mathcal{C}, \mathcal{C}'\right),$$

such that the policy functions and value functions are derived from the consumer and lender problems described above.

Constrained Efficient Allocation By *constrained efficient allocation (CEA)* we will mean a solution to a planning problem in which the planner faces the same resource constraints, information structure and contractual restrictions as the lenders in the decentralized equilibrium, but controls both credit lines and the default decision D of the agent. Conceptually, we think of this problem as the desired allocation by some benevolent legislators who design the bankruptcy law, and thus ex-post, have perfect information regarding borrowers

state (revealed in front of bankruptcy court). In our setup, this implies that the legislators should be able to condition the availability of the default option on the actual realized income y, or equivalently design the schedule of θ_y.

Formally, CEA is thus given by:

$$\max_{D(y),\mathcal{C},\mathcal{C}'(s)} E\mathcal{W}(D; y, \mathcal{C}, \mathcal{C}'(s)) \tag{12}$$

subject to resource feasibility

$$E\left[\pi(y, \mathcal{C}, \mathcal{C}'(s))\right] + E_s\left[\pi'(y, \mathcal{C}, \mathcal{C}'(s))\right] = 0, \tag{13}$$

where π, π' are defined analogously to (11) and (9).

Comparing to the problem of initial lenders, we note that the constraint efficient allocation not only internalizes all externalities, but also allows for cross-subsidization of the two credit lines. This last feature will turn out unimportant, and the distortionary effect of the decentralized setup will solely come from the first feature.

III. Analytic Characterization

In this section, we prove analytically the key results implied by our model and explain the mechanism behind them. To this end, we introduce the following set of simplifying assumptions that allow us to obtain a complete analytical characterization, and also illustrate the results graphically. Most of these assumptions we relax in the numerical section at the end [incomplete in this draft].

ASSUMPTION 1. *The intertemporal aggregator introduced in (2) is given by a quadratic Taylor approximation:*

$$G(c_1, c_2) = c_1 + c_2 - \mu\left(c_1 - c_2\right)^2, \tag{14}$$

where $0 < \mu < \infty$.

This assumption (Taylor approximation) allows us to fully characterize centralized allocation by simplifying the algebra of the model, and yet preserving a parameterized notion of intertemporal smoothing motive. We do not use it in the decentralized case.

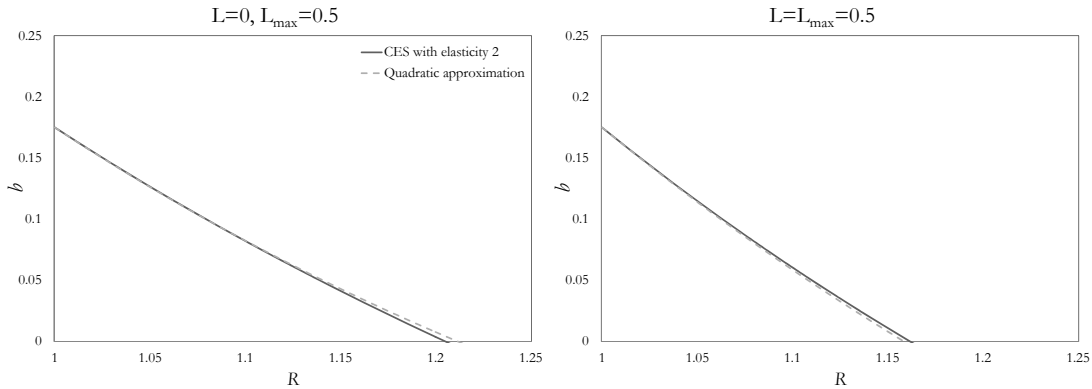

Figure 2: Difference in policies: fitted quadratic G versus CES G ($\epsilon = 2$).

To give a sense how well the quadratic approximation fits the usual the case of CES utility aggregation, Figure 3 presents a sample approximation of a symmetric CES aggregator with an assumed elasticity of substitution 2. The corresponding fitted value of μ in this case is about 0.1, and other parameters are $L' = L - \mathcal{L}_{max}$, $\mathcal{L}_{max} = 0.5$, $y = 1.35$[15].

ASSUMPTION 2. *The signal $s \in S$ can be either fully revealing ($\pi = 1$), or only if explicitly noted, non-revealing ($\pi = 0$).*

Assumption 2 focuses attention on two polar cases of information revelation possible in our model: $\pi = 0$, and $\pi = 1$. These cases yield analytical tractability and at the same time offer a valuable insight into the key mechanism of our model. In the numerical analysis, we cover all intermediate cases and show that, to a large extent, we should expect to see outcomes in-between of these extremes.

ASSUMPTION 3. *There are two income states: $Y = \{y_H, y_L\}$, where $y_L < 1 < y_H$. The probability of y_L occurring is p.*

Two income states allow us to graphically illustrate the intuition behind the results. Under this assumption, we interpret y_L as a catastrophic income loss (expense shock) that is intended to be insured through the institution of personal bankruptcy by the legislators,

[15]This class of quadratic approximations does not preserve homotheticity, and so the approximation involving large income effects is not as precise as the approximation of the substitution effects. Since we consider only 2 income states, and the agent defaults in the low state making policy there almost irrelevant, a good approximation can be warranted. We have experimented with homothetic functions, but the algebra of the problem did not simplify as well.

and y_H stands for normal times in which there is a need to borrow for consumption life-cycle smoothing purposes. Most of the results hold in the case of multiple income states, but there are additional effects introduced by this case that greatly complicate the analytics of the problem. We explore it in the numerical analysis of the model only.

ASSUMPTION 4. *Low state is normalized to be always the default state:* $\theta_L = 1$. *Some repayment capacity is assumed sustainable in the high state:* $\theta_H < 1$, *implying* $\mathcal{L}_{\max}(y_H) > 0$.

The above assumption is introduced for illustrative purposes, and is not crucial to obtain our results. By assuming that the agent always chooses to default in the low income state, we simply give an unambiguous role to the insurance through bankruptcy, and focus attention on the only case that can be affect by non-exclusivity. Otherwise, we would need to restrict parameters to essentially guarantee incentive compatibility of default decision, and would be saying that in case it is incentive compatible, certain effects will be observed. In the numerical section, we present an experiment that covers the entire parameter space of (θ_L, θ_H) to give a sense of the parametric range affected by non-exclusivity [not included in this draft].

DEFINITION 2. *Given the above assumption, we simplify notation by defining:* $\mathcal{L}_{\max} \equiv \mathcal{L}_{\max}(y_H)$, $\theta_H \equiv \theta$.

Finally, to avoid some complications implied by borrowing in the low state under default (due to repayment of interest on rolled-over debt), we impose an assumption that restricts the range of the repayment capacity in the high state so that borrowing in the low state under default never arises. Again, we relax this condition in the numerical analysis.

ASSUMPTION 5. *Income in low state is low enough and satisfies the following restriction:*

$$0 < y_L < 1 - \mathcal{L}_{\max},\ y_H > 1,$$

where $\mathcal{L}_{\max}$ *stands for repayment capacity on high income path—as defined in (2).*

Formally, this assumption is a sufficient condition that guarantees saving in the low state. Clearly, the maximum amount of insurance that can be provided in equilibrium, and thus defaulted on, is $\mathcal{L}_{\max}$. If the sum of credit limits exceeds this number, the agent defaults in

all states, and the lenders can not possibly break even. In the worst case scenario, the agent faces a flat income profile $(1, 1)$, which given the symmetry of intertemporal aggregator G (no discounting), yields a non-binding no-saving/borrowing outcome in low state. The economics of the problem strongly supports this assumption: The institution of bankruptcy is a form of insurance against severe income/expense shocks (e.g. persistent job loss or a serious illness), and so an anticipation of such event should not prompt borrowing against this state before the shock actually occurs.

In what follows next, we start with the characterization of the constraint efficient allocation (CEA) as defined by (12), and then turn to the characterization of the equilibrium allocation under non-exclusivity. To the end of the first part, we also mention how CEA relates to exclusivity regime.

A. Constrained Efficient Allocation

Here, we characterize the constrained efficient allocation (CEA). All our results are illustrated in Figure 3. In our analysis, we focus on the space of intertemporally aggregated consumption $c = G(c_1, c_2)$, as implied by (2) and (14) in the high and in the low income state, respectively. This space is particularly convenient for us, as it directly focuses on the insurance aspects of our model (distribution of consumption across states of the world). To find CEA in this space, we first characterize the preferences (indifference curves), and then the shape of the Resource Feasible Consumption Frontier (RFCF) as defined below.

DEFINITION 3. Resource Feasible Consumption Frontier (RFCF) *is the efficient frontier of all* (c_L, c_H) *consistent with (13).*

DEFINITION 4. *By* indifference curves *in the space of intertemporally aggregated consumption, we mean all* (c_L, c_H) *such that* $E\{u(c)\} = U$, *where* $U \geq 0$ *gives the level of utility corresponding to a selected indifference curve.*

Characterization of preferences is fairly straightforward. In the space of aggregated consumption, the assumed properties of the utility function immediately guarantee a map of nicely behaved indifference curves, which are increasing towards north-east and strictly convex. In addition to these standard properties, as a consequence of A5, we additionally observe

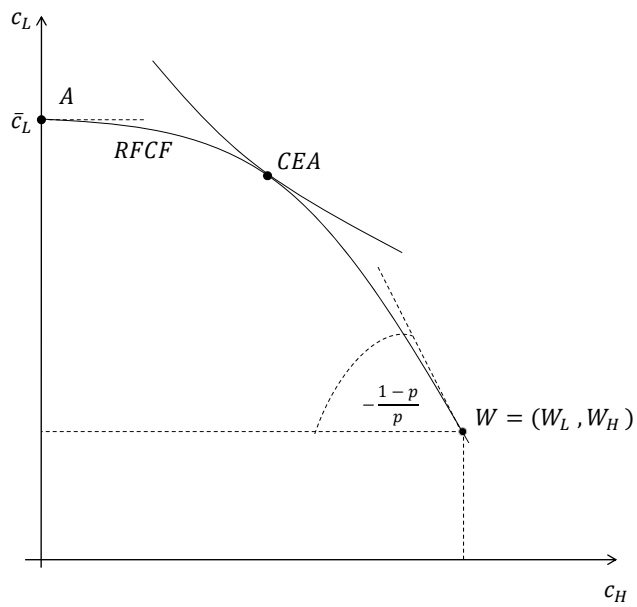

Figure 3: RFCF and CEA.

that the slope of each indifference curve in the relevant domain is strictly below the ratio of probabilities of the the two states $-(1-p)/p$—a number corresponding to the slope of the *Actuarially Fair Transformation Line (AFTL)*. This number will turn out important, as we will later show that, while planner's frontier is flatter than this ratio, the decentralized profit feasible frontier is steeper. As a result, while the planner's solution will be an interior one, the decentralized equilibrium will always be corner and thus independent from consumer preferences for insurance.

The reason why assumption 5 guarantees that the preferences are flatter than the actuarially fair ratio is as follows. Recall that A5 has been imposed to essentially guarantee that for $y = y_L$, we have $b(y_L) \leq 0$ (non-binding) for any interest rate charged on borrowing. However, since $y_H > 1 > y_L$, aggregated consumption on the low income path $y = y_L$ must necessarily be lower than the aggregated consumption on the high income path (i.e. agent is underinsured in best case scenario). The result is that MRS between $c_L = G(c_{1L}, c_{2L})$ and $c_H = G(c_{1H}, c_{2H})$ is thus strictly below the actuarially fair transformation ratio $-(1-p)/p$.

The above properties of the preferences are summarized in the lemma below.

LEMMA 2. *Under assumptions 1-5, the indifference curves implied by $u(\cdot)$ in the space of c_L, c_H are: (i) strictly increasing towards NE, (ii) strictly convex, and (iii) downward-*

sloping, and their slope is strictly lower than the slope of the actuarially fair transformation ratio: $-(1-p)/p$.

Proof. The proof follows closely the argument sketched in text above. See the appendix. □

We next proceed with the characterization of the Resource Feasible Consumption Frontier (RFCF), as defined in (3). We first state a technical result that allows us to restrict attention to only a subset of contracts, and then in the following lemma, we characterize RFCF.

LEMMA 3. *Under assumptions 2-5, the constrained efficient allocation (CEA) features* $R' = R$, *unless* $L = 0$ *or* $L' = 0$. *Moreover, borrowing constraints are never binding in the high state, and in the low state the agent either does not borrow, or a faces marginal interest rate equal to one.*

Proof. In the appendix. □

LEMMA 4. *Under assumptions 1-4, the* Resource Feasible Consumption Frontier (RFCF) *defined in (3) is a decreasing function* $c_L(c_H)$, *obeying the following basic properties: (i) it is defined on a non-empty, closed and connected interval* $[W_H, \bar{c}_H]$ *(*$\bar{c}_H > W_H$*), (ii) it is continuously differentiable, and (iii) its slope monotonically increases from* $-(1-p)/p$ *at* W_H *to* 0 *at* $\bar{c}_H$.

Proof. In the appendix. □

REMARK 1. *The property that RFCF is globally flatter than AFTL can be established without functional form given by A1. [To be completed]*

The above lemma fully characterizes the shape of RFCF, which looks as illustrated in Figure 3. Looking from the bottom, the frontier starts from a no-insurance point (W_L, W_H), where $W_i = 1 + y_i$ is the ex-post wealth discounted at the cost of funds. Here, the consumer is instructed not to default and credit is provided at the cost of funds to guarantee undistorted intertemporal smoothing within each income path. Above this point are all the points that the planner delivers by instructing the consumer to default in the low state but not in the high state. By setting a positive interest rate in the high state, this way the planner can transfer resources from the high state to the low state, and thus the frontier extends upwards.

However, since higher points on the frontier must correspond to progressively higher interest rate levels, as we move up the frontier, the interest rate distortion of intertemporal smoothing increases progressively. As a result, the slope of the frontier progressively flattens to the top, departing from the actuarially fair slope -(1-p)/p achieved at no-insurance point W. Finally, at the very top, the frontier ends with a plateau, reflecting the fact that for sufficiently high levels of R (possibly infinite), any further increases of R only lower interest revenue for the planner (or keep it constant).

Intuitively, the intertemporal distortion due to the interest rate is related to the fact that if higher interest rate involves a substitution effect, that lowers intertemporal borrowing. In the parametric case considered by us under A14, the distortion can be explicitly calculated. It is captured by the quadratic term that imposes penalty for a departure from a non-smooth consumption at the end. Given optimal choice of the agent, with borrowing in high state, saving in low state, and non-binding constraints, the distortion is given by:

$$\mu(c_2 - c_1)^2 = \frac{1}{4\mu}\frac{(R-1)^2}{(R+1)^2}.$$

As we can see, the distortion is convex in R, implying that higher R must result in progressive distortion as we move up. For most preferences, including the standard CES case, the same property would hold—and it is closely related to the standard 'Laffer curve effect' that arises in case of proportional income taxation.

The properties of RFCF established above, when combined with the properties of the consumer preferences given in Lemma 2, allow us to establish the existence of a unique, interior CEA in the interval $[W_L, \bar{c}_L]$ (see Figure 3). This final conclusion is summarized below.

PROPOSITION 1. *Under assumptions 1-4, the constraint efficient allocation (CEA) exists, is unique, and, as illustrated in Figure 3, lies on the interior of the domain of the RFCF given by Lemma 4 part (i).*

Proof. Follows from Lemma 4 and 2. □

In the next paragraph, we establish how our constrained efficient allocation relates to equilibrium allocation with full exclusivity.

Relation to Equilibrium under Exclusivity By equilibrium under exclusivity (EA-E), we mean a situation in which the borrowers commit to a particular lender for a specified period of time. In the model, it implies that there is no second round of competition whenever the first round results in a positive credit limit. As we show below, the equilibrium under exclusivity may involve some distortion, but compared to non-exclusivity regime, this distortion is unambiguously smaller[16].

Why exclusivity can be welfare improving? Formally, this follows from the fact that the allocation in this case solves a problem that is very similar to the planner's problem: i.e. it solves (10), but with an exogenously fixed null second-round contract ($L' = 0$) which makes it similar to (12).

It should be clear that the conditions under which EA-E implements CEA are: (1) in EA-E the credit limits are non-binding in the high state, and (2) in EA the punishment for defaulting in states that the planner chooses to be non-default states is sufficiently high to avoid the problem of 'strategic' default (i.e. make these stated non-default state endogenously).

The first requirement comes from the fact that the planner uses second credit line to always relax the borrowing constraint, and in EA we assume that there is no second round. However, once we take into account the possibility that a committed initial lender can sweeten the terms by raising credit limit, such case will automatically be taken care of in CEA as initial lenders, to avoid binding constraints, will raise credit limit. This is because relaxed borrowing constraint unambiguously raises profits. Thus, we can conclude that, after taking into account the possibility of sweetening of terms during the second round, the two problems are exactly equivalent as long as (2) holds. This is summarized by the proposition below.

PROPOSITION 2. *Exclusivity implements CEA exactly iff default in non-default states of corresponding CEA is sufficiently costly for the consumer, i.e.* $\theta_y > \bar{\theta}$ *is sufficiently large so that* y *is endogenous non-default state under exclusivity (value of* θ_y *in all default states is identical).*

Proof. Follows from the argument in text above. □

[16]This should be fairly obvious. In terms of the underlying contracting problem, non-exclusivity adds a voluntary participation constraint which worsens outcomes.

We should stress that the above proposition applies also to cases that violate our assumptions, and it is a reminiscent of a fairly standard fact in this literature: As mentioned in the introduction of the paper, bankruptcy can, under certain conditions, implement CEA when two-sided commitment of contracts are allowed, and default is punished sufficiently to discourage strategic non-repayment in normal times (high income states). An analog of this result holds in the standard Eaton & Gersovitz (1981) framework.

Moreover, the restriction imposed by the requirement of sufficiently high punishment is detectable in practice (at least in severe cases), and in our view is not a serious limitation of this implementation of state contingency. Clearly, if a severe problem of insufficient punishment arises in equilibrium, since binding credit constraints will be observed in some cases. The legislators will likely learn about it[17], and correct the problem by modifying the bankruptcy accordingly. In fact, an attempt to do so in a clean under exclusivity is means testing regulation similar to the one introduced in 2005. Under exclusivity, it will always work as intended, and only under non-exclusivity can have undesirable side effects that we show next.

Finally, before we turn to the characterization of the equilibrium allocation under non-exclusivity, we develop here a condition that guarantees that CEA lies in the range consistent with assumption A5, i.e. the agent saves at CEA. This will potentially create a situation in which overinsurance through bankruptcy can arise under the assumptions we have imposed. It is clear that saving in CEA is possible, but to make our results unambiguous, here we develop a condition under which saving in CEA under default arises independently from preferences.

LEMMA 5. *The sufficient condition for the agent to save in CEA is:*

$$y_H \leq \min\left\{1+\frac{1}{\mu}, 1+\sqrt{\frac{2p}{(1-p)\mu}}\right\},\ y_L \leq 1-\mu\frac{(1-p)(y_H-1)^2}{2p}.$$

Proof. It is straightforward to verify that this is the case when the above conditions are

[17]The result above does not imply that efficient outcome will be implemented when θ_y can not be varied by state. Nevertheless, the standard parametric cases of 'stigma' type of punishment (additive utility cost) often can implement CEA.

satisfied. □

The above condition essentially requires that there is some separation of the high income state and the low income state from unity—as it would be implied, for example, by a parametric case: $y_H = 1 + \varepsilon$, $y_L = 1 - \varepsilon$, $\varepsilon > 0$. Clearly, if the shock corresponding to the low state is particularly severe—as it would make sense to assume in our case—the interesting parameter range is likely to be preserved by this condition. We illustrate it by an example below.

EXAMPLE 1. *Let* $y_H = 1.35$, $p = 2\%$, *and* $\mu = 0.1$. *Then, 5 gives:* $y_H \leq 1.64$, $y_L \leq 0.7$. *Since the upper bound on* y_H *is not particularly restrictive, by setting* y_H *anywhere in the range between 1 to 1.5, we obtain the upper bound on* y_L *that ranges monotonically from* 1, *for* $y_H = 1$, *to about* .4, *for* y_H *close to 1.5.*

B. Equilibrium Under Non-Exclusivity

In this section, we characterize the equilibrium allocation under fully revealing signals and non-exclusive contracts (EA). At the end of the section, we also extend our results to fully non-revealing signals. We proceed by analogy to the previous section. In order to expose the insurance aspects of our model, we focus attention on the space of intertemporally aggregated consumption ($c_i = G(c_{1i}, c_{2i}), i = H, L$), and characterize the profit feasible consumption frontier (PFCF). PFCF is defined as the locus of intertemporally aggregated consumption allocations implied by the zero profit contracts of the initial lenders, given the best response of the second-round lenders. The formal definition of PFCF is stated below.

DEFINITION 5. *PFCF is defined as the efficient frontier of all* (c_L, c_H) *consistent with (11), given that* $\mathcal{C}'(\mathcal{C}, s)$ *solves (8).*

It turns out that, except for a very narrow range of parameter values, PFCF is globally steeper than the indifference curves of the consumer and the RFCF characterized in the previous section. This is the main result of our paper. It implies that the equilibrium point must lie at the top of PFCF, as illustrated in Figures 4a and 4b. Since the exact position of this point is a corner determined by profit feasibility, and thus is independent from borrower's

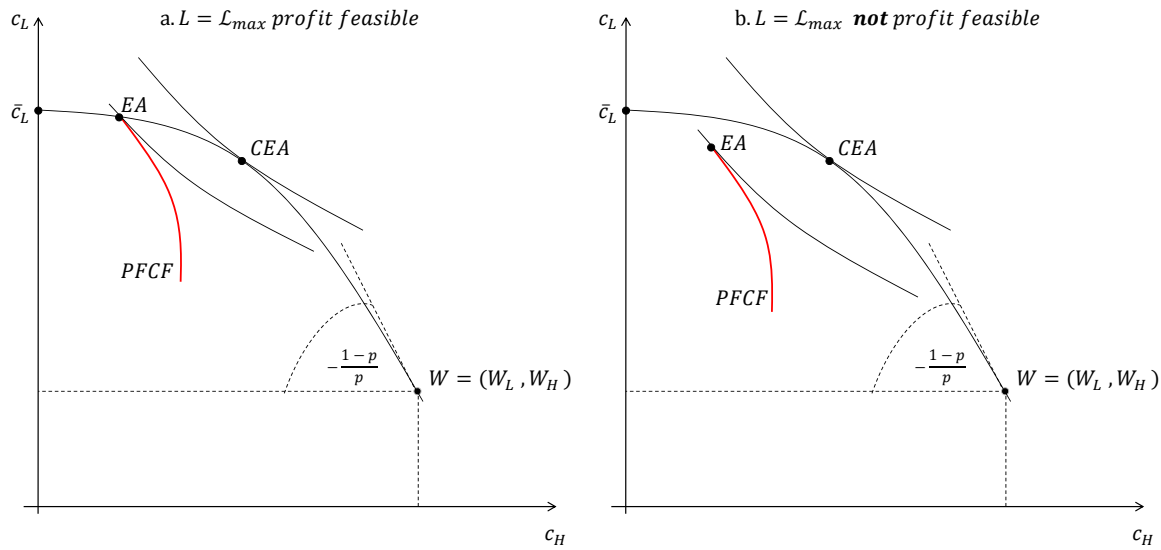

Figure 4: Equilibrium allocation under non-exclusivity.

preferences for insurance, it implies that non-exclusivity regime can potentially be highly distortionary.

In terms of the exact outcome observed in equilibrium, three cases are possible depending on the value of the total repayment capacity $\mathcal{L}_{max}$ of the borrower:

1. For moderate levels of $\mathcal{L}_{max}$, it may be feasible for the initial lender to break even at $L = \mathcal{L}_{max}$, and because at this point there is no entry in the second round ($L' = 0$), this point lies on RFCF. This first type of equilibria is illustrated in panel a of Figures 4.
2. The second possibility, illustrated in panel b of the same figure, is a case in which $L = \mathcal{L}_{max}$ is either very low or very high and thus yields strictly negative profits for the lender. In such case, the highest point on the PFCF will be an equilibrium, and this point may either lie at the top of the planner's frontier (in case $\mathcal{L}_{max}$ is very high) or on the strict interior of the planner's feasibility set as illustrated in panel b of the figure. In this second case, it is an interior point because at $L = \mathcal{L}_{max}$ credit constraint in the high state is binding, and planner can relax this constraint. This solution would align with exclusivity regime as long as it involves underinsurance (c_L below CEA), but not when it involves overinsurance (c_L above CEA)
3. Finally, the third case is a complete shutdown of insurance, which arises if the candidate

equilibria of the type mentioned above feature so severe overinsurance or underinsurance that the borrower may prefer the no-insurance point W. In this equilibrium, no insurance is provided through bankruptcy.

Which case actually arises crucially depends on the punishment for defaulting θ, with only the switch between insurance and no-insurance being dependent on consumer preferences (indifference curves). In particular, within case 1-2, the amount of insurance provided will be fully independent from preferences for insurance. The formal results leading to the above conclusions are summarized below.

LEMMA 6. *If $\mathcal{L}_{max} \notin \left[\underline{\mathcal{L}}, \overline{\mathcal{L}}\right]$, where $\overline{\mathcal{L}} = \frac{y_h - 1}{2} - \frac{p}{8(1-p)\mu}$ and $\underline{\mathcal{L}} = (1 - \frac{1}{2}\frac{p}{1-p})\overline{\mathcal{L}}$, then, under A1-5, the PFCF has the following properties:*

(i) it shares at most one point with the RFCF: Corresponding to the credit limit of the initial lender fully exhausting the repayment capacity of the consumer on high income path, i.e. $L = \mathcal{L}_{max}$, implying $L' = 0$,

(ii) it is differentiable, and

(iii) its slope is globally flatter than than the actuarially fair ratio $-(1-p)/p$

Given the above lemma, together with Lemma 2 and 4 assuring that RFCF and indifference curves are flatter than the actuarially fair transformation line, we next establish our main result: The equilibrium allocation is a corner solution of the default decision in the high state: i.e. the initial lender sets $L = \mathcal{L}_{max}(y_H)$. We summarize this in the proposition below.

PROPOSITION 3. *Under the restrictions of Lemma 6, the equilibrium under non-exclusivity (EA-NE) exists, is unique and features* full entry deterrence *by the initial lenders, in the sense that either $L = \mathcal{L}_{max}(y_H)$ or if this point is not profit feasible L is equal to the highest level that is. The degree of insurance provided by the institution of personal bankruptcy thus crucially depends on the punishment for defaulting in high state θ_H, and is independent from the consumer's preferences.*

As we can see, the above cover entire parameter range with an exception of a tiny interval of $\mathcal{L}_{\max}$. To give a sense how significant this restriction is, consider the following reasonable

parameterization of the model: probability of default state $p = 2\%$, intertemporal elasticity γ =2, high income $y_H = 1.35$. In such case, the excluded size of the range of $\mathcal{L}_{max}$ to which our results do not apply is as small as: $.012\overline{\mathcal{L}} = 0.0018$, where $\overline{\mathcal{L}} = .15$. We conclude that this restriction is very slack.

We next turn to the discussion of the intuition behind the above results.

Intuition behind the main result It turns out that RFCF and PFCF are almost always separated by the actuarially fair ratio. This property implies that while CEA is an interior solution, and the decentralized equilibrium is corner solution largely determined by $\mathcal{L}_{\max}$ (if positive insurance is to be provided).

The intuition behind this result is as follows. The actuarially fair ratio is the relative price of consumption in the low state expressed in terms of consumption in the high state, in the case when consumption can freely be transferred between the two states. This would, for example, be possible under complete contracts. Now, the reason why the implementation of state contingency with bankruptcy and state non-contingent contracts may depart from this fair ratio comes from the fact that this implementation involves a change of the distortion of the intertemporal margin.

Specifically, in the case of the planning solution, the frontier turns out flatter than the actuarially fair ratio. This is because every marginal unit of consumption transferred to the low state through bankruptcy not only requires from the planner to take an actuarially equivalent amount of resources from the high state, but also implies a higher distortion of the intertemporal margin in high state due to increased interest rate.

An exact opposite phenomenon arises in the decentralized equilibrium. The reason is an additional crowding out effect of future lenders. Intuitively, in the decentralized setup, an increase of credit limit L of initial lenders by some $\Delta L > 0$ raises the amount borrowed from the initial lender by $\Delta L(1 + \frac{\partial b}{\partial L})$, as it crowds out second round lenders who steal business from them. This not the case in the planning solution[18], in which the borrowing level is independent from credit limits.

[18]The term $\frac{db}{dL} < 0$ accounts for the fact that the transfer of borrowing from the second round lender to the initial lender is less than dollar for dollar since the overall repayment amount would go up for fixed b.

Formally, given that the profit function of the initial lenders in equilibrium is

$$\pi = (1-p)(R-1)(b-(\mathcal{L}_{\max}-L)) - pL,$$

the aforementioned crowding out effect creates an additional revenue for initial lenders in high state given by:

$$(R-1)\Delta L(1+\frac{\partial b}{\partial L}).$$

Moreover, as illustrated in Figure 5, given that a higher credit of the initial lenders L only involves an income effect, such change allows for a transfer resources towards the low state in a fully non-distortionary way — i.e. at the actuarially fair rate -(1-p)/p.

Now, the crucial question is whether additional revenue in high state suffices to cover additional losses incurred in the low state. If yes, the interest rate can be lowered, and the trade-off will be more favorable than actuarially fair rate. If not, it will just like in the case of the planner. Turns out, the first case holds.

The additional revenue from crowding out second round lenders weakly exceeds the actuarial cost of an additional transfer under default iff

$$(1-p)(R-1)\Delta L(1+\frac{\partial b}{\partial L}) > p\Delta L.$$

After simplifications, it gives

$$-\frac{\partial b}{\partial L} < 1 - \frac{1}{R-1}\frac{p}{1-p}. \tag{15}$$

It turns out that, except for the tiny interval identify in the lemma, this condition holds.Consequently, the interest rate can be lowered and distortion can be reduced. The result a frontier that is steeper than the actuarially fair ratio $-(1-p)/p$.

What is the intuition behind the interval on which (15) does not hold? A careful analysis of the condition reveals that the exception is related to the fact that lower levels of $\mathcal{L}_{max}$ generally correspond to higher utilization rates of credit limits in the high state, i.e. they raise the ratio:

$$\mathcal{U} = \frac{b(y_H)-(\mathcal{L}_{\max}-L)}{L}.$$

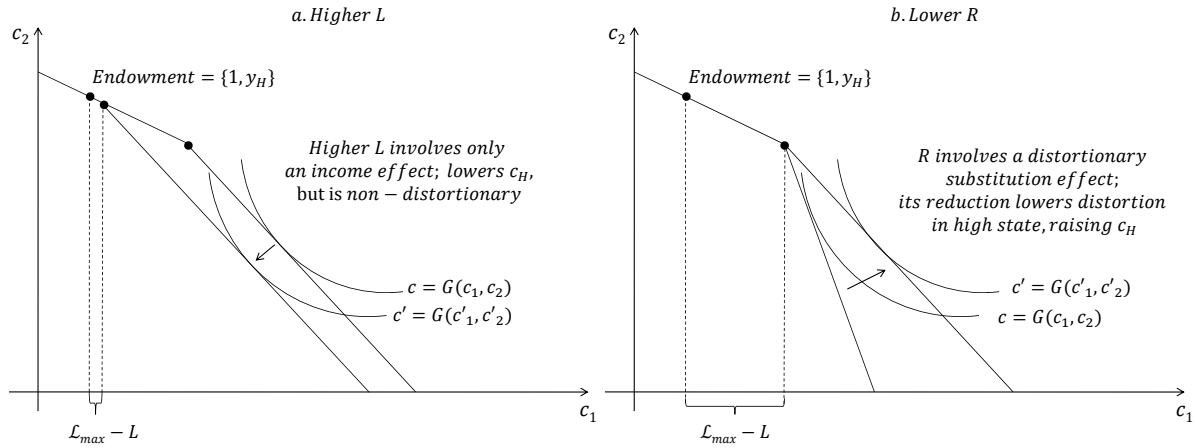

Figure 5: Transfer of resources from high to low state in EA.

The level of utilization pins down the range of the interest rate level R at which lenders can break even (i.e. R characterizing the initial point we started with in the reasoning above), and at sufficiently low level of R, a violation of (15) is possible (given $\frac{db}{dL} < 0$). However, the interval of $\mathrm{L_{max}}$ for which this happens turns out also bounded from below. At 100% utilization level, borrowing constraint starts to bind, implying $\frac{db}{dL} = 0$, which trivially restores (15) for all levels of R.

Our conclusion from the above analysis is that the main results of our paper are implied by a robust economic mechanism. We should expect this mechanism to characterize a much broader class of economic environments. Note that the only link to the specifics of our model is how change of credit limit affects the condition (15) through the derivative $\frac{db}{dL}$. None of the assumption determining this object seem to us non-standard or excessively simplistic.

C. Extension: Multiple contracts within each round

As our first extension, we modify the assumption of single contract per round and allow for a sequential competition with multiple contract. The exact setup is summarized in assumption below.

ASSUMPTION 6. *Multiple contracts per round are allowed in the following sense: Within each round, lenders (publicly) post their contracts at no cost, and the consumers sequentially choose the contracts that suit their needs best. Consumers can sign multiple contracts within each round (one from each lender). The competition is sequential, i.e. after any contract is*

signed, this information is revealed to all lenders, and they can adjust terms on the posted contracts (cannot change terms on accepted contracts). The process continues until the consumers choose to exist the market.

The proposition below shows that in the analyzed case of perfectly revealing signals this modification of the baseline setup is without loss of generality. In the case of non-revealing signal, it is also true, but with an additional restriction on the space of feasible contracts. This actually partially extends our results to this case, as we show next. We do not have a characterization of a partially revealing signal at this point, and the argument below does not generalize to such case for the reasons explained in the remark below.

PROPOSITION 4. *If the consumers may accept multiple contracts in each round of competition, and competition is sequential as described in A6, after they accept the first contract, it is without loss of generality to assume that no further contracts are accepted beyond the first one when signal is fully revealing* $(\pi = 1)$. *When signal is non-revealing* $(\pi = 0)$, *same property holds, but the space of contracts must be additionally restricted so that either* $L + L'(s) = \mathcal{L}_{\max}(y_h)$ *for at least one* $s \in S$, *or in case* $L + L'(s) < \mathcal{L}_{\max}(y_H)$ *for all* $s \in S$, L, L' *there is full utilization, i.e.* $L + L' = b(y_H)$.

Proof. In the appendix.

In the case of perfectly revealing signals, given the above discussion, it is sufficient to consider possibility of multiple contracts only in the first round, and assume that in the second-round there is a unique contract that satisfies: $L' = \mathcal{L}_{\max} - L$, if $s = H$ (and $y = H$ due to revealing signal), and $L' = 0$ otherwise. Going back to the first round, by contradiction, we assume two contracts are signed in the first round R_1, L_2 and a later contract, R_2, L_2. As above, this contract can be improved by offering $\mathcal{C}_{dev}$ listed above. The rest is analogous to the arguments given above, and will be omitted. □

REMARK 2. *The above argument does not readily generalize to the case of* $0 < \pi < 1$. *The following example illustrates the problem that arises in this case. A counterexample can be constructed. Suppose the second-round lender enters only if signal is H, and two lines are offered in the first round, with* $R_1 < R_2$. *Furthermore, suppose both lines are utilized when* $s = L$, *and only the one characterized by lowest interest rate* R_1 *is utilized if the signal is H.*

In this case, unlike in the fully revealing signal case, both credit lines can theoretically break even because y_H is still possible when $S = L$. As a result, the argument showing that deviation contract considered in the proof is better can not be applied.

Finally, we show that an analogous statement applies to the planning problem that defines CEA.

PROPOSITION 5. *Under the same assumptions as in Proposition 4, without loss of generality, the planner will choose to use only one contract per round when signal is fully informative or non-informative.*

Proof. Follows an analogous reasoning involving the same deviation contract. □

IV. Numerical Examples

To illustrate our analytic results, here we consider a concrete numerical example with revealing, partially revealing and non-revealing signals. To accommodate the case of partially revealing signals, in contrast to previous sections, we analyze the allocations in the space of expected utility for each income realization. This space is more convenient with partially revealing signals, as depending on the realization of the signal and entry, consumption may vary in each state by the signal realization. This implies that expectation must be taken in order to evaluate the outcomes, and the only meaningful expectation from the welfare point of view is over the utility from intertermporally aggregated consumption along each income path. In contrast to the previous case in which we used intertemporally aggregated consumption, preferences in this space are then linear, and the slope of indifference curves is equal to the slope of the actuarially fair ratio -(1-p)/p. Formally, the space of allocations is represented by tuples (u_L, u_H) such that $u_i = E_s[u(G(c_{1i}(s), c_{2i}(s))]$, $i = y_H, y_L$, $s = H, L$ stands for the signal (E_s is conditional expectation on signal realization s).

In terms of parameters, we follow here the leading example from the analytic section. Specifically, we assume two income states, $y_H = 1.35, y_L = 0.75$, no punishment for default in low state $\theta_L = 1$, some punishment in high state $\theta_H = 0.8$, probability of low income p set equal to 2%, and a smoothing parameter $\mu = 0.1$ that approximately corresponds to intertemporal elasticity 2. The utility function u is assumed standard: CRRA with elasticity of substitution $\sigma = 2$.

Our goal is to compare outcomes under exclusivity and non-exclusivity, and also show the effects of less noisy information revelation to potential entrant—which was a likely to occur in the early 90s due to the widespread implementation of an automatic credit scoring system and gradual improvement of this system (due to inflow of information).

Table 1: EA and CEA in benchmark case: numerical example with revealing signal.

	$\pi = 1$		$\pi = 0.5$	
Variable	y_L	y_H	y_L	y_H
A. Equilibrium under Non-exclusivity				
L_{max}	0.00	0.27	0.00	0.27
R	1.05	1.05	1.05	1.05
L	0.27	0.27	0.27	0.27
R'	-	-	-	-
L'	0.0	0.0	0.0	0.0
Default decision	1	0	1	0
Borrowing (b)	0.0	0.112	0.0	0.112
B. Equilibrium under Exclusivity				
L_{max}	0.00	0.27	0.00	0.27
R	1.034	1.034	1.034	1.034
L	0.217	0.217	0.217	0.217
L'	0.0	0.0	0.0	0.0
Default decision	1	0	1	0
Borrowing (b)	-.0163	0.1312	-.0163	0.1312
C. Constrained Efficient Allocation				
L_{max}	0.00	0.27	0.0000	0.27
R	1.034	1.034	1.034	1.034
L	0.217	0.217	0.217	0.217
L'	0.0	0.0	0.0	0.0
Default decision	1	0	1	0
Borrowing (b)	-.0163	0.131	-.0163	0.131

The value of the parameters is given in text, with fully revealing signal.

A. Case 1: Perfectly revealing signal ($\pi = 1$)

The numerical solution of the model with perfectly revealing signal is given in Table 1 and, in the space defined above, the outcomes are additionally illustrated in the top panel

in Figure 6. The figure shows: (i) resource feasible allocations (blue dots), (ii) constraint efficient allocation (CEA) with an indifference curve passing through CEA, (iii) profit feasible equilibrium set under non-exclusivity (red stars), and (iv) equilibrium allocation under non-exclusivity (EA), with the indifference curve passing through EA.

As we can see, the profit feasible set, comparing to exclusivity, is severely restricted to only very high levels of credit limits that are still feasible. Essentially, there are points at the top, and one point corresponding to no-entry by initial lenders at the bottom-right. Since this case obeys all the assumptions imposed by us in the analytical section, not surprisingly, the selected equilibrium point in this case is $\mathcal{L}_{max}$, and features significant overinsurance. Specifically, we can calculated based on the numbers from Table 1 below that the credit limits under non-exclusivity are in excess by about 25% of what is optimal, while exclusivity delivers an efficient outcome.

B. Case 2: Partially revealing signal ($\pi < 1$)

We next study how the model is affected by lower signal precision. We consider two cases, labeled, 'signal 0' and 'signal 0.5'. Both are shown in the bottom panels of Figure 6. The first case represents non-revealing signal ($\pi = 0$). In the second one, signal has precision $\pi = 0.5$.

As we can see, the level of overinsurance is drastically mitigated in the case of perfectly non-revealing signals, but the outcome turns out exactly identical with the noisy signal ($\pi = 0.5$). Moreover, in the case of perfectly non-revealing signal, as we have demonstrated in the previous section (Proposition 4), this outcome may not be robust to the restriction of one contract per round, and in fact, it is not. When we introduce multiple contract per round, by introducing a contractual restriction implied by the equivalence result given by Proposition 4, we find that such modification again distorts allocation and result in extreme underinsurance. For comparison, we include an additional Figure 7 analogous to the previous one in which we allow for multiple contracts to be signed. As we can see, the equilibrium outcome changes to an inefficient one, as multiple contract remove all points that do not imply full utilization (compare to the bottom right panel in Figure 6). Again, as stressed in the introduction, in this case the level of insurance turns out independent from the consumer's preferences for insurance, as it is determined here by the optimal borrowing in the high state (non-default

rate) at the actuarially fair rate.

As the figures show, the impact of noise is that it becomes more and more possible for the second-round lender to provide insurance. Note from the included figures that a set of points gradually appears at the bottom right corner of the set as the noise is increased (up from no-insurance point). In these points, entry of the insurer is non-existent or limited. In the latter case, initial lenders only count on the probability that signal is low and second-round lender do not enter. Then, they make make profits, but otherwise they are exposed to losses. Due to noisy signal, however, second-round lenders start to provide insurance instead. Such outcomes start to appear, however, only for very low levels of π. In the extreme case of $\pi = 0$, second round lenders take over and provide efficient insurance, that however is not robust to multiple contracts as explained above.

Another effect of noisy information, which turns unimportant in our numerical solution, is that with noisy signal it is no longer feasible for lender to separate credit provision for the purpose of intertemporal smoothing and insurance. Consequently, as precision of information worsens, points at the bottom of the resource feasible set are gradually wiped out. This should be intuitive: With noisy signals, all lenders (or the planner), regardless whether they want it or not, are exposed to default probability. Consequently, very low levels of insurance result in lower consumption in high state because they requires low credit limits, which hurt intertemporal smoothing (lowering c_H).

Independently, the figures also illustrate how informational advantage can lead to a transition from underinsurance to overinsurance. Interpreted loosely, the effects illustrated are suggestive of how improvements in screening technology that occurred during the 90s, might have possibly altered outcomes in the unsecured credit market towards higher charge-off rates. In this example, we raise π from 0 to 0.5, and calculate the charge-off rate on credit card debt (fraction of debt discharged due to bankruptcy). In our example, the charge-off rate increases between the two cases from 2% to about 4.9%—as it roughly did between late 80s and the end of 90s. The example is obviously very stylized, but reveals some potential of our model to generate plausibly looking bankruptcy trends.

V. Robustness

In this section, we study the robustness of our analytic results to modifications of the baseline setup. [To be completed]

VI. Conclusions

In this paper, we have provided a detailed study of the effects of non-exclusivity on the provision of credit through the institution of personal bankruptcy. Our conclusion is that, in this environment, exclusivity would be desirable. Exclusivity means that borrowers commit to lenders for a specified period of time, or alternatively, regulators allow lenders to impose a switching fee that is paid only if a borrower decides to switch to another lender before the original contract's expiration data. Such regime is unambiguously welfare improving in our model as it eliminates the potential overinsurance that non-exclusivity can give rise to.

Finally, using a numerical example, we have demonstrated the effects of improved information on bankruptcy statistics in our model. This stylized example showed us that better information regarding borrower's future creditworthiness can potentially lead to higher charge-off rates—a feature of the data that existing quantitative models of bankruptcy have difficulty to reproduce.

Appendix

Proof of Lemma 2: (i)-(ii) is trivially implied by the properties of u and G. (iii) is a direct consequence of A5. To see this, note that this assumption has been imposed to guarantee that on the low income path y_L, the agent saves, i.e. $b(y_L) \leq 0$. Given $y_H > 1 > y_L$, this implies that the intertemporally aggregated consumption on the low income path must neccessarily be lower than the intertemporally aggregated consumption on the high income path ($c_L < c_H$). The consequence is that the MRS between $c_L = G(c_{1L}, c_{2L})$ and $c_H = G(c_{1H}, c_{2H})$ must be lower than the actuarially fair ratio $-(1-p)/p$. □

Proof of Lemma 3: The first few properties are trivial. First, the interest rate distortion can only arise in the high state (non-default state). This is because whenever the planner dictates the agent to default in the low state, it is optimal for the planner to use the second credit line ($L = 0$, $L' > 0$), and while setting $R' > 1$ on the high income path, set $R' = 0$ on the low income path (default path). Only this way the planner can eliminate a wasteful intertemporal distortion in the state in which the transfer is meant to be paid. It is also trivial to establish that the planner will always set L' in high state so that the borrowing constraint is non-binding.

To establish the last property ($R = R'$), consider by contradiction that the agent has two contracts (with positive limit) and the interest rate on them does differ. Two cases can arise: (i) there is no borrowing on one of the credit lines or (ii) there is borrowing using both credit lines. Since (i) is trivial, consider (ii) and denote the marginal interest rate by $R_m \equiv \max\{R, R'\}$.

To show that the hypothesis leads to a contradiction, consider instead one deviation contract that sets the same credit limit ($L + L'$), but makes the interest rate charged on the entire range equal to some $\bar{R}$. Let $\bar{R}$ be the lowest possible rate that is resource feasible.

First, note that $\bar{R}$ is obviously strictly lower than R_m and strictly higher than $\min\{R, R'\}$. Clearly, by increasing the interest rate of the lower interest rate credit line the revenue associated with this line goes up, allowing the marginal interest rate to actually fall. Since borrowing b in high state only goes up for a lower marginal R, higher revenue is assured for R sufficiently close to R_m.

Second, note that the deviation contract must imply actually higher consumption level in high state (and same in low state). To this end, observe that the present value of income $1 + y_1$ *less* the present value of consumption $c_1 + c_2$ on the high income path must equal to $p(L + L'(s = y_L))$ (expected amount defaulted on in the low state). Next, define an auxiliary level of income $y' = y - p(L + L'(s = y_L)$, and by setting $R = 1$, evaluate consumption at the optimal choices of b in the high state both under the deviation contract b** and the original set of 2 contracts b*** from the formula: $c = G(1 + b, y' - b)$. Clearly, by definition of y', consumption at these points is exactly equal to the one from the original problem, but the deviation contract must lie closer to the maximizing value of b defined by $b^* = \arg\max_b G(1 + b, y' - b)$: i.e. $b^{***} < b^{**} < b^*$. This ranking is implied by the consumer's first order condition $G_1(1 + b, y' - b) = RG_2(1 + b, y' - b)$, resource feasibility of the original contracts implying $y' > 0$, non-binding borrowing constraint in the high state, and the strict concavity of the symmetric aggregator G. By the Fundamental Theorem of Calculus, under these same conditions, the ranking also implies $G(b^{***}) - G(b^{**}) = -\int_{b^{***}}^{b^{**}} G_b(1+b, y'-b) < 0$. This completes the proof. □

Proof of Lemma 4: By Lemma 3, borrowing constraint never binds in the high state, and without loss of generality we can assume that the planner only uses the second round CL, with R', L' possibly dependent on the realization of the signal (to relax borrowing constraint in high state when needed), and interest rate R' neccessarily equal one in the low state.

We start by calculating the maximum interest revenue that the planner can possibly raise in the high state. The total interest revenue TIR in the high state is defined by $TIR = (R - 1)b(y_H; R, L)$. This revenue, by resource feasibility, determines the maximum transfer the planner can give to the agent in the low state.

By Lemma 3, credit limit is never binding in the high state, and because $R = R'$, even if both credit lines are used by the planner, we can without loss of generality think of one credit line instead. Thus, in what follows next, assume that the planner does not use the first round credit line at all, and only uses the second credit line conditionally on the signal realization (perfectly revealing here)—with the interpretation that L stands for the total credit limit committed in the low state when it is a default state, and R is interest charged in the high state, with a non-binding credit limit. (Obviously, it is suboptimal for the planner to let the

agent default in the high state.)

Using consumer's policy for the non-binding case $b(y_H; R, L)$ and the laid out notational convention above, we calculate the total interest revenue in high state as follows:

$$TIR = \frac{(R-1)(2(R+1)(y_H-1)\mu - R + 1)}{2(R+1)^2\mu}, \tag{A1}$$

where under the assumptions imposed on the problem, we note $b(y_H; R, L) \geq 0$, and so there is no need to make any distinction between saving and borrowing case. Since $y_H > 1$, we observe that the numerator of TIR is strictly positive, except for $R = 1$, when it is zero. Moreover, we note that as long as $y_H < 1 + 1/\mu$, as a function of $R \geq 1$, TIR is first a strictly increasing and then strictly decreasing function—achieving a unique global maximum at

$$\bar{R} = \frac{1 + \mu(y_H - 1)}{1 - \mu(y_H - 1)}.$$

The maximum attainable TIR is thus given by:

$$\overline{TIR} = \frac{1}{2}(y_H - 1)^2\mu.$$

In case $y_H \geq 1 + 1/\mu$, TIR is always increasing wrt R and reaches a limit at $R \to \infty$ given by $\overline{TIR} = 1 - y_H - \frac{1}{2\mu}$. In any case, TIR is bounded from above. To give a precise meaning to TIR, suppose that the agent receives $\overline{TIR}$ in the second period of the low state. If the agent decides to save in such case, we know that by (13) $TIR = pL$.

Summarizing, thus far, we have established the following. First, the domain of the resource feasible interest rates is $R = [1, \bar{R}]$, where $\bar{R}$ is infinite for $y_H \geq 1 + 1/\mu$. As for each R from this interval, by monotonicity of A1, and given $TIR = pL$, we know that we can always solve for highest L that satisfies (13). Moreover, it is not possible to do so outside this domain (or the same L could correspond to a lower R, contradicting the fact that this is meant to be a point on the efficient frontier). Second, using (A1), since credit limits never bind, intertemporally aggregated consumption $c = G(c_1, c_2)$ of the agent can be expressed as a continuously differentiable function of the interest rate in both states; for later use we denote this function by $c_L(R), c_H(R)$. At the end of this proof (see supplement), we establish

the following properties of these function: $dc_L(R)/dR > 0$ on $[1, \bar{R})$, $dc_L(R)/dR = 0$ at $\bar{R}$, and $dc_H(R)/dR < 0$ on $[1, \bar{R}]$.Given strict monotonicity of $c_H(R)$, this proves that the implicitly defined RFCF by the these two functions is a continually differentiable function on a non-empty (W_L, W_H always feasible), compact (note TIR is bounded) and connected set.

To establish (iii-iv), using $c_L(R), c_H(R)$ from above, we calculate that the slope of the RFCF at c_H, corresponding to $c_H = c_H(R)$ (unambiguously by strict monotonicity of $c_H(R)$):

$$\frac{dc_L(R)}{dc_H(R)} = -\frac{1-p}{p}\frac{(y_H-1)(R+1)\mu-(R-1)}{(R+1)(y_H-1)\mu-(R-1)+(R-1)/2}, \text{ all } R \in [1, R_{max}].$$

Given that our domain condition $R \leq \bar{R}$ implies $(y_H-1)(R+1)\mu-(R-1) \geq 0$ (with '='at $R = \bar{R}$), and $(R-1)/2 \geq 0$, it is easy to see that the slope of RFCF monotonically falls from 0, that is attained at $\bar{R}$, to the slope of the actuarially fair ratio: $-\frac{1-p}{p}$, attained at $R = 1$. This finishes the proof of parts (iii) and (iv).

Supplement:

Functions $c_L(R), c_H(R)$ are given by:

$$c_L = \frac{(p-1)(R-1)^2 - 2(R+1)\mu(R(p(y_H-y_L-2)+1-y_H)-p(y_H+y_L)+y_H-1)}{2p(R+1)^2\mu},$$
$$c_H = \frac{8(R+1)\mu(R+y_H)+(R-1)^2}{4(R+1)^2\mu}.$$

The underlying derivatives are:

$$\frac{dc_L(R)}{dR} = \frac{2(p-1)(R-1-(R+1)(y_H-1)\mu)}{p(R+1)^3\mu},$$
$$\frac{dc_H(R)}{dR} = \frac{R-1-2(R+1)(y_H-1)\mu}{(R+1)^3\mu}.$$

1) $dc_L(R)/dR > 0$ for all $1 \leq R < \bar{R}$ and $dc_L(R)/dR = 0$ at $\bar{R}$.

Pf. Clearly, $dc_L(R)/dR = 0$ iff the expression that appears in numerator: $R-1-(y_H-1)\mu = 0$. Plugging in $\bar{R}$ (see in text),we verify that this is true. Furthermore, the numerator $2(p-1)(R-1-(1+R)(y_H-1)\mu)$ is always negative on the relevant domain. Clearly, it is strictly negative for $R = 1$, and as shown below, it is zero at $\bar{R}$. Since it is a linear function, and negative at the two end points of the domain, it is negative on the entire domain.

2) $dc_H(R)/dR < 0$ for all $1 \leq R \leq \bar{R}$.

Pf. First, consider the case $y_H < 1 + \frac{1}{\mu}$. To establish the sign, we evaluate the numerator at $\bar{R}$. Plugging in and simplifying, we obtain that at $\bar{R}$ the expression that determines sign in the numerator equals to

$$2 - \frac{2}{1 - y_H\mu + \mu}.$$

Since $\bar{R} > 1$ iff $1 < y_H < 1 + \frac{1}{\mu}$, the expression is unambiguously negative for all parameter values. The numerator $R - 1 - 2(1 + R)(y_H - 1)\mu$ is also unambiguously negative at $R = 1$. Since it is a linear function, and negative at the two end points of the domain, it is negative on the entire domain. In the case of $y_H < 1 + \frac{1}{\mu}$, $\frac{dc_H(R)}{dR}$ is clearly negative because the numerator is negative for all values of R. □

Proof of Lemma 6: *[incomplete old version.]*

Property (i) follows from the fact that, conditional on $L = \mathcal{L}_{max}$, the initial lender, by choosing R, effectively solves a constrained maximization problem similar to the planner's (but with additional restriction of $L' = 0$ imposed on the planner). Thus, the point corresponding to zero profit contract at $L = \mathcal{L}_{max}$ must lie on RFCF as long as the credit constraint L is non-binding in the high state, and on its interior when it is binding. If borrowing constraint is binding, the planner would relax it using L'. Clearly, for any $L < \mathcal{L}_{max}$, second round lenders enter lowering profits of the initial lenders. Thus, these points must also lie on the interior of the planner's feasibility set.

Property (ii) follows from properties of the functional forms that can be derived using the initial lender's profit function and the consumer's policy function b. [to be completed]

To prove (iii), we need to first characterize the candidate equilibrium contracts given insurance level L. It is easy to check that the only contract combinations that lead to points on the PFCF are $\mathcal{C} = (L, R_0(L, \mathcal{L}_{max}))$ and $\mathcal{C}' = (\mathcal{L}_{max} - L, 1)$, where $R_0(L, \mathcal{L}_{max})$ is the lowest interest rate satisfying $E[\pi(y, \mathcal{C}, \mathcal{C}')] = 0$, i.e. the initial lender's zero profit interest rate. Clearly, Bertrand competition implies that no positive profits can be made, and so $R = R_0(L, \mathcal{L}_{max})$ and $R' = 1$; moreover, the second round lender will extend as much credit as possible without inducing default on the repayment path, implying $L' = \mathcal{L}_{max} - L$.

Having identified the candidate eq. contracts, we use the following key result:

LEMMA 7. *Let* $\mathcal{C} = (L, R_0(L, \mathcal{L}_{max}))$ *and* $\mathcal{C}' = (L', 1)$, *where* $L' = \mathcal{L}_{max} - L$. *Then, the slope of the PFCF is strictly lower than the actuarially fair ratio* $-(1-p)/p$ *under the following conditions: (i) there is partial utilization of total credit limit, i.e.* $b(y_H; \mathcal{C}, \mathcal{C}') < L + L'$ *and* $\frac{\partial R_0(L, \mathcal{L}_{max})}{\partial L} < 0$ *or (ii) there is full utilization, i.e.* $b(y_H; \mathcal{C}, \mathcal{C}') = L + L'$, *with the borrowing constraint being binding.*

Proof. (Part i) Due to partial utilization, we know that the first order condition for consumer borrowing on the high income path holds with equality, and thus we can use it to derive the optimal borrowing policy as a function of L and R. The zero profit condition of the first round lender evaluated at this policy function then implicitly defines $R_0(L, \mathcal{L}_{max})$, or equivalently, the zero profit credit limit for a fixed R, which happens to be more convenient to work with.[19] It is straightforward to see that this function is well defined given the formulation of the zero profit condition (11). Using this zero-profit credit limit, we can then express intertemporally aggregated consumption levels c_L and c_H as a function of R, by substituting for L (in expression for $L' = \mathcal{L}_{\max} - L$). The slope of the PFCF is then implicitly given by $\frac{\partial c_L}{\partial R} / \frac{\partial c_H}{\partial R}$. This expression involves both in the numerator and the denominator are complicated polynomial expressions of R of degree 3 and 4. However, both the numerator and the denominator of the simplified expression have the same sign as $\frac{\partial c_L}{\partial R}$ and $\frac{\partial c_H}{\partial R}$, respectively. Furthermore, they satisfy

$$\text{numerator} \times \frac{p}{1-p} + \text{denominator} = -(R-1)\big(2(R-1) + p(1-3R)\big)^2,$$

which turns out unambiguously strictly less than zero for $R > 1$. We thus conclude the slope of the PFCF is lower than $-(1-p)/p$ whenever $\frac{\partial c_L}{\partial R} < 0$ and $\frac{\partial c_H}{\partial R} > 0$, which is implied by the other condition in (i): $\frac{\partial R_0(L, \mathcal{L}_{max})}{\partial L} < 0$.

(Part ii) The zero profit interest rate must be equal to $\frac{1}{1-p}$, as both credit lines are fully utilized in this case. Using this interest rate, and substituting for $b(y_H; \mathcal{C}, \mathcal{C}') = \mathcal{L}_{max}$ into the expression for c_H, we can compute the slope of the PFCF using the fact that: $\frac{\partial c_L}{\partial L} / \frac{\partial c_H}{\partial L} = 1 / \frac{\partial c_H}{\partial L}$

[19] $R_0(L, \mathcal{L}_{max})$ is the lowest root of a quadratic expression that leads to a quite complex expression of L and $\mathcal{L}_{max}$.

(recall 5: borrower always saves in low state). An analogous argument gives:

$$\text{numerator} \times \frac{p}{1-p} + \text{denominator} = -2p(1-p)\mu(1 + 2\mathcal{L}_{max} - y_h).$$

It turns out that the numerator is positive while the denominator is negative. Thus, the slope of the PFCF is lower than $-(1-p)/p$ whenever $\mathcal{L}_{max} < \frac{y_H - 1}{2}$. We show that this is always the case whenever utilization is 100%. (This finishes the proof of the lemma.) □

Given the above results, we next need to show that under the conditions stated in the first lemma, either there is full utilization or $\frac{\partial R_0(L,\mathcal{L}_{max})}{\partial L} < 0$. The following lemma establishes that this is true whenever the zero profit interest rate above a well-defined threshold that only depends on p. Thus, when the insurer raises L she must lower her R to satisfy the zero profit condition, leading to $\frac{\partial R_0(L,\mathcal{L}_{max})}{\partial L} < 0$.

LEMMA 8. *Fix any $L < \mathcal{L}_{max}$ and R. Let $\mathcal{C}' = (\mathcal{L}_{max} - L, 1)$. Then, if $b(y_H) < \mathcal{L}_{max}$, $\frac{\partial E[\pi(y,\mathcal{C},\mathcal{C}')]}{\partial L} > (<)\, 0$ for $R >$ $(<) \frac{2-p}{2-3p}$.*

Proof. Follows trivially from differentiation of profit function after substituting consumer's policy function. □

The following result completes the proof since it states that the restriction on the range of $\mathcal{L}_{max}$ guarantees that either credit limits are binding or zero profit interest rates (if they exist) must lie above the threshold given in Lemma 8.

LEMMA 9. *If $\mathcal{L}_{max} < (1 - \frac{1}{2}\frac{p}{1-p})\overline{\mathcal{L}}$, then credit limits are strictly binding. If $\mathcal{L}_{max} > \overline{\mathcal{L}}$ then profits are negative for all $R \leq \frac{2-p}{2-3p}$ and all $L > 0$.*

Proof. [To be completed] □

□

Proof of Proposition 4: In the case of non-revealing signal, our model is essentially equivalent to an environment with just the first round of competition, and so make things simpler, assume there is only one round and potentially two contracts possible (R_1, L_1), and signed later, (R_2, L_2). We will show that wlog we can assume $L_2 = 0$, with an additional restriction

that $L_1 = \mathcal{L}_{\max}(y_H)$ or $L_1 = b(y_H)$. So, by the way of contradiction, assume this is not the case: $L_2 > 0$, and $L_1 < \mathcal{L}_{\max}(y_H)$. To the end of the proof, we will consider $L_1 < L_{\max}$ with $L_2 = 0$ to show $L_1 = b(y_H)$. If default does not happen for $L_1 + L_2$, Bertrand competition will require that $R_1 = R_2 = 1$. Thus, we can wlog assume only one contract is offered instead of two. Suppose default does happen in low state. Three cases are possible: (i) $R_1 = R_2$, (ii) $R_1 > R_2$, (iii) $R_1 < R_2$, which is analogous to (ii). Case (i) is analogous to non-default case: no decisions will be affected and, if both lenders make zero profits, so does one cumulative single lender. Case (ii) must imply that $b(y_H) > L_2$. Since default happens, the initial lender could not make zero profits otherwise. This means that the marginal interest rate the borrower faces on high income path is necessarily R_1. It is trivial to show that it is possible to find one credit line $\mathcal{C}_{dev}$ =$(R_2 < R < R_1, L = L_1 + L_2)$, such that the borrower would be strictly better off, and (R, L) will give zero profit to the single lender. To see this, just note that the total revenue raised by the zero profit lender on the high income path on the contract (R, L) must be pL, and so it is the same as the total revenue with two contracts. Now, this means that the total amount of wealth used for consumption on high income path is the same in two cases, and since the marginal interest rate is lower, the borrower must be better off. Moreover, since the interest rate R is lower than marginal one in case with two contracts, the consumer must borrow more — allowing (R, L) to unambiguously break even at $R < R_2$. To show that the single contract in this particular case must feature $L_1 = b(y_H)$, note that $L_1 > b(y_H)$ leaves space for entry that gives exactly zero profits at the same interest rate as long as $L_2 = b(y_H) - L_1-$ thus, for one contract assumption to be wlog, we must restrict space of contracts to force $L_1 = b(y_H)$. $\square$

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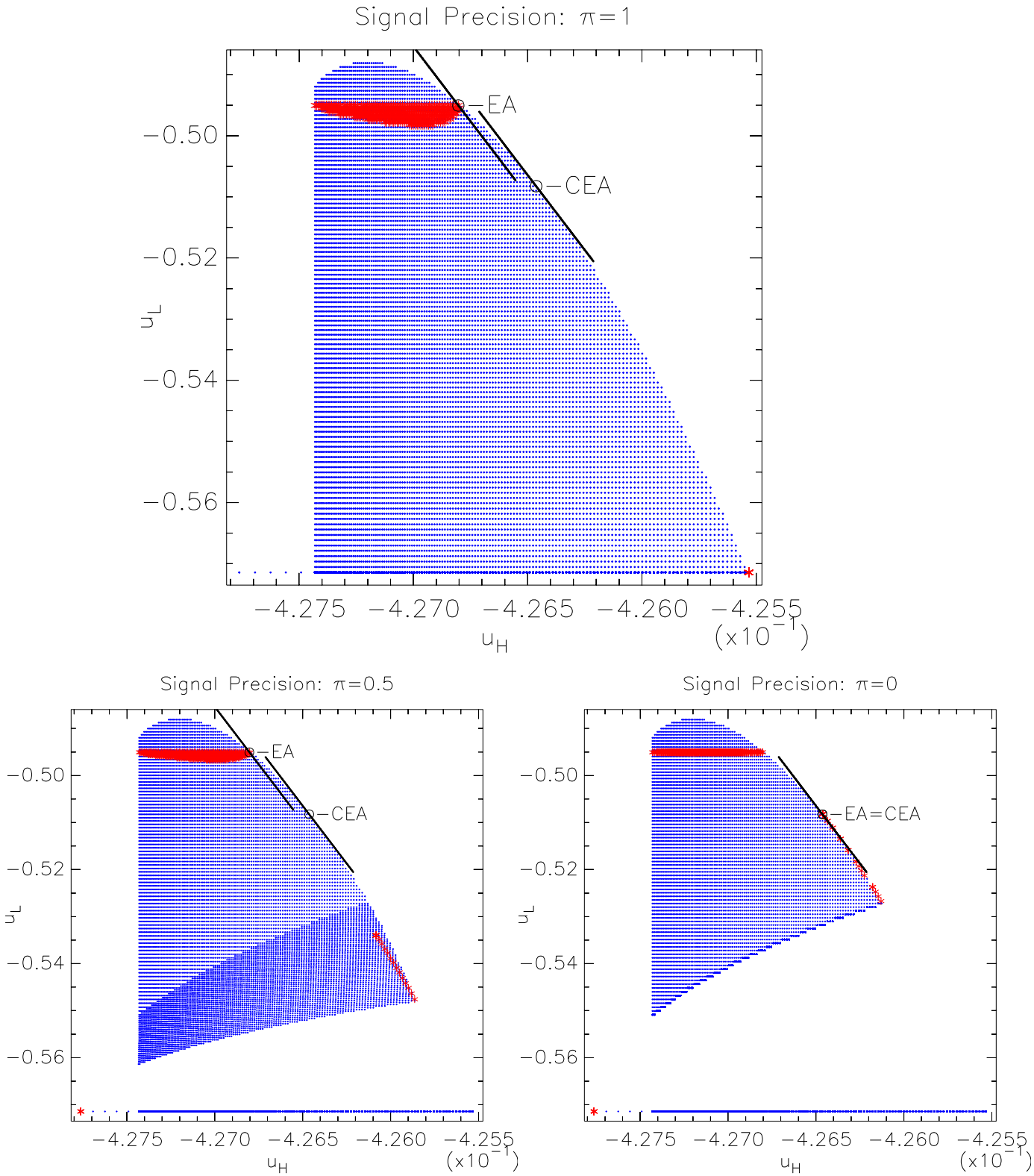

Figure 6: Feasibility sets and equilibria for different signal precisions.

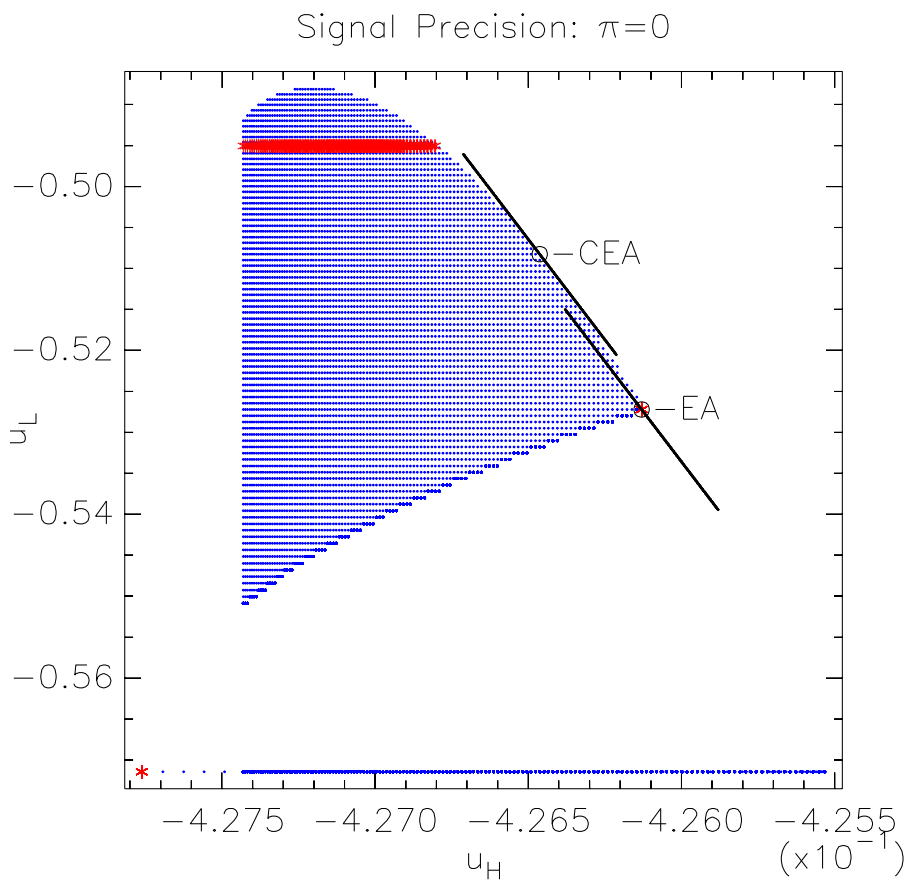

Figure 7: Equilibrium under non-revealing signals w/ multiple contracts.

Citibank (South Dakota), N.A.
c/o Customer Service Center
P.O. Box 6148
Sioux Falls, SD 57117-6148

Regarding your Citibank account ending in

Notice of Change in Terms and Right to Opt Out

The Changes. We are changing your Card Agreement. The changes will be effective for all billing periods beginning on or after December 3, 2008. The changes will be effective whether or not you receive a billing statement.

I. We are changing how we calculate your variable APR for purchases:

We are increasing your variable APR for purchases. Your purchase APR will equal the LIBOR Rate plus 11.99%, with a minimum APR of 14.99%. As of September 17, 2008, this purchase **APR** is 14.99%. This APR equals a daily periodic rate of 0.0411%.

II. We are changing how we calculate your variable APR for cash advances:

We are increasing your variable APR for cash advances. Your cash advance APR will equal the LIBOR Rate plus 19.99%, with a minimum APR of 21.99%. As of September 17, 2008, this cash advance **APR** is 22.73%. This APR equals a daily periodic rate of 0.0623%.

III. We are changing the following sections regarding APRs:

Default APR. All your APRs (including promotional APRs) on all balances may automatically increase to the default APR if you default under any Card Agreement you have with us because you

- do not make the minimum payment when due,
- go over the credit line, or
- make a payment to us that is not honored.

The default APR equals the greater of (1) the LIBOR Rate plus up to 26.99% or (2) up to 29.99%. We set your default APR by reviewing the seriousness of your default with us and your credit history. The default APR takes effect as of the first day of the billing period in which you default. We may lower the APR for new purchases and/or cash advances if you meet the terms of all Card Agreements that you have with us for six billing periods in a row. However, the default APR will continue to apply to your existing balances until they are paid in full, unless we tell you otherwise.

IV. We are changing how we calculate the fee for foreign purchases. This fee will be called the Transaction Fee for Foreign Purchases. The description in your Card Agreement of this fee will be as follows:

Transaction Fee for Foreign Purchases. We add a fee of 3% of the U.S. dollar amount of each purchase made outside the U.S., whether made in U.S. dollars or in a foreign currency. This fee is a **FINANCE CHARGE.**

Right to Opt Out. To opt out of these changes, you must call or write us by January 31, 2009. When you do, you must tell us that you are opting out. Call us toll-free at **1-866-565-7030.** (Please have your account number available.) Write us at Customer Service Center, P.O. Box 6218, Sioux Falls, South Dakota, 57117-6218. Include your name, address, and account number on your letter.

If you opt out of these changes, you may use your account under the current terms until the end of your current membership year or the expiration date on your card, whichever is later. We will close your account at that time. You must then repay the balance under the current terms.

We hope you choose to take full advantage of your Citi Card revolving line of credit and all the benefits and services we offer you.
Please save this notice for future reference.

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Figure 8: Example of an opt-out option offered voluntarily by a major credit card lender before the Credit CARD Act of 2009 went into effect.