Financial Heterogeneity and Monetary Union

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Abstract

In this paper, we analyze the business cycle and welfare consequences of monetary union among countries that face heterogeneous financial market frictions. We show that facing financial distress in the absence of devaluation, the firms in financially weak countries countries have an incentive to raise their prices to cope with liquidity shortfalls. At the same time, firms in countries with greater financial slack poach from the customer base of the former countries by undercutting their prices, without internalizing the detrimental effects on union-wide aggregate demand. Thus, a monetary union among countries with heterogeneous degrees of financial frictions may create a tendency toward internal devaluation for core countries with greater financial resources, leading to chronic current account deficits of the peripheral countries. A risk-sharing arrangement between the core and the periphery can potentially undo the distortion brought about by the currency union. However, such risk sharing requires unrealistic amounts of wealth transfers from the core to the periphery. We show that unilateral fiscal devaluation carried out by the peripheral countries can substantially improve the situation not only for themselves but also for the core countries if there exists an important degree of pecuniary externality not internalized by the predatory pricing strategies of individual firms.

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1 Introduction

Despite numerous debates in both policy circle and academia, it becomes clearer that the root of the current European crisis was the classic balance-of-payment problem of the peripheral countries such as Greece, Italy, Ireland, Portugal and Spain (GIIPS). The origin of the crisis should be found in the mix of overvalued currency and cheap credit supply. Panel (a) of figure 1 shows that the current account deficits of the GIIPS expanded to more than 6 percent of their GDPs on the eve of the global financial crisis.

In panel (b) and (c) of figure 1, one can clearly spot the evidence of overheating in the periphery. The weighted average unemployment rate of the peripheral economy declined almost 5 percentage points until mid-2008. Meanwhile, the consumer price index grew much faster in the periphery than in the core during the same period. As a consequence, the real exchange rates of the peripheral countries appreciated substantially, eroding the competitiveness and producing large trade deficits, which were easily financed by international capital inflow on the backdrop of the convergence of the European interest rates.

It is clear that the peripheral situation called for a sufficient monetary tightening, both for macroeconomic stabilization purpose in a traditional sense and for macroprudential reasons. However, the actual monetary policy of the European Central Bank (ECB), rightfully aimed at stabilizing the 'average' fundamental of the Euro-area, failed to deliver the sufficient tightening.[1] It is somewhat ironic that the insufficient monetary tightening was at least partially responsible for the inefficient credit boom, for the currency union was sold to the periphery on the premise that it would eliminate the inflationary bias of the periphery, so called "the advantage of tying one's hands" (Giavazzi and Pagano [1988]). The flip side of the same irony is that it may fail to deliver a sufficient monetary policy easing this time.[2]

The resolution of the crisis requires a major adjustment in overvalued real exchange rates of the peripheral countries. Given the fixed exchange rate, a necessary condition for this is the mix of reflation of the core and disinflation of the periphery. However, as shown by panel (c) of figure 1, the gap between price levels of the core and the periphery has not been reduced until the end of 2013. As a consequence, the real exchange rates of the peripheral countries has remained overvalued until recently. In figure 2, we show how the bilateral real exchange rates of Euro-zone countries (based on export prices) with respect to Germany has evolved over time since the adoption of the Euro. We find that the real exchange rates of Greece, Italy, Portugal and Spain have appreciated by 10% - 30% with respect to Germany until the end of 2013. In contrast, the real exchange rates of France, Netherlands and Finland have maintained their levels of late 1990s.

[1] For instance, Nechio [2011] shows that Taylor 1993 rule calls for 7 percent of monetary policy rate for the periphery, but only 2 percent for the core.

[2] In contrast to other major central banks such as the Federal Reserve System and the Bank of Japan, the ECB has been reluctant to expand its balance sheet by purchasing soverign debts.

Figure 1: Eurozone Current Account, Unemployment and Inflation

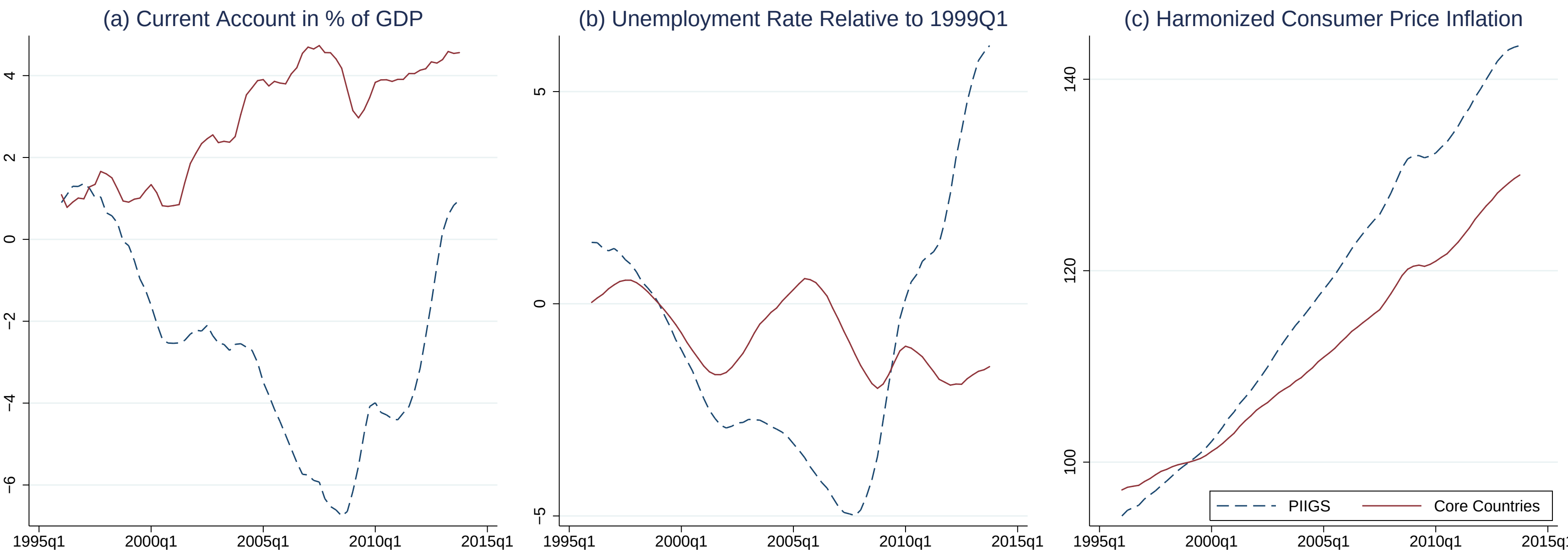
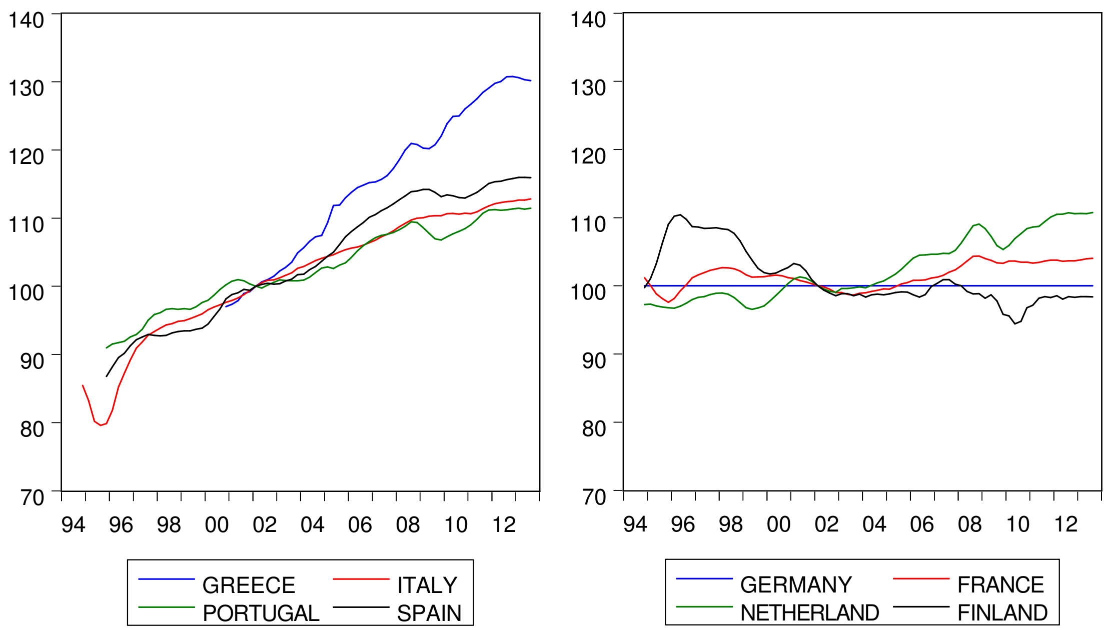


Figure 2: Bilateral Real Exchange Rate With Respect To Germany



Both the core and the periphery are responsible for slow adjustment in the real exchange rates. Why have the firms of the periphery been so slow to cut their prices? Why have the firms of the core countries been reluctant to increase their prices fast enough despite relatively good aggregate demand and employment situation? In fact, some commentators have argued that it is the core countries that are exporting deflationary pressure (Krugman [2014]).

In this paper, we study the macroeconomic consequences of creating a monetary union among countries with heterogenous financial conditions. Our goal is two-fold. First, building upon our earlier work (Gilchrist, Schoenle, Sim, and Zakrajsek [2013]), we show that the firms with relatively large financial slack in the core countries have a strong incentive to use the financial crisis to expand their market shares in both the core and the peripheral economies by undercutting the prices of the peripheral firms, while the peripheral firms have a strong incentive to raise their prices to secure cash flow, even though doing so implies cannibalizing their own future market shares.

The idea that there can be a trade-off for a firm facing liquidity problem between the current cash flow maximization and future market share maximization has a firm ground in macroeconomic literature (see Gottfries [1991], Bucht, Gottfries, and Lundin [2002] and Chevalier and Scharfstein [1996]). When applied to the European situation, this mechanism explains why peripheral countries may manage to avoid outright Fisherian debt-deflation in a financial crisis even with unemployment rates of 20-25 percent, but face chronic stagnation instead with persistent erosion of competitiveness. The following quote eloquently summarizes the strategic dilemma facing the firms in the periphery in the middle of international price war:

"Mr. Marchionne and other auto executives accuse Volkswagen of exploiting the crisis to gain market share by offering aggressive discounts. "It's a bloodbath of pricing and it's a bloodbath on margins," he said." (The New York Times, July 25, 2012)

We show that under a floating exchange rate regime, the central banks of the peripheral countries can absorb the impact of the shock at least partially by slashing their policy rates to sufficiently low levels, which then generate large depreciations of their nominal exchange rates through uncovered interest parity condition. The large devaluations, despite higher domestic inflation rates in the periphery, cause the real exchange rates to depreciate, helping the firms export out their financial problems.

Under a fixed exchange rate regime, however, this policy option is not available. The nature of the price war described above then implies that the real exchange rates depreciate for the core countries, which undergo small, export-driven boom at the cost of the peripheral recession. This is consistent with the stylized facts of the Euro-zone crisis experience. Since the core and the peripheral economies follow divergent paths, the common monetary policy become impotent. One size fits no one in this situation. We show that the volatilities of macroeconomic variables can be endogenously increased up to 2 times under the common monetary policy even the volatilities of the shocks hitting the economies are identical, resulting in a sizable welfare deterioration especially for the periphery.

Second, given that the monetary policy cannot be tailored for each member countries, we explore fiscal policy options. In particular, we consider two alternatives: fiscal union and fiscal devaluation. We first show how a complete risk-sharing arrangement among the member countries can remedy the distortion brought about by the fixed exchange rate. Such a risk-sharing arrangement can in principle be achieved by developing a fiscal union. However, our analysis indicates that such an arrangement requires unrealistic amounts of wealth transfers from the core to the periphery, making its political realism doubtful.

Alternatively, we consider the effects of fiscal devaluation policy. Recent literature explores the stabilization effects of certain fiscal policy mix to replicate the effects of nominal devaluation in a fixed exchange rate environment. What makes the fiscal devaluation policy promising is that in contrast to monetary policy, it can be implemented unilaterally by the governments of the peripheral countries (see Adao, Correia, and Teles [2009] and Farhi, Gopinath, and Itskhoki [2014]). Even so, it is not clear why the core countries should welcome such policy intervention by the periphery if the core countries joined the currency union precisely because they did not like the manipulation of nominal exchange rates by the central banks of the periphery, and why the peripheral countries can carry out the policy without worrying about retaliatory reaction from the core countries.

In this paper, we show that fiscal devaluation by the periphery may be beneficial even to the core if there is demand externality working behind international price wars and left unexploited by the Euro-zone governments. When the firms of the core countries lower their prices to expand their market shares, they do not internalize the pecuniary externality in which driving out the competitors by undercutting their prices to an excessive degree can also reduce aggregate demand for their own products. In this situation, a distortionary taxation can help them internalize the externality, and fiscal devaluation provides an effective means to achieve this goal. Furthermore, we show that the potential benefits to the core increase with the degree of financial market friction that generates the pecuniary externality.

Our framework allow us to understand the strategic interaction between the heterogeneous liquidity conditions of different firms and their pricing choices, we fundamentally extend the analysis of the financial accelerator literature (for example Bernanke, Gertler, and Gilchrist [1999]). Our analysis also expands the scope of New Keynesian theory on real exchange rate dynamics, such as Obstfeld and Rogoff [2000] and Steinsson [2008] by analyzing the issue under both complete and incomplete risk-sharing arrangements under alternative currency regimes in an environment where firms face financial market frictions in their optimal pricing decisions.

Is there empirical evidence supporting the positive relationship between the degree of financial market friction and inflation pressure especially in the European context? We find supporting evidence for the link between financial market frictions and pricing pressure, using macroeconomic data of the member countries of the Euro zone. In particular, we estimate both backward-looking and forward-looking Phillips curves of the member countries using only data prior to the crisis, and construct prediction errors of inflation rates out of sample during the crisis. We then regress these errors onto CDS spreads of the member countries, natural measures of financial market frictions that will also affect price-setting firms in the respective countries. We find strong evidence for a positive, statistically significant relationship between financial frictions and price setting: countries with greater financial market frictions experience higher inflation rates, even after we control for other fundamental determinants of empirical Phillips curves. In contrast, we find no evidence for such a relationship for the core Euro zone countries.

These results confirm our new New Keynesian pricing theory and can help explain why the real exchange rate misalignment in the Euro zone was not corrected, but in fact aggravated by the crisis. In our theory, the adjustment dynamics during the recovery from the financial crisis will be such that the peripheral countries will face a stronger disinflationary pressure than the core countries as their financial stress lessens. This seems to be consistent with the current situation in Europe in which the risk of deflation looms larger as the continent comes closer to the resolution of crisis.

The rest of the paper is organized as follows. We start by summarizing our empirical findings in Section 2 regarding the positive correlation between inflation and financial market friction, and their implications for market share dynamics. Section 3 develops a two-countires general equilibrium model that can be used to analyze the effects of financial and real shocks under alternative arrangements for exchange rates and risk sharing environment. Section 4 analyzes the business cycle and welfare consequences of currency union. Section 5 studies the costs and benefits of fiscal union and analyzes the potential benefits of unilateral fiscal devaluation. Section 6 concludes.

2 Financial Crisis and Inflation: Evidence

In this section, we present evidence on the relationship between price-setting and financial frictions in the Euro Zone.

What motivates our analysis are the findings of Gilchrist, Schoenle, Sim, and Zakrajsek [2013] for U.S. firms during the Great Recession: During the height of the crisis in late 2008, firms with "weak" balance sheets increased prices significantly relative to industry averages, whereas firms with "strong" balance sheets lowered prices, a response consistent with an adverse demand shock. These stark differences in price-setting behavior clearly contradict the standard price-adjustment mechanism emphasized by the New Keynesian literature, where firms financial conditions play no role in determining their price-setting behavior.

Inflation dynamics and financial turmoil in the Euro Crisis mirror this pattern exactly when we compare financially strong and financial weak crisis countries: Anecdotally, it is well known that prices continued to rise in financially weak countries during the Euro Crisis, whereas financially strong, Northern European countries saw only very little inflation or deflation. Here, we systematically quantify this relationship between price-setting and financial frictions in the Euro Zone. We demonstrate that these two are indeed both economically and statistically significantly related.

2.1 Data

To do so, we construct a dataset on inflation and financial frictions in the Euro Zone from two sources. First, we use data from Eurostat on inflation, real marginal cost and unemployment. In particular, we use the all-item harmonized inflation price index (HICP) data to measure inflation, which we seasonally adjust, starting in 1996Q1 and running through 2014Q1. Our measure of real marginal cost is given by seasonally adjusted real unit labor costs from Eurostat. These are defined as total compensation of employees to GDP in market prices. In the case of Greece, the seasonally adjusted series end in the first quarter of 2011, so we link a four-quarter moving average of the available non-adjusted series to the existing series. We complement the real marginal cost data with a series for the unemployment rate, seasonally adjusted, from Eurostat for the same period.

Second, we measure financial frictions using CDS data from Markit. Our main measure of financial frictions is the quarterly average CDS spread of EUR-denominated contracts. To check robustness, we also use the annualized realized volatility of the daily CDS spreads, and the annualized realized volatility of the daily difference in CDS spreads.

We collect data for the following countries: Austria, Finland, France, Germany, Netherlands to which we refer to as "Core Europe"; and Greece, Ireland, Italy, Portugal, Spain, to which we refer to as "GIIPS" countries. All countries taken together account for approximately 94.33% of the Euro Zone's GDP (World Bank, 2009).

2.2 Measuring the Impact of Financial Frictions on Inflation in the Euro Zone

We establish a relationship between price-setting and financial frictions in the Euro zone in two steps: First, we estimate country-specific Phillips curves through the end of 2008.[3] Second, we predict inflation during the Euro crisis based on these estimates, and relate residuals of actual less predicted inflation to financial frictions. We expect larger actual inflation than inflation predicted based on pre-crisis estimates of a Phillips curve if financial frictions in fact – as in our model – are associated with revenue stabilization through higher inflation.

First, we estimate country-specific Phillips curves through 2008Q4:

$$\pi_{i,t} = \beta_i \mathbb{E}_t[\pi_{i,t+1}] + \lambda_i mc_{i,t} \tag{1}$$

where $E_t[\pi_{i,t+1}]$ denotes expectations of future inflation and $mc_{i,t}$ is our measure of marginal cost. We employ GMM as described in Gali et al. (2001), with the same moment condition:

$$\mathbb{E}_t[(\pi_{i,t} - \beta_i \mathbb{E}_t[\pi_{i,t+1}] - \lambda_i mc_{i,t})\mathbf{z}_{i,t}] = 0 \tag{2}$$

where $\mathbf{z}_{i,t}$ is a country-specific set of instruments. We include five lags of inflation, and two lags of marginal cost into the set of instruments. Alternatively, we have estimated a backward-looking Phillips curve specification, as well as one with deviations of unemployment from the natural rate of unemployment. We prefer the marginal cost specification because it yields – as in Gali et al. (2001) – coefficients of the right signs. However, both alternative specifications leave results qualitatively unchanged.

Second, we use these pre-crisis estimates to predict country-specific Phillips-curve residuals for the Euro crisis from 2009Q1 through 2014Q1. These residuals will systematically vary with financial frictions if financial variables play a role for inflation, but are not included in the pre-crisis estimates of the Phillips curve: Since these Phillips curve estimates are from a "normal" economic environment where financial frictions do not influence inflation dynamics, any predicted inflation movements based on these estimates will only capture dynamics that we would expect if financial frictions are not important. At the same time, the residuals of predicted less actual inflation movements should reflect the effect of financial frictions.

We define residuals as the difference between observed and predicted inflation rates: $\epsilon^{\pi}_{i,t} = \pi_{i,t} - \hat{\pi}_{i,t}$. We follow Gali et al. (2001) and Sbordone (1999) in constructing predicted, "fundamental" inflation $\hat{\pi}_{i,t}$ as a function of the discounted stream of expected future real marginal costs. Using $\hat{\beta}_i$ and $\hat{\lambda}_i$, predicted inflation is given as follows:

$$\hat{\pi}_{i,t} = \lambda_i \sum_{k=0}^{\infty} \beta^k \mathbb{E}_t[\widehat{mc}_{t+k}] \tag{3}$$

To obtain estimates of expected future real marginal costs, we estimate a VAR(1) specification for

[3] Our choice of date is motivated by the fact that the crisis started with a delay only in 2009Q1 in the Euro zone

$\mathbf{z_{i,t}} = [mc_{i,t}, mc_{i,t-1}, mc_{i,t-2}, ..., mc_{i,t-q}]$ with $q = 5$ lags. This implies that $E_t[\widehat{mc}_{t+k}|\mathbf{z_t}] = e_1' A_i^k \mathbf{z_{i,t}}$ and $\hat{\pi}_{i,t} = \lambda_i e_1'(I - \beta_i A_i)^{-1}\mathbf{z_{i,t}}$. Note that our VAR(1) is based solely on marginal costs and excludes observed inflation as Gali et al. (2001). Unlike Gali et al. (2001), we thus do not use any information in the VAR(1) on the effect of financial frictions that may be directly contained in observed inflation rates.

We can then directly relate these residuals to observed financial frictions, and test whether they are indeed related. We do so by estimating the following regression specification:

$$\pi_{i,t} - \hat{\pi}_{i,t} = \beta_0 + \beta_1 CDS_{i,t} + \nu_{i,t} \tag{4}$$

where $CDS_{i,t}$ denotes country-specific CDS data. Motivated by the empirical evidence in our earlier work Gilchrist, Schoenle, Sim, and Zakrajsek [2013], we estimate the above regression separately for financially strong "core" countries, and the set of financially weak, GIIPS countries. If financial frictions play a role for inflation during the crisis, then we would expect a positive relationship between residuals and CDS spreads. We also present our findings for each set of countries graphically. As robustness check, we estimate eqn.(3) using our alternative measures of country-level financial frictions in the Euro zone.

2.3 Results

To highlight the role of financial distortions in determining inflation dynamics during the Euro crisis, we first plot the predicted inflation residuals obtained from estimating equation (3). Figure 3 shows our main result: the financially weak GIIPS countries have a very strong, positive relationship with average CDS spreads during the Euro crisis. Panel (a) summarizes this relationship graphically. When we only consider Italy and Spain, the two largest amongst the GIIPS countries, the relationship continues to hold and is even more pronounced and downwards-sloping. Panel (b) summarizes this result. By contrast, there is no evidence for a significantly upwards- or downwards-sloping relationship between average CDS spreads and predicted Phillips curve residuals for core European countries. Panel (c) summarizes this finding.

These results from Figure 3 are difficult to reconcile with the standard price-adjustment mechanism emphasized by the New Keynesian literature, a paradigm where financial conditions play no role in determining price-setting behavior. In general, we would expect that adverse demand shocks – the kind that hit the GIIPS countries between 2009 and 2012 – should induce firms to lower prices. Moreover, if our proxies used to measure financial distortions are also indicative of the weakness in demand, we would expect financially vulnerable firms in the GIIPS countries to lower prices even more relative to financially strong firms in the Northern countries. However, we observe exactly the opposite reaction in the data.

Interestingly, most of the residuals have a positive sign for both the GIIPS countries, and Italy and Spain taken separately. This is what one would expect if financial frictions worsened during the

Table 1: **Sign of Predicted Residuals in PIIGS Countries**

Sign of Residuals	Year												
	02	03	04	05	06	07	08	09	10	11	12	13	14
> 0	10	4	13	14	7	13	7	12	16	20	20	15	-
< 0	10	16	7	6	13	7	13	8	4	0	0	0	5

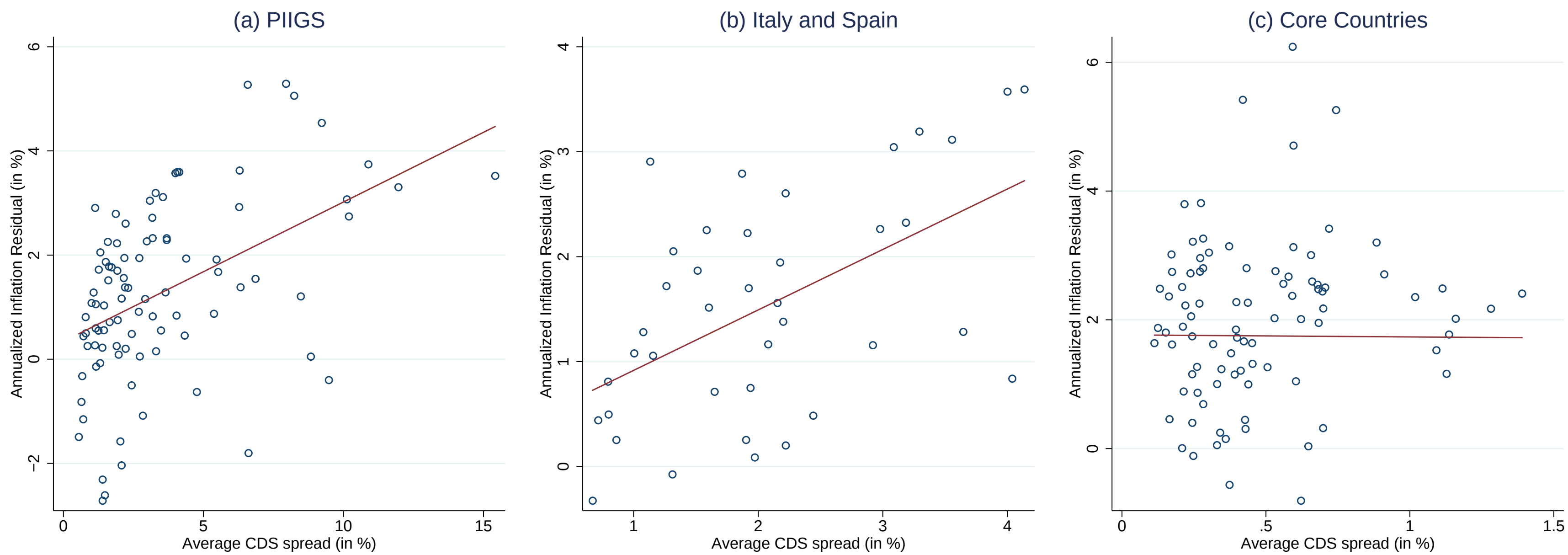
Euro crisis, and induced firms in the GIIPS countries to stabilize their cash flow by setting higher prices than they would have had financial frictions not affected them. Even more strikingly, we find that the majority of inflation residuals during the Euro crisis becomes almost perfectly positive during the crisis, in 2010 through 2012. However, in the pre-crisis period, there is no discernible pattern, and again in 2013, some residuals become positive again. This latter reversal coincides with the "whatever it takes" announcement of Mario Draghi in the summer of 2013 that reduced CDS spreads and calmed markets. Table **??** summarizes the trends by year.

What is the magnitude of the effect of financial distortions on inflationary movements? We present estimates of equation (3) in Tables 2 - 4. For the set of GIIPS countries, we find that a one percentage point increase in the average quarterly CDS spread – ceteris paribus – is associated with a 0.10 percentage point increase in the inflation residual. This relationship is statistically significant different from 0 at the 5% level. In economic terms, this estimate implies a positive 1.20% quarterly inflation residual associated with a one standard deviation increase in CDS spreads. This is summarized in specification (I) of Table 2. For Italy and Spain, we find a statistically significant coefficient of 0.44. At the height of the Euro crisis, this is equivalent to a striking 1.8% predicted quarterly inflation residual. The corresponding regression findings are shown in specification (I) of Table 3. By contrast, there is no statistically significant relationship for the Northern European countries at all. Table 4 presents this finding.

We find that our result are robust in two ways. First, we estimate the regression specifications using our alternative measures of financial distortions in the Euro crisis, namely the annualized realized volatility of the daily CDS spreads, and the annualized realized volatility of the daily difference in CDS spreads. We find that using either measure leaves our results unchanged. Columns (II) and (III) of Tables 2-4 summarize these findings. Second, we take log transformation of CDS spreads to mitigate the effect of the very large CDS spreads of Ireland and Greece during the Euro crisis. Columns (IV) and (VI) of Tables 2-4 show that this also leaves our results economically and statistically unchanged.

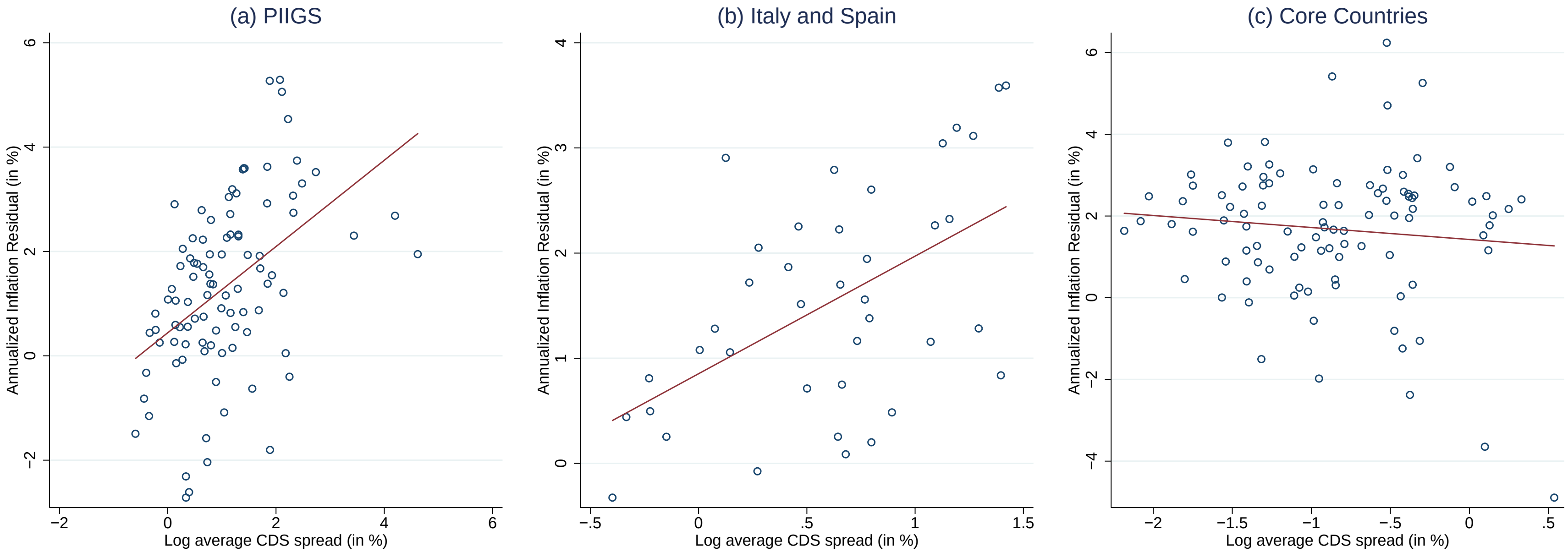
Finally, while we do not have sufficient evidence to statistically quantify the recent impact of financial frictions for deflationary tendencies in the Euro zone, the evidence that we have presented for a positive relationship between financial distortions and predicted inflation residuals is consistent with these recent trends: as financial conditions have started to improve in the GIIPS countries, this allows firms to finally lower prices, generating deflationary trends in the aggregate.

Figure 3: Inflation Forecast Errors and Financial Friction in Euro Zone: Level



Note: The panels show the relationship of predicted inflation residuals and CDS spreads for the respective countries during the Euro Crisis at a quarterly frequency. Residuals are obtained from estimating a Phillips curve, (1), from 1996Q1 through 2008Q4, and predicting residuals for 2009Q1 through 2013Q4. The solid line represents the estimate of (**??**) relating residuals and CDS spreads.

Figure 4: Inflation Forecast Errors and Financial Friction in Euro Zone: Log



Note: The panels show the relationship of predicted inflation residuals and the log of CDS spreads for the respective countries during the Euro Crisis at a quarterly frequency. Residuals are obtained from estimating a Phillips curve, (1), from 1996Q1 through 2008Q4, and predicting residuals for 2009Q1 through 2013Q4. The solid line represents the estimate of (??) relating residuals and the log of CDS spreads.

Table 2: Inflation and Financial Frictions: PIIGS Countries

	(I)	(II)	(III)	(IV)	(V)	(VI)
average CDS spread	0.094**					
	(0.04)					
annualized realized volatility of the daily CDS spreads		0.097**				
		(0.04)				
annualized realized volatility of the daily difference in CDS spreads			0.362**			
			(0.14)			
Log average CDS spread				0.365***		
				(0.11)		
Log annualized realized volatility of the daily CDS spreads					0.297**	
					(0.09)	
Log annualized realized volatility of the daily difference in CDS spreads						0.311**
						(0.1)
Intercept	0.002	0.028	0.001	-0.025	0.088	0.442***
	(0.16)	(0.15)	(0.16)	(0.15)	(0.13)	(0.1)

The table shows results obtained in two steps: First, we estimate a Phillips Curve $\pi_{i,t} = \beta_{0,i} + \beta_{1,i}\pi_{i,t-1} + \beta_{2,i}(u_{i,t} - u_i^n) + \beta_3 TIME + \epsilon_{i,t}$ for Portugal, Ireland, Italy, Greece and Spain through the end of 2008Q4. Second, we use predicted residuals $\hat{\epsilon}_{i,t}$ for $2009Q1 \leq t \leq 2014Q1$ to estimate $\hat{\epsilon}_{i,t} = \gamma_0 + \gamma_1 CDS_{i,t-1} + \nu_{i,t}$ where $CDS_{i,t}$ is either the average CDS spread (pps.), the annualized realized volatility of the daily CDS spreads (pps.), the annualized realized volatility of the daily difference in CDS spreads (pps.) or the natural log of either of them. We report standard errors in brackets.

Table 3: Inflation and Financial Frictions: Italy and Spain

	(I)	(II)	(III)	(IV)	(V)	(VI)
average CDS spread	0.437**					
	(0.16)					
annualized realized volatility of the daily CDS spreads		0.218*				
		(0.12)				
annualized realized volatility of the daily difference in CDS spreads			0.819*			
			(0.41)			
Log average CDS spread				0.828**		
				(0.32)		
Log annualized realized volatility of the daily CDS spreads					0.611**	
					(0.26)	
Log annualized realized volatility of the daily difference in CDS spreads						0.557**
						(0.26)
Intercept	-0.392	0.049	-0.034	0.007	0.176	0.842***
	(0.37)	(0.29)	(0.32)	(0.25)	(0.21)	(0.22)

The table shows results obtained in two steps: First, we estimate a Phillips Curve $\pi_{i,t} = \beta_{0,i} + \beta_{1,i}\pi_{i,t-1} + \beta_{2,i}(u_{i,t} - u_i^n) + \beta_3 TIME + \epsilon_{i,t}$ for Italy and Spain through the end of 2008Q4. Second, we use predicted residuals $\hat{\epsilon}_{i,t}$ for $2009Q1 \leq t \leq 2014Q1$ to estimate $\hat{\epsilon}_{i,t} = \gamma_0 + \gamma_1 CDS_{i,t-1} + \nu_{i,t}$ where $CDS_{i,t}$ is either the average CDS spread (pps.), the annualized realized volatility of the daily CDS spreads (pps.), the annualized realized volatility of the daily difference in CDS spreads (pps.) or the natural log of either of them. We report standard errors in brackets.

Table 4: Inflation and Financial Frictions: Northern Europe

	(I)	(II)	(III)	(IV)	(V)	(VI)
average CDS spread	0.2061					
	(0.28)					
annualized realized volatility of the daily CDS spreads		-0.122				
		(0.15)				
annualized realized volatility of the daily difference in CDS spreads			0.325			
			(0.36)			
Log average CDS spread				0.020		
				(0.11)		
Log annualized realized volatility of the daily CDS spreads					-0.037	
					(0.08)	
Log annualized realized volatility of the daily difference in CDS spreads						0.073
						(0.06)
Intercept	0.181	0.239**	0.114	0.195	0.141	0.326**
	(0.151)	(0.10)	(0.09)	(0.13)	(0.10)	(0.14)

The table shows results obtained in two steps: First, we estimate a Phillips Curve $\pi_{i,t} = \beta_{0,i} + \beta_{1,i}\pi_{i,t-1} + \beta_2(u_{i,t} - u_i^n) + \beta_3 TIME + \epsilon_{i,t}$ for Austria, Finland, France, Germany, and the Netherlands through the end of 2008Q4. Second, we use predicted residuals $\hat{\epsilon}_{i,t}$ for $2009Q1 \leq t \leq 2014Q1$ to estimate $\hat{\epsilon}_{i,t} = \gamma_0 + \gamma_1 CDS_{i,t-1} + \nu_{i,t}$ where $CDS_{i,t}$ is either the average CDS spread (pps.), the annualized realized volatility of the daily CDS spreads (pps.), the annualized realized volatility of the daily difference in CDS spreads (pps.) or the natural log of either of them. We report standard errors in brackets.

3 Model

3.1 Preferences

We restrict our attention to a two-country setting, where foreign country variables are denoted by asterisks. In each country, there exists a continuum of households, indexed by $j \in N_c \equiv [0,1]$. Each household consumes two types, h and f of different varieties of consumption goods, indexed by $i \in N_h \equiv [1,2]$ in the home country and by $i \in N_f \equiv [2,3]$ in the foreign country. As is standard, we assume that the home country only produces $h-$ type and the foreign country only produces $f-$ type. For instance, $c^j_{i,f,t}$ denotes home country consumer j's consumption of product i of type f whereas $c^{j*}_{i,f,t}$ denotes the foreign counterpart. Note that $c^j_{i,f,t}$ is consumption of an imported good by a home country consumer and $c^{j*}_{i,f,t}$ is consumption of a domestically produced good by a foreign consumer.

For simplicity, we assume the labor is perfectly immobile. The preferences of the home country are given by

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s U(x^j_{t+s} - \delta_{t+s}, h^j_{t+s}) \text{ for } j \in [0,1]. \tag{5}$$

The period utility function $U(\cdot,\cdot)$ is strictly increasing and concave in its first argument, and is strictly decreasing and concave in its second argument. The consumption/habit aggregator x^j_t is defined as an Armington aggregator,

$$x^j_t \equiv \left\{ \sum_{k=h,f} \omega_k \left[\int_{N_k} (c^j_{i,k,t}/s^{\theta}_{i,k,t-1})^{1-1/\eta} dk \right]^{\frac{1-1/\varepsilon}{1-1/\eta}} \right\}^{1/(1-1/\varepsilon)}, \quad \sum_{k=h,f} \omega^{\varepsilon}_k = 1 \tag{6}$$

where η and ε are the elasticities of substitution within a type and between types. ω_k is a parameter that governs the consumption home bias in consumption basket in the steady state. $s_{i,k,t}$ denotes good-specific habit, which evolves according to

$$s_{i,k,t} = \rho s_{i,k,t-1} + (1-\rho) c_{i,k,t} \text{ for } k = h, f, \tag{7}$$

where $c_{i,k,t}$ is defined as an average consumption level of good i, that is, $c_{i,k,t} \equiv \int_0^1 c^j_{i,k,t} dj$ for $k = h, f$. In other words, it is not the individual level of consumption that matters for the habit, but the level of average consumption. Hence, the preferences can be considered "Catching Up with Jonesses" at the goods level rather than at the aggregate consumption level. δ_t is a demand shock that alters the marginal utility of consumption.

The cost minimization associated with (5) implies the following demand functions:

$$c^j_{i,k,t} = \left(\frac{P_{i,k,t}}{\tilde{P}_{k,t}} \right)^{-\eta} s^{\theta(1-\eta)}_{i,k,t-1} x^j_{k,t} \tag{8}$$

where the welfare-based price index and consumption basket are defined as

$$\tilde{P}_{k,t} \equiv \left[\int_{N_k} (P_{i,k,t} s^{\theta}_{i,k,t-1})^{1-\eta} di \right]^{1/(1-\eta)} \tag{9}$$

$$\text{and } x^j_{k,t} \equiv \left[\int_{N_k} (c_{i,k,t}/s^{\theta}_{i,k,t-1})^{1-1/\eta} di \right]^{1/(1-1/\eta)} \tag{10}$$

for $k = h, f$. $\tilde{P}_{k,t}$ and $x^i_{k,t}$ should be interpreted as habit adjusted price index, and habit adjusted consumption basket of goods of type k in the home country. The appendix presents the details of the derivations of (8) to (10). The appendix also shows that the consumption basket (10) in equilibrium is determined as

$$x^j_{k,t} = \omega^{\varepsilon}_k \left(\frac{\tilde{P}_{k,t}}{\tilde{P}_t} \right)^{-\varepsilon} x^j_t \text{ for } k = h, f \tag{11}$$

$$\text{where } \tilde{P}_t = \left[\sum_{k=h,f} \omega_k \tilde{P}^{1-\varepsilon}_{k,t} \right]^{1/(1-\varepsilon)}, \tag{12}$$

denotes the welfare-based aggregate price index of the home country. Due to the symmetric structure of the two countries, the foreign counterparts can be expressed simply with asterisks placed in (8)∼(12). For later use, we also define the CPI as

$$P_t \equiv \left[\sum_{k=h,f} \omega_k P^{1-\varepsilon}_{k,t} \right]^{1/(1-\varepsilon)} \tag{13}$$

where

$$P_{k,t} \equiv \left[\int_{N_k} (P_{i,k,t})^{1-\eta} di \right]^{1/(1-\eta)} \text{ for } k = h, f$$

is defined as a type-specific CPI.

3.2 Technology

Following our earlier work (Gilchrist, Schoenle, Sim, and Zakrajsek [2013]), we assume that the production technology is specified as

$$y_{i,t} = \left(\frac{A_t}{a_{i,t}} h_{i,t} \right)^{\alpha} - \phi, \quad 0 < \alpha \leq 1 \tag{14}$$

$$\text{and } y^*_{i,t} = \left(\frac{A^*_t}{a^*_{i,t}} h^*_{i,t} \right)^{\alpha} - \phi^*, \quad 0 < \alpha \leq 1 \tag{15}$$

where A_h and A^*_t are country specific aggregate technology shocks, possibly correlated with each other, and $a_{i,t}$ and $a^*_{i,t}$ are idiosyncratic technology shocks to home and foreign firms. We assume that the idiosyncratic shocks follow symmetric i.i.d. log-normal distributions, $\log a_{i,t} \sim N(-0.5\sigma^2, \sigma^2)$ and $\log a^*_{i,t} \sim N(-0.5\sigma^2, \sigma^2)$.

In this setup, ϕ and ϕ^* are country-specific fixed costs of operation, which make it possible for the firms to have negative income, and hence a liquidity problem if external financing is costly. As shown by Gilchrist, Schoenle, Sim, and Zakrajsek [2013], the fixed costs can work as a parsimonious proxy for heterogeneous degrees of financial friction, including fixed payments to long term bonds, etc. However, to make ϕ and ϕ^* important for liquidity concerns of firms, we need to introduce some frictions to the flow of funds constraint of the firms.

3.3 Pricing Frictions and Financial Distortions

To allow for nominal rigidities, we assume that the firms face a quadratic cost of adjusting nominal prices as specified in (Rotemberg [1982]):

$$\frac{\gamma}{2}\left(\frac{P_{i,h,t}}{P_{i,h,t-1}}-\bar{\pi}\right)^2 c_t+\frac{\gamma^*}{2}\frac{S_t P_t^*}{P_t}\left(\frac{P_{i,h,t}^*}{P_{i,h,t-1}^*}-\bar{\pi}\right)^2 c_t^* \tag{16}$$

where S_t is the nominal exchange rate. We allow the degree of nominal rigidity to differ in home and foreign countries as indicated by separate notations for γ and γ^*. We assume that price adjustment costs are proportional to local consumptions, that is, c_t and c_t^*. After dividing the numerators and denominators by type specific price indices $P_{h,t}$ and $P_{h,t}^*$, we can express the price adjustment costs as

$$\frac{\gamma}{2}\left(\frac{p_{i,h,t}}{p_{i,h,t-1}}\pi_{h,t}-\bar{\pi}\right)^2 c_t+\frac{\gamma^*}{2}q_t\left(\frac{p_{i,h,t}^*}{p_{i,h,t-1}^*}\pi_{h,t}^*-\bar{\pi}^*\right)^2 c_t^*.$$

where $p_{i,h,t} \equiv P_{i,h,t}/P_{h,t}$, $p_{i,h,t}^* \equiv P_{i,h,t}^*/P_{h,t}^*$ and $q_t \equiv S_t P_t^*/P_t$, the real exchange rate.

To introduce financial frictions in a tractable manner we assume that the firms must commit to pricing and hence output decisions based on all aggregate information available within the period, but prior to the realization of their firm-specific idiosyncratic shock to productivity. Based on this aggregate information, the firms post prices, take orders from customers and plan production based on expected marginal cost. The firms then realize actual marginal cost and hire labor to meet demand. Ex-post, profits may be too low to cover the total cost of production in which case the firms must raise external finance. Without loss of generality, we focus on equity finance.[4] We assume that ex-post equity finance involves a constant per-unit dilution cost φ, $\varphi^* \in (0,1)$.

[4]As shown by Gomes [2001] and Stein [2003], other forms of costly external financing can be replicated by properly parameterized equity dilution costs.

3.4 Profit Maximization Problem

The firm problem is to maximize the present value of dividend flows, $\mathbb{E}_t[\sum_{s=0}^{\infty} m_{t,t+s} d_{i,t+s}]$ where $d_{i,t} \equiv D_{i,t}/P_t$ denotes real dividend payouts when positive, and equity issuance when negative. We assume that the firms are owned by the households, and discount future cashflows with the stochastic discounting factor of a representative household, $m_{t,t+s}$. The dilution cost implies that when a firm issues a notional amount of equity $d_{i,t}(< 0)$, actual cash inflow from the issuance is reduced to $-(1-\varphi)d_{i,t}$. An implicit assumption is that the equity markets in the two countries are segmented: only domestic households invest in the shares of domestic firms.

This assumption can be justified if the information asymmetry underlying the dilution phenomenon is disproportionately larger for cross-border equity holdings. The home bias in equity holdings is well documented by empirical researchers in fiance: French and Poterba [1991], Tesar and Werner [1995] and Obstfeld and Rogoff [2000]. In fact, in our model, even an arbitrarily small disadvantage associated with cross-border financing is large enough to generate perfect home bias because the perceived supply of domestic funding at the given cost of dilution is infinitely elastic. Hence the stream of dividends is discounted with the stochastic discounting factor of the representative household in the local country.

Another important assumption we adopt is that the two countries are different in terms of the degree of capital market imperfection. In particular, we assume that the dilution cost is strictly greater in the home country than in foreign country: $0 \leq \varphi^* < \varphi$. Despite this difference, we assume that the firms in home country cannot avoid the higher external financing cost by issuing equities overseas. Together with the higher operating costs in the home country, the higher external financing cost exposes the firms in the home country to a greater degree to liquidity risk. We explore the implication of such financial vulnerability on pricing under various macroeconomic environments such as fixed vs floating exchange rate on the one hand, and complete and incomplete risk sharing between the two countries on the other hand. To save space, we describe the firm problem from the viewpoint of home country.

The firm problem is subject to the following flow of funds constraint:

$$\begin{aligned} d_{i,t} = {} & p_{i,h,t} p_{h,t} c_{i,h,t} + q_t p^*_{i,h,t} p^*_{h,t} c^*_{i,h,t} - w_t h_{i,t} + \varphi \min\{0, d_{i,t}\} \\ & - \frac{\gamma}{2}\left(\frac{p_{i,h,t}}{p_{i,h,t-1}}\pi_{h,t} - \bar{\pi}\right)^2 c_t - \frac{\gamma^*}{2} q_t \left(\frac{p^*_{i,h,t}}{p^*_{i,h,t-1}}\pi^*_{h,t} - \bar{\pi}^*\right)^2 c^*_t \end{aligned} \tag{17}$$

where $p_{h,t} \equiv P_{h,t}/P_t$, $p^*_{h,t} \equiv P^*_{h,t}/P^*_t$, $c_{i,h,t} = \int_{N_c} c^j_{i,h,t} dj$ and $c^*_{i,h,t} = \int_{N_c} c^{j*}_{i,h,t} dj$. $w_t \equiv W_t/P_t$ is real wage rate. In a symmetric equilibrium, all households in the same country make the same consumption decision, and hence, $c^j_{i,h,t} = c_{i,h,t}$ and $c^{j*}_{i,h,t} = c^*_{i,h,t}$ for all j. The firm problem is also subject to a demand constraint:

$$\left(\frac{A_t}{a_{i,t}} h_{i,t}\right)^{\alpha} - \phi \geq c_{i,h,t} + c^*_{i,h,t} \tag{18}$$

The firm problem can then be expressed with the following Lagrangian:

$$
\begin{aligned}
\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} m_{0,t} \Bigg\{ & d_{i,t} + \kappa_{i,t} \left[\left(\frac{A_t}{a_{i,t}} h_{i,t} \right)^{\alpha} - \phi - (c_{i,h,t} + c^*_{i,h,t}) \right] \\
& + \xi_{i,t} \Bigg[p_{i,h,t} p_{h,t} c_{i,h,t} + q_t p^*_{i,h,t} p^*_{h,t} c^*_{i,h,t} - w_t h_{i,t} - d_{i,t} + \varphi \min\{0, d_{i,t}\} \\
& \qquad - \frac{\gamma}{2} \left(\frac{p_{i,h,t}}{p_{i,h,t-1}} \pi_{h,t} - \bar{\pi} \right)^2 c_t - \frac{\gamma^*}{2} q_t \left(\frac{p^*_{i,h,t}}{p^*_{i,h,t-1}} \pi^*_{h,t} - \bar{\pi}^* \right)^2 c^*_t \Bigg] \\
& + \nu_{i,h,t} \left[(p_{i,h,t})^{-\eta} \tilde{p}^{\eta}_{h,t} s^{\theta(1-\eta)}_{i,h,t-1} x_{h,t} - c_{i,h,t} \right] \\
& + \nu^*_{i,h,t} \left[(p^*_{i,h,t})^{-\eta} \tilde{p}^{*\eta}_{h,t} s^{*\theta(1-\eta)}_{i,h,t-1} x^*_{h,t} - c^*_{i,h,t} \right] \\
& + \lambda_{i,h,t} \left[\rho s_{i,h,t-1} + (1-\rho) c_{i,h,t} - s_{i,h,t} \right] \\
& + \lambda^*_{i,h,t} \left[\rho s^*_{i,h,t-1} + (1-\rho) c^*_{i,h,t} - s^*_{i,h,t} \right] \Bigg\}
\end{aligned}
$$

where $\tilde{p}_{h,t} \equiv \tilde{P}_{h,t}/P_{h,t}$, $\tilde{p}^*_{h,t} \equiv \tilde{P}^*_{h,t}/P^*_{h,t}$, and $\kappa_{i,t}$, $\xi_{i,t}$, $\nu_{i,h,t}$, $\nu^*_{i,h,t}$, $\lambda_{i,h,t}$ and $\lambda^*_{i,h,t}$ are shadow values of the constraints (18), (17), (8) and (7).

3.5 Efficiency Conditions

The efficiency conditions for the firm problem in the home country are given by the following:

$$
d_{i,t} : \xi_{i,t} = \begin{cases} 1 & \text{if } d_{i,t} \geq 0 \\ 1/(1-\varphi) & \text{if } d_{i,t} < 0 \end{cases} \tag{19}
$$

$$
h_{i,t} : \xi_{i,t} w_t = \alpha \kappa_{i,t} \left(\frac{A_t}{a_{i,t}} h_{i,t} \right)^{\alpha - 1} \tag{20}
$$

$$
\text{where} \quad h_{i,t} = \frac{a_{i,t}}{A_t} \left(\phi + c_{i,h,t} + c^*_{i,h,t} \right)^{1/\alpha} \tag{21}
$$

$$
c_{i,h,t} : \nu_{i,h,t} = \mathbb{E}^a_t[\xi_{i,t}] \; p_{i,h,t} p_{h,t} - \mathbb{E}^a_t[\kappa_{i,t}] + (1-\rho)\lambda_{i,h,t} \tag{22}
$$

$$
c^*_{i,h,t} : \nu^*_{i,h,t} = \mathbb{E}^a_t[\xi_{i,t}] \; q_t p^*_{i,h,t} p^*_{h,t} - \mathbb{E}^a_t[\kappa_{i,t}] + (1-\rho)\lambda^*_{i,h,t} \tag{23}
$$

$$
\begin{aligned}
s_{i,h,t} : \lambda_{i,h,t} = {} & \rho \mathbb{E}_t[m_{t,t+1} \lambda_{i,h,t+1}] \\
& + \theta(1-\eta) \mathbb{E}_t \left\{ m_{t,t+1} \mathbb{E}^a_{t+1} \left[\nu_{i,h,t+1} \frac{c_{i,h,t+1}}{s_{i,h,t}} \right] \right\}
\end{aligned} \tag{24}
$$

$$
\begin{aligned}
s^*_{i,h,t} : \lambda^*_{i,h,t} = {} & \rho \mathbb{E}_t[m_{t,t+1} \lambda^*_{i,h,t+1}] \\
& + \theta(1-\eta) \mathbb{E}_t \left\{ m_{t,t+1} \mathbb{E}^a_{t+1} \left[\nu^*_{i,h,t+1} \frac{c^*_{i,h,t+1}}{s^*_{i,h,t}} \right] \right\}
\end{aligned} \tag{25}
$$

$$p_{i,h,t}: 0 = \mathbb{E}_t^a[\xi_{i,t}]\left[p_{h,t}c_{i,h,t} - \gamma\frac{\pi_{h,t}}{p_{i,h,t-1}}\left(\pi_{h,t}\frac{p_{i,h,t}}{p_{i,h,t-1}} - \bar{\pi}\right)c_t\right] - \eta\frac{\nu_{i,h,t}}{p_{i,h,t}}c_{i,h,t} \tag{26}$$

$$+ \gamma\mathbb{E}_t\left[m_{t,t+1}\mathbb{E}_{t+1}^a[\xi_{i,t+1}]\pi_{h,t+1}\frac{p_{i,h,t+1}}{p_{i,h,t}^2}\left(\pi_{h,t+1}\frac{p_{i,h,t+1}}{p_{i,h,t}} - \bar{\pi}\right)c_{t+1}\right]$$

$$p_{i,h,t}^*: 0 = \mathbb{E}_t^a[\xi_{i,t}]\left[q_t p_{h,t}^* c_{i,h,t}^* - \gamma^*\frac{q_t\pi_{h,t}^*}{p_{i,h,t-1}^*}\left(\pi_{h,t}^*\frac{p_{i,h,t}^*}{p_{i,h,t-1}^*} - \bar{\pi}^*\right)c_t^*\right] - \eta\frac{\nu_{i,h,t}^*}{p_{i,h,t}^*}c_{i,h,t}^* \tag{27}$$

$$+ \gamma^*\mathbb{E}_t\left[m_{t,t+1}\mathbb{E}_{t+1}^a[\xi_{i,t+1}]q_{t+1}\pi_{h,t+1}^*\frac{p_{i,h,t+1}^*}{p_{i,h,t}^{*2}}\left(\pi_{h,t+1}^*\frac{p_{i,h,t+1}^*}{p_{i,h,t}^*} - \bar{\pi}^*\right)c_{t+1}^*\right]$$

Note they (21) is the labor demand conditional on the level of output, which, in turn, is determined by demand given price. We combine (20) and (21), and express the efficiency condition for labor hours as

$$\kappa_{i,t} = \xi_{i,t}a_{i,t}\frac{w_t}{\alpha A_t}\left(\phi + c_{i,h,t} + c_{i,h,t}^*\right)^{\frac{1-\alpha}{\alpha}}. \tag{28}$$

3.6 Symmetric Equilibrium and International Price Wars

The last six FOCs describe the efficiency conditions for the decisions made prior to the realization of the idiosyncratic cost shock. These first-order conditions involve the expected value of internal funds $\mathbb{E}_t^a[\xi_{i,t}] \equiv \int_0^\infty \xi_{i,t}(a_{i,t})dF(a)$ where the information set of the expectations operator includes all aggregate information up to time t except the realization of the idiosyncratic shock. In contrast, the realized values $\xi_{i,h,t}$ and $a_{i,h,t}$ enter the efficiency conditions (19) and (20) without the expectation operator since equity issuance and labor hiring decisions are made after the realization of the idiosyncratic shock.

With risk-neutrality and i.i.d. idiosyncratic shocks, the timing convention adopted above implies that firms are identical ex ante. Hence we focus on a symmetric equilibrium whereby all monopolistically competitive firms choose identical relative prices ($p_{i,h,t} = 1$ and $p_{i,h,t}^* = 1$), production scales ($c_{i,h,t} = c_{h,t}$ and $c_{i,h,t}^* = c_{h,t}^*$), habit stocks ($s_{i,h,t} = s_{h,t}$ and $s_{i,h,t}^* = s_{h,t}^*$), and shadow values of habit ($\lambda_{i,h,t} = \lambda_{h,t}$ and $\lambda_{i,h,t}^* = \lambda_{h,t}^*$). However, the distributions of labor hours, dividend payouts, equity issuance and the realized shadow value of internal funds ($\xi_{i,t}$) are non-degenerate and depend on the realization of idiosyncratic shocks.

Note that the symmetric equilibrium condition $p_{i,h,t} = 1$ and $p_{i,h,t}^* = 1$ imply that the firms in the home country choose the same price levels in home and in foreign markets vis a vis other competitors from the same origin. However, this symmetric equilibrium condition does not imply that the firms in foreign country make the same pricing decision as home country firms in the same market, even when they share the same fundamentals, that is, preference and aggregate technology level. In other words, foreign firms make the same pricing decisions among themselves both in domestic and export markets such that $p_{i,f,t} = 1$ and $p_{i,f,t}^* = 1$, but the price levels chosen by home firms and and foreign firms in a given market differ from each other, and as a result, $p_{h,t} = P_{h,t}/P_t \neq 1$, $p_{h,t}^* = P_{h,t}^*/P_t^* \neq 1$, $p_{f,t} = P_{f,t}/P_t \neq 1$, $p_{f,t}^* = P_{f,t}^*/P_t^* \neq 1$, and $p_{h,t} \neq p_{f,t}$ and $p_{h,t}^* \neq p_{f,t}^*$ in general. This is because the firms in two countries face different degrees of capital market distortions, which create different liquidity conditions for home and foreign firms and lead these two groups of firms to follow different mark-up strategies. In particular, as will be shown below, the relatively poorer financial conditions lead home firms to maintain higher markups and higher prices in the neighborhood of the non-stochastic steady state such that $p_h > p_f$ and $p_h^* > p_f^*$.

From the vantage point of foreign firms, the same phenomenon can be thought of as them engaging in a price war, exploiting the financial vulnerability of home firms to expand market shares both in their home and in their export markets. The stronger the long-run relationship in customer markets, the greater the incentive of foreign firms to undercut the prices of home firms. If the exchange rate is floating, however, the depreciation of home currency, assisted by easy monetary policy of home country, can greatly improve the liquidity conditions for home country firms, providing an effective defense against the foreign firms' aggressive invasion on their markets. By joining a currency union, the home country essentially surrenders this armor.

3.7 Financial Friction and Phillips Curves

Imposing the symmetric equilibrium conditions to the FOCs for pricing decisions yields the following Phillips curves:

$$p_{h,t}\frac{c_{h,t}}{c_t} = \gamma\pi_{h,t}(\pi_{h,t}-\bar{\pi}) + \eta\frac{\nu_{h,t}}{\mathbb{E}_t^a[\xi_{i,t}]}\frac{c_{h,t}}{c_t} - \gamma\mathbb{E}_t\left[m_{t,t+1}\frac{\mathbb{E}_{t+1}^a[\xi_{i,t+1}]}{\mathbb{E}_t^a[\xi_{i,t}]}\pi_{h,t+1}(\pi_{h,t+1}-\bar{\pi})\frac{c_{t+1}}{c_t}\right] \tag{29}$$

$$\text{and } q_t p_{h,t}^*\frac{c_{h,t}^*}{c_t^*} = \gamma q_t\pi_{h,t}^*(\pi_{h,t}^*-\bar{\pi}^*) + \eta\frac{\nu_{h,t}^*}{\mathbb{E}_t^a[\xi_{i,t}]}\frac{c_{h,t}^*}{c_t^*} - \gamma^*\mathbb{E}_t\left[m_{t,t+1}\frac{\mathbb{E}_{t+1}^a[\xi_{i,t+1}]}{\mathbb{E}_t^a[\xi_{i,t}]}q_{t+1}\pi_{h,t+1}^*(\pi_{h,t+1}^*-\bar{\pi})\frac{c_{t+1}^*}{c_t^*}\right] \tag{30}$$

The left side of the Phillips curve is different from 1 since $p_{h,t} \neq 1$ and $c_{h,t} \neq c_t$. This is because (29) and (30) describe 'sectoral' inflation dynamics rather than aggregate inflation. More importantly, (29) and (30) show an important departure from New Keynesian Phillips curve in that the dynamic liquidity condition plays an essential role in inflation dynamics. This can be seen from the fact that the ratio of the shadow value of internal funds today vs tomorrow works as an additional discounting factor.

The presence of the marginal value of sales $\nu_{h,t}$ on the right side is due to the customer market feature (deep habit a la Ravn, Schmitt-Grohé, and Uribe [2006]) of the model. Given the long run relationship between the firm and the customer base, it can be easily seen that such marginal value captures not only the value of sales today, but also future sales of the firm. What is different from a deep habit model is that the marginal valuation crucially depends on the dynamic liquidity condition of the firm. Such valuation is determined by two elements: (i) future stream of profits created by sales today and the long-run customer relationship; (ii) the dynamic liquidity condition represented by the future shadow value of internal funds. More formally (see the appendix for derivation), it can be shown that if we define *financially adjusted* markup as

$$\tilde{\mu}_t \equiv \frac{\mathbb{E}_t^a[\xi_{i,t}]}{\mathbb{E}_t^a[\xi_{i,t}a_{i,t}]}\frac{\alpha A_t}{w_t}\left(\phi + c_{h,t} + c_{h,t}^*\right)^{\frac{\alpha-1}{\alpha}} = \frac{\mathbb{E}_t^a[\xi_{i,t}]}{\mathbb{E}_t^a[\xi_{i,t}a_{i,t}]}\mu_t,$$

the marginal value of sales normalized by the marginal value of internal funds is given by

$$\frac{\nu_{h,t}}{\mathbb{E}_t^a[\xi_{i,t}]} = p_{h,t} - \frac{1}{\tilde{\mu}_t} + \chi\mathbb{E}_t\left[\sum_{s=t+1}^{\infty}\tilde{\beta}_{t,s}\frac{\mathbb{E}_s^a[\xi_{i,s}]}{\mathbb{E}_t^a[\xi_{i,t}]}\left(p_{h,s} - \frac{1}{\tilde{\mu}_s}\right)\right] \tag{31}$$

$$\text{and } \frac{\nu_{h,t}^*}{\mathbb{E}_t^a[\xi_{i,t}]} = q_t p_{h,t}^* - \frac{1}{\tilde{\mu}_t} + \chi\mathbb{E}_t\left[\sum_{s=t+1}^{\infty}\tilde{\beta}_{t,s}^*\frac{\mathbb{E}_s^a[\xi_{i,s}]}{\mathbb{E}_t^a[\xi_{i,t}]}\left(q_s p_{h,s}^* - \frac{1}{\tilde{\mu}_s}\right)\right] \tag{32}$$

where $\chi \equiv (1-\rho)\theta(1-\eta)$ and the composite discounting factors are given by

$$\tilde{\beta}_{t,s} \equiv m_{s,s+1}g_{h,s+1}\cdot\prod_{j=1}^{s-t}(\rho + \chi g_{h,t+j})m_{t+j-1,t+j} \text{ with } g_{h,t} \equiv \frac{s_{h,t}/s_{h,t-1} - \rho}{1-\rho}$$

$$\tilde{\beta}_{t,s}^* \equiv m_{s,s+1}g_{h,s+1}^*\cdot\prod_{j=1}^{s-t}(\rho + \chi g_{h,t+j}^*)m_{t+j-1,t+j} \text{ with } g_{h,t}^* \equiv \frac{s_{h,t}^*/s_{h,t-1}^* - \rho}{1-\rho}.$$

Hence, to analyze how financial market frictions interact with pricing decisions we need to study how the value of internal funds $\mathbb{E}_t^a[\xi_{i,h,t}]$ is determined, and how it affects the markup decision. To that end, we impose the symmetric equilibrium condition, and define the equity issuance trigger as the idiosyncratic productivity level that satisfies the flow of funds constraint when dividend payouts are exactly zero:

$$a_t^E = \frac{A_t}{w_t(\phi + c_{h,t} + c_{h,t}^*)^{1/\alpha}}\Bigg\{c_t\left[\frac{P_{h,t}c_{h,t}}{P_t c_t} - \frac{\gamma}{2}(\pi_{h,t} - \bar{\pi})^2\right] + q_t c_t^*\left[\frac{P_{h,t}^* c_{h,t}^*}{P_t^* c_t^*} - \frac{\gamma^*}{2}(\pi_{h,t}^* - \bar{\pi}^*)^2\right]\Bigg\} \tag{33}$$

Investigation of the external financing trigger (33) provides a great opportunity to expose the mechanics of a "price war" in our setup, and how manipulation of exchange rates provides an effective tool to counteract such price war. Using the equity issuance trigger, we can rewrite the FOC for dividends as

$$\xi(a_{i,t}) = \begin{cases} 1 & \text{if } a_{i,t} \leq a_{h,t}^E \\ 1/(1-\varphi) & \text{if } a_{i,t} > a_{h,t}^E \end{cases}. \tag{34}$$

This condition simply states that the realized shadow value of internal funds jumps to $1/(1-\varphi)$ due to the costly external financing when the idiosyncratic cost shock is greater than the threshold value. Let z_t^E denote the standardized value of a_t^E, that is, $z_t^E = \sigma^{-1}(\log a_t^E + 0.5\sigma^2)$. From (34), the expected shadow value of internal funds is

$$\mathbb{E}_t^a[\xi_{i,t}] = \int_0^{a_t^E} 1 dF(a) + \int_{a_t^E}^{\infty} \frac{1}{1-\varphi} dF(a) = 1 + \frac{\varphi}{1-\varphi}[1 - \Phi(z_t^E)] \geq 1 \tag{35}$$

where $\Phi(\cdot)$ denotes the cdf of the standard normal distribution. The expected shadow value is strictly greater than unity as long as equity issuance is costly ($\varphi > 0$) and future costs are uncertain ($\sigma > 0$). As emphasized in Gilchrist, Schoenle, Sim, and Zakrajsek [2013], this makes the firms de facto risk averse in their pricing decision: setting the price too low and taking an imprudently large number of orders by lowering the price too much exposes the firm to liquidity risk under costly external finance. Note that if $\varphi = 1$, this means that outside funding is impossible. This is the case of Brunnermeier and Sannikov [2014].

Note that the expected shadow value, which is what matters for the pricing decision, is decreasing in a_t^E. Other things being equal, the threshold value is increasing in market shares of home country firms in both markets, that is, $P_{h,t}c_{h,t}/P_t c_t$ and $P_{h,t}^* c_{h,t}^*/P_t^* c_t^*$. If the elasticity of substitution is sufficiently large enough, which will be satisfied in any realistic calibration of ε, foreign firms increase their market shares, $1 - P_{h,t}c_{h,t}/P_t c_t$ and $1 - P_{h,t}^* c_{h,t}^*/P_t^* c_t^*$ by aggressively slashing their prices both in domestic and export markets. The drop in the market share creates financial distress for the home country firms as can be seen in the drop of the external financing threshold and the resulting increase of the expected shadow value of internal funds. This means that home country firms have to lower their prices in defense to maintain their expected shadow cost of financing low. However, this strategy back-fires because doing so increases production scale and wage bills and expose them to a greater liquidity risk.

This is costly because it elevates the expected shadow cost of external financing. However, if the real exchange rate can depreciate (q_t goes up), the dilemma of firms in the home country can be solved: without changing the nominal prices of home country firms, depreciation can lower their export prices and increases the import prices of foreign competitors. This helps home country firms maintain market shares, and keep expected shadow prices low at a time of price war. In contrast, under a currency union or fixed exchange rate regime, the absence of a nominal exchange rate and independent monetary policy can make the situation much worse for a country with a greater degree of financial market distortion. Losing market share today, which persists in the customer market setting, aggravates tomorrow's liquidity condition, leading to a further upward pressure on the pricing of home country firms, expanding foreign firms' market share even more.

3.8 Household Problem

To analyze how different risk sharing arrangements between the two countries affect macroeconomic allocations, we consider the problem of household under two different arrangements for international finance: (i) complete risk sharing through trading of state contingent bonds; (iii) incomplete risk sharing with only non-contingent bonds.

3.8.1 Complete Risk Sharing Under Floating

To streamline notation, we omit the household index j, anticipating the symmetric equilibrium. The representative household in the home country works h_t, It saves by investing in the shares of home-country, state-contingent bonds that are traded internationally and non-contingent government bonds that are available in zero net supply. The household budget constraint is given by

$$
\begin{aligned}
0 = {} & W_t h_t + B(s^t) + R_{t-1} B^G_t + \int_{N_h} [\max\{D_{it}, 0\} + P^S_{i,t-1,t}] s^S_{i,t} di \\
& - \sum_{k=h,f} \int_{N_k} P_{i,k,t} c_{i,k,t} di - B^G_{t+1} - \int_{\mathcal{S}} M_t(s_{t+1}|s^t) B(s^{t+1}) ds - \int_{N_h} P^S_{i,t} s^S_{i,t+1} di
\end{aligned} \tag{36}
$$

where $s^t \equiv s_0, \dots, s_t$. A unit of state-contingent bond $B(s^{t+1})$ pays out one unit of home currency upon the realization of state s_{t+1} and $M_t(s_{t+1}|s^t)$ is the price of the bond at time t. B^G_{t+1} is the government bond, and r_t is the corresponding interest rate. Using the accounting identity, $\int_{N_k} P_{i,k,t} c_{i,k,t} di = \tilde{P}_{k,t} x_{k,t}$ for $k = h, f$ (see the appendix), we can simplify the last term in the budget constraint as $\sum_{k=h,f} \tilde{P}_{k,t} x_{k,t}$. $s^S_{i,t}$ denotes the outstanding shares of home country firm i, $P^S_{i,t-1,t}$ is the time t value of shares outstanding at time $t-1$ and $P^S_{i,t}$ is the ex-dividend value of shares at time t. The last two terms are related via the accounting identity, $P^S_{i,t} = P^S_{i,t-1,t} + E^S_{i,t}$ where $E^S_{i,t}$ is the value of new shares issued at time t. The costly equity finance assumption implies that $E^S_{i,t} = -(1-\varphi)\min\{D_{i,t}, 0\}$. Using this relationship, we can express the budget constraint only in terms of $P^S_{i,t}$:

$$
\begin{aligned}
0 = {} & W_t h_t + B(s^t) + R_{t-1} B^G_t + \int_{N_h} (\tilde{D}_{i,t} + P^S_{i,t}) s^S_{i,t} di \\
& - \sum_{k=h,f} \tilde{P}_{k,t} x_{k,t} - B^G_{t+1} - \int_{\mathcal{S}} M(s_{t+1}|s^t) B(s^{t+1}) ds - \int_{N_h} P^S_{i,t} s^S_{i,t+1} di
\end{aligned} \tag{37}
$$

where $\tilde{D}_{i,t} \equiv \max\{D_{i,t}, 0\} + (1-\varphi)\min\{D_{i,t}, 0\}$.

The above expression makes it clear that costly equity finance takes the form of sales of new shares at a discount in general equilibrium. Since the owners of old and new shares are the same entity, there is no direct wealth effect associated with costly equity financing: the losses of the old shareholders exactly offset the gains of the new shareholders. Denoting the multiplier for the budget by Λ_t and maximizing (5) subject to (37) yields

$$
x_{h,t} : \Lambda_t \tilde{P}_{h,t} = \omega_h \frac{x_t}{x_{h,t}} U_{x,t} \tag{38}
$$

$$
x_{f,t} : \Lambda_t \tilde{P}_{f,t} = \omega_f \frac{x_t}{x_{f,t}} U_{x,t} \tag{39}
$$

$$
h_t : \Lambda_t W_t = -U_{h,t} \tag{40}
$$

$$
B(s^{t+1}) : \Lambda_t M(s_{t+1}|s^t) = \beta \Pr(s_{t+1}|s^t) \Lambda(s_{t+1}|s^t) \tag{41}
$$

$$B^G_{t+1} : \Lambda_t = \beta \mathbb{E}_t[\Lambda_{t+1} R_t] \tag{42}$$

$$s^S_{i,t+1} : \Lambda_t = \beta \mathbb{E}_t \left[\Lambda_{t+1} \left(\frac{\mathbb{E}^a_{t+1}[\tilde{D}_{i,t+1}] + P^S_{t+1}}{P^S_t} \right) \right] \tag{43}$$

where we use $P^S_t = P^S_{i,t}$ in our symmetric equilibrium. Note that (38) and (39) imply $\Lambda_t \sum_{k=h,f} \tilde{P}_{k,t} x_{k,t} = x_t U_{x,t}$. Since $\sum_{k=h,f} \tilde{P}_{k,t} x_{k,t} = \tilde{P}_t x_t$ holds from an accounting identity (see the appendix), (38) and (39) are equivalent to $\tilde{P}_t \Lambda_t = U_{x,t}$. Combining this condition with (41) yields

$$M_t(s_{t+1}|s^t) = \beta \frac{U_x(s^{t+1})/\tilde{P}_{t+1}}{U_{x,t}/\tilde{P}_t} \Pr(s_{t+1}|s^t) = \beta \frac{U_x(s^{t+1})/\tilde{p}_{t+1}}{U_{x,t}/\tilde{p}_t} \frac{P_t}{P_{t+1}} \Pr(s_{t+1}|s^t) \tag{44}$$

In symmetric equilibrium, $s_{i,k,t-1} = s_{k,t-1}$, and thus it is straightforward to show that $\tilde{p}_t = [\sum_{k=h,f} \omega_k p^{1-\varepsilon}_{k,t} s^{\theta(1-\varepsilon)}_{k,t-1}]^{1/(1-\varepsilon)}$(see the appendix, (A.16)). A condition similar to (44) holds for the foreign representative household, that is,

$$M_t(s_{t+1}|s^t) = \frac{S_t P^*_t}{S(s^{t+1}) P^*_{t+1}} \beta \frac{U^*_x(s^{t+1})/\tilde{p}^*_{t+1}}{\tilde{U}^*_{x,t}/\tilde{p}^*_t} \Pr(s_{t+1}|s^t) \tag{45}$$

Then, equations (44) and (45) jointly imply the risk sharing condition:

$$q_t = \kappa \frac{\tilde{U}^*_{x,t}}{\tilde{U}_{x,t}} \text{ where } \kappa \equiv q_0 \frac{\tilde{U}_{x,0}}{\tilde{U}^*_{x,0}} \text{ and } \tilde{U}_{x,t} \equiv U_{x,t} \left[\sum_{k=h,f} \omega_k p^{1-\varepsilon}_{k,t} s^{\theta(1-\varepsilon)}_{k,t-1} \right]^{-1/(1-\varepsilon)} \tag{46}$$

Note that summing the prices of all state-contingent bonds yields the discount factor of firms

$$\int_{\mathcal{S}} M(s_{t+1}|s^t) ds = \mathbb{E}_t[m_{t,t+1}/\pi_{t+1}] = R^{-1}_t \text{ where } m_{t,t+1} \equiv \beta \tilde{U}_{x,t+1}/\tilde{U}_{x,t}.$$

We assume that the monetary authorities of the two countries control the prices of the government bonds using an identical Taylor-type rule:

$$R_t = R^{1-\rho_R} \left[R_{t-1} \left(\frac{y_t}{y} \right)^{\rho_c} \left(\frac{\pi_t}{\pi} \right)^{\rho\pi} \right]^{\rho_R} \tag{47}$$

where a straightforward algebra can show that the inflation rate π_t is determined as

$$\pi_t = \left[\sum_{k=h,f} \omega_k (p_{k,t-1} \pi_{k,t})^{1-\varepsilon} \right]^{1/(1-\varepsilon)} \tag{48}$$

The foreign counterpart of (47) is symmetrically specified as

$$R^*_t = R^{1-\rho_R} \left[R^*_{t-1} \left(\frac{y^*_t}{y^*} \right)^{\rho_c} \left(\frac{\pi^*_t}{\pi^*} \right)^{\rho\pi} \right]^{\rho_R}. \tag{49}$$

The risk sharing condition implies that the FOC of the home investor for the government bond can be rewritten as follows:

$$1 = \beta \mathbb{E}_t \left[\frac{\tilde{U}_{x,t+1}}{\tilde{U}_{x,t}} \frac{R_t}{\pi_{t+1}} \right] = \beta \mathbb{E}_t \left[\frac{\tilde{U}^*_{x,t+1}}{\tilde{U}^*_{x,t}} \frac{R_t}{\pi_{t+1}} \frac{q_t}{q_{t+1}} \right] \tag{50}$$

Similarly, for the FOC of the foreign investor for the foreign government bond, the following holds:

$$1 = \beta \mathbb{E}_t \left[\frac{\tilde{U}^*_{x,t+1}}{\tilde{U}^*_{x,t}} \frac{R^*_t}{\pi^*_{t+1}} \right] = \beta \mathbb{E}_t \left[\frac{\tilde{U}_{x,t+1}}{\tilde{U}_{x,t}} \frac{R^*_t}{\pi^*_{t+1}} \frac{q_{t+1}}{q_t} \right] \tag{51}$$

(50) and (51) show that the assumption of non cross-border holdings of government bonds are innocuous because the risk sharing condition makes the assumption irrelevant.

3.8.2 Incomplete Risk Sharing Under Floating Exchange Rate

To analyze the effects of incomplete risk sharing, we consider an alternative environment where the two countries trade state-noncontingent bonds. As in Ghironi and Melitz [2005], we assume that there are portfolio rebalancing costs associated with changing the level of capital accounts. These costs are a short-cut to real frictions that hinder efficient borrowing/lending across borders.[5] We denote home country's holdings of international bonds issued in home and foreign currency units by $B_{h,t+1}$ and $B_{f,t+1}$.[6] $B^*_{h,t+1}$ and $B^*_{f,t+1}$ denote the foreign counterparts. The interest rates are denoted by R_t and R^*_t, respectively.[7]

The portfolio rebalancing costs are specified as $(\tau/2)P_t[(B_{h,t+1}/P_t)^2 + q_t(B_{f,t+1}/P^*_t)^2]$, which implies that any deviation from zero is costly, creating inelastic supply of international funds for borrowing. Under these assumptions, the marginal cost of borrowing in home currency is given by $R_t/(1+\tau B_{h,t+1}/P_t)$, which is strictly greater than R_t if $B_{h,t+1} < 0$. The marginal return on foreign lending in home currency is given by $R_t(S_t/S_{t+1})/(1 + \tau B^*_{h,t+1}/P^*_t)$, which is strictly less than $R_t(S_t/S_{t+1})$ if $B^*_{h,t+1} > 0$. Thus, $(1+\tau B_{h,t+1}/P_t)^{-1}$ represents a welfare loss not only to borrowers but also for lenders. By varying the value of τ, one can approximate a range of international borrowing/lending relationships, including one that is fairly close to autarky. The intertemporal budget constraint of home country household is now given by

$$\begin{aligned} 0 = {} & W_t h_t + R_{t-1} B_{h,t} + S_t R^*_{t-1} B_{f,t} + \int_{N_h} (\tilde{D}_{i,t} + P^S_{i,t}) s^S_{i,h,t} di \\ & - \tilde{P}_t x_t - B_{h,t+1} - S_t B_{f,t+1} - \frac{\tau}{2} P_t \left[\left(\frac{B_{h,t+1}}{P_t} \right)^2 + q_t \left(\frac{B_{f,t+1}}{P^*_t} \right)^2 \right] - \int_{N_h} P^S_{i,t} s^S_{i,t+1} di \end{aligned} \tag{52}$$

[5] One of the fundamental barriers to efficient allocation of international funds is sovereign default. Thus, employing a nonlinear framework such as Eaton and Gersovitz [1981] type would be ideal. However, given the large state space, such a nonlinearity is too costly for the current analysis. We leave that for future work.

[6] $B_{h,t+1} + B^*_{h,t+1} = 0$, where $B_{h,t+1}$ and $B^*_{h,t+1}$ are issued in home currency as denoted by the subscripts, and are held by home and foreign countries as denoted by asterisk or lack thereof.

[7] For the imcomplete risk sharing model, we assume away government bond. However, this does not create any material difference in equilbrium because we assume zero net supply of such bond even for the complete risk sharing environment. Such bond is employed just for valuation purpose.

The efficiency conditions of the household problem do not change from (38), (39) and (40) except for international bond holdings, which are given by

$$B_{h,t+1} : \Lambda_t(1 + \tau b_{h,t+1}) = \beta \mathbb{E}_t \Big[\Lambda_{t+1} R_t \Big] \tag{53}$$

$$B_{f,t+1} : \Lambda_t(1 + \tau b_{f,t+1}) = \beta \mathbb{E}_t \Big[\Lambda_{t+1} R_t^* \frac{S_{t+1}}{S_t} \Big] \tag{54}$$

where $b_{h,t+1} \equiv B_{h,t+1}/P_t$ and $b_{f,t+1} \equiv B_{f,t+1}/P_t^*$. The monetary policy is specified in an identical way to (47). Substituting $\tilde{P}_t \Lambda_t = U_{x,t}$ in (53) and (54), one can rewrite them as

$$1 + \tau b_{h,t+1} = \beta \mathbb{E}_t \left[\frac{U_{x,t+1}/\tilde{p}_{t+1}}{U_{x,t+1}/\tilde{p}_{t+1}} \frac{R_t}{\pi_{t+1}} \right] \tag{55}$$

$$\text{and } 1 + \tau b_{f,t+1} = \beta \mathbb{E}_t \left[\frac{U_{x,t+1}/\tilde{p}_{t+1}}{U_{x,t+1}/\tilde{p}_{t+1}} \frac{q_{t+1}}{q_t} \frac{R_t^*}{\pi_{t+1}^*} \right] \tag{56}$$

The bond market clearing conditions are given by

$$0 = b_{h,t+1} + b_{h,t+1}^* \tag{57}$$

$$\text{and } 0 = b_{f,t+1} + b_{f,t+1}^* \tag{58}$$

where foreign holdings of international bonds in home and foreign currencies $b_{h,t+1}^*$ and $b_{f,t+1}^*$ satisfy

$$1 + \tau b_{h,t+1}^* = \beta \mathbb{E}_t \left[\frac{U_{x,t+1}^*/\tilde{p}_{t+1}^*}{U_{x,t+1}^*/\tilde{p}_{t+1}^*} \frac{q_t}{q_{t+1}} \frac{R_t}{\pi_{t+1}} \right] \tag{59}$$

$$\text{and } 1 + \tau b_{f,t+1}^* = \beta \mathbb{E}_t \left[\frac{U_{x,t+1}^*/\tilde{p}_{t+1}^*}{U_{x,t+1}^*/\tilde{p}_{t+1}^*} \frac{R_t^*}{\pi_{t+1}^*} \right]. \tag{60}$$

for foreign investors. Assuming that the portfolio rebalancing cost is transferred back to the household in lump sum, imposing the stock market equilibrium condition $s_{i,h,t}^S = s_{i,h,t+1}^S = 1$, and finally dividing the budget constraint through by P_t, one can rewrite it as a law of motion for bond holdings, that is,

$$b_{h,t+1} + q_t b_{f,t+1} = \frac{R_{t-1}}{\pi_t} b_{h,t} + \frac{R_{t-1}^*}{\pi_t^*} q_t b_{f,t} + w_t h_t + \tilde{d}_t - \tilde{p}_t x_t. \tag{61}$$

where $\tilde{d}_t \equiv \tilde{D}_t/P_t$ and $\tilde{d}_t^* \equiv \tilde{D}_t^*/P_t^*$. One can derive a similar law of motion for the foreign country,

$$q_t^{-1} b_{h,t+1}^* + b_{f,t+1}^* = \frac{R_{t-1}}{q_t \pi_t} b_{h,t}^* + \frac{R_{t-1}^*}{\pi_t^*} b_{f,t}^* + w_t^* h_t^* + \tilde{d}_t^* - \tilde{p}_t^* x_t^*. \tag{62}$$

After multiplying (62) by q_t, subtracting (62) from (61) and imposing the bond market clearing conditions (57) and (58) yields

$$b_{h,t+1}+q_t b_{f,t+1}=\frac{R_{t-1}}{\pi_t}b_{h,t}+\frac{R^*_{t-1}}{\pi^*_t}q_t b_{f,t}+\frac{1}{2}(w_t h_t-q_t w^*_t h^*_t)+\frac{1}{2}(\tilde{d}_t-q_t\tilde{d}^*_t)-\frac{1}{2}(\tilde{p}_t x_t-q_t\tilde{p}^*_t x^*_t). \quad (63)$$

This condition replaces the risk sharing condition (46) in equilibrium.

3.8.3 Complete Risk Sharing Under Currency Union

We now consider the situation where the two countries bilaterally agree to adopt a single currency. One can think of the situation in the following way. Once the two countries adopt a single currency, all products and financial assets are denominated in the single currency unit. As a result, the nominal exchange rate is not defined. Also as a result, a single monetary authority sets the monetary policy rate, denoted by R^U_t, and all investors, regardless of their countries of origin and current locations, earn the same nominal return.

However, depending on the reference location of investors, the real return on international bond holdings differs. This is due to two factors: first, the two countries have different consumption baskets in the long-run owing to heterogeneous home biases; second, at any point in given time, the law of one price is violated as any two consumers residing in different countries have accumulated heterogeneous degrees of habits for an identical product, and as a consequence, firms, in general, price their products to markets, so called "pricing to habits" (see Ravn, Schmitt-Grohé, and Uribe [2007]). Hence, inflation rates are not equalized in the two countries despite the adoption of a single currency and monetary policy, and the real returns on international bonds are not equalized either.

We assume that the common monetary policy is specified so as to reflect the economic fundamentals of both countries:

$$R^U_t=(R^U)^{1-\rho_R}\left[R^U_{t-1}\left(\frac{y^U_t}{y^U}\right)^{\rho_c}\left(\frac{\pi^U_t}{\pi^U}\right)^{\rho_\pi}\right]^{\rho_R} \quad (64)$$

where the union-wide variables are constructed as a weighted average with the weights given by the steady state share of output,[8] that is,

$$y^U_t=y_t\left(\frac{y}{y+qy^*}\right)+q_t y^*_t\left(\frac{qy^*}{y+qy^*}\right) \quad (65)$$

$$\text{and } \pi^U_t=\pi_t\left(\frac{y}{y+qy^*}\right)+\pi^*_t\left(\frac{qy^*}{y+qy^*}\right). \quad (66)$$

As a complete risk sharing regime, the currency union continues to ensure that the risk sharing condition (46) holds that prevails under the floating exchange rate regime. However, under the currency union, only one of the two consumption Euler equations (50) and (51) can be included in the system of equations that constitute the equilibrium. This is because the combination of the

[8]We have tried different formulae, real time or lagged weights, and have found no material difference in model dynamics.

single monetary policy and the risk sharing condition makes the two consumption Euler equations linearly dependent. Hence, only the following efficiency condition enters the system with (46):[9]

$$1 = \beta \mathbb{E}_t \left[\frac{\tilde{U}_{x,t+1}}{\tilde{U}_{x,t}} \frac{R_t^U}{\pi_{t+1}} \right] \tag{67}$$

3.8.4 Incomplete Risk Sharing Under Currency Union

Under the combination of incomplete risk sharing and currency union, the combined law of motion for international bond holdings (63) is replaced with

$$b_{h,t+1} = \frac{R_{t-1}}{\pi_t} b_{h,t} + \frac{1}{2}(w_t h_t - q_t w_t^* h_t^*) + \frac{1}{2}(\tilde{d}_t - q_t \tilde{d}_t^*) - \frac{1}{2}(\tilde{p}_t x_t - q_t \tilde{p}_t^* x_t^*). \tag{68}$$

as there is no longer any distinction between international bonds issued in home or foreign currency. In addition, the bond market clearing condition drops out of the system of equations. Furthermore, there are only two, instead of four, Euler equations characterizing the efficiency in the international bond markets:

$$b_{h,t+1} \ : \ 1 + \tau b_{h,t+1} = \beta \mathbb{E}_t \left[\frac{U_{x,t+1}/\tilde{p}_{t+1}}{U_{x,t+1}/\tilde{p}_{t+1}} \frac{R_t^U}{\pi_{t+1}} \right] \tag{69}$$

$$\text{and } b_{h,t+1}^* \ : \ 1 + \tau b_{h,t+1}^* = \beta \mathbb{E}_t \left[\frac{U_{x,t+1}^*/\tilde{p}_{t+1}^*}{U_{x,t+1}^*/\tilde{p}_{t+1}^*} \frac{q_t}{q_{t+1}} \frac{R_t^U}{\pi_{t+1}} \right] \tag{70}$$

Note that using the definition of the real exchange rate, we have as an identity

$$\frac{q_t}{q_{t+1}} = \frac{S_t}{S_{t+1}} \cdot \frac{\pi_{t+1}}{\pi_{t+1}^*}. \tag{71}$$

However, the currency union implies $S_t/S_{t+j} = 1$ for $j \geq 1$ permanently,[10] and (70) becomes equivalent to

$$1 + \tau b_{h,t+1}^* = \beta \mathbb{E}_t \left[\frac{U_{x,t+1}^*/\tilde{p}_{t+1}^*}{U_{x,t+1}^*/\tilde{p}_{t+1}^*} \frac{R_t^U}{\pi_{t+1}^*} \right] \tag{72}$$

While an identical nominal return enters (69) and (72), their real and nominal discounting factors differ.

4 Results

4.1 Calibration

Our calibration strategy closely follows that of Gilchrist, Schoenle, Sim, and Zakrajsek [2013], expanding the set of parameters as needed for the international environment. There are three sets of

[9] Otherwise, the two consumption Euler equations held withing the system together with the common monetary policy and risk sharing condition would imply $\pi_{t+s} = \pi_{t+s}^*$ for all s, which cannot be satisfied.

[10] We assume that it is impossible to exit the union unilaterally. See Alvarez and Dixit [2014]'s real option approach for a theoretical consideration of the break up of a currency union.

Table 5: Baseline Calibration

Description	Calibration
Preferences and production	
Time discounting factor, β	0.99
Constant relative risk aversion, γ_x	2.00
Deep habit, θ	$-$ 0.80
Persistence of deep habit, ρ	0.95
Elasticity of labor supply, $1/\gamma_h$	5.00
Elasticity of substitution, η	2.00
Armington elasticity, ε	1.50
Home bias, ω_h^ε	0.60
Persistence of technology shock, ρ_A	0.90
Returns to scale, α	1.00
Fixed operation cost, ϕ, ϕ^*	0.30, 0.00
Nominal rigidity and monetary policy	
Price adjustment cost, γ_p	10.0
Wage adjustment cost, γ_w	30.0
Monetary policy inertia, ρ^R	0.85
Taylor rule coefficient for inflation gap, ρ^π	$0.25/(1-\rho^R)$
Taylor rule coefficient for inflation gap, ρ^π	$1.50/(1-\rho^R)$
Financial Frictions	
Equity issuance cost, φ	0.30
Idiosyncratic volatility (a.r.), σ	0.10
Persistence of financial shock, ρ_φ	0.85

parameters in the model: parameters related to preferences and technology; parameters governing the strength of nominal rigidities and monetary policy; parameters determining the strength of financial market frictions, including portfolio rebalancing costs.

We set the time-discounting factor equal to 0.995. We set the deep habit parameter θ equal to -0.9 similar to the choice of Ravn, Schmitt-Grohé, and Uribe [2005]. The key tension between the market share maximization and cash flow maximization does not exist when $\theta = 0$. In this environment, the financial shock we consider has no effects on real outcome. It is in this sense that the current model owes a lot to customer market settings such as the "deep habit" model. We choose a fairly persistent habit formation such that only 10 percent of the habit stock is depreciated in a quarter. This highlights the firms' incentives to compete on market share. The CRRA parameter is then set equal to one given that the deep habit specification provides a strong motive to smooth consumption. We set the elasticity of labor supply equal to 5. For the aggregate technology process, we assume $\rho_A = 0.90$, a somewhat lower value than employed by real business cycle analysis, given that the model has a number of elements that generate persistent dynamics of the endogenous quantities.

The elasticity of substitution is a key parameter in the customer-markets model as the greater the market power the firm has, the greater the incentive to invest in customer capital. We set the elasticity equal to 2 to be consistent with Broda and Weinstein [2006], who provide a set of point estimates for the elasticity of substitution for the U.S. economy. The estimates hover around 2.1∼4.8, depending on the characteristics of products (commodities vs differentiated goods) and sub-samples (before 1990 vs after 1990). Our choice is virtually identical with the point estimate of Broda and Weinstein [2006] for the median value of the elasticity of differentiated goods of 2.1 for the differentiated products for a sub-sample period since 1990. Differentiated products are the relevant category for the deep habit model considered in this paper. The choice is also broadly consistent with Ravn, Schmitt-Grohe, Uribe, and Uuskula [2010]'s point estimate of 2.48 using their structural estimation method.

Regarding ω_h and ω_f, the weights of home and foreign goods in the utility function, we set these such that the share of imported goods in steady state consumption basket $(p_f c_f / \sum_{k=h,f} p_k c_k)$ is equal to 0.4. 0.4 is in the middle range of import/GDP ratios of European countries since 2000. For instance, Germany has 0.46 while Spain, Italy and Greece have 0.35 on average in 2012. Note that ω_f itself is not equal to the imported goods' share, ω_f is set equal to $(p_f c_f / \sum_{k=h,f} p_k c_k)^{1/\varepsilon}$. As for the Armington elasticity, we choose 1.5 to stay close to the near-unit elasticity estimated by Feenstra, Luck, Obstfeld, and Russ [2014]. However, as long as it is greater than 1, a value lower than our choice does not produce alter our conclusions. For instance, lowering this value to 1.01 reduces the impact of a financial shock on the aggregate output under currency union to 2/3 of the baseline calibration. This is because the lower elasticity of cross-border substitution implies a lower degree of price war among countries. However, even in this polar case, the qualitative features of the equilibrium remain the same.

Another important parameter is the fixed operating cost, ϕ. This parameter is jointly determined with the returns to scale parameter α. We set α first, then choose ϕ such that dividend payout ratio (relative to income) hits the post war mean value 2.5 percent in U.S. Decreasing returns to scale enhances the link between the financial market friction and the pricing decision. For this reason, we chose $\alpha = 0.8$ in our earlier work. However, in the current paper, we choose $\alpha = 1.0$ to be consistent with the convention in international macroeconomics literature. With the chosen α, ϕ and η, the average mark-up is determined as 1.19. We then experimented with various values for ϕ^* to explore the implication of heterogenous financial frictions for the member countries of the currency union. To emphasize this aspect, we set $\phi^* = 0$, that is, the foreign country in the model does not face any fixed operating costs. While extreme, setting $\phi^* = \varpi\phi$ with $\varpi \in [0, 1)$ does not modify the main results of the paper in any important way. To calibrate the financial friction, we set the dilution cost $\varphi = \varphi^* = 0.30$ as in Cooley and Quadrini [2001]. We discuss the influence of these choices on model outcomes below. The volatility of the idiosyncratic shock is calibrated to be 0.05 at a quarterly frequency, a moderate degree of idiosyncratic uncertainty.

For the parameters related to nominal rigidity, we set the adjustment costs of nominal price $\gamma_p = 10.0$. In our model presentation, we proceed as if nominal wages were flexible. In our actual simulation, we introduce nominal rigidity for wages along the line of Bordo, Erceg, and Evans [2000] and Erceg, Henderson, and Levin [2000]. In particular, in symmetry to the nominal price rigidity, we assume market power for households that supply labor to production firms, and a quadratic cost of adjusting nominal wage. In this case, and under the assumption of a separable, constant elasticity of labor supply, $U_{h,t} = -h_t^{1/\zeta}$, the efficiency condition (40) is modified into

$$\eta_w \frac{h_t^{1/\zeta}/U_{x,t}}{w_t/\tilde{p}_t} = \eta_w - 1 + \gamma_w(\pi_{w,t} - \pi_w)\pi_{w,t} - \beta\mathbb{E}_t\left[\frac{U_{x,t+1}/\tilde{p}_{t+1}}{U_{x,t}/\tilde{p}_t}\gamma_w(\pi_{w,t+1} - \pi_w)\pi_{w,t+1}\frac{\pi_{w,t+1}}{\pi_{t+1}}\frac{h_{t+1}}{h_t}\right] \tag{73}$$

where $\pi_{w,t} \equiv W_{t+1}/W_t$, γ_w is the coefficient of nominal wage adjustment cost, and η_w is the elasticity of substitution of labor. We choose $\eta_w = 3$ and $\gamma_w = 30$. Our choice of nominal rigidity for both price and wage are very close to the point estimates of $\gamma_p = 14.5$ and $\gamma_w = 41.0$ by Ravn, Schmitt-Grohe, Uribe, and Uuskula [2010], who show that the deep habit model substantially enhances the persistence of inflation dynamics without the help of implausibly large amount of adjustment friction in nominal prices. While nominal wage rigidity does not modify the main conclusions of the paper in any important way, it does help create a greater volatility of the real exchange rate. This is because the countercyclical markup of the country under a financial crisis, which is essential to the main conclusions of the paper, is achieved more by an immediate fall in nominal wage than by an increase in product price in a flexible wage environment. The relatively more stable final product prices then lead to a less volatile real exchange rate, which runs counter to intention, in addition to being unrealistic.

Finally, we set the inertial Taylor coefficient at a conventional level of 0.85 and the coefficient on the inflation gap as 1.5, following Taylor [1993]. The long run coefficient for the output gap is less obvious. In traditional New Keynesian literature, this coefficient does not play an important role due to so called "divine coincidence". As a consequence, a strong reaction to inflation often makes the response to the output gap redundant or even inefficient. However, this is not the case in our current paper. As shown by Gilchrist, Schoenle, Sim, and Zakrajsek [2013], the specific combination of customer market setting and financial friction breaks the divine coincidence in the sense that a strong negative demand shock under a severe financial strain may lead to higher inflation pressure as firms under financial distress may find it optimal to raise prices in order to secure short-term liquidity, giving up long-run market share. For this reason, we take a 50:50 prior by choosing a middle value between 0 and 0.5 suggested by Taylor [1993].

4.2 Currency Regime and Impacts of External Shocks

In this section, we study the macroeconomic consequences of adopting a common currency, and hence a single monetary policy – in an environment where member countries face heterogeneous degrees of financial frictions. We assume that member countries neither have a complete risk-sharing arrangement through cross-border transfers of funds nor cross-border labor mobility. We compare the international macroeconomic dynamics under two environments: a floating exchange rate regime and a currency union.

4.2.1 Impact of Financial Shocks

To study the effects of financial instability under various currency regimes, we impose a financial shock that elevates the cost of outside equity capital for firms in the member countries. More specifically, we subject the cost of issuing new equities to random shocks, and we call it the cost of capital shock:

$$\varphi_t = \bar{\varphi} f_t, \quad \log f_t = \rho_f \log f_{t-1} + \epsilon_{f,t}, \quad \epsilon_{f,t} \sim N(-0.5\sigma_f^2, \sigma_f^2) \tag{74}$$

To create a financial crisis situation hitting the financially vulnerable region, the "home" country, we calibrate $\epsilon_{t,f}$ such that the cost of capital is elevated to $2\bar{\varphi}$ on impact, and gradually comes down to the normal level $\bar{\varphi}$. In contrast, we shut down the shock to the cost of capital for the financially strong, "foreign" country, that is, $\varphi_t^* = \bar{\varphi}$ for all t.[11] The shock is designed to elevate the expected shadow value of internal funds for the firms in the home country. Without the shock, the expected shadow value of internal funds in the model is calibrated to 1.16. The shock immediately increases the expected shadow value to 1.50.

As we mentioned earlier, when the shadow cost of capital is elevated, firms in the model rebalance the benefit of investing into their customer base by lowering prices and the benefit of increasing current cashflow by temporarily raising prices. The former strategy is profitable in the long run, but in the short run, exposes the firms to liquidity risk.

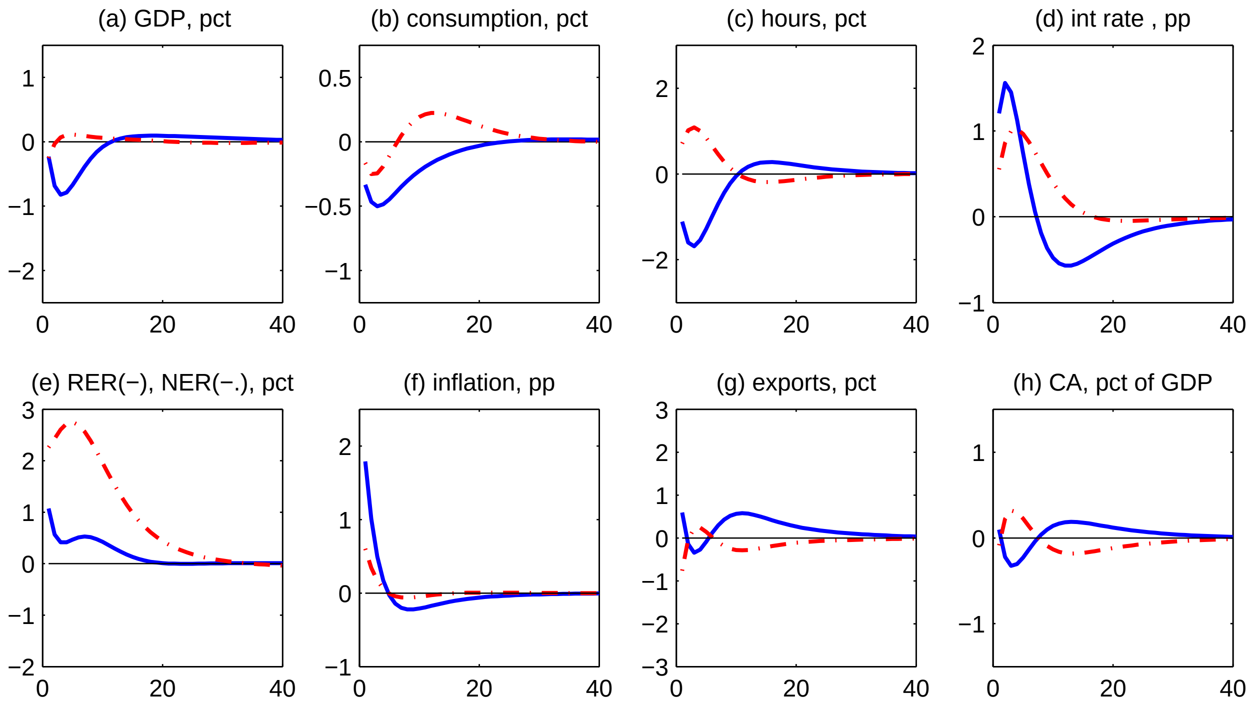
Figure 5 displays the impact of a financial shock given these incentives and under a floating exchange rate regime. Blue, solid line shows the reaction of the home country and red, dash-dotted line the reaction of foreign country. Panel (f) of Figure 5 shows that the firms in the two countries indeed increase their prices in response to the financial shock. These results are consistent with the pattern we demonstrated for a closed-economy setting in our earlier paper (Gilchrist, Schoenle, Sim, and Zakrajsek [2013]). Since the shock here affects the home country directly and only indirectly the foreign country, the response of inflation is disproportionately larger for the home country. In our empirical section, we provide some evidence for exactly this pattern in the data in the context of the European financial crisis during the period of 2008-2012.

What is the reaction of the nominal and real exchange rate? As shown in Panel (3) of Figure 3, the nominal exchange rate, shown by red, dash-dotted line, strongly depreciates. In fact, the depreciation is strong enough that the real exchange rate also depreciates despite the price differential moving in the opposite direction. As in the data, the real exchange rate dynamics in the short run are dominated by the nominal exchange rate rather than price changes. Note that in panel (e), the deviation of nominal exchange rate looks like it disappears in the long run. However, this is simply a coincidence. The New Keynesian framework adopted in the current paper does not have a prediction for the level of nominal exchange rate just as it does not have one for the price level.[12] There is no reason why the levels of the price index and nominal exchange rate should converge to

[11] However, in evaluating the consequences of different currency regimes, we assume that the foreign country is also subject to its own financial shocks given by $\varphi_t^* = \bar{\varphi} f_t^*, \quad \log f_t^* = \rho_f \log f_{t-1}^* + \epsilon_{f,t}^*, \quad \epsilon_{f,t}^* \sim N(-0.5\sigma_f^2, \sigma_f^2)$.

[12] The Figure assumes that the initial value of nominal exchange rate is one, which is an arbitrary, but innocuous assumption. However, the rate of change of the nominal exchange rate is well defined and is one of the model variables.

Figure 5: Financial Shock to Peripheral Country Under Floating



Note: The blue, solid line depicts the peripheral country and red, dash-dotted line the core country. The shock assumes that the dilution cost for the peripheral countries go up by 100% from its normal level.

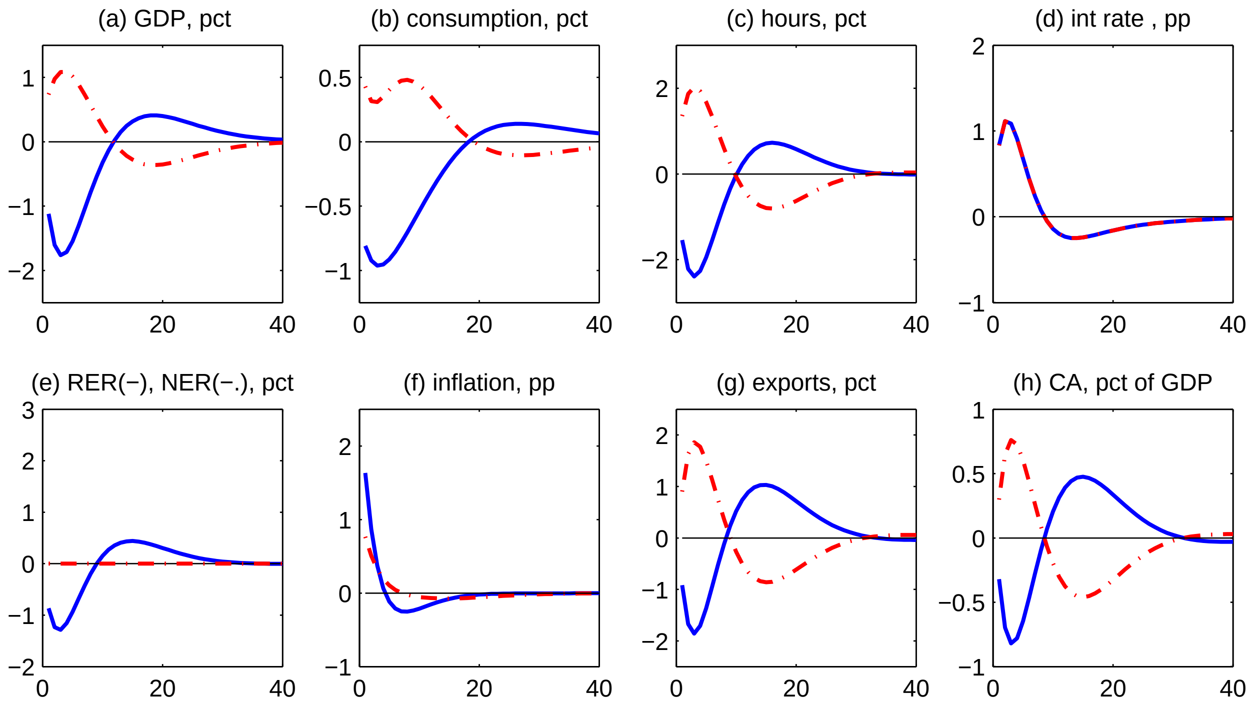
specific levels.

The short-run depreciation of the nominal exchange rate explains why GDP of the home country is affected only mildly despite the large size of the financial shock.[13] Panel (a) shows that GDP of the home country drops about one percent in the trough. The drop in aggregate consumption is even milder: only about a half percent. This is owing to relatively strong, initial gains in exports, shown in panel (g), which is driven by the depreciation of the real exchange rate. While short lived, the depreciation of the nominal exchange rate helps firms avoid having to increase the relative prices of their exports too much to stabilize their cash-flow because the nominal depreciation partly does the job partially. This keeps them from losing too much market shares too much and thus limits the downside risk to the economy.

Under a currency union, adjustment looks dramatically different: Figure 6 shows the fundamentally different pattern of international macroeconomic adjustment ensuing the financial shock. From Panel (a) through (c), one can see that the drop in GDP, consumption and hours of the home country are almost two times greater under the currency union than under the floating exchange rate regime. In panel (g), the trough of exports and the current account deficit (relative to GDP) of the home country are almost 7 times and three times greater under the currency union. More strikingly, the financial crisis of home country is coupled with a modest boom in the foreign country: GDP, consumption and hours go up by 1, 0.5 and 2 percent each. The export and current account

[13]GDP in the model is defined as domestic consumption plus export minus import, that is, $p_t c_t + q_t p^*_{h,t} c^*_{h,t} - p_{f,t} c_{f,t}$. This is not equal to the volume index of aggregate output, y_t.

Figure 6: Financial Shock to Peripheral Country Under Monetary Union



Note: Blue, solid line is the peripheral country and red, dash-dotted line is the core country. The shock assumes that the dilution cost for the peripheral countries go up by 100% from its normal level.

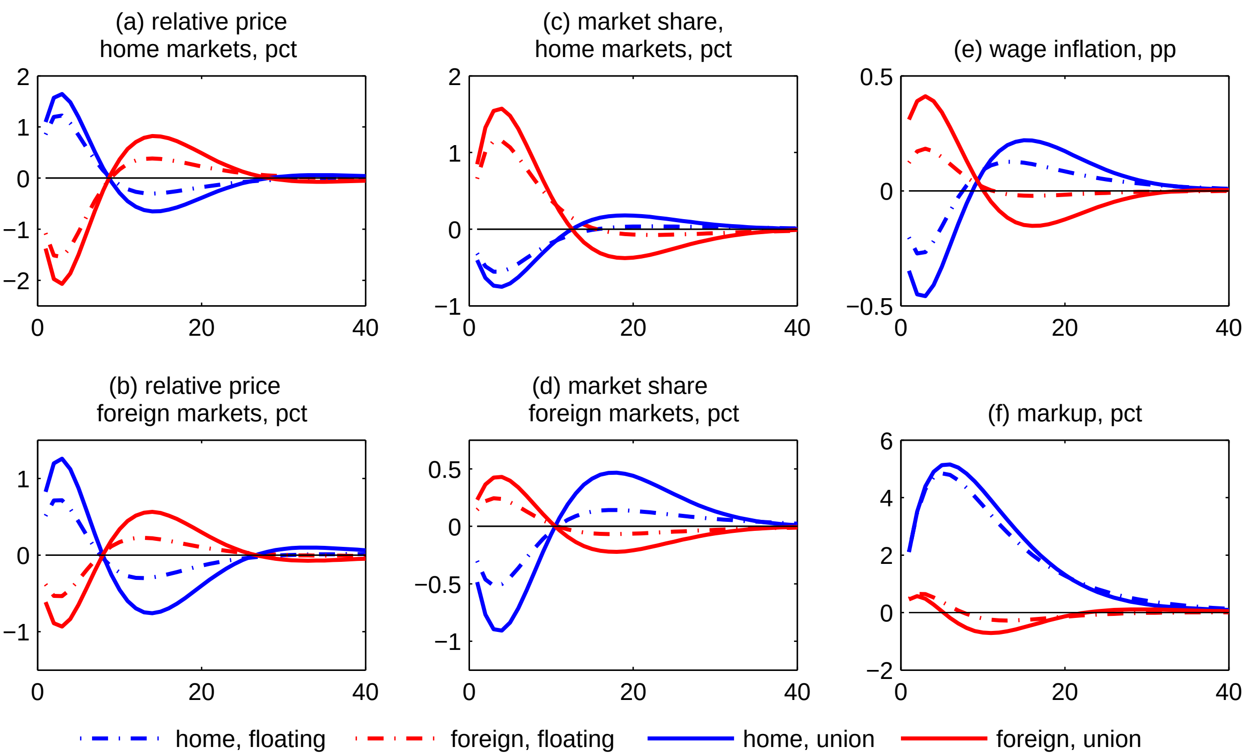
surplus (relative to GDP) of the foreign country to the home country go up by 2 and 3/4 percent, respectively.

The mechanism behind the stark difference in international macroeconomic adjustment patterns can be seen from Panels (d) through (f) in Figure 6. Owing to the financial friction, the firms in the country hit by the financial shock have a greater incentive to raise their prices more than their foreign counterparts. This pattern in the inflation differential following the financial shock is shown in Panel (f) of Figure 5 and 6, and similar across the two currency regimes. However, what is different under the currency union are the real exchange rate dynamics. Under a floating exchange rate regime, the international bond holding conditions (55) and (56) imply the following no-arbitrage condition:

$$\tau(b_{h,t+1} - b_{f,t+1}) = \mathbb{E}_t \left[m_{t,t+1} \left(\frac{R_t}{\pi_{t+1}} - \frac{q_{t+1}}{q_t} \frac{R_t^*}{\pi_{t+1}^*} \right) \right]. \tag{75}$$

In equilibrium with a relatively small cost of portfolio rebalancing, the left side is close to zero up to a first-order approximation. This means $R_t/\pi_{t+1} - (q_{t+1}/q_t)(R_t^*/\pi_{t+1}^*) \approx 0$ in expectation. As shown in panel (d) and (f) of Figure (5). The nominal interest differential between the home and the foreign country is smaller than inflation differential, that is, the real interest rate is lower in the home country than in the foreign country. This is intuitive given the recession in home country. The *absence of capital control* then implies that the real exchange rate should appreciate over time

Figure 7: Price Wars and Market Shares During A Financial Crisis



Note: Solid lines depict the cases of the currency union with blue and red indicating the periphery and the core, respectively. Dash-dotted lines depict the case of the floating exchange rate regime with the same color convention.

$(q_{t+1}/q_t < 1)$ to avoid the exodus of capital from the home to the foreign country. This requires that the nominal exchange rate should *depreciate today* such that $q_{t+1}/q_t < 1$. This restriction from the free capital account is exactly what is missing in the currency union. The bond market efficiency conditions (69) and (72) impose no restriction on the dynamics of the real exchange rate. While the real interest rate differential still exists as in the case of the floating exchange regime, the differential does not have to be compensated by expected changes in nominal exchange rate: one can never exit from one currency to the same currency. (69) and (72) jointly require[14]

$$1 = \mathbb{E}_t \left[\frac{1}{2} \left(\frac{m_{t,t+1}}{\pi_{t+1}} + \frac{m^*_{t,t+1}}{\pi^*_{t+1}} \right) R^U_t \right], \tag{76}$$

This implies that the single policy rate R^U_t should be set according to the *average* fundamental of the two economies regardless of the coefficients of the monetary policy reaction function (64).

As a consequence, any differential in inflation rates is directly translated into a movement of the real exchange rate. Since the model's financial frictions imply that the financially more vulnerable firms optimally choose higher relative prices, the real exchange rate appreciates substantially. This causes the export of the home country to contract severely, and so does GDP. Consumption does

[14]One can derive this condition by simply adding the efficiency conditions and imposing the bond market clearing condition.

not contract as much as GDP since international borrowing, while subject to the costly rebalancing friction, allows consumers in the home country to smooth out the effects of the financial shock to a certain degree. The boom in economic activity of the foreign country is simply a mirror image of home country's economic plight. The contrast between the two countries is quite reminiscent of the European dichotomy between central and peripheral countries during the recent financial crisis.

One perhaps surprising result is that despite the harsher financial environment under the currency union, neither the level of the home country inflation nor the difference from foreign country are very different from the floating exchange rate regime. This is because international price wars are creating offsetting dynamics for overall inflation rates. Figure 7 provides a more detailed account of these international price wars. The solid lines, both blue and red, show the case of the currency union with blue and red lines representing home and foreign countries in the crisis. The dash-dotted lines show the case of the floating exchange rate regime with blue and red lines representing home and foreign countries.

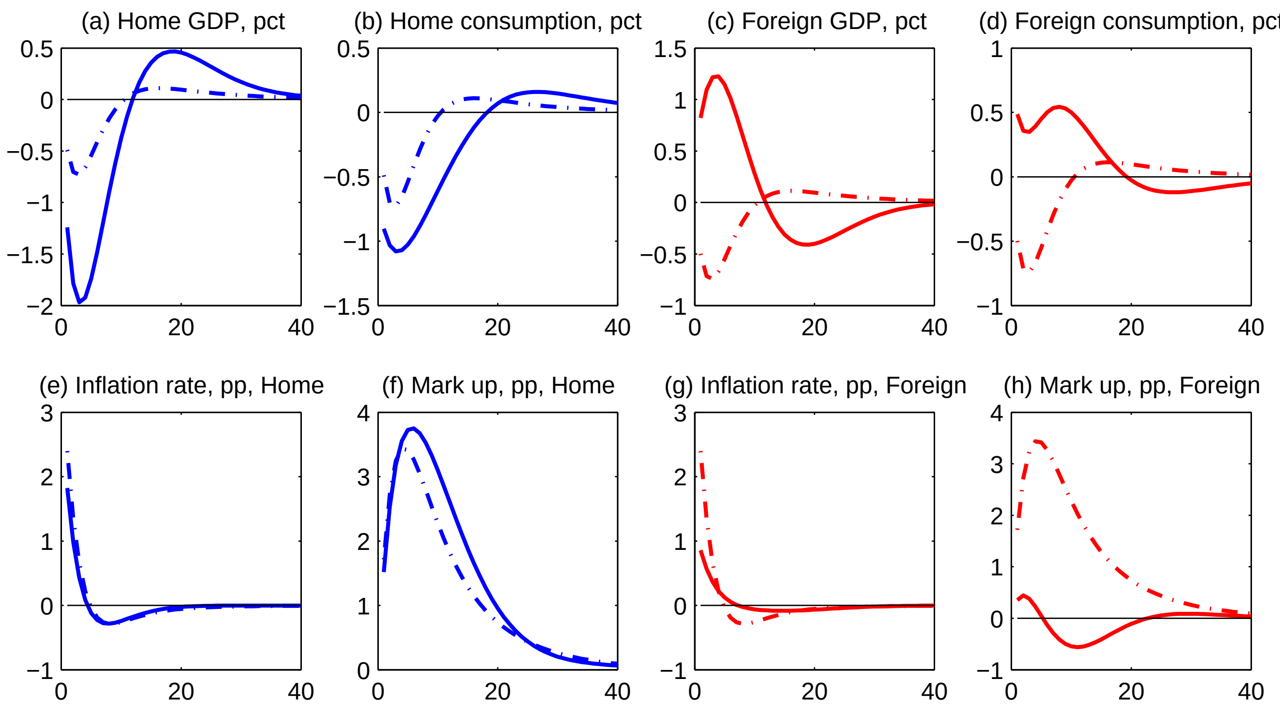
Panel (a) and (b) show the endogenous dispersion of relative prices. Regardless of the exchange rate regime, firms in the country hit by the financial shock increase their relative prices both in their domestic (panel (a)) and export markets (panel (b)). In contrast, the firms in the financially strong country substantially lower their relative prices. Firms in the home country raise their relative prices to secure short-term liquidity, sacrificing market share, both in home and foreign markets (panel (c) and (d)). The firms in the foreign country follow the opposite strategy, and lower their relative prices to gain market share. Interestingly, these firms slash their prices more in the home country (export prices) than in foreign country (domestic prices). Note that the intensity of price war, in terms of the dispersion of prices, is much greater under the currency union. This is because the firms in the home country can no longer rely on the depreciation of their currency to improve their cashflow. Since the higher prices of the home firms (domestic prices in the home country) are offset by the lower prices of the foreign firms (import prices in the home country), the overall inflation rate of the home country is not greatly affected by the currency regime. This explains the seemingly surprising result. Finally, note that the markup is strongly countercyclical in the home country regardless of currency regime during the financial crisis (see panel (e) and (f)).[15]

4.2.2 Financial Heterogeneity and Predatory Price War

The financial heterogeneity plays an essential role in propagating the impact of the financial shock under the currency union because it is the financial strength of the foreign firms that allows them to engage in predatory price wars in order to drive out the competitors in both countries. If all countries are identical–in terms of technology, preferences, financial friction and composition of shocks hitting the economy, one may not generate the same degree of endogenous propagation. To show this, we consider an alternative calibration, in which all countries face the same magnitude

[15]We have not shown the effects of technology shock for the sake of space. However, the appendix shows that the same conclusion is reached: the volatilities of macroeconomic variables under an aggregate technology shock are endogenously increased by the currency union. See figure 15 and 16.

Figure 8: Heterogeneity as a Propagation Channel



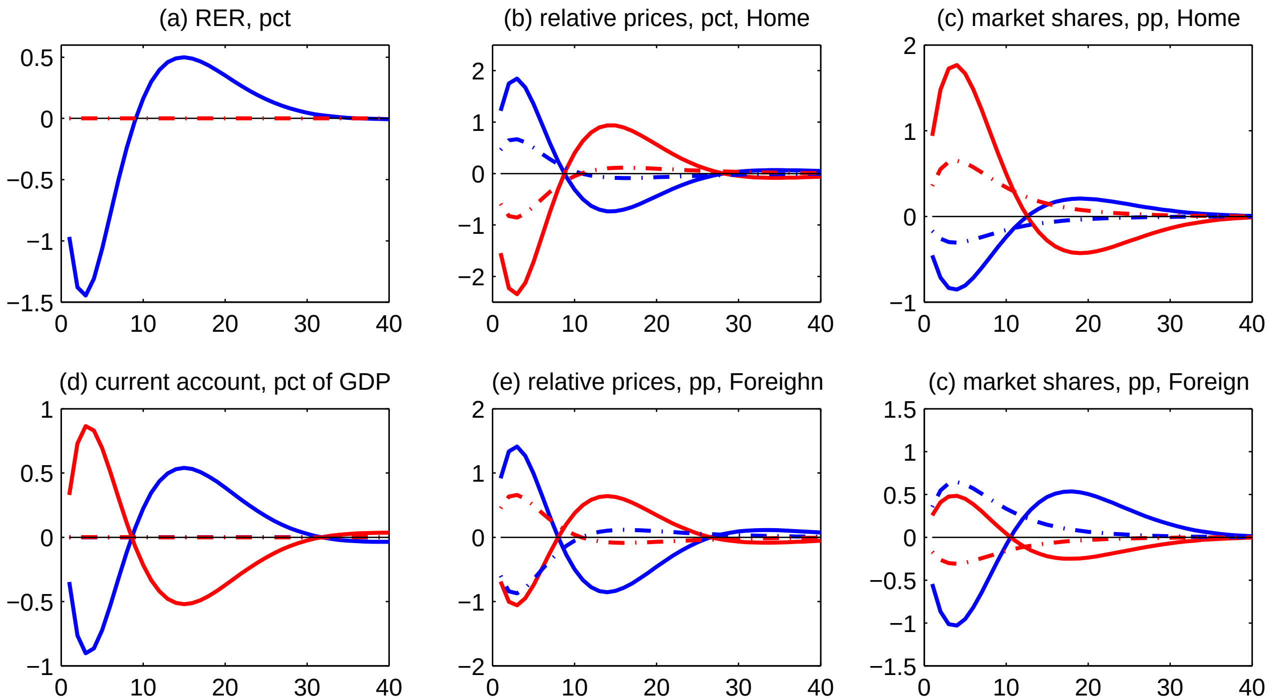
Note: Solid lines describe the baseline case with asymmetric financial condition and shocks and dash-dotted lines show the alternative case with symmetric financial condition and shocks.

of the fixed operation costs and are subject to the same financial shock: $\phi = \phi^* = 0.08$ and $\epsilon_{f,t} = \epsilon^*_{f,t} > 0$.

Figure 8 compares the two cases: the baseline calibration with asymmetric financial friction and financial shocks ($\phi = 0.08$, $\phi^* = 0.00$, $\epsilon_{f,t} > 0$, $\epsilon^*_{f,t} = 0$) and the alternative calibration with symmetric financial friction and financial shocks ($\phi = \phi^* = 0.08$ and $\epsilon_{f,t} = \epsilon^*_{f,t} > 0$). The results are striking in that the home country would prefer the alternative environment in which the foreign country were also subject to the same degree of financial market friction and underwent the same degree of financial shock. The dash-dotted lines of the figure suggest that the degree of the recession is much smaller for the home country in the alternative environment. Since the foreign country now faces the same degree of liquidity crisis, the foreign firms cannot engage themselves in the predatory price war seen in the baseline case.

The foreign country now undergoes the same recession. This is in stark contrast to the baseline case where the foreign country experiences an export-driven boom. In response to its own liquidity shock, the foreign firms also raise their markup endogenously to secure cash flow and inflation rate follows the same route as the home counterpart (see panel (e)∼(f)). Figure 9 shows that with the nominal exchange rate held constant and the price indices of the two countries following the same path, the real exchange rate does not play any role in the macroeconomic dynamics, which is in stark contrast to the baseline case. Consequently, the burden of recession is evenly shared by the two countries and both countries lose equal magnitudes of market shares in their own territories

Figure 9: Financial Heterogeneity, Relative Prices and Market Shares Dynamics



Note: Solid lines describe the baseline case with asymmetric financial condition and shocks and dash-dotted lines show the alternative case with symmetric financial condition and shocks. Blue color is for the home and and red color is for the foreign country.

and gain the same magnitudes of market shares in each other's economy.

4.2.3 Is There Supporting Evidence?

Figure 7 suggests that the market shares of the foreign country expand in both home and foreign countries as a consequence of the financial crisis that the home country is undergoing because the foreign firms with ample liquid resources try to use the financial crisis of the home country to win market share competitions. Is there supporting evidence in the data?

Ideally, it is ideal to look at micro-level data in each member country, and this is what we have done for in our earlier work by matching COMPUSTAT firms and PPI correspondents (Gilchrist, Schoenle, Sim, and Zakrajsek [2013]). However, such good quality data are not available for the Euro-zone economy. Nevertheless, rough and crude measures of aggregate market shares can be constructed based on macroeconomic data. To that end, in figure 10, we compare the nominal value of country A's export to country B with the nominal GDP of country B. The idea is to think of the nominal GDP as the size of an economy and the relative export ratio as rough indicator of shares in relevant markets. In doing so, we delete energy, commodities and agricultural products, which do not fit the notion of customer markets. We normalize the ratios equal to one in 2010Q1, which we take as the onset of the European Balance-of-Payment crisis. Roughly around this period, Greece started finding it hard to finance its current account deficits from private sectors

Figure 10: Euro-zone Market Share Dynamics

Note: Blue lines show the ratios of nominal values of export from Portugal, Italy, Greece and Spain to Germany relative to Germany's nominal GDP. Red lines show the ratios of nominal values of German exports to these countries relative to these countries' nominal GDPs. Export exclude energy, commodities and aggricultural products. The ratios are normalized to one in 2010Q1.

and the CDS spreads on Greek bonds started growing up to unsustainable levels.

Panel (a)∼(d) show the bilateral market shares of Portugal, Italy, Greece and Spain vis a vis Germany. Blue lines show the market shares of these countries in Germany and red lines the German market shares in these countries. Pictures show a remarkably similar pattern: since the beginning of the Balance-of-Payment crisis, German market shares have continued to grow in these countries; in contrast, the market shares of these countries in Germany have continued to shrink since the eruption of the crisis. The sizes of divergences and time series patterns are remarkably similar for all countries, and the time series patterns are consistent with what was shown for the model economy.

4.3 Welfare Consequence of Currency Union

Table 6 summarizes the welfare consequences of adopting a policy union when member countries face heterogeneous financial market friction. To evaluate the effect on welfare, we adopt the following, simple and stylized calibration strategy: we assume that the two countries are subject to aggregate technology shocks ($\epsilon_{A,t}$ and $\epsilon^*_{A,t}$) and financial shocks ($\epsilon_{f,t}$ and $\epsilon^*_{f,t}$) only; we calibrate the standard deviation of aggregate technology shocks as 1 percent each, and set the standard deviations of financial shocks such that they account for 50 percent of variance decomposition of home country output.[16] To evaluate the welfare, we define the value functions of the representative agents of the two countries as

$$W(\mathbf{s}) = U(x,h) + \beta\mathbb{E}[W(\mathbf{s}')|\mathbf{s}]$$
$$\text{and } W^*(\mathbf{s}) = U(x^*,h^*) + \beta\mathbb{E}[W^*(\mathbf{s}')|\mathbf{s}],$$

and approximate them using a second order approximation and report the analytical first moment in table 6.

The first and second rows of table 6 show that the welfare levels of both home and foreign countries deteriorate by adopting a common currency. To put this result in perspective, we also report the consumption equivalent in the third column of the table, which is formally defined as the required increase in average consumption per period to make the agent living in an economy with the common currency indifferent with transitioning to an economy with the floating exchange rate. While the sign of the certainty equivalent change in consumption is intuitive, the degree of welfare deterioration caused by adopting a single currency appears to be small at least in terms of the certainty equivalent changes in consumption.

However, as is standard of welfare cost analysis of business cycle, this may be misleading because the representative agent is clearly a theoretical artifact we use for the sake of conveniency, and the aggregate uncertainty clearly and substantially understates the uncertainty facing individuals without perfect insurance. In this regard, somewhat more useful statistics can be found in standard deviations of output and consumption, which are reported in table 7.[17] The results shown in the table are striking: by agreeing to abolish the currency union and return to the floating exchange rate, home and foreign can reduce the output volatility as much as 28 and 42 percent, respectively.

[16] Obviously, more elaborate strategies can be adopted to provide more realistic representation of the macroeconomy. However, our main conclusions hold true even when a radically different strategy of calibrating the structure of shocks is employed as we have shown that the main problem associated with the currency union stays the same when the driving force of business cycle is entirely dominated by the aggregate technology shocks.

[17] Since about a half of output variation is due to the financial shock, one can easily map the changes in standard deviation of output into variation in unemployment rate using a variant of so called *Okun's law*.

Table 6: Welfare Consequence of Currency Union

	Welfare		Consumption Equivalent
	Currency Union (A)	Floating Ex. Rate (B)	Percent
Home country	-274.86	-274.37	0.22
Foreign country	-217.86	-217.37	0.38
Joint welfare	-492.82	-491.48	-

Note: The consumption equivalent is the required minimum increase in average consumption per period holding labor hours constant to make the representative agent living in the economy under the floating exchange rate regime no worse off by transitioning to the currency union.

Table 7: Output and Consumption Volatility Under Alternative Environment

	Output (GDP) volatility			Consumption volatility		
	Union (A)	Floating (B)	B/A	Union (A)	Floating (B)	B/A
Home country	.0151	.0108	.72	.0219	.0099	.45
Foreign country	.0149	.0087	.58	.0204	.0093	.46

Note: The consumption equivalent is the required minimum increase in average consumption per period holding labor hours constant to make the representative agent living in the economy under the floating exchange rate regime no worse off by transitioning to the currency union.

Even more strikingly, such a transition would reduce the consumption volatility of both countries more than 50 percent in the baseline calibration.

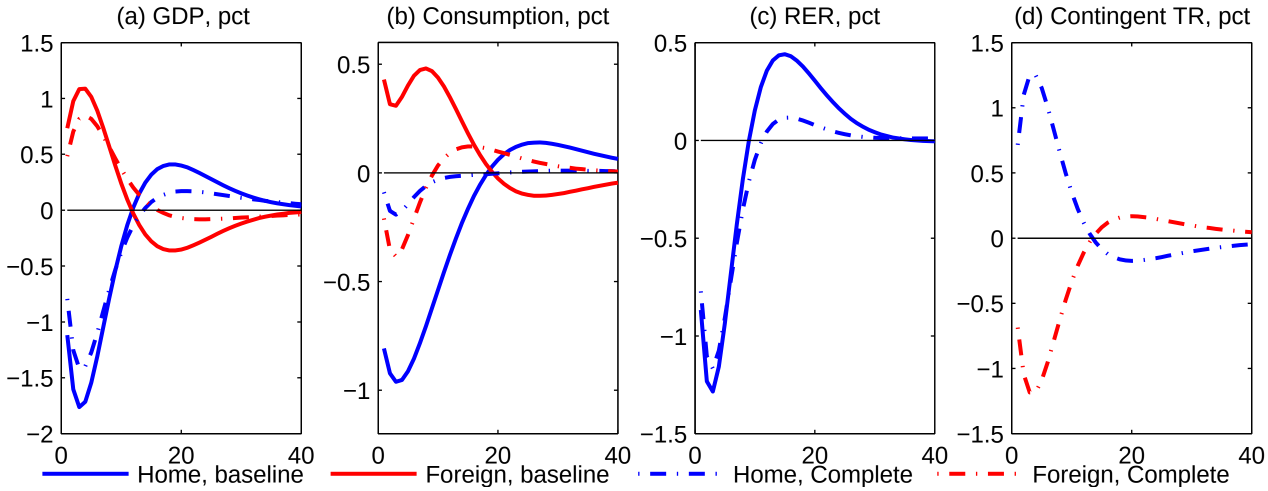
5 Two Fiscal Policy Arrangements

5.0.1 Complete Risk Sharing: Fiscal Union

Assuming that a return to the floating exchange rate is either infeasible or undesirable for whatever non-economic reasons, what could remedy the distortions without breaking the single currency regime? To study this issue, we start by analyzing the nature of real allocation under a currency union with the two countries trading a complete set of state contingent bonds. This provides a natural benchmark against which the efficiency of other policy proposals can be gauged. For the sake of space, we mainly focus on the four aspects of the economy in this section: dynamics of GDP, consumption, real exchange and state-contingent cross-border transfer.

In the international macroeconomics literature, researchers often make the assumption of complete risk sharing. This is because it is often the case that the presence and absence of such risk sharing arrangement do not make substantial differences in the dynamics of endogenous quantities, including real exchange rate (see Steinsson [2008], for example). However, as shown by Figure 11, this is not the case in the current environment. In the Figure, solid lines represent the case of the baseline, that is, currency union without the complete risk sharing arrangement, with blue and

Figure 11: Financial shock and Currency Union with Complete Risk Sharing



Note: Solid lines are the baseline case of the currency union with incomplete risk sharing arrangement with blue and red indicating the periphery and the core countries, respectively. Dashed-dotted lines depict the case of the currency union with complete risk sharing arrangement with the the same color convention.

red being home and foreign cases. Dash-dotted lines show the case of the currency union with the complete risk sharing with the same color convention.

A few conclusions can be easily made from the Figure. First, the state contingent bond trading evenly spreads out the cost of financial shock to the two countries, as shown by panel (b) of the Figure. In terms of the vertical distance from the baseline consumption paths, the two countries undergo the same degree of improvement/sacrifice depending on which country is hit by the shock. Second, the complete risk sharing arrangement works mainly through cross-border wealth transfer, rather than the changes in production share. Panel (a) shows that the production scales of the two allocations are not very different from each other, although the complete risk sharing does reduce the volatility of GDP for both countries. This is owing to the relative inefficiency of home country. To the contrary, the complete risk sharing arrangement organize the production such that the marginal costs are equalized across the countries and then redistribute the proceeds to achieve the risk sharing. Third, as shown by panel (d), this requires a substantial amount of international transfer of wealth, which, in this particular example, amounts to 1.2 percent of GDP. Finally, the complete risk sharing arrangement does not abolish the fluctuations in the real exchange rate, although the volatility is somewhat subdued in this environment. This is because, despite the cross-border transfer of wealth, the ratio of marginal utilities of consumption (more accurately, marginal utilities of consumption habit aggregator, x) declines.

Table 8 compares the welfare measures under the currency union with and without the fiscal union. A policy dilemma of risk sharing arrangement through the fiscal union between the two countries can be easily seen. While it substantially improves the welfare for the home country, it involves nontrivial, cross-border transfer of wealth. As a consequence, the welfare of foreign country deteriorates dramatically. The improvement in the joint welfare, shown in the last low of the table, implies that the gains for home dominates the loss for foreign country. The last column

Table 8: Effects on Welfare of Alternative Environments

	Welfare		Consumption Equiv
	MU (A)	Risk Sharing (B)	Percent
Home country	−274.86	−253.21	10.28
Foreign country	−217.86	−236.96	−9.13
Joint welfare	−492.82	−490.17	–

Note: The consumption equivalent is the required minimum increase in average consumption per period holding labor hours constant to make the representative agent living in the economy under the floating exchange rate regime no worse off by transitioning to the currency union.

shows the certainty equivalent changes in consumptions relative to the baseline. Strikingly enough, the complete risk sharing arrangement increases the steady state consumption level 10 percent for home country, but decreases 9 percent for foreign country. The reason why the currency union cannot become a true union is that there simply is no reason for the residents of foreign country to agree with such transfers.

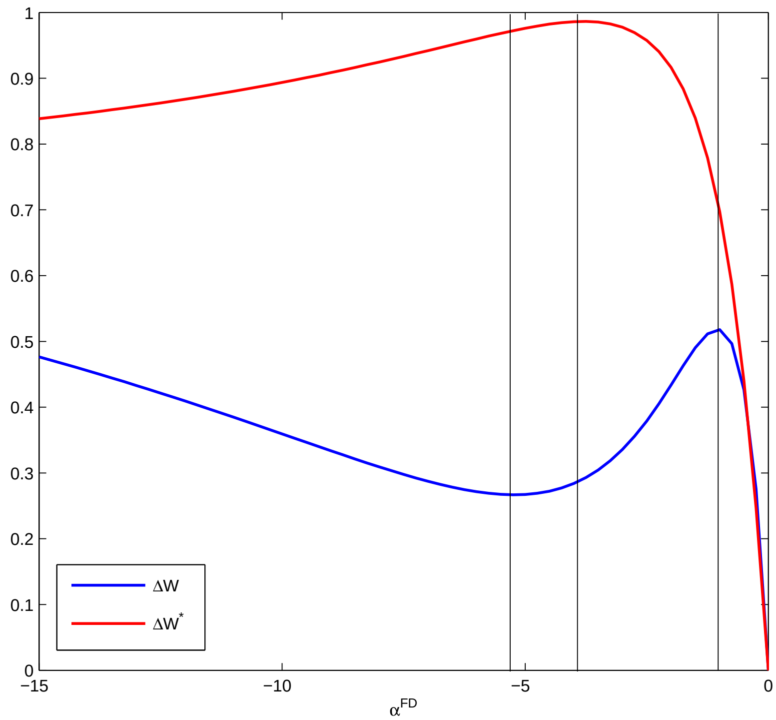
5.1 Fiscal Devaluations

In this section, we consider the effects of fiscal devaluation policy. The idea of fiscal devaluation policy is to replicate the effects of nominal devaluation by a mix of fiscal instruments (see Adao, Correia, and Teles [2009]). For instance, combining import tariff and export subsidy can replicate the effects of nominal devaluation on the competitiveness and terms of trade of a country implementing such a policy mix. Another example can be found in the mix of imposing value added tax (VAT) on all goods domestically sold and providing payroll subsidy to domestic firms. Both VAT and payroll subsidy are discriminatory fiscal tools because the exporters get reimbursement for VAT and foreign exporters are not eligible for payroll subsidy. Farhi, Gopinath, and Itskhoki [2014] provide an in-depth analysis on a wide range of policy mixes that replicate the effects of a given degree of nominal devaluation under various asset market conditions.

There are at least two desirable aspects of fiscal devaluation policies. First, such policies can be implemented unilaterally, and therefore can be tailored for the economic condition of the country implementing the policy. Second, fiscal evaluation policies can be designed is a way that the policies are revenue neutral. For instance import tariff can fund export subsidy and VAT can finance payroll subsidy. This second aspect makes fiscal devaluations particularly attractive for the periphery, which is undergoing fiscal crises.

However, despite the desirability of such policy options, the literature has yet to provide an extensive analysis on the impact of unilateral fiscal devaluations on the trading partners. In particular, if such policies achieve the effects of nominal devaluation by altering international relative prices and terms of trade, they may not be neutral to the trading partners in their effects. It then is unclear if such a unilateral policy move by the periphery will be welcomed by the core, especially

Figure 12: Welfare Differentials from Baseline without Fiscal Devaluation



if the core agreed to form a currency union because it wanted to avoid the manipulation of nominal exchanges rates by the central banks of the periphery. Can the peripheral countries carry out fiscal devaluations without the fear of retaliatory policy feedback from the core?

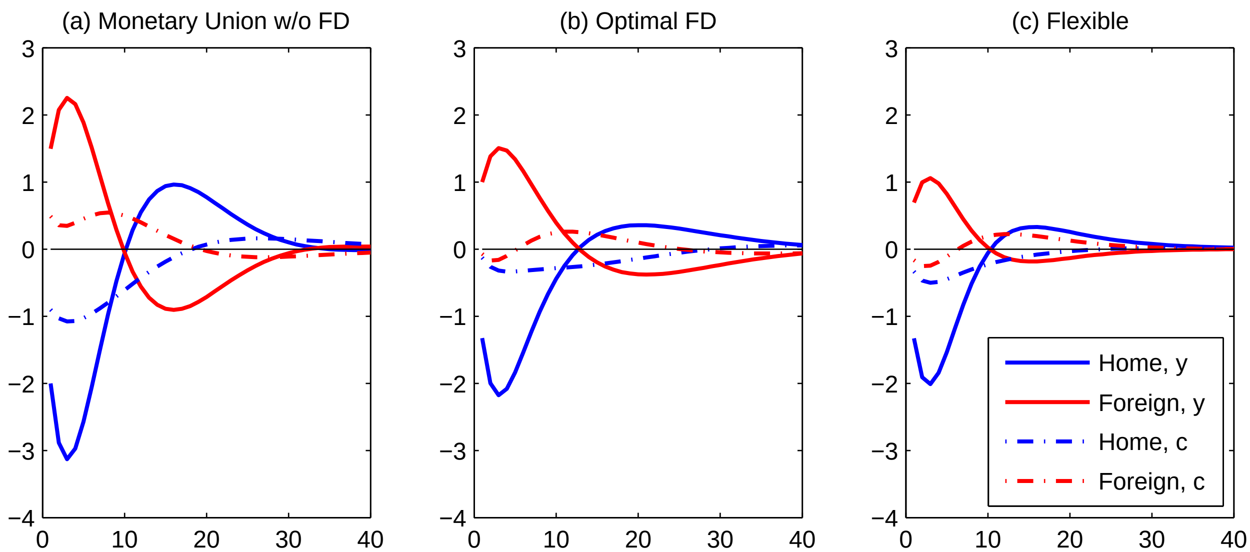
Since aforementioned literature has established equivalency of various policy mixes that generate the same effects of nominal devaluation, we focus on a simple mix of VAT (τ_t^v) and payroll (ς_t^p) subsidy carried out by the home country government. Under these policy, the marginal revenue of a home firm for selling its product in the home country is modified into $(1-\tau_t^\nu)p_{iht}p_{ht}$ where as its marginal cost of hiring becomes $(1-\varsigma_t^p)w_t$. It is assumed that the foreign country does not retaliate, and home firms are not subject to the same VAT in the foreign country. We assume that home government operates the fiscal policies as stabilization tools and follow linear policy rules:

$$\tau_t^v = \varsigma_t^p = \frac{\delta_t}{1+\delta_t}$$
$$\delta_t = \alpha^{FD} \times \log\left(\frac{y_t}{\bar{y}}\right),$$

in which the magnitude of fiscal devaluation δ_t is linearly depends on output gap of the home country. To implement a countercyclical policy, α^{FD} should be nonpositive. We then perform an extensive grid search to calibrate α^{FD} that maximize the second-order approximate welfare of the home country and see what impact such a policy rule brings to the welfare of the foreign country.

Figure 12 shows our surprising finding: The welfare of the home country is maximized around

Figure 13: Monetary Union w/ and w/o optimal FD vs Floating



Note: Solid lines are aggregate output and dash-dotted lines are consumption. Blue color is used for home country and red for foreign country.

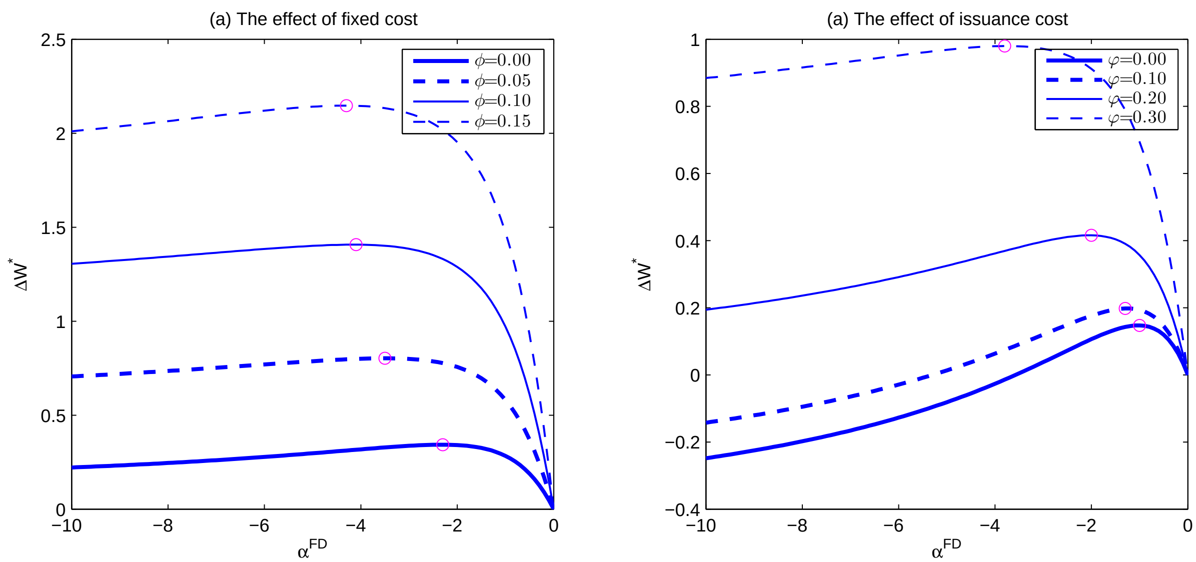
$\alpha^{FD} = -1.0$, at which point the welfare is improved not only for home but also for the foreign country. In fact, the welfare maximizing point is in the welfare increasing region of the foreign country, which means that the foreign country has an incentive to provide a fiscal subsidy to make the home country adopt even stronger fiscal devaluation.

Figure 13 compares three allocations when the financial shock hits the home country: (a) under the currency union without fiscal devaluation; (b) under the currency union with fiscal devaluation optimized for home country welfare; (c) under the flexible exchange rate. Note that the magnitudes of the business cycles ensuing the financial are greatly reduced for both countries under the unilateral fiscal devaluation policy. The stabilization of consumption volatilities is remarkable again for both countries. The quality of resource allocation almost resembles the case of the flexible exchange rate.

What can explain the positive gains for the foreign country? To understand this, it is important to realize that there exists an important pecuniary externality working behind the monopolistic competition when pricing firms are facing financial friction. When the foreign firms take the predatory strategy of slashing the product prices in the middle of their trading partner's financial crisis, they take the general price levels as given, and as a result, they do not internalize the impact of their pricing strategy on the real exchange rate. This makes the foreign firms lower their prices to excessively low levels. Of course, their pricing strategies are individually rational, but they are not collectively in that they do not incorporate the impact on aggregate demand when driving out competitors. In general, to make private agents internalize externality, a distortionary taxation is required, and fiscal devaluation provides such a mechanism.

If our explanation is correct, one should expect that the potential welfare gains for the foreign country from the unilateral fiscal devaluation by the home country would increase as the degree of

Figure 14: Financial Friction and Benefit of Fiscal Devaluation to the Core



financial friction facing the home country, and therefore the strength of the pecuniary externality go up. Figure 14 shows this is indeed the case. In this figure, we show this in two different ways. In the left panel, holding the other parameters constant, we increase the size of the fixed operating costs for the home country. A greater fixed operating cost implies that the home firms have a greater probability of facing liquidity problem, which increases the effective shadow value of internal funds. In the right panel, we hold the size of the fixed operation cost. Instead, we increase the normal level of equity dilution cost $\bar{\varphi}$. Since the shock is given by $\varphi_t = \bar{\varphi} f_t$, this effectively increases the shock variance.

We then check what happens to the welfare differential to the foreign country as the home country policy coefficient for fiscal devaluation increases in absolute value. In the figure, red circles show the location of the policy coefficient that maximizes the welfare differential for the foreign country from the baseline with no fiscal devaluation for the home country. One can see that the welfare differential line generally moves up as the degree of financial market friction goes up and the welfare differential maximization for the foreign country generally calls for a greater policy reaction. This confirms our argument that if there is pecuniary externality working behind predatory price war and left unexploited by the Euro-zone governments, even the core will gain from the fiscal devaluation by the periphery.

6 Conclusion

We have analyzed the business cycle and welfare consequences of forming a currency union among countries facing heterogeneous financial market friction. We have shown that behind the sluggish adjustment of overvalued real exchange rates of the peripheral countries, there exist a mechanism that leads the firms in the core to lower their product prices to gain market shares and force the firms in the periphery to raise their prices to secure current cash flow to cope with liquidity problem. We have also shown that the common monetary policy may not be an effective tool to fight the current financial crisis and unilateral fiscal devaluation policy by the peripheral countries may be beneficial not only for themselves but for the core countries.

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Appendices

A Product Demand and Identities

A.1 Derivation of Product Demands

In the symmetric equilibrium, all households choose the same levels of consumptions. Henceforth, we omit the household superscript. The cost minimization problem is then given by

$$\mathcal{L}_c = \sum_{k=h,f} \int_{N_k} P_{i,k,t} c_{i,k,t} di - \lambda_{c,t} \left[\left\{ \sum_{k=h,f} \omega_k \left[\int_{N_k} (c_{i,k,t}/s^{\theta}_{i,k,t-1})^{1-1/\eta} dk \right]^{\frac{1-1/\varepsilon}{1-1/\eta}} \right\}^{1/(1-1/\varepsilon)} - x_t \right]$$

The efficiency condition for $c_{i,h,t}$ is given by

$$P_{i,h,t} = \omega_h \lambda_{c,t} \frac{(c_{i,h,t}/s^{\theta}_{i,h,t-1})^{1-1/\eta}}{c_{i,h,t}} \left[\int_{N_h} (c_{i,h,t}/s^{\theta}_{i,h,t-1})^{1-1/\eta} dk \right]^{\frac{1-1/\varepsilon}{1-1/\eta}-1} x_t^{1/\varepsilon} \tag{A.1}$$

Similarly, the efficiency condition $c_{j,h,t}$ is given by

$$P_{j,h,t} = \omega_h \lambda_{c,t} \frac{(c_{j,h,t}/s^{\theta}_{j,h,t-1})^{1-1/\eta}}{c_{j,h,t}} \left[\int_{N_k} (c_{j,h,t}/s^{\theta}_{j,h,t-1})^{1-1/\eta} dk \right]^{\frac{1-1/\varepsilon}{1-1/\eta}-1} x_t^{1/\varepsilon} \tag{A.2}$$

Taking the ratio of (A.1) and (A.2) yields

$$\frac{P_{i,h,t}}{P_{j,h,t}} = \frac{c_{j,h,t}}{c_{i,h,t}} \frac{(c_{i,h,t}/s^{\theta}_{i,h,t-1})^{1-1/\eta}}{(c_{j,h,t}/s^{\theta}_{j,h,t-1})^{1-1/\eta}}$$

or equivalently,

$$(c_{j,h,t}/s^{\theta}_{j,h,t-1})^{-1/\eta} = \frac{P_{j,h,t} s^{\theta}_{j,h,t-1}}{P_{i,h,t}} \frac{(c_{i,h,t}/s^{\theta}_{i,h,t-1})^{1-1/\eta}}{c_{i,h,t}}$$

Rasing this expression to the power $1 - 1/\eta$, integrating the resulting expression with respect to j, and finally raising the resulting expression to the power $1/(1 - 1/\eta)$ yields

$$\left[\int_{N_h} (c_{j,h,t}/s^{\theta}_{j,h,t-1})^{1-1/\eta} dj \right]^{1/(1-1/\eta)} = \left[\int_{N_h} (P_{j,h,t} s^{\theta}_{j,h,t-1})^{1-\eta} dj \right]^{1/(1-1/\eta)} c_{i,h,t} (s^{\theta}_{i,h,t-1})^{\eta-1} P^{\eta}_{i,h,t} \tag{A.3}$$

We define two aggregates

$$x_{h,t} \equiv \left[\int_{N_h} (c_{j,h,t}/s^{\theta}_{j,h,t-1})^{1-1/\eta} dj \right]^{1/(1-1/\eta)} \quad \text{and} \tag{A.4}$$

$$\tilde{P}_{h,t} \equiv \left[\int_{N_h} (P_{j,h,t} s^{\theta}_{j,h,t-1})^{1-\eta} dj \right]^{1/(1-\eta)} \tag{A.5}$$

We can then rewrite (A.3) in terms of the two aggregates (A.4) and (A.5) as

$$\begin{aligned} c_{i,h,t} &= \left(\frac{P_{i,h,t}}{\tilde{P}_{h,t}} \right)^{-\eta} s^{\theta(1-\eta)}_{i,h,t-1} x_{h,t} \\ &= \left(\frac{P_{i,h,t}}{P_{h,t}} \right)^{-\eta} \left(\frac{\tilde{P}_{h,t}}{P_{h,t}} \right)^{\eta} s^{\theta(1-\eta)}_{i,h,t-1} x_{h,t}. \end{aligned} \tag{A.6}$$

Following the same steps, one can derive the home demand for foreign product as

$$c_{i,f,t} = \left(\frac{P_{i,f,t}}{\tilde{P}_{f,t}}\right)^{-\eta} s_{i,f,t-1}^{\theta(1-\eta)} x_{f,t} \tag{A.7}$$
$$= \left(\frac{P_{i,f,t}}{P_{f,t}}\right)^{-\eta} \left(\frac{\tilde{P}_{f,t}}{P_{f,t}}\right)^{\eta} s_{i,f,t-1}^{\theta(1-\eta)} x_{f,t},$$

where

$$x_{f,t} \equiv \left[\int_{N_f} (c_{j,f,t}/s_{j,f,t-1}^{\theta})^{1-1/\eta} dj\right]^{1/(1-1/\eta)} \tag{A.8}$$

$$\text{and } \tilde{P}_{f,t} \equiv \left[\int_{N_f} (P_{j,f,t} s_{j,f,t-1}^{\theta})^{1-\eta} dj\right]^{1/(1-\eta)} \tag{A.9}$$

Note that using (A.4) and (A.8), the consumption/habit aggregator x_t can be written as

$$x_t = \left[\sum_{k=h,f} \omega_k x_{k,t}^{1-1/\varepsilon}\right]^{1/(1-1/\varepsilon)} \tag{A.10}$$

We can then think of another cost minimization problem: minimizing the cost of obtaining x_t by choosing $x_{k,t}$ when the unit price of $x_{k,t}$ is given by $P_{k,t}$, that is,

$$\mathcal{L}_x = \sum_{k=h,f} \tilde{P}_{k,t} x_{k,t} - \tilde{P}_t \left[\left(\sum_{k=h,f} \omega_k x_{k,t}^{1-1/\varepsilon}\right)^{1/(1-1/\varepsilon)} - x_t\right]$$

where $\tilde{P}_t$ is the Lagrangian multiplier. The efficiency conditions for this program are given by

$$x_{h,t} = \omega_h^{\varepsilon} \left(\frac{\tilde{P}_{h,t}}{\tilde{P}_t}\right)^{-\varepsilon} x_t \tag{A.11}$$

$$\text{and } x_{f,t} = \omega_f^{\varepsilon} \left(\frac{\tilde{P}_{f,t}}{\tilde{P}_t}\right)^{-\varepsilon} x_t \tag{A.12}$$

Substituting these conditions in (A.10) yields the following condition.

$$1 = \left\{\sum_{k=h,f} \omega_k \left[\omega_k^{\varepsilon} \left(\frac{\tilde{P}_{k,t}}{\tilde{P}_t}\right)^{-\varepsilon}\right]^{1-1/\varepsilon}\right\}^{1/(1-1/\varepsilon)}$$

Solving this expression for $\tilde{P}_t$ results in an expression for a welfare based aggregate price index:

$$\tilde{P}_t = \left[\sum_{k=h,f} \omega_k \tilde{P}_{k,t}^{1-\varepsilon}\right]^{1/(1-\varepsilon)} \tag{A.13}$$

A.2 Accounting Identities

The following accounting identities are used in the main text:

$$\int_{N_k} P_{i,k,t}c_{i,k,t}di = \int_{N_k} P_{i,k,t}\left(\frac{P_{i,k,t}}{\tilde{P}_{k,t}}\right)^{-\eta} s_{i,k,t-1}^{\theta(1-\eta)}x_{k,t}di \tag{A.14}$$

$$= \tilde{P}_{k,t}^{\eta}x_{k,t}\int_{N_k}(P_{i,k,t}s_{i,k,t-1}^{\theta})^{1-\eta}di = \tilde{P}_{k,t}x_{k,t} \text{ for } k = h, f;$$

$$\sum_{k=h,f}\tilde{P}_{k,t}x_{k,t} = \sum_{k=h,f}\tilde{P}_{k,t}\omega_k\left(\frac{\tilde{P}_{k,t}}{\tilde{P}_t}\right)^{-\varepsilon}x_t = \tilde{P}_t^{\varepsilon}x_t\sum_{k=h,f}\omega_k\tilde{P}_{k,t}^{1-\varepsilon} = \tilde{P}_tx_t; \tag{A.15}$$

We define $p_{i,k,t} \equiv P_{i,k,t}/P_{k,t}$, $\tilde{p}_{k,t} \equiv \tilde{P}_{k,t}/P_{k,t}$ and $p_{k,t} \equiv P_{k,t}/P_t$ for $k = h, f$. Similarly, we define $p^*_{i,k,t} \equiv P^*_{i,k,t}/P^*_{k,t}$, $\tilde{p}^*_{k,t} \equiv \tilde{P}^*_{k,t}/P^*_{k,t}$ and $p^*_{k,t} \equiv P^*_{k,t}/P^*_t$ for $k = h, f$. Using these relative prices together with (9) and (12) under symmetric equilibrium, we can derive

$$\tilde{p}_t \equiv \frac{\tilde{P}_t}{P_t} = \left[\sum_{k=h,f}\omega_k\tilde{p}_{k,t}^{1-\varepsilon}p_{k,t}^{1-\varepsilon}\right]^{1/(1-\varepsilon)}$$

$$= \left[\sum_{k=h,f}\omega_k p_{k,t}^{1-\varepsilon}s_{k,t-1}^{\theta(1-\varepsilon)}\right]^{1/(1-\varepsilon)}. \tag{A.16}$$

B Phillips Curve

Using the symmetric equilibrium condition and dividing the FOC for $c_{i,h,t}$ of the firm problem by $\mathbb{E}_t^a[\xi_{i,t}]$, one can express the ratio of the marginal sales to the marginal value of internal funds as

$$\frac{\nu_{h,t}}{\mathbb{E}_t^a[\xi_{i,t}]} = p_{h,t} - \frac{\mathbb{E}_t^a[\kappa_{i,t}]}{\mathbb{E}_t^a[\xi_{i,t}]} + (1-\rho)\frac{\lambda_{h,t}}{\mathbb{E}_t^a[\xi_{i,t}]} \tag{B.1}$$

$$= p_{h,t} - \frac{\mathbb{E}_t^a[\xi_{i,t}a_{i,t}]}{\mathbb{E}_t^a[\xi_{i,t}]}\frac{w_t}{\alpha A_t}\left(\phi + c_{h,t} + c^*_{h,t}\right)^{\frac{1-\alpha}{\alpha}} + (1-\rho)\frac{\lambda_{h,t}}{\mathbb{E}_t^a[\xi_{i,t}]}$$

Define aggregate (marginal) gross mark-up $\mu(s_t)$ as $\mu_t \equiv \dfrac{\alpha A_t}{w_t}\left(\phi + c_{h,t} + c^*_{h,t}\right)^{\frac{\alpha-1}{\alpha}}$. Define also financially adjusted markup $\tilde{\mu}_{h,t}$ as

$$\tilde{\mu}_t \equiv \frac{\mathbb{E}_t^a[\xi_{i,t}]}{\mathbb{E}_t^a[\xi_{i,t}a_{i,t}]}\mu_t = \frac{\mathbb{E}_t^a[\xi_{i,t}]}{\mathbb{E}_t^a[\xi_{i,t}a_{i,t}]}\frac{\alpha A_t}{w_t}\left(\phi + c_{h,t} + c^*_{h,t}\right)^{\frac{\alpha-1}{\alpha}}$$

We can then express (B.1) as

$$\frac{\nu_{h,t}}{\mathbb{E}_t^a[\xi_{i,t}]} = p_{h,t} - \frac{1}{\tilde{\mu}_{h,t}} + (1-\rho)\frac{\lambda_{h,t}}{\mathbb{E}_t^a[\xi_{i,h,t}]} \tag{B.2}$$

Dividing the FOC for $s_{i,h,t}$ through by $\mathbb{E}_t^a[\xi_{i,t}]$ and rearranging terms yields

$$\frac{\lambda_{h,t}}{\mathbb{E}_t^a[\xi_{i,t}]} = \rho\mathbb{E}_t\left[m_{t,t+1}\frac{\mathbb{E}_{t+1}^a[\xi_{i,t+1}]}{\mathbb{E}_t^a[\xi_{i,t}]}\frac{\lambda_{h,t+1}}{\mathbb{E}_{t+1}^a[\xi_{i,t+1}]}\right] \tag{B.3}$$

$$+\theta(1-\eta)\mathbb{E}_t\left[m_{t,t+1}\frac{\mathbb{E}_{t+1}^a[\xi_{i,t+1}]}{\mathbb{E}_t^a[\xi_{i,t}]}\frac{\nu_{h,t+1}}{\mathbb{E}_{t+1}^a[\xi_{i,t+1}]}\frac{c_{h,t+1}}{s_{h,t}}\right]$$

After substituting (B.2) in (B.3) and solving the expression forwardly, one can verify that

$$\frac{\lambda_{h,t}}{\mathbb{E}_t^a[\xi_{i,t}]} = \theta(1-\eta)\mathbb{E}_t\left[\sum_{s=t+1}^{\infty}\tilde{\beta}_{t,s}\frac{\mathbb{E}_s^a[\xi_{i,s}]}{\mathbb{E}_t^a[\xi_{i,t}]}\left(p_{h,s}-\frac{1}{\tilde{\mu}_s}\right)\right] \tag{B.4}$$

where $\tilde{\beta}_{t,s} \equiv m_{s,s+1}g_{h,s+1} \cdot \prod_{j=1}^{s-t}[\rho+\theta(1-\eta)(1-\rho)g_{h,t+j}]m_{t+j-1,t+j}$ with $g_{h,t} \equiv c_{h,t}/s_{h,t-1} = (s_{h,t}/s_{h,t-1}-\rho)/(1-\rho)$ denotes a growth-adjusted discount factor. Hence,

$$\frac{\nu_{h,t}}{\mathbb{E}_t^a[\xi_{i,t}]} = p_{h,t}-\frac{1}{\tilde{\mu}_t}+(1-\rho)\theta(1-\eta)\mathbb{E}_t\left[\sum_{s=t+1}^{\infty}\tilde{\beta}_{t,s}\frac{\mathbb{E}_s^a[\xi_{i,s}]}{\mathbb{E}_t^a[\xi_{i,t}]}\left(p_{h,s}-\frac{1}{\tilde{\mu}_s}\right)\right] \tag{B.5}$$

C Nonstochastic Steady State

To derive the steady state relationship, it is useful to state the problem of foreign firms and derive FOCs first. The firm problem can be expressed as the following Lagrangian:

$$\begin{aligned}
\mathcal{L}^* = \mathbb{E}_0\sum_{t=0}^{\infty} m_{0,t}^*\Bigg\{ & d_{i,t}^* + \kappa_{i,t}^*\left[\left(\frac{A_t^*}{a_{i,t}^*}h_{i,t}^*\right)^{\alpha} - \phi^* - (c_{i,f,t}^* + c_{i,f,t})\right] \\
& + \xi_{i,t}^*\Bigg[p_{i,f,t}^*p_{f,t}^*c_{i,f,t}^* + q_t^{-1}p_{i,f,t}p_{f,t}c_{i,f,t} - w_t^*h_{i,t}^* - d_{i,t}^* + \varphi^*\min\{0, d_{i,t}^*\} \\
& \qquad - \frac{\gamma}{2}\left(\frac{p_{i,f,t}^*}{p_{i,f,t-1}^*}\pi_{f,t}^* - \bar{\pi}^*\right)^2 c_t^* - \frac{\gamma^*}{2}q_t^{-1}\left(\frac{p_{i,f,t}}{p_{i,f,t-1}}\pi_{f,t} - \bar{\pi}\right)^2 c_t\Bigg] \\
& + \nu_{i,f,t}^*\left[(p_{i,f,t}^*)^{-\eta}s_{i,f,t-1}^{*\theta(1-\eta)}x_{f,t}^* - c_{i,f,t}^*\right] \\
& + \nu_{i,f,t}\left[(p_{i,f,t})^{-\eta}s_{i,f,t-1}^{\theta(1-\eta)}x_{f,t} - c_{i,f,t}\right] \\
& + \lambda_{i,f,t}^*\left[\rho s_{i,f,t-1}^* + (1-\rho)c_{i,f,t}^* - s_{i,f,t}^*\right] \\
& + \lambda_{i,f,t}\left[\rho s_{i,f,t-1} + (1-\rho)c_{i,f,t} - s_{i,f,t}\right]\Bigg\}
\end{aligned}$$

C.1 Efficiency Conditions of Foreign Firms

The efficiency conditions for the firm problem in the foreign country are given by the followings:

$$d^*_{i,t} : \xi^*_{i,t} = \begin{cases} 1 & \text{if } d^*_{i,t} \geq 0 \\ 1/(1-\varphi^*) & \text{if } d^*_{i,t} < 0 \end{cases} \tag{C.1}$$

$$h^*_{i,t} : \xi^*_{i,t} w^*_t = \alpha \kappa^*_{i,t} \left(\frac{A^*_t}{a^*_{i,t}} h^*_{i,t} \right)^{\alpha-1} \tag{C.2}$$

$$\text{where } \; h^*_{i,t} \;=\; \frac{a^*_{i,t}}{A^*_t} (\phi^* + c^*_{i,f,t} + c_{i,f,t})^{1/\alpha}$$

$$c^*_{i,f,t} : \nu^*_{i,f,t} = \mathbb{E}^a_t[\xi^*_{i,t}] \; p^*_{i,f,t} p^*_{f,t} - \mathbb{E}^a_t[\kappa^*_{i,t}] + (1-\rho)\lambda^*_{i,f,t} \tag{C.3}$$

$$c_{i,f,t} : \nu_{i,f,t} = \mathbb{E}^a_t[\xi^*_{i,t}] \; q_t^{-1} p_{i,f,t} p_{f,t} - \mathbb{E}^a_t[\kappa^*_{i,t}] + (1-\rho)\lambda_{i,f,t} \tag{C.4}$$

$$s^*_{i,f,t} : \lambda^*_{i,f,t} = \rho \mathbb{E}_t[m^*_{t,t+1} \lambda^*_{i,f,t+1}] + \theta(1-\eta)\mathbb{E}_t \left\{ m^*_{t,t+1} \mathbb{E}^a_{t+1} \left[\nu^*_{i,f,t+1} \frac{c^*_{i,f,t+1}}{s^*_{i,f,t}} \right] \right\} \tag{C.5}$$

$$s_{i,f,t} : \lambda_{i,f,t} = \rho \mathbb{E}_t[m^*_{t,t+1} \lambda_{i,f,t+1}] + \theta(1-\eta)\mathbb{E}_t \left\{ m^*_{t,t+1} \mathbb{E}^a_{t+1} \left[\nu_{i,f,t+1} \frac{c_{i,f,t+1}}{s_{i,f,t}} \right] \right\} \tag{C.6}$$

$$p^*_{i,f,t} : 0 = \mathbb{E}^a_t[\xi^*_{i,t}] \left[p^*_{f,t} c^*_{i,f,t} - \gamma \frac{\pi^*_{f,t}}{p^*_{i,f,t-1}} \left(\pi^*_{f,t} \frac{p^*_{i,f,t}}{p^*_{i,f,t-1}} - \bar{\pi}^* \right) c^*_t \right] - \eta \frac{\nu^*_{i,f,t}}{p^*_{i,f,t}} c^*_{i,f,t} + \gamma \mathbb{E}_t \left[m^*_{t,t+1} \mathbb{E}^a_{t+1}[\xi^*_{i,t+1}] \pi^*_{f,t+1} \frac{p^*_{i,f,t+1}}{p^{*2}_{i,f,t}} \left(\pi^*_{f,t+1} \frac{p^*_{i,f,t+1}}{p^*_{i,f,t}} - \bar{\pi}^* \right) c^*_{t+1} \right] \tag{C.7}$$

$$p_{i,f,t} : 0 = \mathbb{E}^a_t[\xi^*_{i,t}] \left[q_t^{-1} p_{f,t} c_{i,f,t} - \gamma \frac{q_t^{-1} \pi_{f,t}}{p_{i,f,t-1}} \left(\pi_{f,t} \frac{p_{i,f,t}}{p_{i,f,t-1}} - \bar{\pi} \right) c_t \right] - \eta \frac{\nu_{i,f,t}}{p_{i,f,t}} c_{i,f,t} + \gamma \mathbb{E}_t \left[m^*_{t,t+1} \mathbb{E}^a_{t+1}[\xi^*_{i,t+1}] q^{-1}_{t+1} \pi_{f,t+1} \frac{p_{i,f,t+1}}{p^2_{i,f,t}} \left(\pi_{f,t+1} \frac{p_{i,f,t+1}}{p_{i,f,t}} - \bar{\pi} \right) c_{t+1} \right] \tag{C.8}$$

C.2 Symmetric Equilibrium and Relative Prices

Before we move onto the model dynamics, it is useful to discuss how relative prices are determined and how they are related with each other in symmetric equilibrium. The risk neutrality, i.i.d. idiosyncratic shock and the timing convention aforementioned imply that all home country firms choose an identical price level for a given market, that is, $P_{i,h,t} = P_{h,t}$ and $P^*_{i,h,t} = P^*_{h,t}$. Similarly, $P_{i,f,t} = P_{f,t}$ and $P^*_{i,f,t} = P^*_{f,t}$. Due to the pricing to market mechanism, $P_{i,h,t} \neq S_t P^*_{i,h,t}$ and $P_{i,f,t} \neq S_t^{-1} P^*_{i,f,t}$ in general. However, the symmetric equilibrium implies $p_{i,h,t}(= P_{i,h,t}/P_{h,t}) = p^*_{i,h,t}(= P^*_{i,h,t}/P^*_{h,t}) = p_{i,f,t}(= P_{i,f,t}/P_{f,t}) = p^*_{i,f,t}(= P^*_{i,f,t}/P^*_{f,t}) = 1$ always.

In any path of symmetric equilibrium, the relative ratio of type specific, habit adjusted price index ($\tilde{P}_{k,t}$) and CPI index ($P_{k,t}$) satisfy the followings:

$$\tilde{p}_{h,t} = \tilde{P}_{h,t}/P_{h,t} = s^{\theta}_{h,t-1}$$
$$\tilde{p}^*_{h,t} = \tilde{P}^*_{h,t}/P^*_{h,t} = s^{*\theta}_{h,t-1}$$
$$\tilde{p}_{f,t} = \tilde{P}_{f,t}/P_{f,t} = s^{\theta}_{f,t-1}$$
$$\text{and } \tilde{p}^*_{f,t} = \tilde{P}^*_{f,t}/P^*_{f,t} = s^{*\theta}_{f,t-1}.$$

These relative prices can then be used to derive the demands for habit adjusted consumption baskets in the symmetric equilibrium: With a symmetric equilibrium condition $x^j_{h,t} = x_{h,t}$,

$$\begin{aligned} x_{h,t} &= \omega_h^{\varepsilon}\left(\frac{\tilde{P}_{h,t}}{\tilde{P}_t}\right)^{-\varepsilon} x_t \\ &= \omega_h^{\varepsilon}\left(\frac{\tilde{P}_{h,t}}{P_{h,t}}\cdot\frac{P_{h,t}}{P_t}\cdot\frac{P_t}{\tilde{P}_t}\right)^{-\varepsilon} x_t \\ &= \omega_h^{\varepsilon} p_{h,t}^{-\varepsilon}\left(\frac{\tilde{p}_{h,t}}{\tilde{p}_t}\right)^{-\varepsilon} x_t \end{aligned}$$

where

$$\begin{aligned} \tilde{p}_t &= \left[\sum_{k=h,f}\omega_k\left(\frac{\tilde{P}_{k,t}}{P_t}\right)^{1-\varepsilon}\right]^{1/(1-\varepsilon)} \\ &= \left[\sum_{k=h,f}\omega_k\left(\frac{\tilde{P}_{k,t}}{P_{k,t}}\cdot\frac{P_{k,t}}{P_t}\right)^{1-\varepsilon}\right]^{1/(1-\varepsilon)} \\ &= \left[\sum_{k=h,f}\omega_k s_{k,t-1}^{\theta(1-\varepsilon)} p_{k,t}^{1-\varepsilon}\right]^{1/(1-\varepsilon)} \end{aligned}$$

Similarly, it can be shown that

$$\begin{aligned} x_{f,t} &= \omega_f^{\varepsilon} p_{f,t}^{-\varepsilon}\left(\frac{\tilde{p}_{f,t}}{\tilde{p}_t}\right)^{-\varepsilon} x_t, \\ x^*_{h,t} &= \omega_h^{*\varepsilon}(p^*_{h,t})^{-\varepsilon}\left(\frac{\tilde{p}^*_{h,t}}{\tilde{p}^*_t}\right)^{-\varepsilon} x^*_t, \\ x^*_{f,t} &= \omega_f^{*\varepsilon}(p^*_{f,t})^{-\varepsilon}\left(\frac{\tilde{p}^*_{f,t}}{\tilde{p}^*_t}\right)^{-\varepsilon} x^*_t, \end{aligned}$$

and

$$\tilde{p}^*_t = \left[\sum_{k=h,f}\omega^*_k s_{k,t-1}^{*\theta(1-\varepsilon)} p_{k,t}^{*(1-\varepsilon)}\right]^{1/(1-\varepsilon)}.$$

As is usual, only relative prices are determined in equilibrium: $p_{h,t}$, $p^*_{h,t}$, $p_{f,t}$, $p^*_{f,t}$, $\tilde{p}_{h,t}$, $\tilde{p}^*_{h,t}$, $\tilde{p}_{f,t}$, $\tilde{p}^*_{f,t}$, $\tilde{p}_t$, $\tilde{p}^*_t$ and q_t.

C.3 Equilibrium Relative Prices and Quantities

The Phillips curves in the steady state is given by

$$p_h = \eta\frac{\nu_h}{\mathbb{E}^a_t[\xi_i]} \tag{C.9}$$

$$qp^*_h = \eta\frac{\nu^*_h}{\mathbb{E}^a_t[\xi_i]} \tag{C.10}$$

$$p^*_f = \eta\frac{\nu^*_f}{\mathbb{E}^a_t[\xi^*_i]} \tag{C.11}$$

$$\text{and } p_f q^{-1} = \eta\frac{\nu_f}{\mathbb{E}^a_t[\xi^*_i]}. \tag{C.12}$$

(C.9)∼(C.12) are the steady state Phillips curves of home good in the home country, home good in the foreign country, foreign good in the home country and foreign good in the foreign country, respectively. The notational convention is that h and f indicate the origin of the good, and asterisks and the absence thereof indicate the destination of the good with asterisks indicating the foreign country. For instance, p_f^* is the (relative) price of good produced by and sold in the foreign country, whereas p_f is the price of good produced by the foreign country, but sold in the home country in home currency unit, and hence $1/q$ attached to it to convert it to foreign currency unit.

The symmetric equilibrium and the law of motion for habit stock imply $c_h = s_h$, $c_h^* = s_h^*$, $c_f = s_f$ and $c_f^* = s_f^*$. Using these conditions together with the FOCs for habit stocks, one can derive

$$\frac{\lambda_h}{\mathbb{E}^a[\xi_i]} = \frac{\theta(1-\eta)\beta}{1-\rho\beta}\frac{\nu_h}{\mathbb{E}^a[\xi_i]}, \tag{C.13}$$

$$\frac{\lambda_h^*}{\mathbb{E}^a[\xi_i]} = \frac{\theta(1-\eta)\beta}{1-\rho\beta}\frac{\nu_h^*}{\mathbb{E}^a[\xi_i]}, \tag{C.14}$$

$$\frac{\lambda_f^*}{\mathbb{E}^a[\xi_i^*]} = \frac{\theta(1-\eta)\beta}{1-\rho\beta}\frac{\nu_f^*}{\mathbb{E}^a[\xi_i^*]}, \tag{C.15}$$

$$\text{and } \frac{\lambda_f}{\mathbb{E}^a[\xi_i^*]} = \frac{\theta(1-\eta)\beta}{1-\rho\beta}\frac{\nu_f}{\mathbb{E}^a[\xi_i^*]}. \tag{C.16}$$

Combining (C.9)∼(C.12) and (C.13)∼(C.16) yields

$$\frac{\lambda_h}{\mathbb{E}^a[\xi_i]} = p_h\frac{\theta(1-\eta)\beta}{\eta(1-\rho\beta)}, \tag{C.17}$$

$$\frac{\lambda_h^*}{\mathbb{E}^a[\xi_i]} = qp_h^*\frac{\theta(1-\eta)\beta}{\eta(1-\rho\beta)}, \tag{C.18}$$

$$\frac{\lambda_f^*}{\mathbb{E}^a[\xi_i^*]} = p_f^*\frac{\theta(1-\eta)\beta}{\eta(1-\rho\beta)}, \tag{C.19}$$

$$\text{and } \frac{\lambda_f}{\mathbb{E}^a[\xi_i^*]} = \frac{p_f}{q}\frac{\theta(1-\eta)\beta}{\eta(1-\rho\beta)}, \tag{C.20}$$

which imply $qp_h^*/p_h = \lambda_h^*/\lambda_h$ and $qp_f^*/p_f = \lambda_f^*/\lambda_f$. Combining the FOCs (20), (22), (23), (C.2), (C.3) and (C.4), we have

$$\frac{\nu_h}{\mathbb{E}^a[\xi_i]} = p_h - \frac{\mathbb{E}^a[\xi_i a_i]}{\mathbb{E}^a[\xi_i]}\frac{w}{\alpha A}\left(\phi + c_h + c_h^*\right)^{\frac{1-\alpha}{\alpha}} + (1-\rho)\frac{\lambda_h}{\mathbb{E}^a[\xi_i]}, \tag{C.21}$$

$$\frac{\nu_h^*}{\mathbb{E}^a[\xi_i]} = qp_h^* - \frac{\mathbb{E}^a[\xi_i a_i]}{\mathbb{E}^a[\xi_i]}\frac{w}{\alpha A}\left(\phi + c_h + c_h^*\right)^{\frac{1-\alpha}{\alpha}} + (1-\rho)\frac{\lambda_h^*}{\mathbb{E}^a[\xi_i]}, \tag{C.22}$$

$$\frac{\nu_f^*}{\mathbb{E}^a[\xi_i^*]} = p_f^* - \frac{\mathbb{E}^a[\xi_i^* a_i^*]}{\mathbb{E}^a[\xi_i^*]}\frac{w^*}{\alpha A^*}\left(\phi^* + c_f^* + c_f\right)^{\frac{1-\alpha}{\alpha}} + (1-\rho)\frac{\lambda_f^*}{\mathbb{E}^a[\xi_i^*]}, \tag{C.23}$$

$$\text{and } \frac{\nu_f}{\mathbb{E}^a[\xi_i^*]} = \frac{p_f}{q} - \frac{\mathbb{E}^a[\xi_i^* a_i^*]}{\mathbb{E}^a[\xi_i^*]}\frac{w^*}{\alpha A^*}\left(\phi^* + c_f^* + c_f\right)^{\frac{1-\alpha}{\alpha}} + (1-\rho)\frac{\lambda_f}{\mathbb{E}^a[\xi_i^*]}. \tag{C.24}$$

Substituting (C.9)∼(C.12) and(C.17)∼(C.20) in (C.21)∼(C.24) and solving for p_k and p_k^* yields

$$p_h = \frac{\eta(1-\rho\beta)}{(\eta-1)[(1-\rho\beta)-\theta\beta(1-\rho)]}\frac{\mathbb{E}^a[\xi_i a_i]}{\mathbb{E}^a[\xi_i]}\frac{w}{\alpha A}(\phi+c_h+c_h^*)^{\frac{1-\alpha}{\alpha}} \tag{C.25}$$

$$p_h^* = \frac{\eta(1-\rho\beta)}{(\eta-1)[(1-\rho\beta)-\theta\beta(1-\rho)]}q^{-1}\frac{\mathbb{E}^a[\xi_i a_i]}{\mathbb{E}^a[\xi_i]}\frac{w}{\alpha A}(\phi+c_h+c_h^*)^{\frac{1-\alpha}{\alpha}} \tag{C.26}$$

$$p_f^* = \frac{\eta(1-\rho\beta)}{(\eta-1)[(1-\rho\beta)-\theta\beta(1-\rho)]}\frac{\mathbb{E}^a[\xi_i^* a_i^*]}{\mathbb{E}^a[\xi_i^*]}\frac{w^*}{\alpha A^*}(\phi^*+c_f^*+c_f)^{\frac{1-\alpha}{\alpha}} \tag{C.27}$$

$$\text{and } p_f = \frac{\eta(1-\rho\beta)}{(\eta-1)[(1-\rho\beta)-\theta\beta(1-\rho)]}q\frac{\mathbb{E}^a[\xi_i^* a_i^*]}{\mathbb{E}^a[\xi_i^*]}\frac{w^*}{\alpha A^*}(\phi^*+c_f^*+c_f)^{\frac{1-\alpha}{\alpha}} \tag{C.28}$$

Note that the law of one price holds in the non-stochastic steady state: $p_h = qp_h^*$ and $p_f^* = p_f/q$, which also imply $\lambda_h^*/\lambda_h = 1$ and $\lambda_f^*/\lambda_f = 1$. This is simply because we assume the symmetry of the two markets in terms of elasticity of substitution, strength of customer relationship, etc. However, the law of one price is generally violated in stochastic simulation as two countries undergo different histories of asymmetric shocks, which affect the intensity of customer relationships, and hence the demand elasticities in the two countries, and different financing conditions. Firms in general exploit any discrepancies in customer relationship and discriminate prices across the border.

The external financing triggers in the steady state are given by

$$a^E = \frac{A}{w(\phi+c_h+c_h^*)^{1/\alpha}}(p_h c_h + qp_h^* c_h^*) \tag{C.29}$$

$$a^{E*} = \frac{A^*}{w^*(\phi^*+c_f^*+c_f)^{1/\alpha}}(p_f^* c_f^* + q^{-1}p_f c_f), \tag{C.30}$$

which can be used to compute $\mathbb{E}^a[\xi_i]$, $\mathbb{E}^a[\xi_i a_i]$, $\mathbb{E}^a[\xi_i^*]$ and $\mathbb{E}^a[\xi_i^* a_i^*]$:

$$\mathbb{E}^a[\xi_i] = 1 + \frac{\varphi}{1-\varphi}[1-\Phi(z^E)] \tag{C.31}$$

$$\mathbb{E}^a[\xi_i a_i] = 1 + \frac{\varphi}{1-\varphi}[1-\Phi(z^E-\sigma)] \tag{C.32}$$

$$\mathbb{E}^a[\xi_i^*] = 1 + \frac{\varphi^*}{1-\varphi^*}[1-\Phi(z^{*E})] \tag{C.33}$$

$$\mathbb{E}^a[\xi_i^* a_i^*] = 1 + \frac{\varphi^*}{1-\varphi^*}[1-\Phi(z^{*E}-\sigma)] \tag{C.34}$$

where

$$z^E \equiv \sigma^{-1}(\log a^E + 0.5\sigma^2) \tag{C.35}$$

$$\text{and } z^{*E} \equiv \sigma^{-1}(\log a^{*E} + 0.5\sigma^2). \tag{C.36}$$

(8) and (11) and their foreign counterparts imply that the following ratios should be satisfied in the steady state

$$\frac{c_{i,h}}{c_{i,f}} = \frac{p_{i,h}^{-\eta}\tilde{p}_h^{\eta}s_{i,h}^{\theta(1-\eta)}x_h}{p_{i,f}^{-\eta}\tilde{p}_f^{\eta}s_{i,f}^{\theta(1-\eta)}x_f} = \frac{p_{i,h}^{-\eta}\tilde{p}_h^{\eta}s_{i,h}^{\theta(1-\eta)}\omega_h^{\varepsilon}\tilde{p}_h^{-\varepsilon}p_h^{-\varepsilon}\tilde{p}^{\varepsilon}x}{p_{i,f}^{-\eta}\tilde{p}_f^{\eta}s_{i,f}^{\theta(1-\eta)}\omega_f^{\varepsilon}\tilde{p}_f^{-\varepsilon}p_f^{-\varepsilon}\tilde{p}^{\varepsilon}x}$$

$$\frac{c_{i,h}^*}{c_{i,f}^*} = \frac{p_{i,h}^{*-\eta}\tilde{p}_h^{*\eta}s_{i,h}^{*\theta(1-\eta)}x_h^*}{p_{i,f}^{*-\eta}\tilde{p}_f^{*\eta}s_{i,f}^{*\theta(1-\eta)}x_f^*} = \frac{p_{i,h}^{*-\eta}\tilde{p}_h^{*\eta}s_{i,h}^{*\theta(1-\eta)}\omega_h^{\varepsilon}\tilde{p}_h^{*-\varepsilon}p_h^{*-\varepsilon}\tilde{p}^{*\varepsilon}x^*}{p_{i,f}^{*-\eta}\tilde{p}_f^{*\eta}s_{i,f}^{*\theta(1-\eta)}\omega_f^{\varepsilon}\tilde{p}_f^{*-\varepsilon}p_f^{*-\varepsilon}\tilde{p}^{*\varepsilon}x^*}$$

Imposing the symmetric equilibrium conditions and using $\tilde{p}_k = s_k^\theta$, we have

$$\frac{c_h}{c_f} = \left(\frac{\omega_h}{\omega_f}\right)^{\varepsilon}\left(\frac{p_h}{p_f}\right)^{-\varepsilon}\left(\frac{s_h^\theta}{s_f^\theta}\right)^{1-\varepsilon} \tag{C.37}$$

$$\text{and } \frac{c_h^*}{c_f^*} = \left(\frac{\omega_h}{\omega_f}\right)^{\varepsilon}\left(\frac{p_h^*}{p_f^*}\right)^{-\varepsilon}\left(\frac{s_h^{*\theta}}{s_f^{*\theta}}\right)^{1-\varepsilon}. \tag{C.38}$$

Since $c_{i,k} = c_k = s_k = s_{i,k}$ and $c_{i,k} = c_k^* = s_k^* = s_{i,k}^*$, (10) and its foreign counterpart imply

$$x = \left[\sum_{k=h,f} \omega_k (c_k^{1-\theta})^{1-1/\varepsilon}\right]^{1/(1-1/\varepsilon)} \tag{C.39}$$

$$x^* = \left[\sum_{k=h,f} \omega_k (c_k^{*1-\theta})^{1-1/\varepsilon}\right]^{1/(1-1/\varepsilon)}. \tag{C.40}$$

Aggregate (conditional) labor demand in home and foreign markets satisfy

$$h = \left[\frac{\phi + c_h + c_h^*}{A^\alpha \exp[0.5\alpha(1+\alpha)\sigma^2]}\right]^{1/\alpha}, \tag{C.41}$$

$$\text{and } h^* = \left[\frac{\phi^* + c_f + c_f^*}{A^{*\alpha} \exp[0.5\alpha(1+\alpha)\sigma^2]}\right]^{1/\alpha}, \tag{C.42}$$

which also implies goods market clearing conditions. The FOCs of households for labor hours can be expressed as

$$h = U_h^{-1}\left[-\frac{w}{\tilde{p}}\frac{\eta_w - 1}{\eta_w}U_x\right], \tag{C.43}$$

$$\text{and } h^* = U_h^{-1}\left[-\frac{w^*}{\tilde{p}^*}\frac{\eta_w - 1}{\eta_w}U_x^*\right], \tag{C.44}$$

The labor market clearing conditions in home and abroad can then be given by

$$U_h^{-1}\left[-\frac{w}{\tilde{p}}\frac{\eta_w - 1}{\eta_w}U_x\right] = \left[\frac{\phi + c_h + c_h^*}{A^\alpha \exp[0.5\alpha(1+\alpha)\sigma^2]}\right]^{1/\alpha} \tag{C.45}$$

$$\text{and } U_h^{-1}\left[-\frac{w^*}{\tilde{p}^*}\frac{\eta_w - 1}{\eta_w}U_x^*\right] = \left[\frac{\phi^* + c_f + c_f^*}{A^{*\alpha} \exp[0.5\alpha(1+\alpha)\sigma^2]}\right]^{1/\alpha}, \tag{C.46}$$

which characterize the labor market clearing conditions in home and abroad, and can be used to equilibrium wages in both markets. Finally, equilibrium consistency requires

$$1 = \left[\sum_{k=h,f} \omega_k p_k^{1-\varepsilon}\right]^{1/(1-\varepsilon)} \tag{C.47}$$

$$\text{and } 1 = \left[\sum_{k=h,f} \omega_k p_k^{*1-\varepsilon}\right]^{1/(1-\varepsilon)} \tag{C.48}$$

C.4 Real Exchange Rate

In the case of complete risk sharing between the two countries, the real exchange rate at any point in time should satisfy (46).

$$q = \kappa \frac{U_x^*}{U_x} \left[\frac{\sum_{k=h,f} \omega_k (p_k^* s_k^{*\theta})^{(1-\varepsilon)}}{\sum_{k=h,f} \omega_k (p_k s_k^{\theta})^{(1-\varepsilon)}} \right]^{-1/(1-\varepsilon)} \tag{C.49}$$

We assume that the equilibrium interest rates are determined by time preferences: $r = r^* = \beta^{-1} - 1$. This condition, in the case of incomplete risk sharing, pins down the equilibrium holdings of international bonds: $B_h = B_f = 0$, which, via the bond market clearing conditions, $B_h + B_h^* = 0$ and $B_f + B_f^* = 0$, pins down $B_h^* = B_f^* = 0$. In the case of incomplete risk sharing, the real exchange rate is determined such that $b_h = b_f = 0$, which, together with (63) implies

$$0 = wh - qw^*h^* + \tilde{d} - q\tilde{d}^* - (\tilde{p}x - q\tilde{p}^*x^*)$$

or equivalently,

$$q = \frac{wh + \tilde{d} - \tilde{p}x}{w^*h^* + \tilde{d}^* - \tilde{p}^*x^*}. \tag{C.50}$$

D System of Equations

There are total 71 equations for 71 endogenous variables in the system in the case with a floating exchange rate under the complete risk sharing arrangement. We provide these equations in the symmetric equilibrium forms:

$$
\begin{aligned}
0 = & -\frac{h_t^{1/\zeta}/U_{x,t}}{w_t/\tilde{p}_t} + \frac{\eta_w - 1}{\eta_w} + \frac{\gamma_w}{\eta_w}(\pi_{w,t} - \pi_w)\pi_{w,t} \\
& -\beta \frac{\gamma_w}{\eta_w} \mathbb{E}_t \left[\frac{U_{x,t+1}/\tilde{p}_{t+1}}{U_{x,t}/\tilde{p}_t} (\pi_{w,t+1} - \pi_w)\pi_{w,t+1} \frac{\pi_{w,t+1}}{\pi_{t+1}} \frac{h_{t+1}}{h_t} \right]
\end{aligned} \tag{D.1}
$$

$$
\begin{aligned}
0 = & -\frac{h_t^{*1/\zeta}/U_{x,t}}{w_t^*/\tilde{p}_t^*} + \frac{\eta_w - 1}{\eta_w} + \frac{\gamma_w}{\eta_w}(\pi_{w,t}^* - \pi_w)\pi_{w,t}^* \\
& -\beta \frac{\gamma_w}{\eta_w} \mathbb{E}_t \left[\frac{U_{x,t+1}^*/\tilde{p}_{t+1}^*}{U_{x,t}^*/\tilde{p}_t^*} (\pi_{w,t+1}^* - \pi_w)\pi_{w,t+1}^* \frac{\pi_{w,t+1}^*}{\pi_{t+1}^*} \frac{h_{t+1}^*}{h_t^*} \right]
\end{aligned} \tag{D.2}
$$

$$
0 = -\frac{c_{h,t}}{c_{f,t}} + \left(\frac{\omega_h}{\omega_f}\right)^{\varepsilon} \left(\frac{p_{h,t}}{p_{f,t}}\right)^{-\varepsilon} \left(\frac{s_{h,t-1}^{\theta}}{s_{f,t-1}^{\theta}}\right)^{1-\varepsilon} \tag{D.3}
$$

$$
0 = -\frac{c_{h,t}^*}{c_{f,t}^*} + \left(\frac{\omega_h}{\omega_f}\right)^{\varepsilon} \left(\frac{p_{h,t}^*}{p_{f,t}^*}\right)^{-\varepsilon} \left(\frac{s_{h,t-1}^{*\theta}}{s_{f,t-1}^{*\theta}}\right)^{1-\varepsilon} \tag{D.4}
$$

$$
0 = -\tilde{p}_{h,t} + s_{h,t-1}^{\theta} \tag{D.5}
$$

$$
0 = -\tilde{p}_{f,t} + s_{f,t-1}^{\theta} \tag{D.6}
$$

$$
0 = -\tilde{p}_{h,t}^* + s_{h,t-1}^{*\theta} \tag{D.7}
$$

$$
0 = -\tilde{p}_{f,t}^* + s_{f,t-1}^{*\theta} \tag{D.8}
$$

$$
0 = -x_{h,t} + \omega_h^{\varepsilon} p_{h,t}^{-\varepsilon} \left(\frac{\tilde{p}_{h,t}}{\tilde{p}_t}\right)^{-\varepsilon} x_t \tag{D.9}
$$

$$
0 = -x_{f,t} + \omega_f^{\varepsilon} p_{f,t}^{-\varepsilon} \left(\frac{\tilde{p}_{f,t}}{\tilde{p}_t}\right)^{-\varepsilon} x_t \tag{D.10}
$$

$$
0 = -x_{h,t}^* + \omega_h^{*\varepsilon} (p_{h,t}^*)^{-\varepsilon} \left(\frac{\tilde{p}_{h,t}^*}{\tilde{p}_t^*}\right)^{-\varepsilon} x_t^* \tag{D.11}
$$

$$
0 = -x_{f,t}^* + \omega_f^{*\varepsilon} (p_{f,t}^*)^{-\varepsilon} \left(\frac{\tilde{p}_{f,t}^*}{\tilde{p}_t^*}\right)^{-\varepsilon} x_t^* \tag{D.12}
$$

$$
0 = -\tilde{p}_t + \left[\sum_{k=h,f} \omega_k s_{k,t-1}^{\theta(1-\varepsilon)} p_{k,t}^{1-\varepsilon} \right]^{1/(1-\varepsilon)} \tag{D.13}
$$

$$0 = -\tilde{p}_t^* + \left[\sum_{k=h,f} \omega_k^* s_{k,t-1}^{*\theta(1-\varepsilon)} p_{k,t}^{*(1-\varepsilon)} \right]^{1/(1-\varepsilon)} \tag{D.14}$$

$$0 = -\pi_{h,t} + \frac{p_{h,t}}{p_{h,t-1}} \pi_t \tag{D.15}$$

$$0 = -\pi_{h,t}^* + \frac{p_{h,t}^*}{p_{h,t-1}^*} \pi_t^* \tag{D.16}$$

$$0 = -\pi_{f,t} + \frac{p_{f,t}}{p_{f,t-1}} \pi_t \tag{D.17}$$

$$0 = -\pi_{f,t}^* + \frac{p_{f,t}^*}{p_{f,t-1}^*} \pi_t^* \tag{D.18}$$

$$0 = -h_t^S + h_t^D \tag{D.19}$$

$$0 = -h_t^{*S} + h_t^{*D} \tag{D.20}$$

$$0 = -\mathbb{E}_t^a[\kappa_{i,t}] + \mathbb{E}_t^a[\xi_{i,t} a_{i,t}] \frac{w_t}{\alpha A_t} (\phi + c_{h,t} + c_{h,t}^*)^{\frac{1-\alpha}{\alpha}} \tag{D.21}$$

$$0 = -\mathbb{E}_t^a[\kappa_{i,t}^*] + \mathbb{E}_t^a[\xi_{i,t}^* a_{i,t}^*] \frac{w_t^*}{\alpha A_t^*} (\phi + c_{f,t} + c_{f,t}^*)^{\frac{1-\alpha}{\alpha}} \tag{D.22}$$

$$0 = -\nu_{h,t} + \mathbb{E}_t^a[\xi_{i,t}] \; p_{h,t} - \mathbb{E}_t^a[\kappa_{i,t}] + (1-\rho)\lambda_{h,t} \tag{D.23}$$

$$0 = -\nu_{h,t}^* + \mathbb{E}_t^a[\xi_{i,t}] \; q_t p_{h,t}^* - \mathbb{E}_t^a[\kappa_{i,t}] + (1-\rho)\lambda_{h,t}^* \tag{D.24}$$

$$0 = -\lambda_{h,t} + \rho \mathbb{E}_t[m_{t,t+1}\lambda_{h,t+1}] + \theta(1-\eta)\mathbb{E}_t \left\{ m_{t,t+1} \mathbb{E}_{t+1}^a \left[\nu_{h,t+1} \frac{c_{h,t+1}}{s_{h,t}} \right] \right\} \tag{D.25}$$

$$0 = -\lambda_{h,t}^* + \rho \mathbb{E}_t[m_{t,t+1}\lambda_{h,t+1}^*] + \theta(1-\eta)\mathbb{E}_t \left\{ m_{t,t+1} \mathbb{E}_{t+1}^a \left[\nu_{h,t+1}^* \frac{c_{h,t+1}^*}{s_{h,t}^*} \right] \right\} \tag{D.26}$$

$$0 = -\nu_{f,t}^* + \mathbb{E}_t^a[\xi_{i,t}^*] \; p_{f,t}^* - \mathbb{E}_t^a[\kappa_{i,t}^*] + (1-\rho)\lambda_{f,t}^* \tag{D.27}$$

$$0 = -\nu_{f,t} + \mathbb{E}_t^a[\xi_{i,t}^*] \; q_t^{-1} p_{f,t} - \mathbb{E}_t^a[\kappa_{i,t}^*] + (1-\rho)\lambda_{f,t} \tag{D.28}$$

$$0 = -\lambda_{f,t}^* + \rho \mathbb{E}_t[m_{t,t+1}^*\lambda_{f,t+1}^*] + \theta(1-\eta)\mathbb{E}_t \left\{ m_{t,t+1}^* \mathbb{E}_{t+1}^a \left[\nu_{f,t+1}^* \frac{c_{f,t+1}^*}{s_{f,t}^*} \right] \right\} \tag{D.29}$$

$$0 = -\lambda_{f,t} + \rho \mathbb{E}_t[m_{t,t+1}^*\lambda_{f,t+1}] + \theta(1-\eta)\mathbb{E}_t \left\{ m_{t,t+1}^* \mathbb{E}_{t+1}^a \left[\nu_{f,t+1} \frac{c_{f,t+1}}{s_{f,t}} \right] \right\} \tag{D.30}$$

$$0 = -p_{h,t}\frac{c_{h,t}}{c_t} + \gamma\pi_{h,t}(\pi_{h,t} - \bar{\pi}) + \eta\frac{\nu_{h,t}}{\mathbb{E}_t^a[\xi_{i,t}]}\frac{c_{h,t}}{c_t}$$
$$-\gamma\mathbb{E}_t\left[m_{t,t+1}\frac{\mathbb{E}_{t+1}^a[\xi_{i,t+1}]}{\mathbb{E}_t^a[\xi_{i,t}]}\pi_{h,t+1}(\pi_{h,t+1} - \bar{\pi})\frac{c_{t+1}}{c_t}\right] \tag{D.31}$$

$$0 = -q_t p_{h,t}^*\frac{c_{h,t}^*}{c_t^*} + \gamma q_t\pi_{h,t}^*(\pi_{h,t}^* - \bar{\pi}^*) + \eta\frac{\nu_{h,t}^*}{\mathbb{E}_t^a[\xi_{i,t}]}\frac{c_{h,t}^*}{c_t^*}$$
$$-\gamma^*\mathbb{E}_t\left[m_{t,t+1}\frac{\mathbb{E}_{t+1}^a[\xi_{i,t+1}]}{\mathbb{E}_t^a[\xi_{i,t}]}q_{t+1}\pi_{h,t+1}^*(\pi_{h,t+1}^* - \bar{\pi})\frac{c_{t+1}^*}{c_t^*}\right] \tag{D.32}$$

$$0 = -p_{f,t}^*\frac{c_{f,t}^*}{c_t^*} + \gamma\pi_{f,t}^*(\pi_{f,t}^* - \bar{\pi}^*) + \eta\frac{\nu_{i,f,t}^*}{\mathbb{E}_t^a[\xi_{i,t}^*]}\frac{c_{f,t}^*}{c_t^*}$$
$$-\gamma\mathbb{E}_t\left[m_{t,t+1}^*\frac{\mathbb{E}_{t+1}^a[\xi_{i,t+1}^*]}{\mathbb{E}_t^a[\xi_{i,t}^*]}\pi_{f,t+1}^*\left(\pi_{f,t+1}^* - \bar{\pi}^*\right)\frac{c_{t+1}^*}{c_t^*}\right] \tag{D.33}$$

$$0 = -q_t^{-1}p_{f,t}\frac{c_{f,t}}{c_t} + \gamma q_t^{-1}\pi_{f,t}(\pi_{f,t} - \bar{\pi}) + \eta\frac{\nu_{i,f,t}}{\mathbb{E}_t^a[\xi_{i,t}^*]}\frac{c_{f,t}}{c_t}$$
$$-\gamma\mathbb{E}_t\left[m_{t,t+1}^*\frac{\mathbb{E}_{t+1}^a[\xi_{i,t+1}^*]}{\mathbb{E}_t^a[\xi_{i,t}^*]}q_{t+1}^{-1}\pi_{f,t+1}\left(\pi_{f,t+1} - \bar{\pi}\right)\frac{c_{t+1}}{c_t}\right] \tag{D.34}$$

$$0 = -\mu_t + \frac{\alpha A_t}{w_t}\left(\phi + c_{h,t} + c_{h,t}^*\right)^{\frac{\alpha-1}{\alpha}} \tag{D.35}$$

$$0 = -\mu_t^* + \frac{\alpha A_t^*}{w_t^*}(\phi^* + c_{f,t} + c_{f,t}^*)^{\frac{\alpha-1}{\alpha}} \tag{D.36}$$

$$0 = -\tilde{\mu}_t + \frac{\mathbb{E}_t^a[\xi_{i,t}]}{\mathbb{E}_t^a[\xi_{i,t}a_{i,t}]}\mu_t \tag{D.37}$$

$$0 = -\tilde{\mu}_t^* + \frac{\mathbb{E}_t^a[\xi_{i,t}^*]}{\mathbb{E}_t^a[\xi_{i,t}^*a_{i,t}^*]}\mu_t^* \tag{D.38}$$

$$0 = -a_t^E + \frac{A_t}{w_t(\phi + c_{h,t} + c_{h,t}^*)^{1/\alpha}}$$
$$\times\left\{c_t\left[\frac{p_{h,t}c_{h,t}}{c_t} - \frac{\gamma}{2}(\pi_{h,t} - \bar{\pi})^2\right] + q_t c_t^*\left[\frac{p_{h,t}^*c_{h,t}^*}{c_t^*} - \frac{\gamma^*}{2}(\pi_{h,t}^* - \bar{\pi}^*)^2\right]\right\} \tag{D.39}$$

$$0 = -a_t^{*E} + \frac{A_t^*}{w_t^*(\phi^* + c_{f,t} + c_{f,t}^*)^{1/\alpha}}$$
$$\times\left\{c_t^*\left[\frac{p_{f,t}^*c_{f,t}^*}{c_t^*} - \frac{\gamma^*}{2}(\pi_{f,t}^* - \bar{\pi})^2\right] + q_t^{-1}c_t\left[\frac{p_{f,t}c_{f,t}}{c_t} - \frac{\gamma}{2}(\pi_{f,t} - \bar{\pi})^2\right]\right\} \tag{D.40}$$

$$0 = -z_t^E + \sigma^{-1}(\log a_t^E + 0.5\sigma^2) \tag{D.41}$$

$$0 = -z_t^{*E} + \sigma^{-1}(\log a_t^{*E} + 0.5\sigma^2) \tag{D.42}$$

$$0 = -\mathbb{E}_t^a[\xi_{i,t}] + 1 + \frac{\varphi_t}{1-\varphi_t}[1 - \Phi(z_t^E)] \tag{D.43}$$

$$0 = -\mathbb{E}_t^a[\xi_{i,t} a_{i,t}] + 1 + \frac{\varphi_t}{1-\varphi_t}[1 - \Phi(z_t^E - \sigma)] \tag{D.44}$$

$$0 = -\mathbb{E}_t^a[\xi_{i,t}^*] + 1 + \frac{\varphi_t^*}{1-\varphi_t^*}[1 - \Phi(z_t^{*E})] \tag{D.45}$$

$$0 = -\mathbb{E}_t^a[\xi_{i,t}^* a_{i,t}^*] + 1 + \frac{\varphi_t^*}{1-\varphi_t^*}[1 - \Phi(z_t^{*E} - \sigma)] \tag{D.46}$$

$$0 = -h_t^D + \left[\frac{\phi + c_{h,t} + c_{h,t}^*}{A_t^\alpha \exp[0.5\alpha(1+\alpha)\sigma^2]}\right]^{1/\alpha} \tag{D.47}$$

$$0 = -h_t^{*S} + \left[\frac{\phi^* + c_{f,t} + c_{f,t}^*}{A_t^{*\alpha} \exp[0.5\alpha(1+\alpha)\sigma^2]}\right]^{1/\alpha} \tag{D.48}$$

$$0 = -U_{x,t} + (x_t - \delta_t)^{-\gamma_x} \tag{D.49}$$

$$0 = -U_{x,t}^* + (x_t^* - \delta_t^*)^{-\gamma_x} \tag{D.50}$$

$$0 = -y_t + \exp[0.5\alpha(1+\alpha)\sigma^2](A_t h_t)^\alpha - \phi \tag{D.51}$$

$$0 = -y_t^* + \exp[0.5\alpha(1+\alpha)\sigma^2](A_t^* h_t^*)^\alpha - \phi^* \tag{D.52}$$

$$0 = -1 + \mathbb{E}_t\left[\beta \frac{U_{x,t+1}/\tilde{p}_{t+1}}{U_{x,t}/\tilde{p}_t} \frac{R_t}{\pi_{t+1}}\right] \tag{D.53}$$

$$0 = -1 + \mathbb{E}_t\left[\beta \frac{U_{x,t+1}^*/\tilde{p}_{t+1}^*}{U_{x,t}^*/\tilde{p}_t^*} \frac{R_t^*}{\pi_{t+1}^*}\right] \tag{D.54}$$

$$0 = -R_t + R^{1-\rho_R}\left[R_{t-1}\left(\frac{y_t}{y}\right)^{\rho_c}\left(\frac{\pi_t}{\bar{\pi}}\right)^{\rho_\pi}\right]^{\rho_R} \tag{D.55}$$

$$0 = -R_t^* + R^{*1-\rho_R}\left[R_{t-1}^*\left(\frac{y_t^*}{y^*}\right)^{\rho_c}\left(\frac{\pi_t^*}{\bar{\pi}^*}\right)^{\rho_\pi}\right]^{\rho_R} \tag{D.56}$$

$$0 = -c_t + p_{h,t} c_{h,t} + p_{f,t} c_{f,t} \tag{D.57}$$

$$0 = -c_t^* + p_{h,t}^* c_{h,t}^* + p_{f,t}^* c_{f,t}^* \tag{D.58}$$

$$0 = -\pi_t + \left[\sum_{k=h,f} \omega_k (p_{k,t-1}\pi_{k,t})^{1-\varepsilon}\right]^{1/(1-\varepsilon)} \tag{D.59}$$

$$0 = -\pi_t^* + \left[\sum_{k=h,f} \omega_k (p_{k,t-1}^* \pi_{k,t}^*)^{1-\varepsilon} \right]^{1/(1-\varepsilon)} \tag{D.60}$$

$$0 = -x_t + \left[\sum_{k=h,f} \omega_k (c_{k,t}^{1-\theta})^{1-1/\varepsilon} \right]^{1/(1-1/\varepsilon)} \tag{D.61}$$

$$0 = -x^* + \left[\sum_{k=h,f} \omega_k (c_{k,t}^{*1-\theta})^{1-1/\varepsilon} \right]^{1/(1-1/\varepsilon)} \tag{D.62}$$

$$0 = -s_{h,t} + \rho s_{h,t-1} + (1-\rho) c_{h,t} \tag{D.63}$$

$$0 = -s_{f,t} + \rho s_{f,t-1} + (1-\rho) c_{f,t} \tag{D.64}$$

$$0 = -s_{h,t}^* + \rho s_{h,t-1}^* + (1-\rho) c_{h,t}^* \tag{D.65}$$

$$0 = -s_{f,t}^* + \rho s_{f,t-1}^* + (1-\rho) c_{f,t}^* \tag{D.66}$$

D.1 Complete Risk Sharing With Floating Exchange Rate

Under the complete risk sharing arrangement, the real exchange rate is determined by the risk-sharing condition:

$$0 = -q_t + \kappa \frac{U_{x,t}^*/\tilde{p}_t^*}{U_{x,t}/\tilde{p}_t} \tag{D.67}$$

D.2 Incomplete Risk Sharing With Floating Exchange Rate

Under the incomplete risk sharing arrangement, the risk sharing condition (D.71), and the FOCs for risk-free bonds (D.57) and (D.58) should be replaced by the following equations:

$$0 = -(1 + \tau b_{h,t+1}) + \beta \mathbb{E}_t \left[\frac{U_{x,t+1}/\tilde{p}_{t+1}}{U_{x,t}/\tilde{p}_t} \frac{R_t}{\pi_{t+1}} \right] \tag{D.68}$$

$$0 = -(1 + \tau b_{f,t+1}) + \beta \mathbb{E}_t \left[\frac{U_{x,t+1}/\tilde{p}_{t+1}}{U_{x,t}/\tilde{p}_t} \frac{R_t^*}{\pi_{t+1}^*} \frac{q_{t+1}}{q_t} \right] \tag{D.69}$$

$$0 = -(1 + \tau b_{h,t+1}^*) + \beta \mathbb{E}_t \left[\frac{U_{x,t+1}^*/\tilde{p}_{t+1}^*}{U_{x,t}^*/\tilde{p}_t^*} \frac{R_t}{\pi_{t+1}} \frac{q_t}{q_{t+1}} \right] \tag{D.70}$$

$$0 = -(1 + \tau b_{f,t+1}^*) + \beta \mathbb{E}_t \left[\frac{U_{x,t+1}^*/\tilde{p}_{t+1}^*}{U_{x,t}^*/\tilde{p}_t^*} \frac{R_t^*}{\pi_{t+1}^*} \right] \tag{D.71}$$

$$0 = b_{h,t+1} + b_{h,t+1}^* \tag{D.72}$$

$$0 = b_{f,t+1} + b_{f,t+1}^* \tag{D.73}$$

$$0 = -(b_{h,t+1} + q_t b_{f,t+1}) + \frac{R_{t-1}}{\pi_t} b_{h,t} + \frac{R^*_{t-1}}{\pi^*_t} q_t b_{f,t} + \frac{1}{2}(w_t h_t - q_t w^*_t h^*_t) + \frac{1}{2}(\tilde{d}_t - q_t \tilde{d}^*_t) - \frac{1}{2}(\tilde{p}_t x_t - q_t \tilde{p}^*_t x^*_t) \tag{D.74}$$

where

$$0 = -\tilde{d}_t + \tilde{d}^+_t + (1 - \varphi_t)\tilde{d}^-_t, \tag{D.75}$$

$$0 = -\tilde{d}^*_t + \tilde{d}^{*+}_t + (1 - \varphi^*_t)\tilde{d}^{*-}_t, \tag{D.76}$$

$$0 = -\tilde{d}^+_t + \Phi(z^E_t)\Big[p_{h,t}c_{h,t} + q_t p^*_{h,t}c^*_{h,t} - \frac{w_t}{A_t}\frac{\Phi(z^E_t - \sigma)}{\Phi(z^E_t)}(\phi + c_{h,t} + c^*_{h,t})^{1/\alpha} - \frac{\gamma}{2}(\pi_{h,t} - \bar{\pi})^2 c_t - \frac{\gamma^*}{2} q_t (\pi^*_{h,t} - \bar{\pi}^*)^2 c^*_t\Big], \tag{D.77}$$

$$0 = -\tilde{d}^-_t + \frac{1 - \Phi(z^E_t)}{1 - \varphi_t}\Big[p_{h,t}c_{h,t} + q_t p^*_{h,t}c^*_{h,t} - \frac{w_t}{A_t}\frac{1 - \Phi(z^E_t - \sigma)}{1 - \Phi(z^E_t)}(\phi + c_{h,t} + c^*_{h,t})^{1/\alpha} - \frac{\gamma}{2}(\pi_{h,t} - \bar{\pi})^2 c_t - \frac{\gamma^*}{2} q_t (\pi^*_{h,t} - \bar{\pi}^*)^2 c^*_t\Big], \tag{D.78}$$

$$0 = -\tilde{d}^{*+}_t + \Phi(z^{*E}_t)\Big[q_t^{-1} p_{f,t}c_{f,t} + p^*_{f,t}c^*_{f,t} - \frac{w^*_t}{A^*_t}\frac{\Phi(z^{*E}_t - \sigma)}{\Phi(z^{*E}_t)}(\phi^* + c_{f,t} + c^*_{f,t})^{1/\alpha} - \frac{\gamma}{2} q_t^{-1}(\pi_{f,t} - \bar{\pi})^2 c_t - \frac{\gamma^*}{2}(\pi^*_{f,t} - \bar{\pi}^*)^2 c^*_t\Big], \tag{D.79}$$

and

$$0 = -\tilde{d}^{*-}_t + \frac{1 - \Phi(z^{*E}_t)}{1 - \varphi^*_t}\Big[q_t^{-1} p_{f,t}c_{f,t} + p^*_{f,t}c^*_{f,t} - \frac{w^*_t}{A^*_t}\frac{1 - \Phi(z^{*E}_t - \sigma)}{1 - \Phi(z^{*E}_t)}(\phi^* + c_{f,t} + c^*_{f,t})^{1/\alpha} - \frac{\gamma}{2} q_t^{-1}(\pi_{f,t} - \bar{\pi})^2 c_t - \frac{\gamma^*}{2}(\pi^*_{f,t} - \bar{\pi}^*)^2 c^*_t\Big]. \tag{D.80}$$

D.3 Incomplete Risk Sharing With Monetary Union

Under the incomplete risk sharing with monetary union, (D.68)∼(D.71) should be replaced with

$$0 = -(1 + \tau b_{h,t+1}) + \beta \mathbb{E}_t\left[\frac{U_{x,t+1}/\tilde{p}_{t+1}}{U_{x,t+1}/\tilde{p}_{t+1}}\frac{R^U_t}{\pi_{t+1}}\right] \tag{D.81}$$

$$0 = -(1 + \tau b^*_{h,t+1}) + \beta \mathbb{E}_t\left[\frac{U^*_{x,t+1}/\tilde{p}^*_{t+1}}{U^*_{x,t+1}/\tilde{p}^*_{t+1}}\frac{R^U_t}{\pi^*_{t+1}}\right] \tag{D.82}$$

and the bond market clearing condition $0 = b_{f,t+1} + b^*_{f,t+1}$ is deleted. The following identity is added to the system:

$$\frac{\mathbb{E}_t[q_{t+1}]}{q_t} = \frac{\mathbb{E}_t[S_{t+1}]}{S_t} \cdot \frac{\mathbb{E}_t[\pi^*_{t+1}]}{\mathbb{E}_t[\pi_{t+1}]}. \tag{D.83}$$

Note that S_t is not a model variable as the level of nominal exchange rate cannot be determined in the steady state. However, $\pi^S_{t+1} \equiv S_{t+1}/S_t$ is well-defined as a model variable.

D.4 Exogenous Variables

There are 6 exogenous variables:

$$0 = -\log A_t + \rho_A \log A_{t-1} + \epsilon_{A,t} \tag{D.84}$$

$$0 = -\log A_t^* + \rho_A \log A_{t-1}^* + \epsilon_{A,t}^* \tag{D.85}$$

$$0 = -\delta_t + \rho_\delta \delta_{t-1} + \epsilon_{\delta,t} \tag{D.86}$$

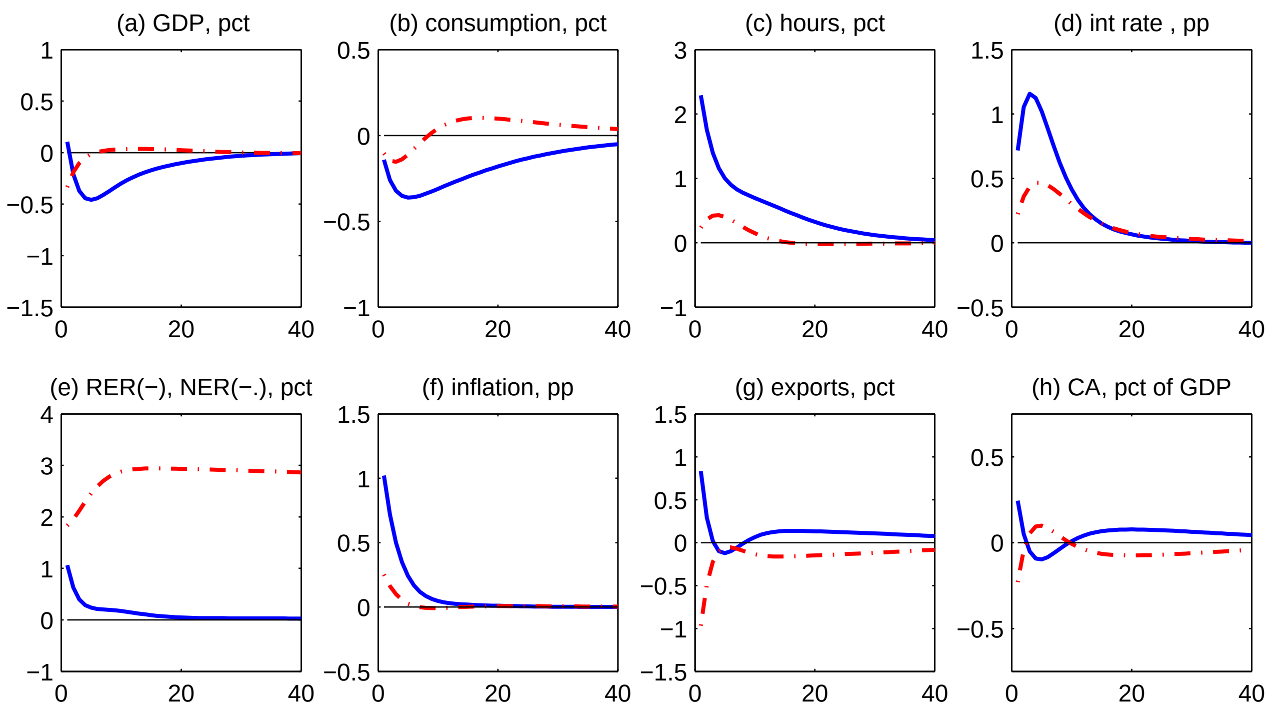
$$0 = -\delta_t^* + \rho_\delta \delta_{t-1}^* + \epsilon_{\delta,t}^* \tag{D.87}$$

$$0 = -\log \varphi_t + (1 - \rho_\varphi) \log \bar{\varphi} + \rho_\varphi \log \varphi_{t-1} + \epsilon_{\varphi,t} \tag{D.88}$$

$$0 = -\log \varphi_t^* + (1 - \rho_\varphi) \log \bar{\varphi}^* + \rho_\varphi \log \varphi_{t-1}^* + \epsilon_{\varphi,t}^* \tag{D.89}$$

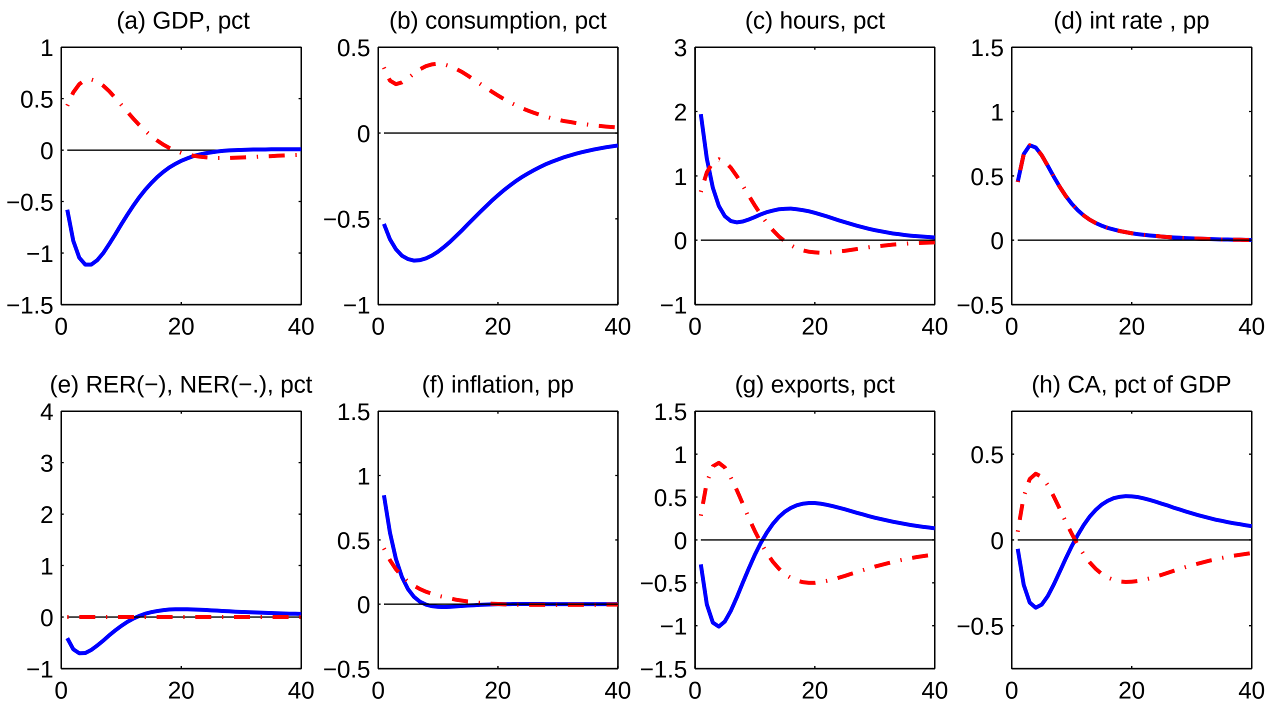
E Impact of Technology Shocks

Figure 15: Technology Shock to Peripheral Country Under Floating



Note: Blue, solid line is the peripheral country and red, dash-dotted line is the core country.

Figure 16: Technology Shock to Peripheral Country Under Monetary Union



Note: Blue, solid line is the peripheral country and red, dash-dotted line is the core country.