Aging and Deflation: Japanese Experience*

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Abstract

This paper explores a causal link between aging of the labor force and the deflationary pressure in Japan. We develop a search/matching model with skill and age heterogeneities, coupled with standard new Keynesian nominal rigidity. We examine long-run implications of the sharp drop in labor force entry during the 1970s in Japan. The model predicts that, as the share of old workers gets higher and higher, the inflation rate falls, eventually turns negative and stays persistently negative before coming back to zero. The key to the mechanism is a high degree of firm specificity of human capital. We show that economies with a smaller degree of firm specificity and a larger flow of labor force entry are less likely to experience deflation.

JEL Classification: E24, E31, E52
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1 Introduction

Japan has experienced small yet prolonged deflation since the late 1990s. At the same time, the Japanese labor force has been rapidly aging. These two phenomena are usually considered orthogonal to each other, although some policymakers contemplated whether there is indeed a causal link between the two or the correlation is simply a coincidence (see, for example, Shirakawa (2012)). This paper takes the former view and examines underlying interactions between demographic transitions and inflation dynamics in Japan. Specifically, we examine how a high degree of firm specificity of human capital, one of the key features of the Japanese labor market, influences long-run behavior of inflation and labor market variables such as wage.

We develop a search and matching model with demographic and skill transitions, embedded in a standard new Keynesian framework. In the model, a worker enters the labor force as a young worker with no experience and thus no specificity of human capital. The worker

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eventually becomes old and experienced at the same time, having higher productivity. The skill the old worker possesses is firm specific in the sense that in case he loses the job, he also loses the skill. He will then need to look for a job in the matching market, where young workers also look for their entry-level job. The model thus features three types of workers, (i) young workers, (ii) old experienced workers, and (iii) old inexperienced workers; there are two types of jobs, (i) the entry-level jobs and (ii) the jobs that only experienced workers can qualify. Firms in the economy produce and sell differentiated goods combining the three types of workers, subject to a standard new Keynesian price setting friction. Real wages are determined by the Nash bargaining as in the standard search/matching framework and thus are affected by each worker's outside option value as well as the worker's productivity.

Our key experiment entails feeding a path of the labor force entry rate to the model economy and computes its perfect foresight equilibrium path. In Japan, labor force entry fell dramatically during the 1970s (see Figure 1 presented in Section 2). This corresponds to the collapse in the birth rate due to the end of the (first) baby boom in the 1950s. The assumed path of the entry rate in the model replicates the observed decline.[1] The perfect foresight equilibrium path replicates many important low-frequency features observed for Japan over the 40 years between 1970 and 2010. In the model, the first decades of this 40-year period are characterized by rising per-capita consumption, real wage, and labor productivity. The real interest rate is also rising in this period. Assuming that the nominal interest rate set by the standard Taylor rule, the inflation rate also rises during this period. The main reason for the growing per-capita real variables in this phase of aging is a rising share of experienced workers, who are more productive. During the second phase of aging (i.e., the second 20-year period), however, the negative effect due to high specificity of human capital becomes dominant: A higher share of old workers itself implies a higher share of inexperienced workers.[2] Average labor productivity and real wage (and thus per-capita consumption) then begin to fall. In our benchmark calibration, the model generates small, yet persistent deflation, starting the late stage of aging (roughly 30 years after the sharp decline in the entry rate). We also find that, as aging progresses, the job finding rate for young workers falls significantly: More and more old workers who lost their firm specific skill flow into the matching market for the entry-level jobs, which negatively impacts young workers' wage.

We show that the deflationary pressure is smaller, when the parameter that captures the degree of human capital specificity is set to a lower value. This result highlights the importance of specific human capital in the Japanese labor market. Higher job loss rates are also shown to increase the magnitude of deflation.[3] It is well known that the pace of job loss considerably increased during the 1990s and 2000s (see Esteban-Pretel et al. (2011) and Lin and Miyamoto (2012)). When job loss rates are calibrated to be a higher value to incorporate

[1]Note that the birth rate and labor force entry kept falling even after the end of the baby boom. The birth rate stopped falling only in 2005. However, our experiment focuses on the end of the first baby boom, given that it is quantitatively most important.

[2]This negative effect surfaces only in the later phase of the 40-year period because job loss that is accompanied by the loss of the firm specific skill occurs with small probability per period.

[3]In our model, separation rates are assumed to be exogenous.

this fact, the adverse effect of aging (i.e., the increase in the share of inexperienced workers) surfaces faster and is magnified.

It is also important to notice that the level of the labor force started falling in the mid 1990s, which represents a serious nature of aging in Japan. Our benchmark calibration (not surprisingly) matches this fact, given that the assumed path of the entry rate replicates its observed decline. However, none of large developed economies have experienced a decline in the labor force size itself. For example, the U.S. labor force grew more than 40% since the mid 1990s, even though entry into the labor force also slowed down during the 1970s. When we feed an alternative path of the entry rate that falls less dramatically (so that the labor force keeps growing albeit at a smaller rate), the deflationary pressure is found to be weaker. Our results thus imply that particular features and the environment facing the Japanese labor market are very important in shaping the long-run behavior of inflation and labor market variables.

The main driving factor behind our results is the time-varying nature of the natural rate of interest due to changes in the demographic structure. In other words, price stickiness itself plays a minor role in the long-run behavior of the inflation rate. However, we find that the marginal cost and thus markup do fluctuate especially in the early stage of aging, because changes in the entry rate influence the hiring cost workers through the search friction. In the next draft, we plan to investigate into this channel more closely and discuss optimal monetary policy.

The model we used in this paper is an extension of standard new Keynesian search/-matching models. Early attempts in this vein include Trigari (2009) and Krause et al. (2008) but these papers feature neither heterogeneity in worker skills nor demographic transitions. Esteban-Pretel and Fraglia (2010) extends the new Keynesian search/matching framework to include skill heterogeneity. However, they do not consider demographic transitions in their model and the evolution of the worker's skill is quite different from the one in our model. Moreover, their research focus is on Spanish economy. Fujiwara and Teranishi (2008) develop a new Keynesian model with a life-cycle structure. Their study focuses on consumption heterogeneity between workers and retirees and consider neither skill heterogeneity nor search frictions. Konishi and Ueda (2013) also point out the possible link between aging and deflation, by embedding the fiscal theory of price level into an overlapping-generations model. Their mechanism is largely orthogonal to ours and their results are mostly qualitative.

The paper is organized as follows. The next section presents some basic facts concerning aging of the Japanese labor force and also review wage and inflation behavior in the last two decades. Section 3 develops the model, which is calibrated in Section 4. Section 5 presents the main results of the paper. Section 6 concludes the paper by discussing some implications of our findings.

2 Some Facts

This section reviews some important facts regarding the demographic structure of the Japanese labor force. We view the Japanese demographic structure as being an important factor be-

(a) Birth Rate (b) Entry into the Labor Force

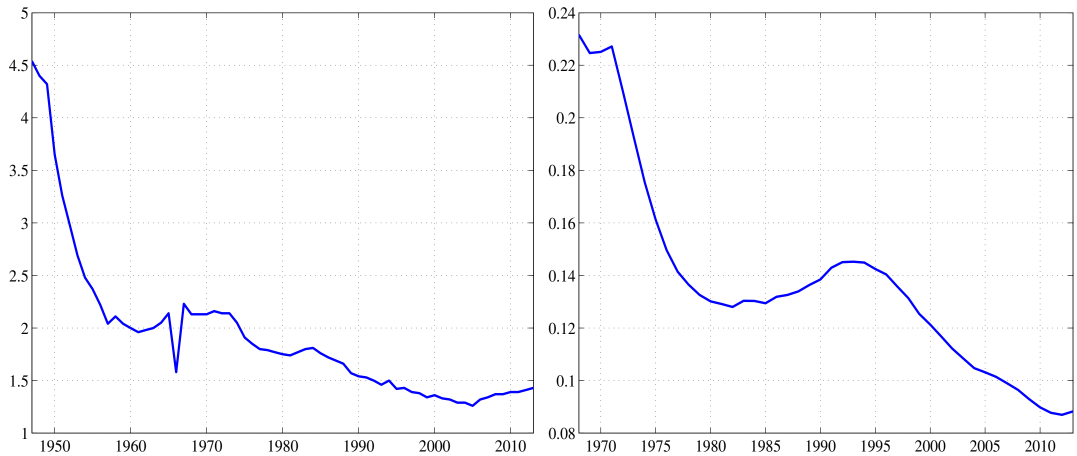

Figure 1: Labor Force Entry in Japan

Notes: The birth rate equals "the total fertility rate" which gives the number of children that are born to each woman in her childbearing years (15-49 years old). Panel (b) plots the share of workers between 15 and 24 in the total labor force (workers between 15 and 64).

hind the observed wage compression and deflationary pressure in Japan for the last 15-20 years.

2.1 Aging of the Labor Force

Panel (a) of Figure 1 plots the birth rate, more specifically, the total fertility rate, starting at 1948.[4] The effect of changes in the birth rate shows up 15-20 years later in the labor force. To see the effect, Panel (b) plots the share of workers between 15 and 24 years old in the total labor force.[5] We view this share as an approximation to entry into the labor force. One can clearly see in Panel (a) that the birth rate was very high in the late 1940s, but fell dramatically in the following 10 years or so. This corresponds to the end of the (first) baby boom in Japan. The birth rate from then on was roughly flat at around 2 until the early 1970s. Since the early 1970s, however, the birth rate kept falling until 2005 when it hit the historical low at around 1.3. Panel (b) attempts to translate the behavior of the birth rate into the entry flow into the labor force by plotting the share of workers between 15-24 years old in the total labor force. This series behaves similarly to the birth rate with roughly a 15- to 20-year lag: It fell sharply in the 1970s, recovered temporarily around the early 1990s, and, since then, has been declining steadily.

In Panel (a) of Figure 2, we compute the average age of the labor force. One can

[4]The total fertility rate gives the number of children that are born to each woman in her childbearing years, which are usually defined between 15 and 49 years old. We use the term the birth rate to mean the total fertility rate throughout the paper.

[5]We drop workers who are 65 or above from the total labor force, so that our analysis is not influenced by the fact that a larger number of 65+ workers are in the labor force.

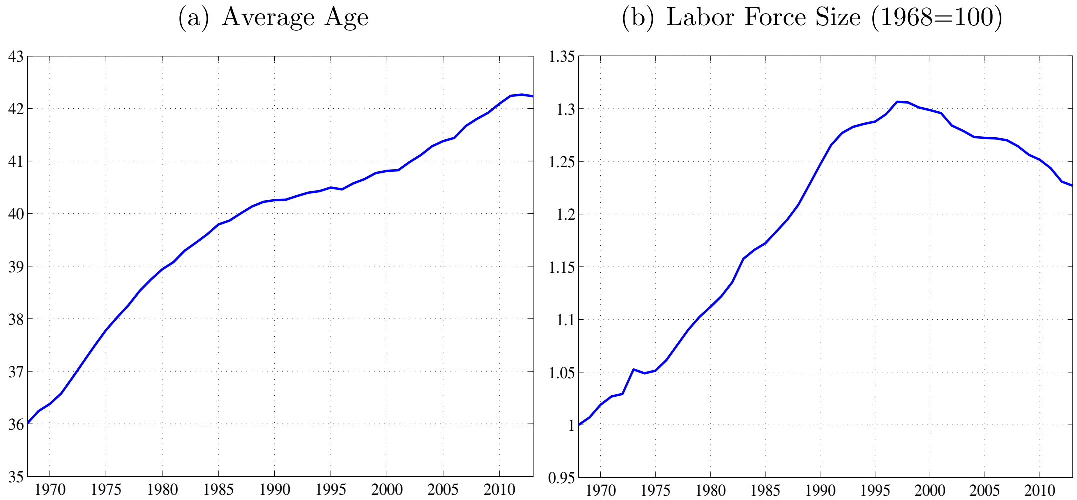

Figure 2: The Average Age and Labor Force Size

see that the average age has increased from 36 years old to 42 years old, demonstrating the significant aging of the Japanese labor force over time. Another consequence of the slowdown in labor force entry is that it eventually leads to decline in the labor force (in the absence of meaningful flows of workers from overseas). Panel (b) shows that the labor force started to shrink in the late 1990s and continued to decline to date. Given that none of the large developed economies have experienced such a sustained decline in the labor force, it represents a serious nature of aging in Japan.

2.2 Wages and Prices

Next, Figure 3 presents hourly nominal and real wages since 1990. Both nominal and real wages increased in the first half of the 1990s. However, since the late 1990s, both series have been on a downward trend. Specifically, nominal wage fell roughly 7 percent between 1996 and 2013. Real wage declined less because of the decline in the price level. However, deflation did not overturn the decline in real wage. The declines in both real and nominal wages are also quite extraordinary from the international perspective. Below we present the comparison to the U.S. data.

Lastly, Figure 4 shows the price level and the inflation rate measured by the CPI, starting at 1978. Not surprisingly, the behavior of the price level and nominal wage is similar to each other. As is well known, Japan has suffered deflation since the late 1990s, as shown Panel (b).

2.3 Comparison with the U.S. Data

Panel (a) of Figure 5 compares the average age of the labor force in Japan and the U.S. One can see that the US labor force has also been getting older since 1980. However, the US labor force is still younger by more than 2 years old as of 2013. A more striking difference

(a) Nominal Hourly Wage (b) Real Hourly Wage

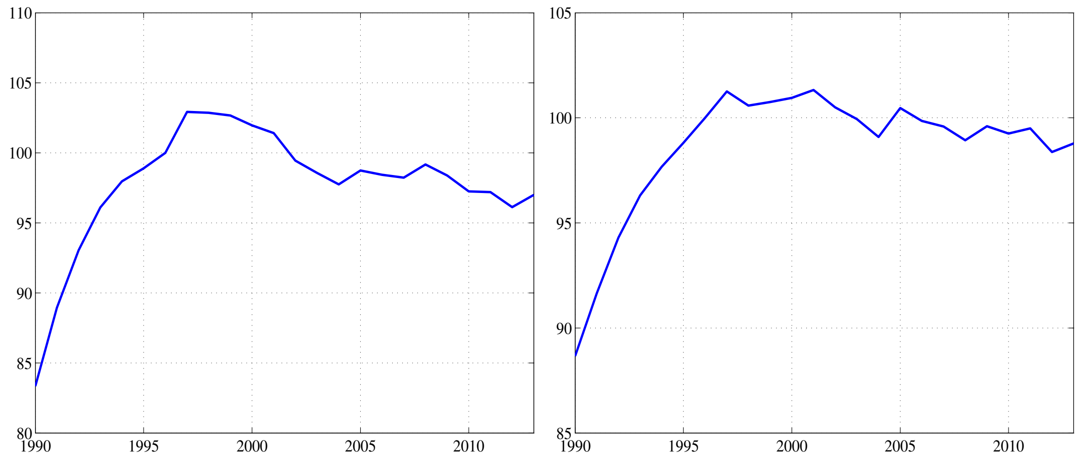

Figure 3: Wages in Japan (1996=100)

(a) Price Level (CPI; 1996=100) (b) Inflation Rate

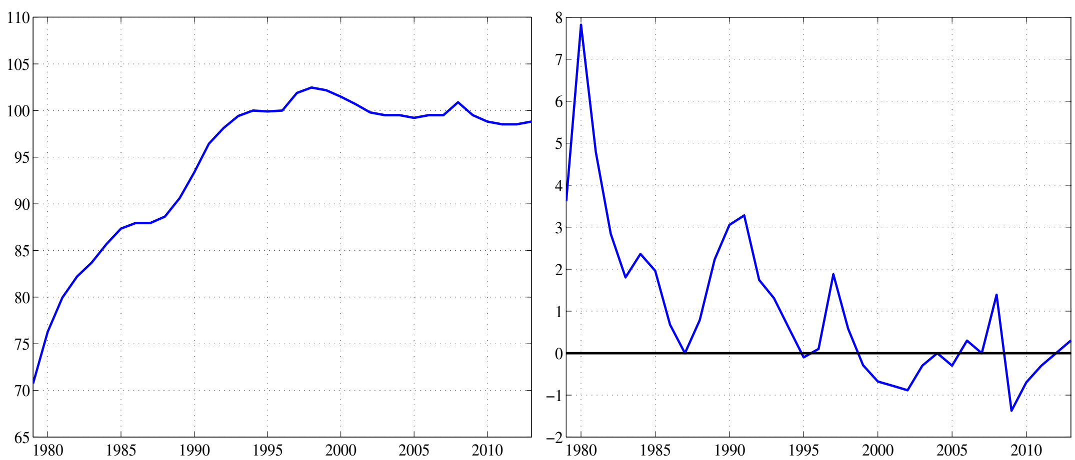

Figure 4: Price Level and Inflation Rate in Japan

can be observed in Panel (b) which shows that the U.S. labor force expanded at a much more rapid pace over the last 45 years. In particular, the U.S. labor force grew 40% between 1996 and 2013, while the Japanese labor force is shrinking. Wages in the two countries have behaved dramatically different in the last 25 years (Panels (c)-(d)). Nominal wage in the U.S. increased 70% between 1996 and 2013 while that in Japan fell more than 5 percent. A Similar pattern can be observed in the comparison of the price level and inflation (Panel (e)-(f)).

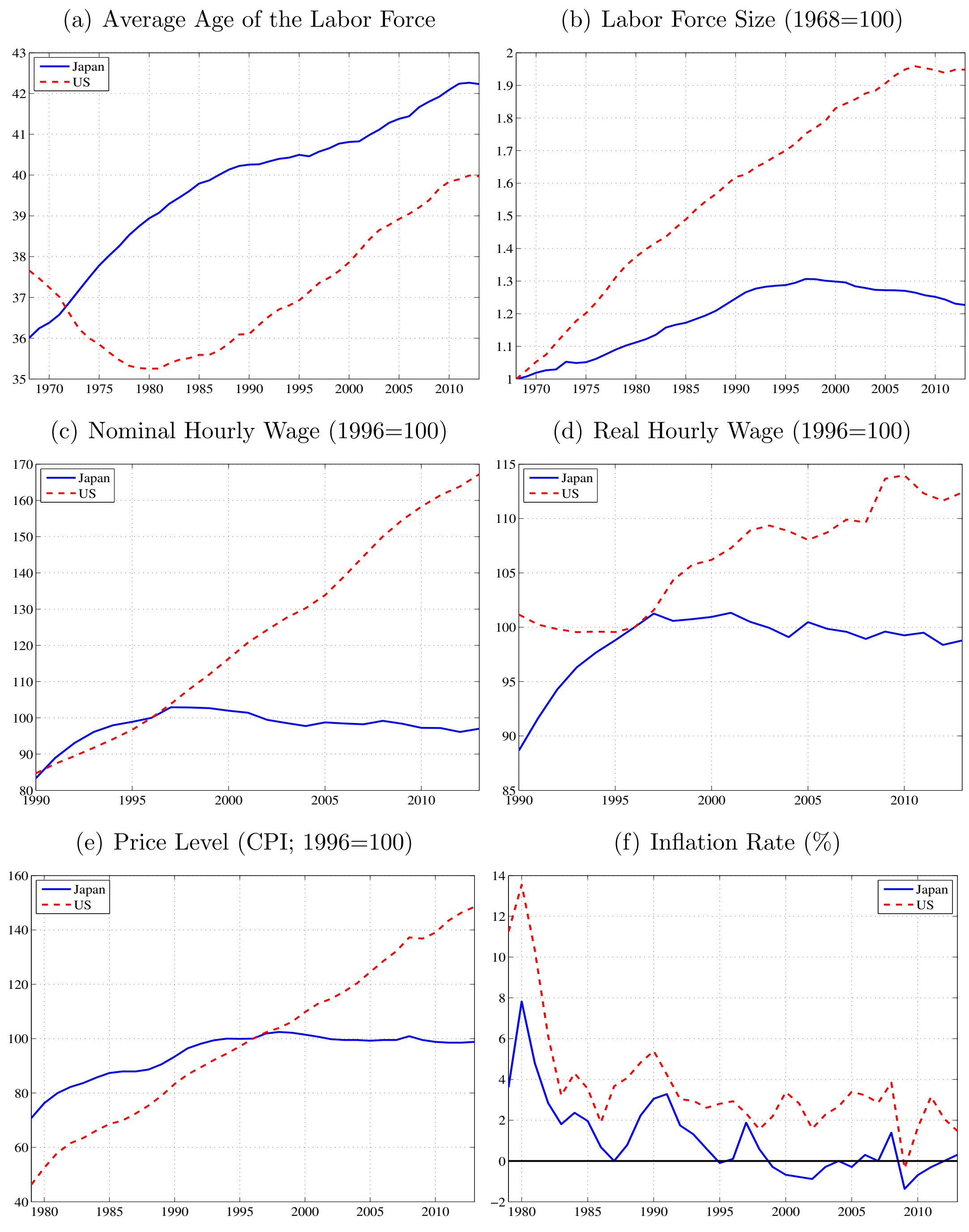

Figure 5: Comparison Between Japan and the U.S.

3 Model

We now lay out the model. The model incorporates what we believe are salient features of the Japanese labor market, aging and firm-specific human capital. The economy consists of three types of agents: households, firms, and a monetary authority. Firms produce and sell differentiated goods to the household. The goods market is characterized by monopolistic competition and price stickiness as in the standard new Keynesian models. Labor is the only input for production; and there are three types of workers, as specified below. Hiring is subject to search frictions. The central bank determines the policy interest rate following the Taylor-type rule.

3.1 Labor Market with Heterogeneous Workers

In the economy, there are young and old workers. A mass of ϕ young workers are born every period and enter the labor market as jobless. Below we consider the effects of the decline in this parameter. Young workers become old with probability μ every period. Old workers die with probability d every period.[6] When young workers become old, they also become "experienced," having higher labor productivity by a factor of $1 + \gamma$. Jobs are subject to exogenous job destruction probabilities. When jobs are destroyed, workers enter the matching market where they look for a new job opportunity. We capture specificity of human capital by assuming that experienced (i.e., old) workers lose their skills at the time of separation. When they lose their skills, they become "inexperienced," losing their experience premium. Note that if human capital is fully firm specific, job separation results in a complete loss of skills. We assume that only a fraction δ of human capital is firm specific. With probability $1 - \delta$, the worker remains experienced and can be hired as an experienced worker retaining the productivity premium. This structure implies that there are three types of workers in the model: (i) young and inexperienced workers, (ii) old and experienced workers, and (iii) old and inexperienced workers. Note that all young workers are inexperienced and thus the terms "young" and "inexperienced" are equivalent in the model. Note also that all experienced workers are old workers. However, there are old workers that used to be experienced but are currently inexperienced due to the skill depreciation at the time of job loss.

We assume that the matching market is divided by the skill level, meaning that experienced workers and inexperienced workers look for a job in the separate labor markets. In other words, firms hire workers separately for different types of jobs: (i) jobs that require previous experience thus suitable only for experienced (and hence old) workers and (ii) entry-level jobs for which any workers can be employed. We call the matching market for the first type of job "E-matching market" and for the latter type of job "I-matching market." Each jobless worker finds a job with probability, either $f^e(\theta_t^e)$ or $f^i(\theta_t^i)$, depending on whether he/she participates in the E-matching market or I-matching market. Job finding probabilities are a function of labor market tightness in the respective matching market (θ_t^e and θ_t^i).[7]

[6] See Cheron et al. (2013) and Esteban-Pretel and Fujimoto (2012) for search/matching models with a more explicit demographic structure.

[7] We exclude the possibility that old workers look for a job in the I-matching market. Such an incentive does not exist under plausible calibrations. In our quantitative exercises below, we ensure that experienced workers are better off in looking for a job in the E-matching market.

Table 1: Summary of Worker Transitions

States at the End of $t-1$	Transition Probabilities	State at the End of t
$n_{y,t-1}$	$(1-\mu)s^i f^i(\theta_t^i)$	$n_{y,t}$
$n_{y,t-1}$	$\mu s^e f^e(\theta_t^e)$	$n_{o,t}^e$
$n_{y,t-1}$	$(1-\mu)s^i(1-f^i(\theta_t^i))$	$u_{y,t}$
$n_{y,t-1}$	$\mu s^e(1-f^e(\theta_t^e))$	$u_{o,t}^e$
$n_{y,t-1}$	$(1-\mu)(1-s^i)$	$n_{y,t}$
$n_{y,t-1}$	$\mu(1-s^e)$	$n_{o,t}^e$
$u_{y,t-1}$	$(1-\mu)f^i(\theta_t^i)$	$n_{y,t}$
$u_{y,t-1}$	$\mu f^e(\theta_t^e)$	$n_{o,t}^e$
$u_{y,t-1}$	$(1-\mu)(1-f^i(\theta_t^i))$	$u_{y,t}$
$u_{y,t-1}$	$\mu(1-f^e(\theta_t^e))$	$u_{o,t}^e$
$n_{o,t-1}^e$	d	$-$
$n_{o,t-1}^e$	$(1-d)s^e\delta f^i(\theta_t^i)$	$n_{o,t}^i$
$n_{o,t-1}^e$	$(1-d)s^e\delta(1-f^i(\theta_t^i))$	$u_{o,t}^i$
$n_{o,t-1}^e$	$(1-d)s^e(1-\delta)f^e(\theta_t^e)$	$n_{o,t}^e$
$n_{o,t-1}^e$	$(1-d)s^e(1-\delta)(1-f^e(\theta_t^e))$	$u_{o,t}^e$
$n_{o,t-1}^e$	$(1-d)(1-s^e)$	$n_{o,t}^e$
$u_{o,t-1}^e$	d	$-$
$u_{o,t-1}^e$	$(1-d)f^e(\theta_t^e)$	$n_{o,t}^e$
$u_{o,t-1}^e$	$(1-d)(1-f^e(\theta_t^e))$	$u_{o,t}^e$
$n_{o,t-1}^i$	d	$-$
$n_{o,t-1}^i$	$(1-d)s^i f^i(\theta_t^i)$	$n_{o,t}^i$
$n_{o,t-1}^i$	$(1-d)s^i(1-f^i(\theta_t^i))$	$u_{o,t}^i$
$n_{o,t-1}^i$	$(1-d)(1-s^i)$	$n_{o,t}^i$
$u_{o,t-1}^i$	d	$-$
$u_{o,t-1}^i$	$(1-d)f^i(\theta_t^i)$	$n_{o,t}^i$
$u_{o,t-1}^i$	$(1-d)(1-f^i(\theta_t^i))$	$u_{o,t}^i$

3.1.1 Timing of Events

We adopt the following timing of events in each period.

1. Demographic transitions occur.
2. Job destruction occurs. If the worker is old, the worker may lose their skill with probability δ at this point.
3. Job search takes place if the worker is jobless; whether the worker finds a job or not is then determined.
4. Production takes place.

Note that workers who lost their job in 2. above can possibly find a new job in the same period. This timing assumption is often used in the new Keynesian literature. We follow this convention.[8]

Table 1 considers all possibilities. The variables n and u respectively denote employment and unemployment. The subscripts y and o indicate the age group, young and old. The superscripts for the old worker e and i indicate the skill group, experienced and inexperienced. Job destruction probabilities are denoted by s with superscripts i and e, indicating the two skill groups are subject to potentially different job destruction probabilities.

3.1.2 Stock-Flow Relationships

Given the transitions summarized in Table 1, we can write the laws of motion for each type of workers as follows.

$$n_{y,t} = (1-\mu)[1-s^i+s^i f^i(\theta_t^i)]n_{y,t-1} + f^i(\theta_t^i)(1-\mu)u_{y,t-1}, \tag{1}$$

$$u_{y,t} = (1-f^i(\theta_t^i))(1-\mu)u_{y,t-1} + s^i(1-f^i(\theta_t^i))(1-\mu)n_{y,t-1} + \phi h_{y,t-1}, \tag{2}$$

$$\begin{aligned} n_{o,t}^e = {} & (1-d)[1-s^e+(1-\delta)s^e f^e(\theta_t^e)]n_{o,t-1}^e \\ & + [1-s^e+s^e f^e(\theta_t^e)]\mu n_{y,t-1} + f^e(\theta_t^e)\left(\mu u_{y,t-1} + (1-d)u_{o,t-1}^e\right), \end{aligned} \tag{3}$$

$$\begin{aligned} u_{o,t}^e = {} & (1-d)(1-f^e(\theta_t^e))u_{o,t-1}^e \\ & + (1-f^e(\theta_t^e))\mu[s^e n_{y,t-1} + u_{y,t-1}] + (1-d)s^e(1-\delta)(1-f^e(\theta_t^e))n_{o,t-1}^e, \end{aligned} \tag{4}$$

$$n_{o,t}^i = (1-d)[1-s^i+s^i f^i(\theta_t^i)]n_{o,t-1}^i + (1-d)f^i(\theta_t^i)\left(s^e\delta n_{o,t-1}^e + u_{o,t-1}^i\right), \tag{5}$$

$$u_{o,t}^i = (1-d)(1-f^i(\theta_t^i))\left(u_{o,t-1}^i + s^i n_o^i + s^e\delta n_{o,t-1}^e\right). \tag{6}$$

The last term in Equation (2) $\phi h_{y,t-1}$ represents the number of workers entering the labor force, expressed as a product of the entry rate ϕ and the total number of young workers, denoted by $h_{y,t}(= n_{y,t}+u_{y,t})$. Given Equations (1) and (2), the total number of young workers h_y can be written as

$$h_{y,t} = (1-\mu)h_{y,t-1} + \phi h_{y,t-1}. \tag{7}$$

The number of old experienced workers ($h_{o,t}^e = n_{o,t}^e + u_{o,t}^e$) and old inexperienced workers ($h_{o,t}^i = n_{o,t}^i + u_{o,t}^i$), respectively, evolve according to:

$$h_{o,t}^e = (1-d)(h_{o,t-1}^e - s^e\delta n_{o,t-1}^e) + \mu_{t-1}h_{y,t-1}, \tag{8}$$

$$h_{o,t}^i = (1-d)(h_{o,t-1}^i + s^e\delta n_{o,t-1}^e). \tag{9}$$

Aggregating these two equations, one gets the evolution of the number of the old workers in the economy:

$$h_{o,t} = (1-d)h_{o,t-1} + \mu h_{y,t-1}. \tag{10}$$

[8] Note also that there are different but equally plausible timing assumptions (for example, we could assume that demographic transitions occur at the end). Those alternative timing assumptions result in a different equilibrium. However, we find that our timing results in somewhat cleaner algebraic expressions than other alternatives and the differences due to alternative timing assumptions are quantitatively unimportant.

Finally, the total number of workers in the economy evolves according to:

$$h_t = h_{t-1} + \phi h_{y,t-1} - d h_{o,t-1}. \tag{11}$$

Note that the number of job seekers in the two matching markets are not equal to us, because those who are separated at the beginning of each period can start looking for a job within the period. Let $\bar{u}_t^i$ and $\bar{u}_t^o$ be the number of job seekers in the I- and E-matching markets, respectively. These variables are written as

$$\bar{u}_t^i = (1-\mu)[s^i n_{y,t-1} + u_{y,t-1}] + (1-d)[\delta s^e n_{o,t-1}^e + s^i n_{o,t-1}^i + u_{o,t-1}^i], \tag{12}$$

$$\bar{u}_t^e = \mu[s^e n_{y,t-1} + u_{y,t-1}] + (1-d)[(1-\delta) s^e n_{o,t-1}^e + u_{o,t-1}^e]. \tag{13}$$

The terms inside the first square brackets in Equation (12) correspond to the number of young workers who were unemployed at the end of the previous period and who have just lost their job at the beginning of this period, respectively. The terms in the second square brackets are the number of old inexperienced job seekers. A similar interpretation applies to Equation (13). We define the share of old inexperienced workers within $\bar{u}_t^i$ as $\omega_{o,t}$:

$$\omega_{o,t} \equiv \frac{(1-d)[\delta s^e n_{o,t-1}^e + s^i n_{o,t-1}^i + u_{o,t-1}^i]}{\bar{u}_t^i}. \tag{14}$$

The matching function in each market, indicated by the superscript i or e, takes the following Cobb-Douglas form:

$$m_t^i = \overline{m}^i \left(\bar{u}_t^i\right)^{\alpha} \left(v_t^i\right)^{1-\alpha}, \tag{15}$$

$$m_t^e = \overline{m}^e \left(\bar{u}_t^e\right)^{\alpha} \left(v_t^e\right)^{1-\alpha}, \tag{16}$$

where the left-hand side gives the number hires in each market, v_t^i and v_t^e gives the number job openings posted in I- and E-matching market, respectively, and $\bar{m}^i$ and $\bar{m}^e$ are scale parameters of the two matching functions. The job finding probabilities in the two job search pools are written as

$$f^i(\theta_t^i) = \overline{m}^i \left(\theta_t^i\right)^{1-\alpha},$$
$$f^e(\theta_t^e) = \overline{m}^e \left(\theta_t^e\right)^{1-\alpha},$$

where θ_t^i and θ_t^e represent labor market tightness in the two matching markets and defined as

$$\theta_t^i = \frac{v_t^i}{\bar{u}_t^i},$$
$$\theta_t^e = \frac{v_t^e}{\bar{u}_t^e}.$$

Let us also define the job filling probability for each job vacancy posted in the two markets:

$$q^i(\theta_t^i) = \overline{m}^i \left(\theta_t^i\right)^{\alpha},$$
$$q^e(\theta_t^e) = \overline{m}^e \left(\theta_t^e\right)^{\alpha}.$$

3.2 Firms

Let us now consider the firm's profit maximization problem. Each firm combines the three types of labor to produce differentiated goods that are sold in the monopolistically competitive market at price $p_t(j)$. The firm is subject to the convex price adjustment cost as in Rotemberg (1982) and search frictions in hiring workers. Regarding the latter, the firm pays κ^i and κ^e per vacancy posted in I- and E-matching markets, respectively.

The following linear production technology is available to the firms, which are indexed by j:

$$y_t(j) = n_{y,t}(j) + n^i_{o,t}(j) + (1+\gamma)n^e_{o,t}(j).$$

where $y_t(j)$ is output of the firm j in period t. Let $\mathbf{s}^f_t(j)$ be the vector of the state variables for the firm j coming into period t. The firm maximizes its value, $\Pi(\mathbf{s}^f_t(j))$, by choosing $p_t(j), n_{y,t}(j), n^i_{o,t}(j), n^e_{o,t}(j), v^i_t(j)$, and $v^e_t(j)$:

$$\begin{aligned}\Pi\big(\mathbf{s}^f_t(j)\big) = \max &\left(\frac{p_t(j)}{P_t}\right)^{1-\epsilon} Y_t - w_{y,t}(j)n_{y,t}(j) - w^i_{o,t}(j)n^i_{o,t}(j) - w^e_{o,t}(j)n^e_{o,t}(j) - T \\ &- \kappa^i v^i_t(j) - \kappa^e v^e_t(j) - \frac{\chi}{2}\left(\frac{p_t(j)}{p_{t-1}(j)} - 1\right)^2 Y_t + (1+g_{t+1})\hat{\beta}_{t,t+1}\Pi\big(\mathbf{s}^f_{t+1}(j)\big), \end{aligned} \tag{17}$$

where Y_t is aggregate output in the economy; P_t is the aggregate price index; χ is the parameter for the price adjustment cost; ϵ is the elasticity of substitution between goods; $\hat{\beta}_{t,t+1}$ is a stochastic discount factor of the representative household, which will be specified below; T is a lump sum tax that will be used to finance unemployment insurance benefits; and $w_{y,t}(j)$, $w^i_{o,t}(j)$, and $w^e_{o,t}(j)$ are wages of young workers, old inexperienced workers, and old experienced workers, respectively. Wages are determined though Nash bargaining between each worker and the firm. We will discuss the wage determination process below.

The vector of the state variables $\mathbf{s}^f_t(j)$ includes $p_{t-1}(j)$, $n_{y,t-1}(j)$, $n^i_{o,t-1}(j)$, and $n^e_{o,t-1}(j)$.[9] Note also that in writing the first term on the right-hand side of (17), we used the demand function for the good j:

$$y_t(j) = \left(\frac{p_t(j)}{P_t}\right)^{-\epsilon} Y_t.$$

The firm is subject to the following constrains:

$$\left[\frac{p_t(j)}{P_t}\right]^{-\epsilon} Y_t = n_{y,t}(j) + n^i_{o,t}(j) + (1+\gamma)n^e_{o,t}(j), \tag{18}$$

$$n_{y,t}(j) = (1-\mu)(1-s^i)n_{y,t-1}(j) + (1-\omega_{o,t})q^i(\theta^i_t)v^i_t(j), \tag{19}$$

$$n^i_{o,t}(j) = (1-d)(1-s^i)n^i_{o,t-1}(j) + \omega_{o,t}q^i(\theta^i_t)v^i_t(j), \tag{20}$$

$$n^e_{o,t}(j) = (1-d)(1-s^e)n^e_{o,t-1}(j) + (1-s^e)\mu n_{y,t-1}(j) + q^e(\theta^e_t)v^e_t(j). \tag{21}$$

[9]The price level drops out of the list in the end but we include it here for completeness.

Recall that $\omega_{o,t}$ is the share of old workers in the I-matching market. When the firm is posting a vacancy in the I-matching market, the probability of finding any worker is given by $q^i(\theta^i_t)$, and conditional on that, he is an old worker with probability $\omega_{o,t}$, which is defined in (14), and a young worker with probability $1-\omega_{o,t}$.

The firm's decision is characterized by the following first order conditions. At this point, we impose the symmetry of the equilibrium and drop the index j. First,the following condition characterizes the evolution of the inflation rate in the presence of the price adjustment cost as in the standard new Keynesian model.

$$1-\epsilon-\chi\left(\pi_t-1\right)\pi_t+(1+g_{t+1})\hat{\beta}_{t,t+1}\chi\left(\pi_{t+1}-1\right)\pi_{t+1}\frac{Y_{t+1}}{Y_t}+\tau_t\epsilon=0, \tag{22}$$

where τ_t is the Lagrange multiplier associated with constraint (18) and π_t represents the gross inflation rate, defined by $\frac{P_t}{P_{t-1}}$. The multiplier τ_t can be interpreted as the marginal cost for the firm as in the standard new Keynesian model. The following two equations govern job creation (vacancy posting) in the two matching market:

$$\omega_{o,t}J^i_{o,t}+(1-\omega_{o,t})J_{y,t}=\frac{\kappa^i}{q^i(\theta^i_t)}, \tag{23}$$

$$J^e_{o,t}=\frac{\kappa^e}{q^e(\theta^e_t)}, \tag{24}$$

where $J_{y,t}$, $J^i_{o,t}$ and $J^e_{o,t}$ are the Lagrange multipliers associated with constraints (19), (20), and (21), respectively and can be interpreted as the marginal gain to the firm from adding each type of labor by one unit. Equations (23) and (24) equate the marginal cost of posting a vacancy (RHS) and the marginal gain (LHS) in the I-matching market and the E-matching market, respectively. The LHS of Equation (23) indicates that the gain from posting a vacancy in the I-matching market is influenced by the composition of the matching market. In particular, if $J_{y,t}>J^i_{o,t}$ holds, then a higher $\omega_{o,t}$ would lower the value of LHS and thus, in equilibrium, market tightness θ^i_t decreases. Lastly, the marginal value of each type of job evolves according to:

$$J_{y,t}=\tau_t-w_{y,t}+\hat{\beta}_{t,t+1}\Big[(1-\mu)(1-s^i)J_{y,t+1}+(1-s^e)\mu J^e_{o,t+1}\Big], \tag{25}$$

$$J^e_{o,t}=(1+\gamma)\tau_t-w^e_{o,t}(j)+(1-d)(1-s^e)\hat{\beta}_{t,t+1}J^e_{o,t+1}, \tag{26}$$

$$J^i_{o,t}=\tau_t-w^i_{o,t}+(1-d)(1-s^i)\hat{\beta}_{t,t+1}J^i_{o,t+1}. \tag{27}$$

The interpretation of these three equations is straightforward. First, a job filled by a young worker yields a flow payoff of $\tau_t-w_{y,t}$. The worker becomes an old and experienced worker with probability μ, while he continues to be inexperienced with probability $1-\mu$. Next, if the worker is experienced, the flow payoff is $(1+\gamma)\tau_t-w^e_{o,t}$. When either the worker exits from the labor force with probability d or the job is destroyed with probability s^e_o, the firm obtains zero due to free entry; otherwise the value is $J^e_{o,t+1}$. Lastly, the flow payoff for the firm when the job is filled by an old but inexperienced worker is given by $\tau_t-w^i_{o,t}$. When either the worker exits from the labor force with probability d or the job is destroyed with probability s^i_o, the firm obtains zero due to free entry; otherwise the value is $J^i_{o,t+1}$.

3.3 Household

The household sector consists of different types of workers in terms of their age, their skill level, and their labor market status. We assume that the representative household pools incomes of all members and allocates consumption across its members cooperatively. The household is also assumed to be fully altruistic towards its future members as well. These assumptions imply that the member's age, skill, and labor market status do not matter for his/her consumption.[10]

The household maximizes the following value function $V(.)$:

$$V(\mathbf{s}_t^h) = \max h_t u\left(c_t\right) + \beta V(\mathbf{s}_{t+1}^h), \tag{28}$$

where c_t is the per-capita consumption index given by:

$$c_t \equiv \left(\int_0^1 c_t(j)^{1-\frac{1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}$$

with $c_t(j)$ representing the quantity of good j consumed by each member of the household. A continuum of goods exist over the interval $[0, 1]$. $\mathbf{s}_t^h$ is the vector of the state variables for the household. Recall that h_t represents population in the economy and thus Equation (28) implies that the household maximizes total welfare of the dynasty into the indefinite future, with the future periods discounted by a common discount factor β per period. All employed workers supply labor inelastically with the same constant disutility level (regardless of their types). We also assume that jobless workers look for a new job with the same constant disutility level, again regardless of their types. Accordingly, hours of work and search do not enter the value function above.[11]

Each member of the household allocates his/her consumption expenditures among the different goods by maximizing c_t for any given level of expenditures $\int_0^1 p(j)c(j)dj$, resulting in the demand equations:

$$c_t(j) = \left(\frac{p_t(j)}{P_t} \right)^{-\epsilon} c_t \tag{29}$$

for all $j \in [0, 1]$ where $P_t \equiv \left(\int_0^1 p_t(j)^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}}$. Under this situation, we can write the household budget constraint as:

$$h_t c_t + \frac{B_{t+1} h_t}{P_t} = (1 + i_{t-1}) \frac{B_t h_{t-1}}{P_t} + w_{y,t} n_{y,t} + w_{o,t}^e n_{o,t}^e + w_{o,t}^i n_{o,t}^i + b_y u_{y,t} + b_o^e u_{o,t}^e + b_o^i u_{o,t}^i + d_t,$$

where B_t represents the per-capita bond holdings that pay nominal interest; b_y, b_o^e, and b_o^i represent unemployment insurance benefits that each type of jobless worker receives at the

[10] We can extend our model by allowing for heterogeneity in the consumption side of the model as in Fujiwara and Teranishi (2008). Our intension of abstracting away from it is to first focus on the effects of labor market heterogeneities on inflation dynamics in the current paper.

[11] This specification is adopted only for the purpose of focusing on the margins we would like to highlight in this paper. We can easily adopt a different formulation, for example, in which unemployed workers enjoy some utility from leisure.

end of the period; and d_t represents a dividend from the firm that the household owns. The state variables for the household $\mathbf{s}_t^h$ consists of $n_{y,t-1}$, $n_{o,t-1}^e$, $n_{o,t-1}^i$, $h_{o,t-1}^e$, and B_t.

The first-oder conditions with respect to consumption and bond holdings result in the following consumption Euler equation:

$$u'(c_t) = \beta \frac{1+i_t}{\pi_{t+1}} u'(c_{t+1}).$$

Note that the stochastic discount factor $\hat{\beta}_{t,t+1} \equiv \frac{u'(c_{t+1})}{u'(c_t)}$ was used in discounting the future profit flows in the firm's problem.

3.3.1 Marginal Values of Employment

We assume that wages are determined through Nash bargaining between each individual worker and the firm. For this purpose, we need to compute the marginal value of each type of employment. Note that evolutions of the marginal values take into account the following laws of motion:

$$n_{y,t} = (1-\mu)[1-s^i+s^if^i(\theta_t^i)]n_{y,t-1} + f^i(\theta_t^i)(1-\mu)(h_{y,t}-n_{y,t-1}), \tag{30}$$

$$\begin{aligned} n_{o,t}^e = (1-d)[1-s^e+(1-\delta)s^ef^e(\theta_t^e)]n_{o,t-1}^e + [1-s^e+s^ef^e(\theta_t^e)]\mu n_{y,t-1} \\ + f^e(\theta_t^e)[\mu(h_{y,t-1}-n_{y,t-1}) + (1-d)(h_{o,t-1}^e - n_{o,t-1}^e)], \end{aligned} \tag{31}$$

$$n_{o,t}^i = (1-d)[(1-s^i+s^if^i(\theta_t^i))n_{o,t-1}^i + f^i(\theta_t^i)(s^e\delta n_{o,t-1}^e + h_{o,t-1} - h_{o,t-1}^e - n_{o,t-1}^i)], \tag{32}$$

$$h_{o,t}^e = (1-d)(h_{o,t-1}^e - s^e\delta n_{o,t-1}^e) + \mu_{t-1}h_{y,t-1}, \tag{33}$$

where we substituted out $u_{y,t-1}$, $u_{o,t-1}^e$, $u_{o,t-1}^i$, and $h_{o,t-1}^i$ from (1), (3), (5), and (8). To simplify the notation, let us introduce the following four variables:

$$M_{y,t} \equiv \frac{\partial V(\mathbf{s}_t^h)}{\partial n_{y,t-1}}, M_{o,t}^e \equiv \frac{\partial V(\mathbf{s}_t^h)}{\partial n_{o,t-1}^i}, M_{o,t}^i \equiv \frac{\partial V(\mathbf{s}_t^h)}{\partial n_{o,t-1}^i}, \text{and } D_t \equiv \frac{\partial V(\mathbf{s}_t^h)}{\partial h_{o,t-1}^e}.$$

Given (30) through (32), the evolutions of these four variables are written as follows:

$$\begin{aligned} M_{y,t} = {} & u'(c_t)(w_{y,t}-b_y) \\ & + \beta\Big[(1-\mu)(1-s^i)(1-f^i(\theta_{t+1}^i))M_{y,t+1} + \mu(1-s^e)(1-f^e(\theta_{t+1}^e))M_{o,t+1}^e\Big], \end{aligned} \tag{34}$$

$$M_{o,t}^i = u'(c_t)(w_{o,t}^e - b_o^e) + (1-d)(1-s^i)(1-f^i(\theta_{t+1}^i))\beta M_{o,t+1}^i, \tag{35}$$

$$\begin{aligned} M_{o,t}^e = u'(c_t)(w_{o,t}^e - b_o^e) + (1-d)\beta\Big[(1-s^e)(1-f^e(\theta_{t+1}^e))M_{o,t+1}^e \\ - \delta s^e\big\{f^e(\theta_{t+1}^e)M_{o,t+1}^e - f^i(\theta_{t+1}^i)M_{o,t+1}^i + D_{t+1}\big\}\Big], \end{aligned} \tag{36}$$

$$D_t = u'(c_t)(b_o^e - b_o^i) + (1-d)\beta\Big[f^e(\theta_{t+1}^e)M_{o,t}^e - f^i(\theta_{t+1}^i)M_{o,t+1}^i + D_{t+1}\Big]. \tag{37}$$

Equations (34) through (35) give the marginal values of each type of employment to the household net of having one more unemployed worker of the corresponding type. In Equation (34), the first term represents the flow surplus of employment for the young worker and the terms that follow capture the future possibilities that the worker stays young with probability $1-\mu$ while the worker becomes old and experienced with probability μ. A similar interpretation applies to Equation (35). Equation (36) looks somewhat different because of the terms in the curly brackets which capture the possibility of loss of human capital which occurs with probability $s^e\delta$. The value D_t whose evolution is described in Equation (37) corresponds to the difference in the values of being unemployed as an old experienced worker and an old inexperienced worker.

3.4 Wages

We assume that each type of worker and the firm engage in Nash bargaining individually. Let $S_{y,t}$ be the joint surplus of the match between the young worker and the firm, i.e., $S_{y,t} = J_{y,t} + \frac{1}{u'(c_t)} M_{y,t}$. This surplus is split between the worker and the firm according to the worker bargaining power η and the firm bargaining power $1-\eta$:

$$\begin{aligned}(1-\eta)S_{y,t} &= J_{y,t},\\ \eta S_{y,t} &= \frac{1}{u'(c_t)} M_{y,t}.\end{aligned}$$

Plugging (25) and (34) in $\eta J_{y,t} = (1-\eta)\frac{1}{u'(c_t)} M_{y,t}$ and solving for the wage, we obtain the following expression:

$$w_{y,t} = \eta\tau_t + (1-\eta)b_y + \eta\hat{\beta}_{t,t+1}\left[(1-\mu)(1-s^i)f^i(\theta^i_{t+1})J_{y,t+1} + \mu(1-s^e)f^e(\theta^e_{t+1})J^e_{o,t+1}\right], \tag{38}$$

Similar algebras applied to the other two types of matches result in the following wage equations:

$$w^i_{o,t} = \eta\tau_t + (1-\eta)b^i_o + (1-d)(1-s^i)\eta\hat{\beta}_{t,t+1}f^i(\theta^i_{t+1})J^i_{o,t+1} \tag{39}$$

$$\begin{aligned}w^e_{o,t} = \eta(1+\gamma)\tau_t + (1-\eta)b^e_o + (1-d)\hat{\beta}_{t,t+1}\Bigg[&(1-s^e)f^e(\theta^e_{t+1})\eta J^e_{o,t+1}\\ &+ \delta s^e\left\{f^e(\theta^e_{t+1})\eta J^e_{o,t+1} - f^i(\theta^i_{t+1})\eta J^i_{o,t+1} + \frac{D_{t+1}}{u'(c_{t+1})}\right\}\Bigg].\end{aligned} \tag{40}$$

3.5 Monetary Policy and Resource Constraint

Imposing zero net supply of the bonds in the aggregate, and assuming that $b_y u_{y,t} + b^e_o u^e_{o,t} + b^i_o u^i_{o,t} = T$, one can obtain the following resource constraint:

$$Y_t = h_t c_t + \kappa^i v^i_t + \kappa^e v^e_t + \frac{\chi}{2}(\pi_t - 1)^2 Y_t.$$

Lastly, monetary policy is characterized by a standard Taylor-type rule:

$$1 + i_t = \frac{1}{\beta} + \psi_\pi(\pi_t - 1).$$

where we assume that $\psi_\pi > 1$.

4 Benchmark Calibration

The calibration below intends to replicate the Japanese economy at the start of the aging process, assuming that the economy is in the steady state at that point. In the quantitative exercises below, we consider the effects of a lower value of ϕ. One period in the model corresponds to one quarter in the actual economy.

4.1 Demographic Transitions

We assume that each worker in the model spends on average 20 years as a "young" worker and 25 years as an old worker. What we have in mind as a career of a typical worker is that he enters the labor force when he is 20 years old, accumulates his experience over the next 20 years, then becomes an experienced worker when he is 40 years old, spends the rest of his career as an old worker, and then retires (or die) at the age of 65 years old. This average demographic transition implies that $\mu = 1/80 = 0.0125$ and $d = 0.01$. Next, we translate the total fertility rate into the entry rate ϕ. Note that the TFR represents the number of children that are born to a woman over her childbearing years, and it was above 4 at the peak in 1948. Suppose that childbearing years correspond to the 20-year period that each worker spends as a young worker.[12] The TFR of 4 implies that each young worker reproduces 2 young workers over this 20-year period, corresponding to $\phi = 0.025(= 2/80)$. But given that our young worker's definition is somewhat narrower than the actual childbearing years, we choose to use $\phi = 0.02$. Note that the steady state of the economy is characterized by a constant population growth. Specifically, Equation (7) implies that this growth rate is given by $g = \phi - \mu = 0.02 - 0.0125 = 0.0075$, where $g = \frac{h_t}{h_{t-1}} - 1$.

4.2 Labor Market Transitions

Next, let us discuss the calibration of the parameter values related to labor market transitions. Note that transition rates are all expressed as quarterly values. First, to set separation rates, we refer to the evidence presented by Lin and Miyamoto (2012) who show that the monthly job separation rate into unemployment was 0.4% between 1980 and 2009. We set both s^i and s^e at 1.2% (per quarter).[13] Next, we target the steady-state job finding rates

[12]This assumption is not exactly correct because childbearing years in the calculation of the TFR are between 15-49 years old. But we are considering only the middle 20-year period for simplicity.

[13]It seems that $s^e < s^i$ is a more appropriate choice. Esteban-Pretel and Fujimoto (2012) indeed show that the separation rate declines as workers get older. But we want keep our calibration as simple as possible.

$f^i(\theta^i)$ and $f^e(\theta^e)$ both at 35%. Again, this value is roughly consistent with the monthly evidence presented by Lin and Miyamoto (2012). We also impose that, in the steady state $q^i(\theta^i) = f^i(\theta^i)$ and $q^i(\theta^i) = f^e(\theta^e)$. These assumptions allow us to determine the scale parameter of the matching functions (see (15) and (16)), $\overline{m}^i$ and $\overline{m}^e$ for a given value of α. By setting $\alpha = 0.5$, we can set both $\overline{m}^i$ and $\overline{m}^i$ to 0.35. The only remaining parameter for calibrating the steady state stock-flow relationships is δ which measures the risk of the loss of human capital. We simply set this parameter at 0.8 in the benchmark calibration and show how the model's behavior changes as we lower δ.

4.3 Remaining Parameters

The discount factor is chosen to be 0.99, which appears to a standard value for a quarterly calibration of macro models. The value of the CRRA is set to 2, a value that is within a plausible range in macro literature. We assume that the worker bargaining power η is 0.5, which is often used in the search/matching literature given the lack of direct evidence. We set the experience premium γ at 60%, which is based on the observed wage profile in Japan. The observed profile shows that old workers (those who are between 40-64) on average make roughly 40% more than young workers (those who are between 20-39). Note that the slope of the observed profile does not measure $1 + \gamma$ because old workers in our model include those who have lost their skills thus have become inexperienced, which lowers the measured premium. By setting $\gamma = 0.6$, we can match the observed slope. The level of unemployment insurance benefits is set by assuming that each worker receives 60% of their productivity while unemployed. This procedure implies that $b_y = 0.6$ and $b_o^e = (1 + \gamma) * 0.6 = 0.96$ given that productivity of the young worker is normalized to 1 and old experienced worker enjoys 60% productivity premium. Regarding the old inexperienced worker, we assume that their benefit level is tied to their productivity level as an experienced worker, meaning that $b_o^i = b_o^e$.[14] Given the parameter values so far, we can compute all continuation values in the steady state of the economy. Using the steady state values of J_y, J_o^e, and J_o^i in the free entry conditions (24) and (23), we can back out the vacancy posting costs in the two matching markets. This procedure yields $\kappa_i = 0.251$ and $\kappa_e = 0.337$.

We set the remaining three parameters ψ_π, ϵ and χ to the values standard in the new Keynesian literature. First, the elasticity of substitution ϵ is set to the conventional value of 6, which implies the steady-state markup at 20%. Given this steady-state markup, the Rotemberg (1982) price adjustment cost parameter χ is chosen to be 77. This value corresponds to the average price resetting frequency being once a year under the Calvo (1983) setup.[15] Finally, we set the policy reaction parameter in the Taylor rule ψ_π at 1.5, the value originally proposed by Taylor (1993).

[14] An alternative is to link it to the productivity level as an inexperienced worker. In light of the actual UI benefit scheme, the procedure we adopted appears more reasonable, given the replacement ratio is usually tied to the worker's wage prior to job loss. However, the alternative calibration does not change our results materially.

[15] As is well known, the aggregate supply equations under the Rotemberg (1982) and Calvo (1983) formulations are observationally equivalent up to the first-order approximation.

4.4 Initial Steady State Equilibrium

Table 2) presents steady-state values of wages, employment levels, and unemployment rates at different levels of aggregation in the initial steady state. Note that employment levels are presented as a share of the total labor force (population) while unemployment rates are expressed as a share of the labor force of each type.

At the aggregate level, the unemployment rate is 5.3%, which is higher than the levels observed in the 1970s and 1980s. The main reason for this is that our model assumes that all young workers enter the labor force as unemployed and in the initial steady state, the entry rate is assumed to be very high. One can see that the young worker's unemployment rate is 7.4% while the old worker's unemployment rate is much lower at 2.3%. Within old workers, the experienced worker's unemployment rate is only 0.8% while inexperienced worker's unemployment rate is 5.1%. The reason for the higher unemployment rate for the latter group is that 80% (δ) of the job loss flow from experienced employment goes into this I-matching pool in our calibration.[16]

The steady-state employment levels indicate that within all employed workers, roughly 43% are old workers and the rest are young workers. Roughly one third of old workers are experienced (14% as a share of the labor force) and two thirds are inexperienced. The average wage is calculated as 0.956. Note that we normalize average productivity of the young worker and old inexperienced worker at 1 and the experienced worker enjoys 60% skill premium. The value 0.956 is the weighted average of the wages of all three types. The wage of the experienced worker is the highest given the large skill premium. There are two reasons why w_o^i and w_y differ from each other even though their productivities are equal. First, young worker's current wage incorporates the possibility that they become experienced in the future (see $J^e_{o,t+1}$ in Equation (38)), which raises w_y relative to w_o^i, while the old worker's continuation value drops to zero when hit by the d shock. However, old worker's wage is pushed up because his flow outside option value b_o^i is higher (remember that we set b_o^i by linking it to their productivity as an experienced worker). The latter effect is larger than the first effect and thus w_o^i and higher than w_y in our calibration. The average wage of old workers (including both experienced and inexperienced) is roughly 40% higher than the young worker's wage. As mentioned above, this measured age premium is consistent with the observed wage profile in Japan.

5 Quantitative Exercises

In this section, we analyze the perfect foresight equilibrium path, assuming that the labor force entry rate falls to a new steady state value. The assumed path of the entry rate mimics the path of the observed data plotted in Figure 1. Demographic transitions take place only gradually and thus it takes a long time for the economy to converge to a new steady state even if ϕ itself reaches a new value relatively quickly.

[16] A larger constant flow into the pool implies a larger stock in the steady state.

Table 2: Initial Steady-State Values

	Wages		Employment Levels		Unemployment Rates	
Aggregate	0.956	$\frac{n_y w_y + n_o^e w_o^e + n_o^i w_o^i}{n_y + n_o^e + n_o^i}$	0.947	$\frac{n_o^i + n_o^e + n_y}{h}$	0.053	$\frac{u_o^i + u_o^e + u_y}{h}$
Young	0.816	w_y	0.540	$\frac{n_y}{h}$	0.074	$\frac{u_y}{h_y}$
Old	1.142	$\frac{n_o^e w_o^e + n_o^i w_o^i}{n_o^e + n_o^i}$	0.407	$\frac{n_o^e + n_o^i}{h}$	0.023	$\frac{u_o^e + u_o^i}{h_o^e + h_o^i}$
Old Exp.	1.303	w_o^e	0.138	$\frac{n_o^e}{h}$	0.008	$\frac{u_o^e}{h_o^e}$
Old Inexp.	0.831	w_o^i	0.269	$\frac{n_o^i}{h}$	0.051	$\frac{u_o^i}{h_o^i}$

(a) Assumed Entry Rate Path

(b) Implied Path of the Labor Force

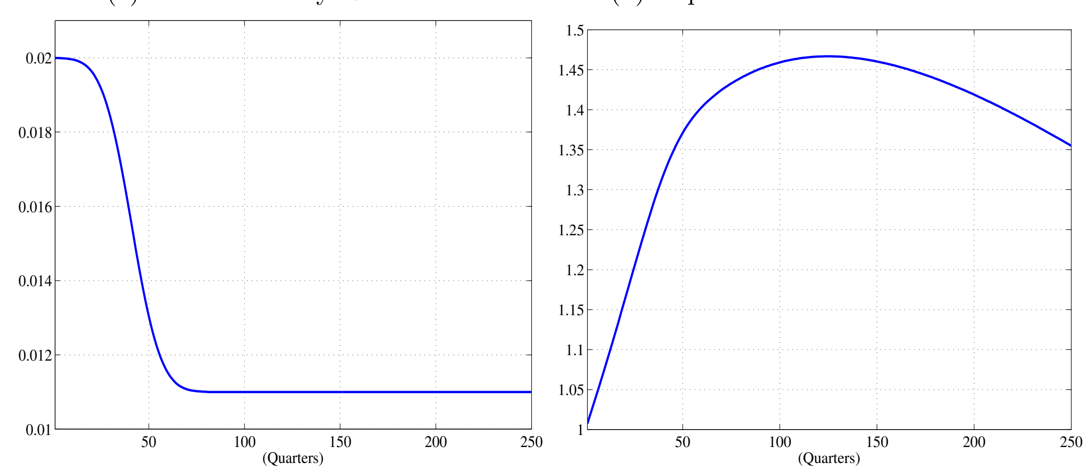

Figure 6: The Entry Rate and Labor Force

Notes: The labor force is normalized to 1 in the initial period.

5.1 Perfect Foresight Equilibrium

Recall that the labor force entry rate ϕ was set to 2% per quarter in the initial steady state. We assume that it falls to 1.1% roughly over the following 15-year period. This assumed path is plotted in Panel (a) of Figure 6. The entry rate of 1.1% per quarter corresponds to the total fertility rate that is somewhat below 1.8. With this level of the entry rate, the labor force begins to fall at roughly the same timing as in the actual data presented (compare Panel (b) of Figures 1 and 6)).

Figure 7 presents the equilibrium paths of the key variables in the model. First, observe that the inflation rate increases quite significantly and then falls from around 50 quarters. The inflation rate drops below zero at around 120th quarter (30 years after the decline in ϕ), and stays persistently negative (before eventually coming back to zero). While the deflationary pressure is not extraordinary, it is indeed present and notable. We can appreciate this result more, when we consider alternative calibrations to highlight the mechanism in the model. In Panel (d), we can also observe that average real wage increases for the first 30 years and then gradually falls. Remember that real wage in Japan started to fall in the late 1990s (see Panel (b) of Figure 3) and thus the timing of the reversal in the model is in line with the observed pattern. Panels (e) and (f) present average wages of old and young workers. We can see that wages of young and old workers, when considered separately, behave differently from the aggregate wage series.[17] This fact suggests that the changing composition of workers over time plays an important role in shaping the behavior of aggregate wage. Overall wage increases initially (except for the first 5 years or so), because of the increases in more productive workers, i.e., old experienced workers. In this initial phase of labor force aging, having more experienced workers in the labor force contributes to higher per-capita output and thus per-capita consumption (Panel (a)). One can also observe that labor productivity increases in a similar manner (Panel (j)) for the same reason. However, as aging progresses further, the share of inexperienced workers increases, pushing down aggregate wage because these inexperienced workers earn much less than experienced workers. In the model, this process occurs through the effect that old workers loses their skills, flowing into the I-matching market. The variable $\omega(=\frac{\bar{u}^i_{o,t}}{\bar{u}^i_t})$ captures this effect and is plotted in Panel (l). In this later phase, per-capita consumption and labor productivity declines.

The path of the real interest rate is plotted in Panel (b) and reflects the behavior of per-capita consumption. The path of the inflation rate is then implied by the paths of nominal and real interest rates. In particular, deflation occurs in the later stage of labor force aging because of the declines in real wage, which, in turn, results from the increase in inexperienced old workers.

The marginal cost (τ) varies over time due to price stickiness (Panel (k)). Specifically, τ oscillates as labor force entry falls. The swing in the marginal cost is indeed reflected in

[17] Note that there are two types of old workers, experienced and inexperienced, and thus the series plotted in Panel (e) is also an weighted average of these two types of workers. Young workers are homogeneous in the model.

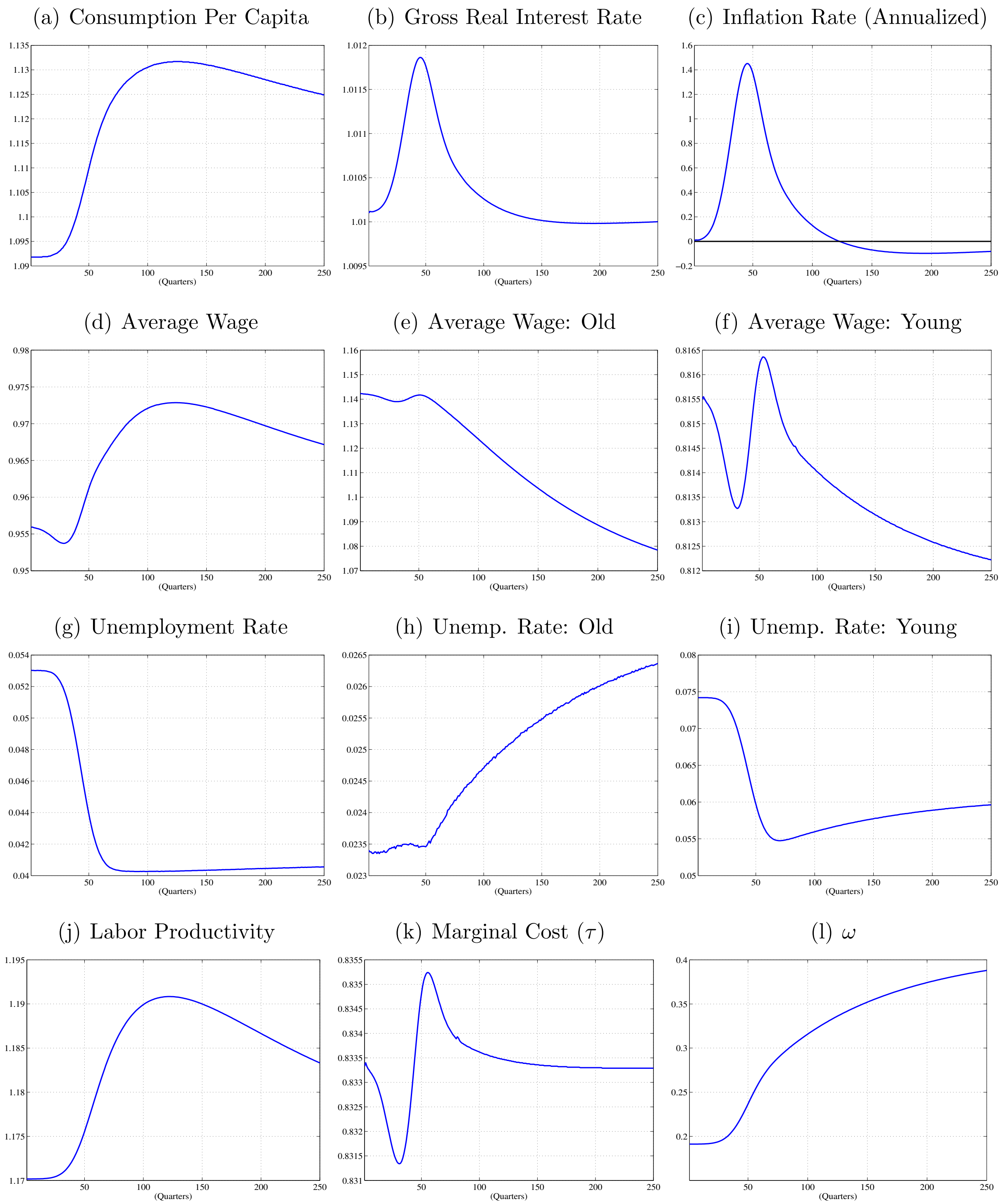

Figure 7: Perfect Foresight Equilibrium Paths (Benchmark Calibration)

individual-level wages: The swing is clearly seen in young worker's wage (Panel (f)); And we can detect the swing in the average wage of old workers, albeit it is small.[18]

The unemployment rate falls rather sharply initially as entry into the labor force drops (Panel (j)). Remember that young workers enter as unemployed into the labor force and thus a smaller volume of this flow directly reduces the unemployment rate. The effect can be seen more clearly in Panel (i), where the unemployment rate among young workers drops almost 2 percentage points. The unemployment rate among old workers tend to increase over time. However, the magnitude of the increase is relatively small (only 0.3 percentage point over the 250 quarters). This small, yet steady increase in the unemployment rate among old workers is due to the higher ω. That is, as the share of old inexperienced workers in the I-matching market (ω) increases, the pace of job creation in this market slows down. As we saw earlier, wage of the old (inexperienced) worker w_o^i tends to be higher than that of the young worker. This means that job creation slows down as ω gets higher (i.e., gain from creating an entry-level job gets smaller because there are more inexperienced workers that are more expensive for the firm). This channel adversely affects the unemployment rate among young workers as well. Recall that the unemployment rate among young workers falls initially due to the direct effect of the lower entry rate. As this direct effect fades away, the adverse effect from higher ω raises the unemployment rate among young workers. One can see that the aggregate unemployment rate also increases slightly in the later phase of aging. The aggregate unemployment rate, however, does not increase as much as the unemployment rates among young and old workers, because the labor force composition shifts toward old workers whose unemployment rate is lower. The path of the unemployment rate in this phase of aging is not consistent with the observed unemployment rate in Japan. However, as we will show later, this result changes when we consider an alternative calibration with a higher separation rate.

5.2 Results Under Alternative Calibrations

To better understand the benchmark results so far, we consider the following three alternative calibrations.

1. Lower δ: The parameter δ is set to a lower value 0.1 instead of 0.8 in the benchmark calibration. This case almost eliminates the effect of specificity of human capital and thus is useful in highlighting the importance of human capital specificity in generating our results.

2. Higher separation rates s_o and s_y: Both separation rates are set to 0.02 (instead of 0.012 under the benchmark calibration). In our model, the separation decision is taken to be exogenous, but the data show that the aggregate job separation rate has increased significantly since the mid 1990s in Japan, raising the unemployment rate in Japan. The value (0.02) is chosen by referring to the evidence presented by Lin and

[18] Panel (e) only plots average wage of old workers but when considered separately, w_o^i and w_o^e show clearer oscillations.

Miyamoto (2012) who show that the employment-to-unemployment transition rate almost doubled in the 1990s and 2000s. The separation rate (especially s_o) has an important implication for the model dynamics since it means a higher risk of human capital loss for the experienced workers.

3. Smaller decline in ϕ: ϕ is assumed to drop to 0.014 (instead of 0.011 in the benchmark calibration) from 0.02. Under this calibration, population keeps growing albeit at a slower pace. With 0.014, the path of the labor force roughly mimics the observed path of the U.S. labor force. The intention with this exercise is to demonstrate that labor force growth itself influences dynamics of inflation and real variables.

Note that in the first and second cases, we recalibrate the initial steady state following the same procedure as in the benchmark calibration. We then simulate the perfect foresight equilibrium path of the economy, assuming that the entry rate follows the same exogenous path as in the benchmark case. In the third case, the economy is assumed to be in the same steady state initially, but ϕ falls to a higher new steady state value (0.014). All results are put together in Figure 8.

5.2.1 Low Human Capital Specificity

Blue solid lines in Figure 8 represent the paths under the lower specificity of human capital. We can see that many of the key results discussed above disappear with this parameter value. First, the inflation rate is always positive in all phases of aging. Per-capita consumption, average wage, and labor productivity all behave similarly: they all increase gradually and eventually reach new steady-state level. The reason is that, with little specificity of human capital, aging only means that there are more productive (i.e., experienced) workers in the economy, raising all of these three variables. The real interest rate follows the path consistent with the path of consumption, and when the nominal interest rate follows the Taylor rule, the inflation rate stays positive over the entire process of aging. Because δ is set to a low level, ω in the initial steady state is also lower than in the benchmark economy. While ω increases as the labor force gets older, the magnitude is small, and thus it has only minor impacts on other variables. Given that this adverse effect of higher ω is quantitatively unimportant, unemployment rates among both young and old workers only decline. The path of the marginal cost is similar to the one in the benchmark calibration.

5.2.2 Higher Separation Rates

With higher separation rates, all the results that hold in the benchmark calibration become more pronounced. The effects are magnified because the higher separation rate s^e implies a higher risk of human capital loss. The unemployment rate in the initial steady state is higher and increases more significantly following the initial decline. Again, the initial decline is due to a lower entry rate but the unemployment rate starts increasing once the adverse effect of aging (due to the larger and lager number of inexperienced old workers in the I-matching market) becomes more evident. The timing at which the unemployment rate

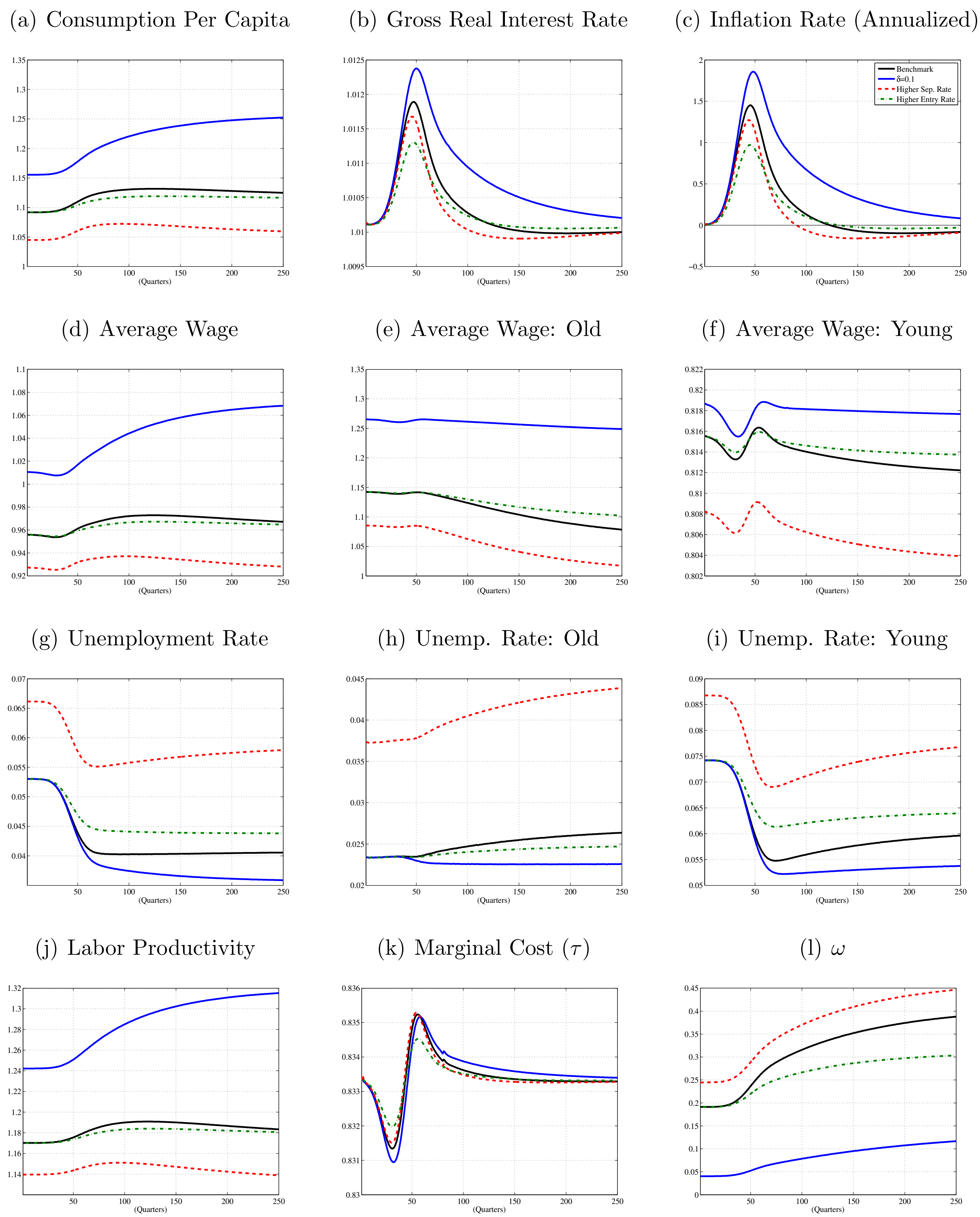

Figure 8: Perfect Foresight Equilibrium Paths (Alternative Calibrations)

Notes: Black solid: benchmark; Blue solid: $\delta = 0.1$; Red dashed: higher separation rates; Green dashed: positive labor force growth.

starts increasing also comes earlier, given that the pace of the increase in ω is faster due to the higher separation rate. Similarly, the decline in real wage is more pronounced and again occurs earlier than in the benchmark calibration.[19] The inflation rate turns negative significantly earlier, and the subsequent deflationary pressure is larger. The lowest level of the inflation rate amounts to -0.2%. While 0.2% seems like insignificant deflation, the differences from the cases under the benchmark calibration and the other two cases are considerable.

This exercise assumes that the separation rate was higher from the beginning and thus not not entirely realistic. However, this exercise implies that when the separation rate increases (which indeed occurred throughout the 1990s), it puts a downward pressure on real wages and thus inflation. In this sense, the benchmark results may be viewed as an underestimate of the impacts of the labor force aging.

5.2.3 Smaller Decline in the Entry Rate

In this hypothetical scenario, the entry rate drops sharply initially in a manner similar to the one assumed in the benchmark calibration, but it is assumed to converge to a higher level (0.014). Note that steady-state labor force growth in the model is given by $\phi - \mu$. Given that μ is set to 0.0125, the benchmark calibration implies that labor force growth in the new steady state is negative ($g = 0.011 - 0.0125 = -0.0015$) whereas, in the alternative scenario, labor force continues to grow even after the economy reaches the new steady state (at a rate equal to 0.0015). Panel (a) of Figure 9 presents the alternative path of ϕ. The green dashed line on Panel (b) corresponds to the path of the labor force in this case. The other lines display the path under the benchmark calibration (also under the other two cases).

Figure 8 indicates that this seemingly innocuous change makes significant differences in the path of the economy. The difference results from the fact that different labor force growth implies a different age composition in the steady state. We can see this in Panel (l) where the composition in the I-matching market increases less rapidly than in the benchmark scenario and reaches a lower level in the new steady state.

The unemployment rate does not decline as much in this case simply because the entry rate does not fall as much. The unemployment does not increase following the decline. Note that the ultimate source of the increase in the unemployment rate (either in the benchmark case or the case with higher separation rates) is due to the deterioration of the composition in the I-matching market, represented by ω. And under the scenario with higher labor force growth, ω does not increase as much and thus the effect on the unemployment rate is mitigated.

Per-capita consumption, aggregate wage and labor productivity do not increase as much initially compared to the case with the benchmark calibration but do not fall as much either in the subsequent periods. Remember that initial increases in these variables in the benchmark calibration come from a rapidly increasing share of old workers. Conversely, in this alternative scenario, the share of old workers do not rise as much, and thus suppressing

[19] It is somewhat difficult see the differences in the timing in the figure. However, the bottom of the unemployment rate and the peak of real wage come indeed earlier.

(a) Alternative Entry Rate Path (b) Implied Path of the Labor Force

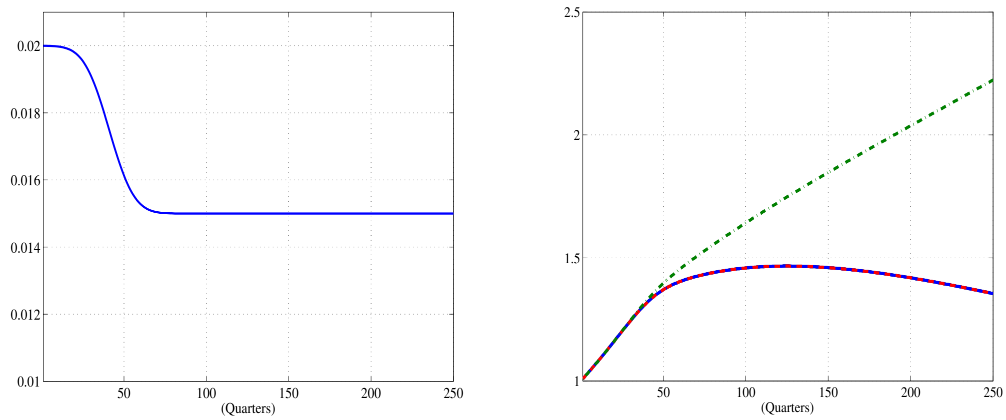

Figure 9: The Entry Rate and Labor Force

Notes: The labor force is normalized to 1 in the initial period. Green dashed line on Panel (b) gives the labor force path under the entry rate path plotted in Panel (a).

upward movements on these variables. However, they do not decline as fast as in the benchmark calibration. The paths of the real interest rate and inflation rate reflect the smoother paths of per-capita consumption and real wage.

6 Conclusion

This paper has studied the implications of aging on the low-frequency behavior of the labor market variables and inflation in Japan. We show that the collapse in labor force entry during the 1970s had material impacts on these variables over the subsequent 40 years. In particular, in the late stage of labor force aging, real wage, labor productivity, and the level of the labor force decline. The natural rate of interest also follows a similar path, and the inflation rate eventually turns negative.

We show that the key mechanism behind our results is a high degree of human capital specificity. Under the alternative calibration with a smaller degree of specificity, the inflation rate never turns negative. We also find that a smaller decline in the entry rate mitigates the deflationary pressure. These findings imply that a country's labor market structure and the pace of aging impact the low-frequency movements of the inflation rate as well as real variables. For example, in the U.S., human capital is said to be occupation specific, implying that the decline in worker's productivity at the time of job loss may not be as large to the extent that the worker can find a job in a similar occupation at a different firm. Also, while the U.S. labor force is also getting older in recent years, the labor force still has grown 40% since the mid 1990s. This comparatively younger labor force in the U.S. could be a result of the higher birth rate as well as a much larger flow of immigration.

In our model, the natural rate of interest evolves along with the demographic structure of the economy and the long-run behavior of inflation is similar to that of the natural rate of interest. However, due to the presence of nominal rigidity, changes in labor force entry do affect the marginal cost, and hence the pace of job creation. We plan to explore this channel more closely in the next draft and study optimal monetary policy.

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