Notes on the Underground:
Monetary Policy in Resource-Rich Economies*

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Abstract

How should monetary policy respond to a commodity price shock in a resource-rich economy? We study optimal monetary policy in a simple model of an oil exporting economy to provide a first answer to this question. The central bank faces a trade-off between the stabilization of domestic inflation and an appropriately defined output gap as in the reference New Keynesian model. But the output gap depends on oil technology, and the weight on output gap stabilization is increasing in the importance of the oil sector. Given substantial spillovers to the rest of the economy, optimal policy therefore calls for a reduction of the interest rate following a drop in the oil price in our model. In contrast, a central bank with a mandate to stabilize consumer price inflation may raise interest rates to limit the inflationary impact of an exchange rate depreciation.

JEL codes: E52, E58, J11

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1 Introduction

The oil price has been very volatile over the past four and a half decades (Figure 1a). Steep price increases in the 1970s were followed by a sharp reversal in the first half of the 1980s. After two decades with oil prices below the USD 40 mark, the price swung sharply again during the first decade of the 2000s, briefly reaching its all-time high of USD 145 in July 2008. Since North Sea oil exploitation began in the early 1970s, the real price of Brent Crude has averaged 2014-USD 53 with a standard deviation of 29. More recently, the price appeared to be relatively stable for a time. In the three years leading up to 19 June 2014, the price moved around an average of USD 110 with a standard deviation of 6 (Figure 1b). But at the end of the year, the price had dropped from 115 to less than USD 60, corresponding to a decline of close to 50% or more than eight standard deviations of the previous three years.

How should monetary policy respond to such a large oil price shock? In the literature, this question has chiefly been addressed from the perspective of countries that are net importers of oil, most notably the United States (Hamilton, 1983; Bernanke et al., 1997). This has naturally led to an emphasis on oil as a consumption good and as an input to production (Finn, 1995; Rotemberg and Woodford, 1996; Leduc and Sill, 2004). A few studies investigate monetary policy issues from the perspective of exporters of oil (Catao and Chang, 2013; Hevia and Nicolini, 2013). But also in these papers, oil is generally introduced as an intermediate and final consumption good, while supply is exogenously given as an endowment.

We depart from this approach and analyze the response of monetary policy to oil price shocks from the perspective of an economy which is dependent on oil exports for foreign currency revenue. Starting from the framework developed by Galí and Monacelli (2005) (henceforth GM), our objective is to establish a benchmark for monetary policy in resource-rich economies. In contrast to the previous literature, we abstract from domestic consumption of natural resources. Instead, we let the extraction of oil be endogenous and reliant on domestic intermediate inputs. This assumption provides a direct demand link from the oil sector to the rest of the economy. We further assume that our economy is a small player in the global market, taking the world price of oil as given. We allow for oil rents to be channeled into a sovereign wealth fund and spent according to a fiscal policy rule. We believe that these features are particularly important for resource-rich economies.

We evaluate optimal monetary policy for this economy in a linear quadratic framework. While the model features substantial spillovers from the oil sector to the rest of the economy, we show that, as in GM, the objective function only penalizes deviations of domestic inflation from its target (normalized to zero) and of non-oil output from its efficient level. However, the presence of the oil

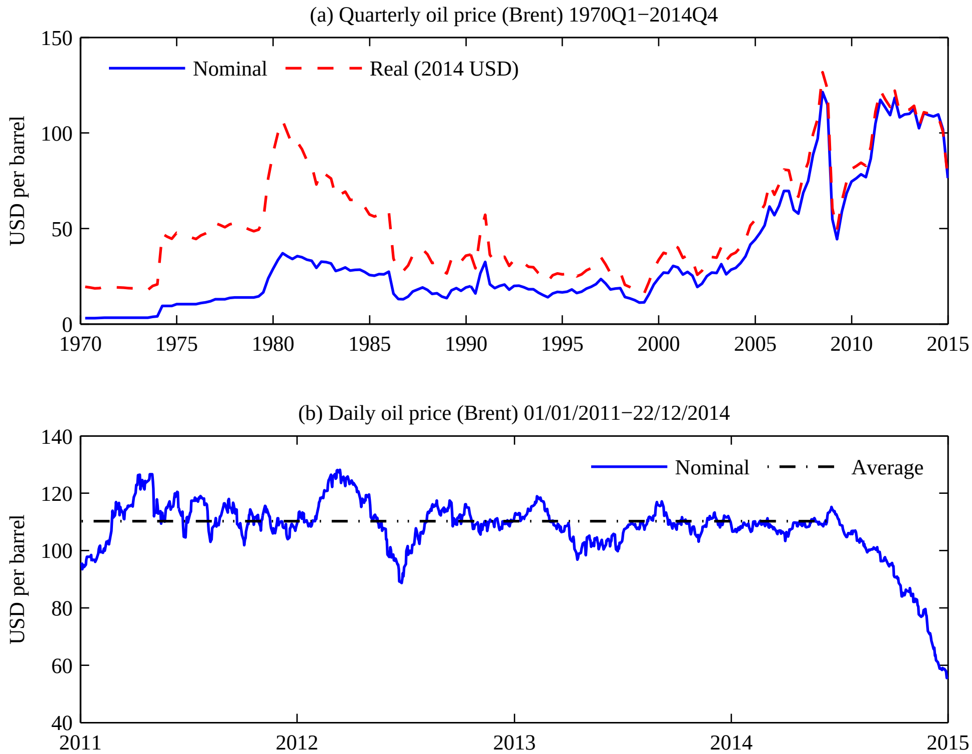

Figure 1: USD price of one barrel of oil (Brent Crude). Sources: OECD and Thomson Reuters/EIA, and own calculations.

sector changes both the relative weight on the output gap in the loss function and the slope of the Phillips curve in addition to the efficient level of production. Ultimately, the weight on stabilization of real activity increases by the importance of the oil sector, and it is higher than in GM under our baseline calibration. Given the spillovers to the rest of the economy, optimal policy therefore calls for a reduction of the interest rate following a drop in the oil price in our model. In contrast, a central bank with a mandate to stabilize consumer price inflation may raise interest rates to limit the inflationary impact of an exchange rate depreciation.

Given our approach, we naturally abstract from a number of features and frictions that may influence the appropriate stance of monetary policy in more complicated models as well as in practice. Here, we focus solely on the implications of introducing a reliance on the export of commodities into the reference model in the New Keynesian literature on monetary policy in small open economies. While for concreteness we focus on oil, our analysis and results apply more broadly to economies which produce and primarily export large quantities of a commodity whose price is determined in world markets.

The paper is organized as follows. In Section 2, we motivate our modeling assumptions by presenting key stylized facts from Norway as an example of a resource-rich economy with an independent monetary policy. We present the model in Section 3 and its equilibrium is characterized in Section 4. Section 5 is devoted to optimal monetary policy. Section 6 presents responses to a negative oil price shock in a parameterized version of the model, and compares responses and welfare under optimal policy with several simple monetary policy rules. Section 7 concludes.

2 A Small Oil-Exporting Economy

While we believe that our results apply broadly to resource-rich economies, Norway presents a particularly clear example of the transmission mechanism that we have in mind for commodity exporters. In 2012, Norway's total production of crude petroleum and natural gas amounted to 214 standard cubic meters of oil equivalent, corresponding to about 3.7 million barrels per day (Tormodsgard, 2014). This Nordic country of five million people was thereby the world's tenth largest exporter of crude oil and third largest exporter of natural gas. Oil and gas exploration began off the Norwegian shore in the early 1970s. Since then, the share of value added generated by the offshore industry has grown to close to 25 per cent of total gross domestic product (GDP), see Figure 2a.[1]

Figure 2b shows how the composition of Norwegian exports have changed in the same period. Crude petroleum and natural gas now comprise about 50 percent of exports. A further 20 percent are exports of petroleum products and services, and other primary or basic secondary goods such as metals and chemicals. In addition, an increasing share of remaining exports of goods and services is related to the oil and gas industry (Mellbye et al., 2012). Up to three quarters of Norway's exports are thus either petroleum or other commodities or commodity-related products. Imports of crude oil and natural gas are negligible compared to exports.

Since 1996, the Norwegian state's cash flow from the offshore sector has been transferred to a sovereign wealth fund. Given high tax rates and the state's direct ownership of production licenses, this means that a large share of the foreign currency revenue generated by exports of oil and gas is channeled into the fund.[2] The objective is to save oil wealth for future generations, while

[1]In addition to the NACE Rev. 2 sector '06 Extraction of crude petroleum and natural gas,' Statistics Norway's aggregate offshore sector comprises '09.1 Support activities for petroleum and natural gas extraction,' '49.50 Transport via pipeline, ' '50.1 Sea and coastal passenger water transport,' and '50.2 Sea and coastal freight water transport.

[2]In addition to an ordinary corporate tax rate of 27 per cent, a special tax of 51 per cent is levied on oil companies. Of the state's aprox. USD 54 bn cash flow from the offshore sector in 2012, 59 per cent was tax revenue, and 41 per cent was income from direct ownership either through the "State's Direct Financial Interest" in production licences awarded to multinational oil companies, or through its 67 per cent stake in the previously fully state-owned national champion Statoil.

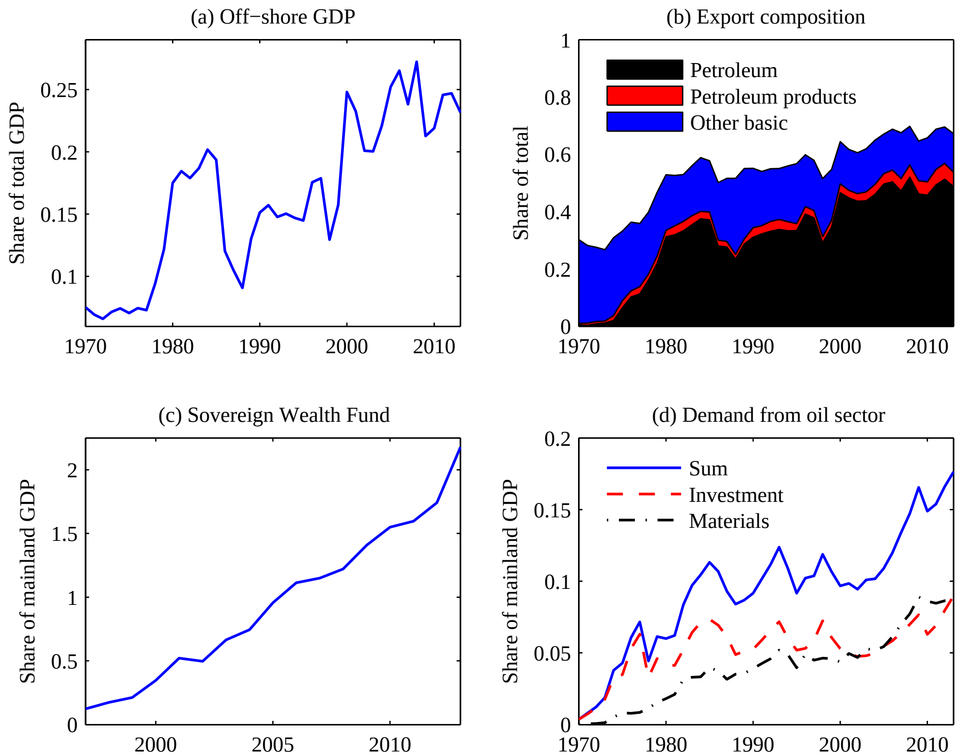

Figure 2: Stylized facts about Norway. Sources: Statistics Norway, Norges Bank Investment Management, and own calculations.

transforming the national asset portfolio in the direction of less oil underground and more financial assets abroad. Figure 2c shows that the fund's market value has grown to more than twice the size of the mainland economy. At the end of 2014, the market value of the fund exceeded USD 800 billion. Since 2001, successive governments have committed to a fiscal policy rule stipulating that a maximum of four percent of the value of the fund—corresponding to the expected average return—can be transferred to the government each year to cover the so-called structural non-oil deficit. This gradually passes part of the export revenue from the offshore sector on to the mainland.

But importantly, this is not the only channel by which activity in the offshore sector affects the mainland economy. Figure 2d illustrates a further transmission channel from oil and gas exploitation to other economic activities. The dashed line shows investment in capital stock offshore as a fraction of mainland GDP. While highly volatile, this investment share has fluctuated around a stable average of about six per cent since the mid-1970s. As the capital stock has been built up and offshore production intensified, intermediate consumption in oil and gas extraction has grown steadily and now comprises about seven percent of value added on the mainland (dashed-dotted line). Together, the oil industry's demand for investment goods and intermediate inputs have grown to constitute approximately 15 percent of mainland GDP in 2013 (solid line).[3]

Such a link from the oil industry to the rest of the economy is central to the model outlined in the next section. While materials and oil investments are equally important in the Norwegian data, we shall emphasize materials in the oil sector in what follows. This allows us to characterize the demand link from the oil sector in a particularly simple form, and we avoid going into the details of oil investment decisions, driven as they are not only by long swings in the oil price, but also by political and administrative processes. Our approach therefore emphasizes a relatively high-frequency transmission of oil price shocks working through the utilization of existing oil fields, marginal investments and the terms of contracts for suppliers. However, as we study a highly persistent oil price shock, we take the responses to be indicative of the direction if not necessarily of the exact timing of the transmission working through oil investments as well.

3 Model

The basic structure of the model corresponds to GM, with the exception of an oil export sector and the presence of a sovereign wealth fund. Our modeling choices are motivated by the stylized facts just described.

Oil firms are located offshore, operate under perfect competition, and sell oil in the word market taking the oil price as given. In contrast, firms on the mainland sell consumption goods to domestic households and to the government, and supply materials to offshore firms to be used as inputs in oil extraction. Mainland firms operate under monopolistic competition and set prices on a staggered basis. For simplicity, we assume that mainland firms do not export to the rest of the world. Similarly, neither offshore nor mainland firms import materials from abroad.[4]

A representative household consumes, supplies labor to mainland firms, and trades in a complete set of state-contingent securities in international financial markets. The consumption bundle consists of home goods produced by mainland firms and imported foreign goods.

Oil profits go into a sovereign wealth fund. The treasury receives a transfer from the fund in each period and cannot issue debt. As a consequence, for a given level of government spending, the transfer endogenously determines taxes. We let the transfer be determined by a simple rule, according to which the government can spend a fixed fraction of the fund value in each period.

[3] In the early part of the sample, import shares were very high. But since a domestic oil supply and services industry developed through the 1970s, direct import shares have fallen to around 20 percent (Eika et al., 2010).

[4] These assumptions do not change the qualitative nature of our results. In Appendix A, we show how to extend the analysis with exported mainland goods. Appendix B outlines two extensions that allow for inputs to be imported.

3.1 Households

At time t, the representative household chooses consumption C_{t+s}, state-contingent securities D_{t+s} and hours at work N_{t+s} for periods $t+s$, where $s = 0, 1, ...$, by solving the intertemporal utility maximization problem

$$\max_{\{C_{t+s}, D_{t+s}, N_{t+s}\}} \mathbb{E}_t \left[\sum_{s=0}^{\infty} \beta^s \left(\ln C_{t+s} - \frac{N_{t+s}^{1+\varphi}}{1+\varphi} \right) \right], \tag{1}$$

subject to

$$P_t C_t + \mathbb{E}_t(Q_{t,t+1} D_{t+1}) = W_t N_t + D_t + \Psi_t - T_t. \tag{2}$$

In the objective, $\mathbb{E}_t$ is the conditional expectation operator, $\beta \in (0,1)$ is the subjective discount factor, and $\varphi > 0$ is the inverse Frisch elasticity of labor supply. The overall consumption basket C_t is a Cobb-Douglas bundle of home mainland goods C_{Ht} and imported foreign goods C_{Ft}, whose price in units of domestic currency is P_t—the consumer price index (CPI)

$$C_t \equiv \frac{C_{Ht}^{1-\alpha} C_{Ft}^{\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}, \tag{3}$$

where $\alpha \in (0,1)$ is a measure of the degree of openness. In the budget constraint, W_t is the nominal wage, Ψ_t represents dividends from the ownership of intermediate goods producing firms, and T_t denotes lump-sum taxes paid to the government.

3.2 Firms

On the mainland, competitive final goods producers assemble intermediate goods $Y_{Ht}(i)$. Their problem is to minimize costs

$$\min_{Y_{Ht}(i)} \int_0^1 P_{Ht}(i) Y_{Ht}(i) di,$$

subject to an aggregation technology with constant elasticity of substitution

$$Y_{Ht} \equiv \left[\int_0^1 Y_{Ht}(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}},$$

where $P_{Ht}(i)$ is the price charged by individual firm i and $\varepsilon > 1$ is the elasticity of substitution between varieties of mainland goods.

Intermediate goods producers set prices on a staggered basis (Calvo, 1983). Each period, a measure $(1-\theta)$ of randomly selected firms get to post a new price $\tilde{P}_{Ht}(i)$ to maximize expected discounted profits

$$\max_{\tilde{P}_{Ht}(i)} \mathbb{E}_t \left\{ \sum_{s=0}^{\infty} \theta^s Q_{t,t+s} \left[(1+\varsigma)\tilde{P}_{Ht}(i) Y_{Ht,t+s}(i) - W_{t+s} N_{t+s}(i) \right] \right\},$$

where $\varsigma > 0$ is a steady-state subsidy, subject to a linear technology

$$Y_{Ht}(i) = A_{Ht} N_t(i),$$

where A_{Ht} is total factor productivity, and the demand from final goods producers

$$Y_{Ht}(i) = \left[\frac{P_{Ht}(i)}{P_{Ht}} \right]^{-\varepsilon} Y_{Ht}, \tag{4}$$

conditional on no further price change in the future.

The main departure from GM is the presence of an oil sector. The production of oil Y_{Ot} uses mainland final goods M_t as input in a diminishing-return technology[5]

$$Y_{Ot} = A_{Ot} M_t^{\eta}, \tag{5}$$

where A_{Ot} is total factor oil extraction technology, and $\eta \in (0,1)$.

A representative producer takes the price of inputs as given and sells any quantity in the world market at the price $P_{Ot} = \mathcal{E}_t P^*_{Ot}$. We assume that the oil producer cannot affect the world price of oil P^*_{Ot}, which is instead determined in the international market. The oil firm's problem is

$$\max_{M_t} P_{Ot} Y_{Ot} - P_{Ht} M_t,$$

subject to (5).

[5] An alternative would be to model the oil-service industry as a separate sector. In the absence of other frictions, such as labor market segmentation or different degrees of price rigidity, this formulation would be equivalent to the one in the text. Bergholt and Seneca (2014) study a richer supply chain channel by developing a model with tradable and non-tradable inputs in oil extraction.

3.3 Government

The fiscal authority takes spending G_t as given and needs to respect the budget constraint

$$P_{Ht}G_t = T_t + R_t, \tag{6}$$

where R_t represents transfers from the sovereign wealth fund. We assume that the government follows the fiscal policy rule

$$R_t = \rho \left(1 + i^*_{t-1}\right) \mathcal{E}_t F^*_{t-1}, \tag{7}$$

where F^*_{t-1} is foreign asset fund holdings at end of the previous period, and $\rho \in (0,1)$. This rule allows the government to spend a fixed fraction ρ of the initial value of the fund each period and is similar to the "bird-in-hand rule" in Wills (2014). Since oil profits are fully taxed, the rule implies that the value of the fund evolves according to

$$\mathcal{E}_t F^*_t = (1-\rho)\left(1 + i^*_{t-1}\right) \mathcal{E}_t F^*_{t-1} + (1-\eta) P_{Ot} Y_{Ot}. \tag{8}$$

To insure that the real value of the fund is stationary, we restrict ρ to be such that

$$(1-\rho)\left(1 + i^*_{t-1}\right) < 1.$$

This restriction ensures that the government spends slightly more than the average yield on the fund each period. In this case, the value of the fund will stabilize in the long run even with a constant stream of oil revenue.

3.4 Goods Market Clearing

Domestic goods can be consumed by the household, as input in oil extraction, and for government spending. Hence, goods market clearing requires

$$Y_{Ht} = C_{Ht} + M_t + G_t. \tag{9}$$

Foreign goods are consumed at home and abroad so that

$$Y^*_{Ft} = C_{Ft} + C^*_{Ft}. \tag{10}$$

By the small open economy assumption, we have $Y_t^* = C_t^*$. Finally, all oil is exported abroad and the world economy is able to absorb any quantity of oil produced domestically at the prevailing price.

3.5 National Accounts

Nominal gross domestic product (GDP) for the mainland economy is the value of the goods produced by mainland firms at home, $GDP_{Ht} = P_{Ht}Y_{Ht}$, while nominal offshore GDP is the value added in the oil sector, $GDP_{Ot} = P_{Ot}Y_{Ot} - P_{Ht}M_t$. Total nominal GDP is the sum of the two: $GDP_t = GDP_{Ht} + GDP_{Ot}$.

Using expenditure minimization and the resource constraint for mainland goods, we can rewrite total GDP as $GDP_t = P_tC_t - P_{Ft}C_{Ft} + P_{Ot}Y_{Ot} + P_{Ht}G_t$. Because the home country fully exports its oil production and no other manufacturing good, the previous expression can be also written in real terms as

$$Y_t \equiv \frac{GDP_t}{P_t} = C_t + \frac{P_{Ht}}{P_t}G_t + NX_t,$$

where the real trade balance is

$$NX_t \equiv \frac{P_{Ot}}{P_t}Y_{Ot} - \frac{P_{Ft}}{P_t}C_{Ft}.$$

Following a similar reasoning, we can also define real mainland GDP as

$$\tilde{Y}_{Ht} \equiv \frac{P_{Ht}}{P_t}Y_{Ht} = C_t + \frac{P_{Ht}}{P_t}G_t + NX_{Ht},$$

where the trade balance for the mainland economy is

$$NX_{Ht} \equiv \frac{P_{Ht}}{P_t}M_t - \frac{P_{Ft}}{P_t}C_{Ft}.$$

4 Optimality Conditions and Equilibrium

In this section we first discuss the optimality conditions for households and firms. We then define an imperfectly competitive equilibrium, and we show how it evolves independently of fiscal policy. Finally, we characterize the efficient allocation.

4.1 First Order Conditions for the Representative Household

Expenditure minimization gives rise to the downward-sloping demand functions for home and foreign goods

$$C_{Ht} = (1-\alpha)\left(\frac{P_{Ht}}{P_t}\right)^{-1} C_t \qquad \text{and} \qquad C_{Ft} = \alpha\left(\frac{P_{Ft}}{P_t}\right)^{-1} C_t,$$

where P_{Ht} and P_{Ft} are the domestic price of domestic and foreign goods, respectively. The associate price consumer price index is

$$P_t = P_{Ht}^{1-\alpha} P_{Ft}^{\alpha}.$$

The terms of trade $\mathcal{T}_t \equiv P_{Ft}/P_{Ht}$ measure the price of imports in terms of the price of domestic goods. Domestic and foreign goods are related to the terms of trade according to

$$\frac{P_{Ht}}{P_t} = \mathcal{T}_t^{-\alpha} \qquad \text{and} \qquad \frac{P_{Ft}}{P_t} = \mathcal{T}_t^{1-\alpha}.$$

We impose that the home country does not export domestic manufacturing goods ($\alpha^* = 0 \Rightarrow P_t^* = P_{Ft}^*$) and that the law of one price holds for foreign goods ($P_{Ft} = \mathcal{E}_t P_{Ft}^*$). Combining these two assumptions gives a relation between the real exchange rate ($S_t \equiv \mathcal{E}_t P_t^*/P_t$) and the terms of trade

$$S_t = \mathcal{T}_t^{1-\alpha}.$$

From the relations between relative prices and the terms of trade, we can rewrite the demand for home and foreign goods as

$$C_{Ht} = (1-\alpha)\mathcal{T}_t^{\alpha} C_t \tag{11}$$

and

$$C_{Ft} = \alpha \mathcal{T}_t^{\alpha-1} C_t. \tag{12}$$

The first-order conditions for state-contingent securities and consumption can be combined to give

$$Q_{t,t+1} = \beta \left(\frac{C_{t+1}}{C_t}\right)^{-1} \frac{1}{\Pi_{t+1}}, \tag{13}$$

where $\Pi_t \equiv P_t/P_{t-1}$ is the gross CPI inflation rate. A similar expression, adjusted for the presence of the nominal exchange rate (the price of foreign currency in units of home currency) $\mathcal{E}_t$ holds for the representative household in the rest of the world. Therefore, perfect risk-sharing implies that the ratio of consumption across countries is proportional to real exchange rate

$$C_t = \vartheta C_t^* S_t = Y_t^* \mathcal{T}_t^{1-\alpha}, \tag{14}$$

where ϑ is a constant that depends on the initial relative net asset position. The second equality follows from assuming symmetric initial conditions ($\vartheta = 1$), using the relation between the real exchange rate and the terms of trade derived above, and imposing the small open economy assumption ($C_t^* = Y_t^*$). By no arbitrage, the nominal net return on a one-period risk-free bond i_t denominated in domestic currency satisfies

$$(1 + i_t)^{-1} = \mathbb{E}_t Q_{t,t+1}, \tag{15}$$

A similar condition holds for a risk-free bond denominated in foreign currency, implying the uncovered interest rate parity condition

$$\mathbb{E}_t \left\{ Q_{t,t+1} \left[(1 + i_t) - (1 + i_t^*) \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \right] \right\} = 0, \tag{16}$$

Finally, the first order condition for labor supply is

$$\frac{W_t}{P_t} = A_{Ht} \mathcal{T}_t^{-\alpha} MC_t = N_t^{\varphi} C_t, \tag{17}$$

where the first part of the equality is the definition of the marginal cost for intermediate goods producers.

4.2 First Order Conditions for Firms

Cost minimization for final goods producers yields the demand for intermediate goods (4) and the associated price index

$$P_{Ht} = \left[\int_0^1 P_{Ht}(i)^{1-\varepsilon} di \right]^{\frac{1}{1-\varepsilon}}. \tag{18}$$

In a symmetric equilibrium, all intermediate goods producing firms that can change their prices make the same choice ($\tilde{P}_{Ht}(i) = \tilde{P}_{Ht},\ \forall i$). The first-order condition for their problem is

$$\mathbb{E}_t \left[\sum_{s=0}^{\infty} \theta^s Q_{t,t+s} Y_{Ht,t+s}(i) \left(\tilde{P}_{Ht} - \frac{1}{1+\varsigma} \frac{\varepsilon}{\varepsilon - 1} \frac{W_{t+s}}{A_{Ht+s}} \right) \right] = 0. \tag{19}$$

Using the demand relation (4) and the labor market clearing condition, we can also write the aggregate production function for the mainland economy as

$$Y_{Ht}\Delta_t = A_{Ht}N_t, \tag{20}$$

where labor market clearing implies

$$N_t = \int_0^1 N_t(i)di,$$

and Δ_t is an index of price dispersion defined as

$$\Delta_t \equiv \int_0^1 \left[\frac{P_{Ht}(i)}{P_{Ht}}\right]^{-\varepsilon} di.$$

From the price index (18) and the assumption of staggered price setting, the law of motion of the index of price dispersion is

$$\Delta_t = \theta\Pi_{Ht}^{\varepsilon}\Delta_{t-1} + (1-\theta)\left(\frac{1-\theta\Pi_{Ht}^{\varepsilon-1}}{1-\theta}\right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{21}$$

where $\Pi_{Ht} \equiv P_{Ht}/P_{Ht-1}$ is the domestic inflation rate. Also from the definition of the price index, the optimal reset price is related to the domestic inflation rate according to

$$\frac{\tilde{P}_{Ht}}{P_{Ht}} = \left(\frac{1-\theta\Pi_{Ht}^{\varepsilon-1}}{1-\theta}\right)^{\frac{1}{1-\varepsilon}} = \frac{X_{1t}}{X_{2t}}, \tag{22}$$

where the second part of the equality follows from the first order condition for firms (19). The variable X_{1t} represents the present discounted value of total costs in real terms, and can be written recursively as

$$X_{1t} = \frac{1}{1+\varsigma}\frac{\varepsilon}{\varepsilon-1}C_t^{-1}MC_t\mathcal{T}_t^{-\alpha}Y_{Ht} + \beta\theta\mathbb{E}_t(\Pi_{Ht}^{\varepsilon}X_{1t+1}). \tag{23}$$

Similarly, the variable X_{2t} is the present discounted value of total revenues in real terms, and can be written as

$$X_{2t} = C_t^{-1}\mathcal{T}_t^{-\alpha}Y_{Ht} + \beta\theta\mathbb{E}_t(\Pi_{Ht}^{\varepsilon-1}X_{2t+1}). \tag{24}$$

The first order condition for the representative oil producing firm gives rise to the demand for intermediate inputs

$$M_t = \left(\eta A_{Ot}\frac{P_{Ot}^*}{P_t^*}\mathcal{T}_t\right)^{\frac{1}{1-\eta}}, \tag{25}$$

which in turn determines oil production by (5) and oil profits as $(1-\eta)P_{Ot}Y_{Ot}$.

4.3 Equilibrium

All prices can be expressed in terms of the CPI and related to the terms of trade. In addition to the expressions for the prices of home and foreign goods in section 3.1, we can write the real oil price as

$$\frac{P_{Ot}}{P_t} = \frac{P^*_{Ot}}{P^*_t}\frac{\mathcal{E}_t P^*_t}{P_t} = \frac{P^*_{Ot}}{P^*_t}\mathcal{T}_t^{1-\alpha},$$

where the real foreign currency price of oil (P^*_{Ot}/P^*_t) is exogenous. The terms of trade is therefore the only relative price that matters for the characterization of the equilibrium. Hence, given monetary policy (pinning down the nominal interest rate i_t), initial conditions (Δ_{-1} and $\mathcal{T}_{-1}$) and exogenous processes for foreign output ($Y^*_t = C^*_t$), interest rates (i^*_t), inflation ($\Pi^*_{Ft} = \Pi^*_t$), productivity (A_{Ht} and A_{Ot}) and the real dollar oil price (P^*_{Ot}/P^*_t), an imperfectly competitive equilibrium is a sequence of quantities

$$\{C_t, C_{Ht}, C_{Ft}, N_t, X_{1t}, X_{2t}, Y_{Ot}, M_t, Y_{Ht}\}_{t=0}^{\infty}$$

and prices

$$\{Q_{t,t+1}, \Pi_t, \mathcal{E}_t, MC_t, \Delta_t, \Pi_{Ht}, \mathcal{T}_t\}_{t=0}^{\infty}$$

such that

1. Households optimize: Expressions (3) and (11) to (17) are satisfied.
2. Firms optimize: Expressions (5) and (20) to (25) are satisfied.
3. All markets clear: Expression (9) is satisfied.

4.4 Sovereign Wealth Fund Irrelevance

Inspection of conditions (5), (9), (11) to (17), and (20) to (25) reveals that the equilibrium is independent of fiscal policy decisions. In particular, the evolution of the sovereign wealth fund and the transfer rule from the sovereign wealth fund to the fiscal authority are irrelevant. Given domestic oil production, equation (8) in real terms pins down the evolution of the sovereign wealth fund, and equation (6) pins down the real value of lump-sum taxes.

The assumptions of lump-sum taxation and complete markets are crucial for the result. Iterating the household budget constraint (2), replacing profits, and imposing the transversality condition gives

$$D_t = \mathbb{E}_t\left[\sum_{s=0}^{\infty} Q_{t,t+s}(P_{t+s}C_{t+s} - P_{Ht+s}Y_{Ht+s} + T_{t+s})\right],$$

where, without loss of generality, we have abstracted from the steady state subsidy. Further, using expenditure minimization, the resource constraint for mainland goods, the production function in the oil sector, and the government budget constraint leads to

$$D_t = \mathbb{E}_t \left[\sum_{s=0}^{\infty} Q_{t,t+s} (P_{Ft+s} C_{Ft+s} - \eta P_{Ot+s} Y_{Ot+s} - R_{t+s}) \right].$$

From the transfer rule (7) and the evolution of the sovereign wealth fund (8), we can then write

$$R_t = -\mathcal{E}_t [F_t^* - (1 + i_{t-1}^*) F_{t-1}^*] + (1 - \eta) P_{Ot} Y_{Ot}.$$

Replacing this relation in the expression for D_t above yields

$$D_t = \mathbb{E}_t \left\{ \sum_{s=0}^{\infty} Q_{t,t+s} [P_{Ft+s} C_{Ft+s} - P_{Ot+s} Y_{Ot+s} + \mathcal{E}_{t+s} F_{t+s}^* - (1 + i_{t+s-1}^*) \mathcal{E}_{t+s} F_{t+s-1}^*] \right\}.$$

Therefore, the state-contingent payment in period t compensates for any future trade imbalance (the difference between the first two terms in square brackets) and for the accumulation/decumulation of future net foreign asset positions via the sovereign wealth fund (the other two terms). In other words, state-contingent securities undo any international wealth transfer associated with the fund.

4.5 Efficient Allocation and Steady State

The efficient allocation corresponds to the outcome of the optimization problem of a benevolent social planner who maximizes the utility of the representative agent in the absence of distortions subject only to technological, resource and international risk-sharing constraints. This problem is static and can be represented as

$$\max_{N_t, \mathcal{T}_t} \log(\mathcal{T}_t^{1-\alpha} Y_t^*) - \frac{N_t^{1+\varphi}}{1+\varphi}$$

subject to

$$A_{Ht} N_t = (1-\alpha) \mathcal{T}_t Y_t^* + \left(\eta A_{Ot} \frac{P_{Ot}^*}{P_t^*} \mathcal{T}_t \right)^{\frac{1}{1-\eta}} + G_t,$$

where we have used the risk-sharing condition to substitute for aggregate consumption, and the demands for domestic goods as well as the production function to replace variables in the resource constraint. The first-order condition for this problem is

$$1-\alpha=(N_t^e)^{1+\varphi}\gamma_{\tau t}, \tag{26}$$

where the superscript "e" denotes the efficient equilibrium and

$$\gamma_{\tau t}\equiv\frac{C_{Ht}}{Y_{Ht}}+\frac{1}{1-\eta}\frac{M_t}{Y_{Ht}}. \tag{27}$$

represents the elasticity of mainland output with respect to the terms of trade. The planner equates the marginal rate of transformation to the representative household's marginal rate of substitution between consumption and leisure.[6] The latter is captured not only by the share of home consumption $1-\alpha$ but also by the effect of a change in the terms of trade on the composition of output (a switch from materials to consumption goods, or vice versa).

In GM, the term $\gamma_{\tau t}$ is constant and equal to one. Consequently, the efficient level of employment is itself constant (and determined by $N^{1+\varphi}=1-\alpha$) at all times. Conversely, in our model, $\gamma_{\tau t}$ depends on the time-varying share of the oil sector's demand for materials in total demand for mainland goods. As a result, the efficient level of employment moves with the exogenous shocks out of the steady state.

In what follows, we consider an approximation of the model about the zero-inflation efficient steady state with the terms of trade equal to one. In this case, from expression (26), we can find a solution for the efficient level of employment

$$N^e=\left[\frac{(1-\alpha)A_H}{(1-\alpha)Y^*+(\eta A_O p_O^*)^{\frac{1}{1-\eta}}/(1-\eta)}\right]^{\frac{1}{\varphi}}.$$

Starting from this result, we can then easily derive the expressions for all other endogenous variables in the efficient steady state.

The flexible price equilibrium of this economy, and hence the steady state, however, are inefficient because firms do not internalize the effect of movements in the terms of trade on consumption when setting prices. In order to restore efficiency of the steady state, we assume that the government can appropriately choose the subsidy ς so as to eliminate the terms of trade externality in addition to the effects of market power.

In steady state, the optimal pricing condition equates the marginal cost to the inverse of the markup gross of the subsidy (equations 23 and 24). Substituting for the marginal cost from the

[6] Note that (26) can also be represented as $[(1-\alpha)/\gamma_{\tau t}]\,(C_t/N_t)=C_tN_t^{\varphi}$.

labor supply condition (17) and rearranging, we obtain

$$\frac{1}{1+\varsigma}\frac{\varepsilon}{\varepsilon-1}N^{1+\varphi}\frac{C_H}{Y_H} = 1-\alpha.$$

Comparing the previous expression with (26) we can see that the subsidy that makes the steady state efficient is

$$\varsigma = \frac{1}{\gamma_\tau}\frac{\varepsilon}{\varepsilon-1}\frac{C_H}{Y_H} - 1.$$

This subsidy differs from the one in GM due to the demand for domestic goods arising from the resource sector and the presence of government spending.

5 Linear-Quadratic Framework

To characterize the optimal monetary policy plan away from steady state, we take a second-order approximation of the utility function of the representative agent and a first-order approximation to the equilibrium conditions about the efficient steady state discussed in the previous section. Appendix C reports the full derivations. In terms of notation, we use lower case letter to represent log-deviations from steady state, i.e. $z_t \equiv \ln(Z_t/Z)$ for any variable Z_t. The resulting linear-quadratic framework allows us to derive a targeting rule for the central bank that implements optimal policy. We focus on the solution under commitment from a timeless perspective (Woodford, 1999).

5.1 Efficient and Natural Output

Up to a first-order approximation, we can solve for the efficient levels of mainland output and the terms of trade using the efficiency condition derived in the previous section, together with the production function, the resource constraint, the risk-sharing condition, and the demand equations for goods and materials (see Appendix D). The terms of trade become

$$\begin{aligned}\left\{\varphi\gamma_\tau^2+\lambda_\tau\right\}\tau_t^e = \\ (1+\varphi)\gamma_\tau a_{Ht} - s_c(1+\varphi\gamma_\tau)y_t^* - \frac{s_m}{1-\eta}\left(\varphi\gamma_\tau+\frac{1}{1-\eta}\right)(a_{Ot}+p_{Ot}^*) - \varphi\gamma_\tau(1-s_c-s_m)g_t,\end{aligned} \quad (28)$$

where $s_c \equiv C_H/Y_H$, $s_m \equiv M/Y_H$, γ_τ is the steady state value of $\gamma_{\tau t}$, and

$$\lambda_\tau \equiv s_c + \frac{s_m}{(1-\eta)^2}.$$

The efficient level of output is

$$\left\{\frac{\lambda_\tau}{\gamma_\tau} + \varphi\gamma_\tau\right) y^e_{Ht} = \\ (1+\varphi)\gamma_\tau a_{Ht} + \frac{\lambda_\tau}{\gamma_\tau}\left(1 - s_c - s_m\right) g_t + \left(\frac{\lambda_\tau}{\gamma_\tau} - 1\right) s_c y^*_t - \frac{s_c}{\gamma_\tau}\frac{\eta}{1-\eta}\frac{s_m}{1-\eta}\left(a_{Ot} + p^*_{Ot}\right). \quad (29)$$

Note that a reduction in the oil price leads to a depreciation of the efficient terms of trade and an increase in the efficient level of mainland output. As oil production becomes less profitable, the offshore economy demands less inputs, thus reducing the relative price of home mainland goods. Under complete markets, however, the depreciation of the terms of trade corresponds to higher domestic consumption because of international risk sharing. This transfer allows the planner to increase production of domestic goods to meet consumption demand without adverse effects on welfare.

For comparison, the approximate flexible-price equilibrium level of output is (see Appendix F)

$$y^n_{Ht} = \frac{1}{1+\varphi\gamma_\tau}\left[(1+\varphi)\gamma_\tau a_{Ht} + \frac{s_m}{1-\eta}(p^*_{Ot} + a_{Ot} - y^*_t) + (1 - s_c - s_m)g_t\right], \quad (30)$$

while the flexible-price level of the terms of trade solves

$$\tau^n_t = (1+\varphi)a_{Ht} - y^*_t - \varphi y^n_{Ht}. \quad (31)$$

A negative oil price shock also leads to a depreciation of the flexible-price terms of trade. But the natural level of output falls as the market does not internalize the welfare effects of the depreciation working through consumption. Notice that the natural and efficient levels of output coincide if $\gamma_\tau = \lambda_\tau = s_c$, which holds when the resource sector does not demand any resources from the mainland so that $\eta = s_m = 0$. If, in addition, government spending is zero so that $\gamma_\tau = s_c = 1$, we have $y^n_{Ht} = y^e_{Ht} = a_{Ht}$ as in GM.

5.2 Quadratic Loss Function

To derive the quadratic loss function, we use the expression for the aggregate production function to rewrite the utility function of the representative household as

$$\mathcal{W}_t = \mathbb{E}_t \left\{ \sum_{s=0}^{\infty} \beta^s \left[\ln C_{t+s} - \frac{(Y_{Ht+s}/A_{Ht+s})^{1+\varphi}}{1+\varphi} \Delta_{t+s}^{1+\varphi} \right] \right\}.$$

In Appendix E, we show that a second order approximation of this expression about a steady state with zero inflation and relative prices equal to one yields

$$\mathcal{W}_t = -\frac{\Omega}{2} \mathbb{E}_t \left[\sum_{s=0}^{\infty} \beta^s (\pi_{Ht+s}^2 + \lambda_x x_{Ht+s}^2) \right] + t.i.p. + \mathcal{O}(\|\epsilon_t\|^3), \tag{32}$$

where $t.i.p.$ stands for "terms independent of policy" (i.e. exogenous shocks) and $\mathcal{O}(\|\epsilon_t\|^3)$ collects the terms of order three or higher that we neglect by taking a second order approximation. The constants in the previous expression are functions of the structural parameters of the model

$$\Omega \equiv \frac{(1-\alpha)\varepsilon}{\kappa\gamma_\tau},$$
$$\lambda_x \equiv \frac{\kappa}{\varepsilon}\left(\frac{\lambda_\tau}{\gamma_\tau^2} + \varphi\right),$$

where

$$\kappa \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}.$$

The welfare-relevant output gap is defined as the deviation of mainland output from its efficient level

$$x_{Ht} = y_{Ht} - y_{Ht}^e.$$

The form of the loss function in our model coincides with the one in GM but the relative weight on the efficient output gap is different. In GM, the absence of an oil sector and government spending implies that $s_m = 0$ and $s_c = 1$ so that $\lambda_\tau = \gamma_\tau^2 = 1$. Effectively, in that model, the deviation of the level of employment from its efficient level determines the inefficiency gap between the marginal rate of substitution and the marginal rate of transformation. A higher value of φ leads to a higher inefficiency gap for a given output gap so that the weight in the loss function is increasing in this parameter. Here, $\lambda_\tau > \gamma_\tau^2$ and the weight on the output gap is larger. This outcome is a consequence of the terms-of-trade externality, which leads to a further opening of the inefficiency gap whenever output deviates from its efficient level.

5.3 Linear Constraints

Expressions (28) and (29) characterize the efficient equilibrium away from steady state up to the first order. We now find a representation for the Phillips curve in terms of the efficient output gap that can be used to derive the optimal policy rule in our model.

A first-order approximation of the firm price-setting condition gives the New Keynesian Phillips curve

$$\pi_{Ht} = \kappa mc_t + \beta \mathbb{E}_t \pi_{Ht+1}, \tag{33}$$

where marginal cost is given as

$$mc_t = \varphi y_{Ht} - (1 + \varphi) a_{Ht} + y_t^* + \tau_t. \tag{34}$$

As we show in Appendix F, we can rewrite the previous expression in terms of the efficient output gap only as

$$\pi_{Ht} = \xi x_{Ht} + \beta \mathbb{E}_t \pi_{Ht+1} + u_t, \tag{35}$$

where

$$u_t \equiv \kappa[\varphi y_{Ht}^e + \tau_t^e - (1 + \varphi) a_{Ht} + y_t^*]$$

and

$$\xi \equiv \frac{\kappa(1 + \varphi\gamma_\tau)}{\gamma_\tau}.$$

The term u_t consists of a weighted sum of shocks and is generally different from zero away from the steady state. It measures the extent to which contemporaneous stabilization of inflation and the efficient output gap is impossible as a consequence of demand spillovers from the oil sector. Without the resource and government sectors the coefficient γ_τ equals one, and u_t drops out of the Phillips curve since in this case $\tau_t^e = a_{Ht} - y_t^*$ and $y_{Ht}^e = a_{Ht}$. In this case, the "divine coincidence" holds, and monetary policy does not face a trade-off between domestic inflation and output gap stabilization.

Note that, differently from models in which oil is an input in the production stage, oil prices have inflationary consequences only through an indirect impact on marginal costs (see equation 33 and 34). The price of oil does not enter directly in the aggregate supply relation. An increase in the price of oil leads to an appreciation of the terms of trade (τ_t falls) and higher demand for intermediate inputs from the offshore sector. With nominal stickiness in mainland prices, production of mainland goods increases (y_{Ht} rises). The two effects on marginal costs go in opposite directions, and their relative strength depends on the inverse Frisch elasticity of labor supply.

The Phillips curve can also be written in terms of the flexible-price output gap as

$$\pi_{Ht} = \xi(y_{Ht} - y^n_{Ht}) + \beta \mathbb{E}_t \pi_{Ht+1}.$$

Comparing this representation with the one in terms of the efficient output gap, it follows that the term that prevents contemporaneous stabilization of inflation and the efficient output gap is proportional to the difference between efficient and flexible-price level of output

$$u_t = \xi(y^e_{Ht} - y^n_{Ht}).$$

Therefore, the term u_t captures the extent of the distortions in the economy that arise because of a terms of trade externality.

Up to the first order, the consumption Euler equation expressed in terms of efficient output gap reads as

$$x_{Ht} = -\sigma_\alpha(i_t - \mathbb{E}_t \pi_{t+1} - r^e_t) + \mathbb{E}_t x_{Ht+1}, \tag{36}$$

where $\sigma_\alpha \equiv (1-\alpha)/\gamma_\tau$ and the efficient real interest rate is defined implicitly by $r^e_t = \mathbb{E}_t c^e_{t+1} - c^e_t$ and

$$y^e_{Ht} = \frac{\gamma_\tau}{1-\alpha} c^e_t + \left(s_c - \frac{\gamma_\tau}{1-\alpha}\right) y^*_t + \frac{s_m}{1-\eta}(p^*_{Ot} + a_{Ot}) + (1 - s_c - s_m) g_t.$$

For a given choice of monetary policy, the relation between CPI and domestic inflation ($\pi_t = \pi_{Ht} + \alpha(\tau_t - \tau_{t-1})$) and the relation between the efficient output gap and the terms of trade ($x_{Ht} = \gamma_\tau(\tau_t - \tau^e_t)$) complete the description of the equilibrium up to a first-order approximation.

5.4 Optimal Monetary Policy

The linear-quadratic framework then consists of maximizing the second-order approximation of the objective function $\mathcal{W}_t$ in (32). This corresponds to solving the welfare loss minimization problem

$$\min_{\{\pi_{Ht+s}, x_{Ht+s}, i_{t+s}, \pi_{t+s}, \tau_{t+s}\}} \frac{\Omega}{2} \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s (\pi^2_{Ht+s} + \lambda_x x^2_{Ht+s})$$

subject to the aggregate supply equation (35), the aggregate demand equation (36) and the relations between CPI and domestic inflation as well as between the output gap and the terms of trade:

$$\begin{aligned}
\pi_{Ht} &= \xi x_{Ht} + \beta \mathbb{E}_t \pi_{Ht+1} + u_t \\
x_{Ht} &= -\sigma_\alpha (i_t - \mathbb{E}_t \pi_{t+1} - r_t^e) + \mathbb{E}_t x_{Ht+1} \\
\pi_t &= \pi_{Ht} + \alpha(\tau_t - \tau_{t-1}) \\
x_{Ht} &= \gamma_\tau (\tau_t - \tau_t^e).
\end{aligned}$$

Given the representation of the loss function and the constraints, the problem is equivalent to minimizing the loss function subject to the aggregate supply equation only, thus obtaining a solution for domestic inflation and the output gap. The remaining variables (interest rate i_t, CPI inflation π_t, and terms of trade τ_t) are then the solution to the remaining three equations given the optimal values of π_{Ht} and x_{Ht}.

The first-order conditions for the simplified problem (under commitment from a timeless perspective) are

$$\pi_{Ht} - \mu_t + \mu_{t-1} = 0,$$

and

$$\lambda_x x_{Ht} + \xi \mu_t = 0,$$

where μ_t is the Lagrange multiplier on the constraint. Combining the two first-order conditions to eliminate the Lagrange multiplier yields a standard optimal targeting rule

$$\pi_{Ht} + \frac{\lambda_x}{\xi}(x_{Ht} - x_{Ht-1}) = 0. \tag{37}$$

The optimal targeting rule takes the same form as in a closed-economy model with exogenous cost-push shocks (Clarida et al., 1999; Woodford, 2003). The same result would also hold in GM. In our model, however, the term u_t is not a cost-push shock per se, but rather a linear combination of disturbances arising from the demand side of the economy. This convolution of demand shocks prevents contemporaneous stabilization of inflation and the output gap. Its presence in the Phillips curve depends on the reallocation of resources between the domestic and the offshore economy due to terms of trade fluctuations that affect the marginal cost for mainland firms.

Another difference with the standard model is that the coefficient that governs the optimal policy trade-off is a function of the size of the oil sector through its effect on the composite parameters γ_τ

and λ_τ

$$\frac{\lambda_x}{\xi} = \frac{(\lambda_\tau + \varphi\gamma_\tau^2)}{(\gamma_\tau + \varphi\gamma_\tau^2)\varepsilon}.$$

In the absence of the oil sector, $\lambda_\tau = \gamma_\tau = 1$, and the weight on real activity in the optimal targeting rule equals the inverse of the elasticity of substitution among varieties. This special case encompasses both the closed and open economy counterparts (Clarida et al., 1999, and GM).

6 Quantitative results

In this section, we assign values to parameters to derive quantitative results from the model. We first compare the propagation of oil price shocks under optimal policy with the dynamics implied by three simple rules for monetary policy. Second, we compare the conditional welfare losses following oil price shocks when monetary policy follows these simple rules with the minimum loss achieved under optimal policy, and we provide a sensitivity analysis to the size of the demand impulse from the offshore sector. Impulse responses and welfare comparisons for an additional set of simple monetary policy rules are given in Appendix G.

6.1 Parameterization

We consider a period to be one quarter and set $\beta = 0.9963$. This value implies that the real interest rate is about 1.5% in the steady state. As in GM, we calibrate the expected duration of price contracts to one year by setting $\theta = 0.75$, and the net desired mark-up of prices over marginal costs 20% by setting $\epsilon = 6$. By implication, the slope of the Phillips curve is $\kappa = 0.0843$. Also as in GM, we set the degree of openness α equal to 0.4 (close to the Norwegian ratio of imports to mainland GDP), and the inverse of the labor supply elasticity φ to 3.

We set the value for the material share in oil production, η, along with values for the steady-state values of the exogenous variables A_{Ht}, A_{Ot} and Y_t^* to match a set of targets for the steady-state ratios of consumption to mainland GDP s_c, materials to mainland GDP s_m, and oil production to total GDP Y_O/Y. Specifically, we set $Y_O/Y = 0.15$, $s_m = 0.07$, and $1 - s_c - s_m = 0.33$ based on the data for Norway presented in Section 2. These restrictions imply that $\eta = 0.28$. Finally, we assume that the Treasury can spend a fraction equal to 4% of annualized GDP of the value of the sovereign wealth fund each period ($\rho = 0.01$). Given the level of foreign interest rates, this assumption means that the fund will stabilize at a bit more than six times annual mainland GDP in steady state.

We parameterize the shock process for the global oil price by estimating a simple first-order

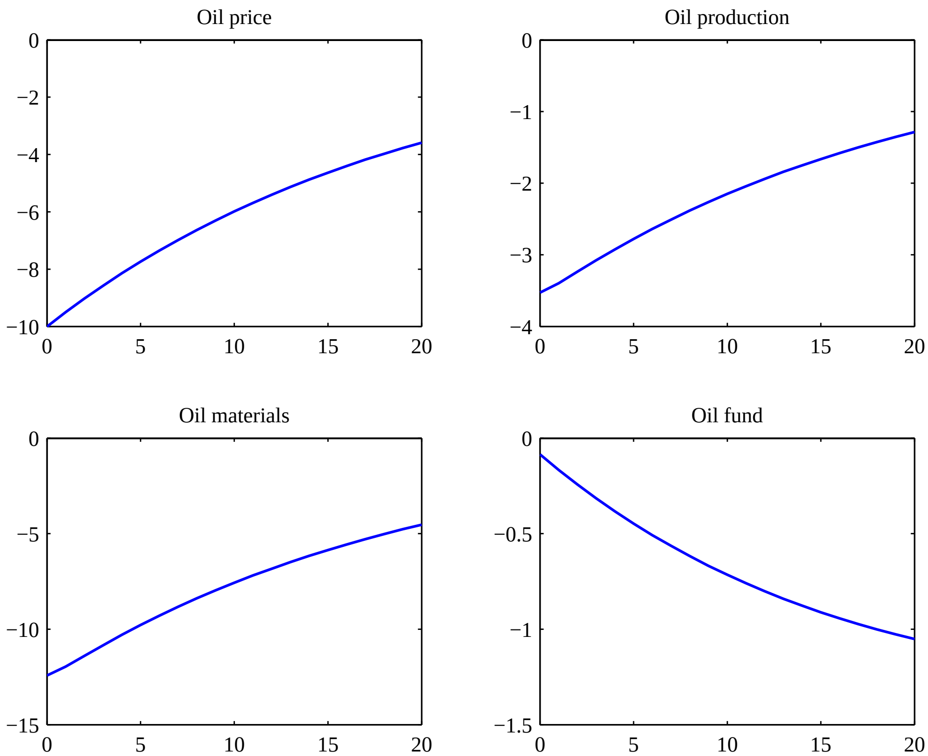

Figure 3: Impulse responses in the offshore economy to a negative shock to the USD oil price under optimal monetary policy.

autoregression for the demeaned log real oil price shown in (1) over the sample 1970Q1-2015Q1

$$p_{Ot}^{*} = \rho_O p_{Ot-1}^{*} + \varepsilon_t,$$

where $\varepsilon_t \sim (0, \sigma_O^2)$. Not surprisingly, the estimates suggest that oil price movements are highly persistent ($\hat{\rho}_O = 0.95$) and that oil price shocks are large ($\hat{\sigma}_O = 0.15$).

6.2 Impulse Responses to an Oil Price Shock

Consider a shock to the dollar oil price in the world market as in the top-left panel of figure 3. For ease of interpretation of the impulse responses, we show a 10% fall rather than a one standard-deviation shock of 15%. With $\rho_O = 0.95$, the shock is highly persistent and has a half-life of more than three years. For a decade, the shock reduces the profitability of oil production. Oil producers respond by reducing extraction (top-right panel of figure 3). Hence, the demand for intermediate input from the mainland declines, and both the volume of oil extracted and the profits generated in the offshore economy fall. The value of the sovereign wealth fund drops since payments into the fund fall short of transfers to the government. Only very slowly will the fund revert to its initial size as oil revenues recover. These responses in the offshore sector are driven by the fall in the oil

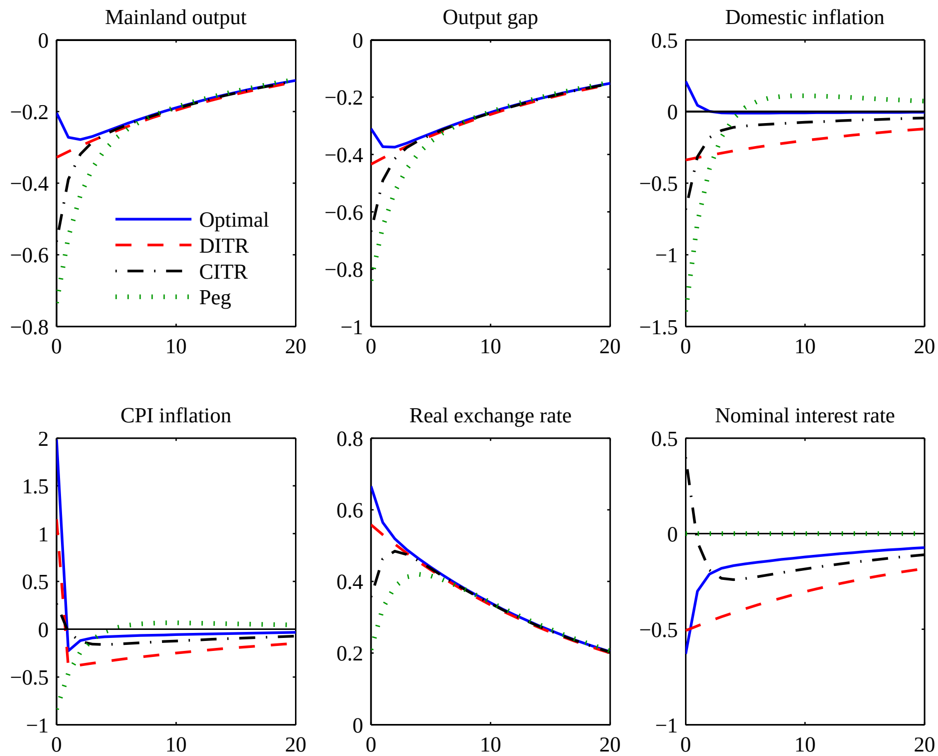

Figure 4: Impulse responses to a 10% negative shock to the USD oil price in the mainland economy with optimal monetary policy (solid lines) and an inertial Taylor rule (dashed line)

price and are only very marginally affected by monetary policy. In contrast, monetary policy shapes the propagation of the oil price shock through the mainland economy, thus affecting the optimal monetary policy response.

Figure 4 compares the responses under optimal policy (Optimal) with those under three simple rules

Domestic inflation targeting (DITR)	$i_t = \phi_\pi \pi_{Ht}$
CPI inflation targeting (CITR)	$i_t = \phi_\pi \pi_t$
Exchange rate peg (Peg)	$\Delta e_t = 0$

In all cases, the fall in the demand for intermediate goods offshore leads to a contraction of output on the mainland, and the efficient output gap falls. As firms cut production, the demand for labor also falls. This effect works to drive down the real wage and marginal costs. For a given policy stance, firms therefore also want to cut prices and domestic inflation tends to fall. But with optimal policy (solid line), the central bank will not allow the output gap and domestic prices to move in the same direction, as the targeting rule in (37) implies. The monetary authority therefore reduces the interest rate enough to induce an increase in the real wage despite the contraction in output and employment. By further stimulating private consumption, the policy contracts labor supply enough to more than offset the effect on the real wage from a fall in labor demand. In addition, a stronger real exchange rate depreciation works to increase the real product wage. With higher marginal costs, firms set higher prices and domestic inflation rises in equilibrium. By reducing the interest rate by more than half a percentage point on impact, the central bank reduces the contraction in output to about 0.2% with our baseline calibration.

Under the DITR (dashed line), the central bank simply leans against the fall in domestic inflation. A lower interest rate stimulates aggregate demand and thus reduces labor supply at given wages, and a real exchange rate depreciation reduces the relative price of home goods. But both the output gap and domestic inflation fall. The weaker response to domestic inflation means that, in equilibrium, monetary policy has to keep interest rates low for longer to bring about the required reduction in the real interest rate. For the first year after the shock, the mainland recession is larger than under optimal policy.

In contrast to the previous two regimes, the CITR (dashed-dotted line) leads to an increase in the interest rate on impact of the shock. As the mainland terms of trade deteriorates with lower prices of mainland goods, the exchange rate depreciates. This leads to inflationary pressure through imported goods. With the CITR, monetary policy leans against this rise in CPI inflation, and the initial spike in CPI inflation is limited to about 0.7 percentage points. This comes at the cost of making the recession in the mainland economy more severe. By increasing the interest rate by about 0.4 percentage points, mainland output falls by about 0.6% following the oil price shock with our calibration. At the same time, domestic inflation falls by more than half a percentage point.

With an exchange rate peg (dotted line), monetary policy is restricted to keep the interest rate in line with the foreign rate. The real exchange rate depreciation now takes the form of consumer price deflation, while dynamics in the real economy are similar to those under the CITR. But both the recession and the fall in domestic inflation are larger under this regime.

6.3 Welfare Comparisons

Table 1 shows standard deviations of key variables under the four monetary policy regimes when the economy is subjected to oil price shocks with the estimated standard deviation of 15%. Also in the table are the unconditional expectations of one-period welfare losses expressed in deviations from the minimum loss achieved under optimal policy, both in terms of percentages of steady-state consumption and in relative terms.

The optimal policy regime is characterized by relatively low variability in both domestic inflation and the output gap. In contrast, optimal policy allows for higher fluctuations in both CPI inflation

Table 1: Fluctuations and welfare conditional on oil price shocks

	Standard deviations (in %)						Losses	
Policy regime	i_t	π_{Ht}	π_t	Δe_t	y_{Ht}	x_t	Dev.	Rel.
Optimal	1.38	0.33	3.03	1.78	1.46	1.97	0.00	1.00
DITR	2.44	1.63	2.57	1.41	1.58	2.09	0.19	1.75
CITR	1.42	1.28	0.94	0.73	1.75	2.25	0.17	1.68
Peg	0.00	2.56	1.54	0.00	2.06	2.54	0.57	3.26

Note: Welfare losses are unconditional expectations of one-period welfare losses in deviation from the minimum loss achieved under optimal policy in terms of percentages of steady-state consumption (dev.) and in relative terms (rel.).

and the exchange rate than any of the other regimes considered. Hence, as in GM, the simple rules are characterized by excess smoothness of the nominal exchange rate. The exchange rate peg is therefore also the worst performer amongst the simple rules. Nominal exchange-rate stability comes at the cost of high volatility in domestic inflation and the output gap, and the loss under the peg is more than three times higher than the minimum loss achieved under optimal policy, that is an additional loss of approximately 0.6% of steady-state consumption. The DITR and the CITR perform substantially better with losses respectively 1.7 and 1.8 times higher than under optimal policy. The CITR slightly outperforms the DITR because it actually delivers more domestic inflation stability than the DITR at a small cost in terms of higher output gap volatility.[7]

By determining the optimal trade-off between domestic inflation and the output gap, the slope of the targeting rule in (37) is critical for the performance of the simple rules. Compared to GM, our baseline calibration implies a somewhat steeper targeting rule. Monetary policy allows domestic inflation to absorb more—and the output gap less—of the adjustment after cost-push shocks. This outcome is a consequence of a larger weight on output stabilization in the loss function as the Phillips curve becomes steeper with our baseline calibration.[8] That is, even if the sacrifice ratio (the output cost of reducing inflation) is lower in our model than in GM, the planner penalizes output fluctuations more because of the higher welfare consequences of deviating from the efficient level of output.

[7]The relative performance of the DITR and the CITR is highly sensitive to the value of ϕ_π. As we show in Appendix G, a strict domestic inflation target (corresponding to the DITR with $\phi_\pi \to \infty$) comes very close to achieving the minimum loss, while a strict CPI target performs worse than the CITR with $\phi_\pi = 1.5$.

[8]This result follows even in the absence of a resource sector since $s_c < 1$ with government spending. For values of s_m close to 0.25, γ_τ equals one, and the slope of the Phillips curve coincides with the one in GM. But in this case λ_τ is about 1.8 and the weight on output stabilization is higher than in GM.

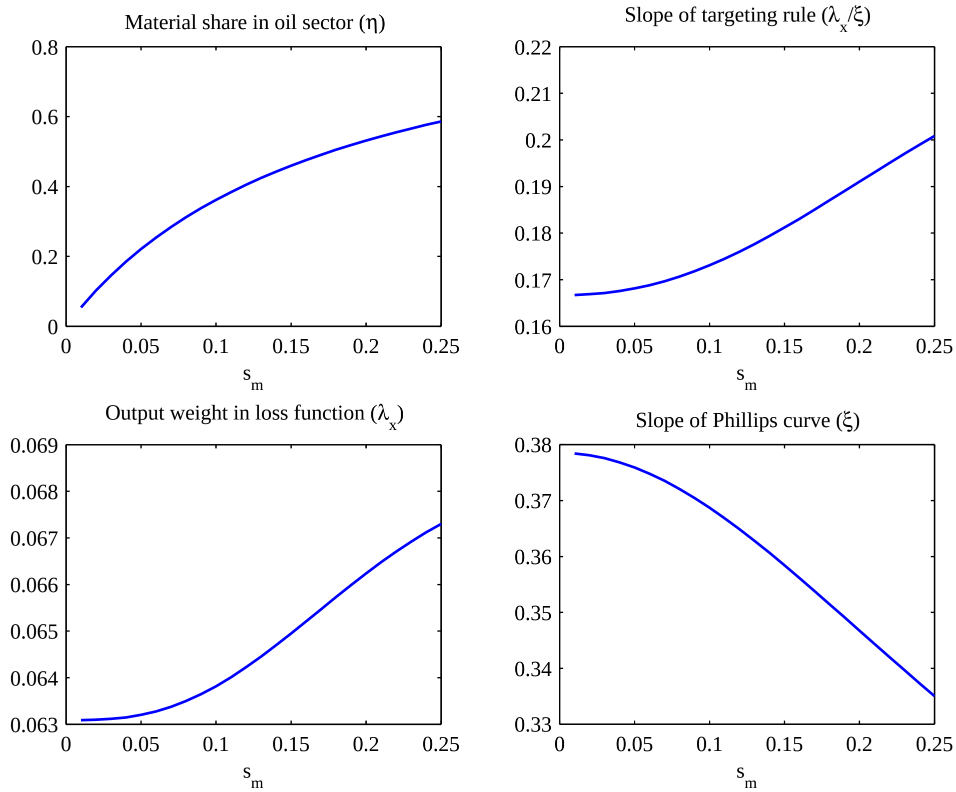

Figure 5: Sensitivity of the policy trade-off to the size of the demand impulse from the oil sector

Figure 5 illustrates how the slope of the targeting rule changes with the size of the demand impulse from the oil sector. For completeness, the top left panel shows the value of η consistent with each value of s_m considered. As shown in the top right panel, the targeting rule is increasing in the share of demand originating offshore. This correspondence follows from both an increasing weigh on the output gap in the loss function (bottom left) and a declining slope of the Phillips curve (bottom right).[9]

Table 2 shows the sensitivity of the relative welfare losses to the steady-state share of demand for mainland goods originating offshore as well as to the labor supply elasticity. As in GM, a lower Frisch elasticity (i.e. a higher φ) generally leads to higher costs of following suboptimal policy. For all the regimes, a larger steady-state demand impulse from the oil sector reduces the relative loss of suboptimal policy. But the ranking of the simple rules is not unaffected.

[9]In this exercise, we keep $1 - s_m - s_c$ fixed at 0.33. Therefore, the calibration target for s_c declines as we increase s_m.

Table 2: Relative welfare losses conditional on oil price shocks

	$\varphi = 1$			$\varphi = 3$			$\varphi = 5$		
	s_m			s_m			s_m		
Policy regime	0.02	0.07	0.12	0.02	0.07	0.12	0.02	0.07	0.12
Optimal	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
DITR	1.32	1.22	1.16	2.14	1.75	1.49	3.06	2.36	1.89
CITR	1.29	1.21	1.15	2.03	1.68	1.44	3.07	2.32	1.83
Peg	1.77	1.52	1.34	4.57	3.26	2.39	8.62	5.79	3.92

Note: Welfare losses are unconditional expectations of one-period welfare losses relative to the minimum loss achieved under optimal policy

7 Conclusion

We have studied monetary policy in a simple New Keynesian model of a resource-rich economy. Given substantial spillovers from the commodity sector to the rest of the economy, optimal policy calls for a significant reduction of the interest rate following a drop in the commodity price. While this prescription is clear in our model, the results also illustrate that a central bank with a flexible consumer price inflation target may find itself in a dilemma after a shock to the commodity price. A fall in the price will lead to a slowdown in the domestic economy. But a sharp depreciation of the exchange rate may lead to inflationary pressure. Given its mandate, the central bank may therefore have to increase interest rates at the cost of deepening the domestic recession further.

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A Exported Consumption Goods

In the text, we have assumed that the small open economy only exports the commodity. In this section, we show how to extend the model to the case in which the small open economy also exports some consumption goods.

For simplicity, we follow Paoli (2009) and assume that the world consists of two countries, Home and Foreign, respectively of size n and $1 - n$. The Cobb-Douglas consumption bundle becomes

$$C_t \equiv \frac{C_{Ht}^{\lambda} C_{Ft}^{1-\lambda}}{\lambda^{\lambda}(1-\lambda)^{1-\lambda}},$$

which implies the CPI price index

$$P_t \equiv P_{Ht}^{\lambda} P_{Ft}^{1-\lambda}.$$

The weight on consumption of Home and Foreign goods is a function of the size of the country and the degree of openness $\alpha \in (0, 1)$ according to $1 - \lambda \equiv (1 - n)\alpha$. Notice that in the limiting case of a small open economy ($n \to 0$), the consumption bundle corresponds to the baseline case in the text. The representative household in the Foreign country has similar preferences, but the weight on consumption of Home goods is $\lambda^* \equiv n\alpha$.

Expenditure minimization implies domestic demand functions for Home and Foreign goods similar to the ones obtained in the baseline case, after replacing λ with $1 - \alpha$. In this version of the model, a continuum of retailers of measure n packages intermediate goods so that market clearing requires

$$Y_{Ht} = C_{Ht} + \frac{1-n}{n} C_{Ht}^{*} + M_t + G_t,$$

where variables are expressed in per-capita terms. After substituting for demand, we can rewrite the previous expression as

$$Y_{Ht} = \lambda \left(\frac{P_{Ht}}{P_t} \right)^{-1} C_t + \frac{1-n}{n} \lambda^* \left(\frac{P_{Ht}^*}{P_t^*} \right)^{-1} C_t^* + M_t + G_t.$$

Replacing for λ and λ^* and taking the limit for n that goes to zero we obtain

$$Y_{Ht} = (1-\alpha) \left(\frac{P_{Ht}}{P_t} \right)^{-1} C_t + \alpha \left(\frac{P_{Ht}^*}{P_t^*} \right)^{-1} C_t^* + M_t + G_t.$$

Given the maintained assumption that the law of one price holds, we can rewrite the last expression as

$$Y_{Ht} = \left(\frac{P_{Ht}}{P_t}\right)^{-1} [(1-\alpha)C_t + \alpha S_t C_t^*] + M_t + G_t.$$

From the price index, the relation between the relative price of Home goods and the terms of trade is

$$\frac{P_{Ht}}{P_t} = \mathcal{T}_t^{-(1-\lambda)},$$

while the one between the real exchange rate and the terms of trade is

$$S_t = \mathcal{T}_t^{\lambda}.$$

Taking the limit for n that goes to zero implies that the expressions above coincide with the ones in the text. Therefore, market clearing for Home goods becomes

$$Y_{Ht} = \mathcal{T}_t^{\alpha}[(1-\alpha)C_t + \mathcal{T}_t^{1-\alpha}\alpha C_t^*] + M_t + G_t.$$

Using the risk-sharing condition $C_t = Y_t^* \mathcal{T}_t^{1-\alpha}$ finally gives

$$Y_{Ht} = \mathcal{T}_t Y_t^* + M_t + G_t.$$

We can then define the planner problem as in the text as

$$\max_{N_t, \mathcal{T}_t} \log(\mathcal{T}_t^{1-\alpha} Y_t^*) - \frac{N_t^{1+\varphi}}{1+\varphi}$$

subject to

$$A_{Ht} N_t = \mathcal{T}_t Y_t^* + \left(\eta A_{Ot} \frac{P_{Ot}^*}{P_t^*} \mathcal{T}_t\right)^{\frac{1}{1-\eta}} + G_t,$$

where we used the demand for materials, which is unchanged, to substitute out for M_t. The first order condition for this problem can be written as

$$1 - \alpha = (N_t^e)^{1+\varphi} \tilde{\gamma}_{\tau t}$$

where now

$$\tilde{\gamma}_{\tau t} \equiv \frac{\mathcal{T}_t Y_t^*}{Y_{Ht}} + \frac{1}{1-\eta} \frac{M_t}{Y_{Ht}}.$$

The presence of exported consumption goods, therefore, gives a similar efficiency condition as in the main text. The expression for the terms of trade externality is slightly, but this extension does not change significantly the qualitative nature of our results. The same steps to derive a linear-quadratic framework can be repeated in this context.

B Imported Inputs

In the baseline model, the oil sector exclusively uses inputs produced domestically in the mainland. In this section, we consider two extensions that allow for inputs to be imported. The first extension considers the case of inputs directly purchased by oil-producing firms. The second one assumes that foreign goods are an input in the production of domestic goods, hence indirectly affecting the oil sector.

B.1 Imports in Oil Production

We assume that oil-producing firms combine materials produced domestically and abroad. The technology becomes

$$Y_{Ot} = A_{Ot} M_t^{\eta_1} X_t^{\eta_2},$$

where X_t represents imported inputs in oil extraction, $\eta_1,\ \eta_2 \in (0,1)$, and $\eta_1 + \eta_2 < 1$.

Profit maximization defines the demand for the two inputs

$$\begin{aligned} P_{Ht} &= P_{Ot}\eta_1 A_{Ot} M_t^{\eta_1 - 1} X_t^{\eta_2} \\ P_{Ft} &= P_{Ot}\eta_2 A_{Ot} M_t^{\eta_1} X_t^{\eta_2 - 1}. \end{aligned}$$

Taking the ratio between the two first order conditions yields

$$\frac{\eta_1}{\eta_2}\frac{X_t}{M_t} = \frac{1}{\mathcal{T}_t}.$$

Solving for X_t and plugging back this expression into the first order condition for M_t gives

$$M_t = \left[\eta_1 \eta_2^{\frac{\eta_2}{1-\eta_2}} \left(A_{Ot} p_{Ot}^*\right)^{\frac{1}{1-\eta_2}} \mathcal{T}_t\right]^{\frac{1-\eta_2}{1-\eta_1-\eta_2}}.$$

As evident from the previous expression, if $\eta_2 = 0$ and $\eta_1 = \eta$, we are back in the case considered in the text.

Given the modifications in the expression for the demand of domestic inputs, the rest of the derivation in the text is unchanged. We can derive the efficient allocation and the linear-quadratic framework once again following the same steps as in the baseline model. The only additional difference is that the expression for profits of oil-producing firms, given by $(1-\eta_1-\eta_2)P_{Ot}Y_{Ot}$, is affected by the presence of imported inputs. But since the oil fund is irrelevant for the equilibrium allocation, this change influences the dynamics of the fund without real consequences.

B.2 Imports as Factor of Production for Intermediate Goods

Next, we consider the case in which imports are a factor of production for intermediate goods producers. The production function for this class of firms becomes

$$Y_{Ht}(i) = A_{Ht}N_t(i)^{\chi}X_t(i)^{1-\chi},$$

with $\chi \in (0,1)$.

In an efficient equilibrium, all firms are identical (hence, we can drop the index i), and the demand for imported inputs is

$$X_t = (1-\chi)\frac{Y_{Ht}}{\mathcal{T}_{\sqcup}}.$$

Plugging this expression back in into the production function yields

$$Y_{Ht} = (1-\chi)^{\frac{1-\chi}{\chi}} A_{Ht}^{\frac{1}{\chi}} \mathcal{T}_t^{\frac{\chi-1}{\chi}} N_t.$$

The planner now maximizes the utility of the representative household subject to the resource constraint that takes into account the use of imported imports (and hence the effect of the terms of trade) in production. The efficiency condition in this case becomes

$$1-\alpha = N_t^{1+\varphi}\breve{\gamma}_{\tau t},$$

where

$$\breve{\gamma}_{\tau t} \equiv \frac{1-\chi}{\chi} + \frac{C_{Ht}}{Y_{Ht}} + \frac{1}{1-\eta}\frac{M_t}{Y_{Ht}}.$$

The terms of trade externality, as summarized by the wedge from constant the employment in GM, coincides with our baseline case except for the constant $(1-\chi)/\chi$.

The presence of imported imports in production also changes the marginal cost for firms ($MC_t(i)$) in the dynamic equilibrium, and hence the forcing term in the New Keynesian Phillips curve. Cost minimization implies firms set the factor price as a mark-up over the marginal product of each factor. The first order conditions are

$$\begin{aligned}\frac{W_t}{P_{Ht}} &= \chi MC_t(i) A_{Ht} N_t(i)^{\chi-1} X_t(i)^{1-\chi} \\ \frac{P_{Ft}}{P_{Ht}} &= (1-\chi) MC_t(i) A_{Ht} N_t(i)^{\chi} X_t(i)^{-\chi}.\end{aligned}$$

Taking the ratio between the two first order conditions yields

$$\frac{W_t}{P_{Ft}} = \frac{\chi}{1-\chi}\frac{X_t(i)}{N_t(i)}.$$

Because the ratio of inputs is independent of firm-specific factors, so is the real marginal cost ($MC_t(i) = MC_t$). Plugging the first order conditions back into the production function gives an expression for marginal costs in terms of factor prices

$$MC_t = \frac{\left(\frac{W_t}{P_t}\right)^{\chi} \mathcal{T}_t^{1-\chi+\alpha\chi}}{\chi^{\chi}(1-\chi)^{1-\chi} A_{Ht}},$$

where we have used the relation between the relative price of domestic goods and the terms of trade to express real marginal cost as a function of the real wage in terms of the CPI.

Up to a first order approximation, the expression for marginal costs becomes

$$mc_t = \chi w_t + [1-\chi(1-\alpha)]\tau_t - a_{Ht},$$

while the aggregate production function is

$$y_{Ht} = a_{Ht} + \chi n_t + (1-\chi)x_t.$$

Combining these two expression with the optimal labor supply and the risk-sharing conditions, which are unchanged compared to the baseline case, we obtain

$$mc_t = \varphi y_{Ht} - (1+\varphi)a_{Ht} + \tau_t + \chi y_t^* - \varphi(1-\chi)x_t.$$

The first order approximation of the demand for imported inputs yields

$$x_t = mc_t + y_{Ht} - \tau_t.$$

Replacing into marginal costs finally gives

$$mc_t = \frac{\varphi\chi}{1+\varphi(1-\chi)} y_{Ht} - \frac{1+\varphi}{1+\varphi(1-\chi)} a_{Ht} + \tau_t + \frac{\chi}{1+\varphi(1-\chi)} y_t^*.$$

Not surprisingly, if $\chi = 1$, so that the only input in production is domestic labor, the expression for marginal costs coincides with our baseline case.

C Linear Model

This section derives a first order log-linear approximation of the equilibrium about a symmetric efficient steady state with zero inflation and relative prices normalized to one.

C.1 Households

Combining equations (13) and (15), we can obtain the Euler equation for a nominal risk-free asset, which can be approximated as

$$c_t = -\left(i_t - \mathbb{E}_t\pi_{t+1}\right) + \mathbb{E}_t c_{t+1}.$$

The international risk-sharing condition (14) becomes

$$c_t = y_t^* + (1-\alpha)\tau_t.$$

The UIP condition (16) is

$$i_t = i_t^* + \mathbb{E}_t e_{t+1} - e_t.$$

The labor supply condition (17) is

$$a_{Ht} - \alpha\tau_t + mc_t = \varphi n_t + c_t.$$

The demand for home and foreign goods (equation 11 and 12 respectively) are

$$c_{Ht} = \alpha\tau_t + c_t \qquad\qquad c_{Ft} = -(1-\alpha)\tau_t + c_t.$$

C.2 Firms

Mainland production technology is

$$y_{Ht} = a_{Ht} + n_t.$$

The price-setting first-order condition results in the New Keynesian Phillips curve

$$\pi_{Ht} = \beta E_t \pi_{Ht+1} + \kappa mc_t$$

with $\kappa \equiv= (1 - \beta\theta)(1 - \theta)/\theta$, marginal costs given as

$$mc_t = w_t - p_{Ht} - a_{Ht}$$

and

$$\pi_{Ht} - \pi_t = p_{Ht} - p_{Ht-1}.$$

Offshore technology is given as

$$y_{Ot} = a_{Ot} + \eta m_t.$$

Input demand in the oil sector is

$$p_{Ht} - p_{Ot} = a_{Ot} + (\eta - 1)\, m_t$$

with

$$p_{Ot} = s_t + p^*_{Ot}.$$

C.3 Government

The government budget constraint is

$$g_t + p_{Ht} = \frac{G - T/P}{G} r_t + \frac{T/P}{G} t_t$$

and the fiscal policy rule

$$r_t = s_t + f^*_{t-1} + i^*_{t-1} - \pi^*_t.$$

The fund evolves according the process

$$f^*_t + s_t = (1 - \rho)(1 + i^*)\left(f^*_{t-1} + s_t + i^*_{t-1} - \pi^*_t\right) + \left[1 - (1 - \rho)(1 + i^*)\right](y_{Ot} + p_{Ot}).$$

Monetary policy may follow a simple rule like

$$i_t = \rho i_{t-1} + (1-\rho)\left(\phi_\pi \pi_t^T + \phi_y y_t^T\right)$$

where π_t^T is the inflation target and y_t^T is the output target, or an optimal targeting rule like (37).

C.4 Market Clearing and Definitions

Market clearing requires

$$y_{Ht} = \frac{C_H}{Y_H} c_{Ht} + \frac{M}{Y_H} m_t + \frac{G}{Y_H} g_t.$$

The relation between the real exchange rate and the terms of trade is

$$s_t = (1-\alpha)\tau_t,$$

where

$$\tau_t = p_{Ft} - p_{Ht}.$$

Total GDP is

$$y_t = \frac{\tilde{Y}_H}{Y} y_{Ht} + \frac{\tilde{Y}_O}{Y} y_{Ot}.$$

D Efficient Output

Log-linearizing the efficiency condition (26) gives

$$(1+\varphi)n_t^e = -\gamma_\tau^{-1}\left(s_c c_{Ht}^e + \frac{s_m}{1-\eta} m_t^e\right) + y_{Ht}^e. \tag{38}$$

To rewrite this, note that the demand relation $c_{Ht} = \alpha\tau_t + c_t$ and the risk sharing condition $c_t = (1-\alpha)\tau_t + y_t^*$ can be combined to give $c_{Ht} = \tau_t + y_t^*$, and that the demand for materials can be written as $m_t = (1-\eta)^{-1}(a_{Ot} + p_{Ot}^* + \tau_t)$. Inserting these expressions along with the production function in (38), gives

$$\varphi\gamma_\tau y_{Ht}^e = -\lambda_\tau \tau_t^e + (1+\varphi)\gamma_\tau a_{Ht} - s_c y_t^* - \frac{1}{(1-\eta)^2} s_m \left(a_{Ot} + p_{Ot}^*\right). \tag{39}$$

Similarly, the resource constraint can be written as

$$y_{Ht}^e = \gamma_\tau \tau_t^e + s_c y_t^* + \frac{1}{1-\eta} s_m \left(a_{Ot} + p_{Ot}^*\right) + (1 - s_c - s_m) g_t. \tag{40}$$

Equations (39) and (40) represent a system of two equations in the two unknowns y_{Ht}^e and τ_t^e. Solving this system gives (28) and (29) in the text.

E Loss Function

Using the expression for the aggregate production function, we can rewrite the utility function of the representative household as

$$\mathcal{W}_t = \mathbb{E}_t \left\{ \sum_{s=0}^{\infty} \beta^s \left[\ln C_{t+s} - \frac{(Y_{Ht+s}/A_{Ht+s})^{1+\varphi}}{1+\varphi} \Delta_{t+s}^{1+\varphi} \right] \right\}.$$

A second order approximation of this expression around a steady state with zero inflation and relative prices equal to one yields

$$\mathcal{W}_t = \mathbb{E}_t \left(\sum_{s=0}^{\infty} \beta^s \mathcal{L}_{t+s} \right) + t.i.p. + \mathcal{O}(\|\epsilon_t^3\|)$$

where $t.i.p.$ stands for "terms independent of policy" (i.e. exogenous shocks), $\mathcal{O}(\|\epsilon_t^3\|)$ collects terms of order three or higher that we neglect by taking a second order approximation, and the per-period loss function is

$$\mathcal{L}_t = c_t - N^{1+\varphi} \left\{ y_{Ht} + \frac{1}{2} \left[(1+\varphi)(y_{Ht}^2 - 2 a_{Ht} y_{Ht}) + \frac{\varepsilon}{\kappa} \pi_{Ht}^2 \right] \right\}.$$

In order to obtain a purely quadratic approximation of the utility function, we need to eliminate the linear terms in consumption and output. To this end, we take a second order approximation of the resource constraint, which yields

$$y_{Ht} + \frac{1}{2} y_{Ht}^2 = s_c \left(c_{Ht} + \frac{1}{2} c_{Ht}^2 \right) + s_m \left(m_t + \frac{1}{2} m_t^2 \right) + (1 - s_c - s_m) \left(g_t + \frac{1}{2} g_t^2 \right).$$

We combine the last expression with (i) the risk sharing conditions

$$c_t = y_t^* + (1-\alpha) \tau_t, \tag{41}$$

(ii) the demand for home goods

$$c_{Ht} = \alpha\tau_t + c_t, \tag{42}$$

and (iii) the demand for intermediate inputs

$$m_t = \frac{1}{1-\eta}(a_{Ot} + p_{Ot}^* + \tau_t). \tag{43}$$

Notice that, since the last three expressions are exactly log-linear, their first and second order approximations coincide.

We can then rearrange the linear terms in the right-hand side of the previous expression for the period loss function as a function of the terms of trade and terms independent of policy

$$c_t - N^{1+\varphi} y_{Ht} = \left[(1-\alpha) - N^{1+\varphi}\gamma_\tau\right]\tau_t - \frac{N^{1+\varphi}}{2}(s_c c_{Ht}^2 + s_m m_t^2 - y_{Ht}^2) + t.i.p.$$

But now note that, in the steady state of the efficient allocation, the term in square brackets of the previous expression is actually zero. A benevolent policymaker can replicate the efficient level of employment (and, hence, output) by appropriately choosing the subsidy ς. Therefore, we have now managed to express the linear terms in the second order approximations of the welfare objective as a function of second order terms only, and hence obtained a purely quadratic per-period loss function

$$\mathcal{L}_t = -\frac{N^{1+\varphi}}{2}\left[s_c c_{Ht}^2 + s_m m_t^2 + \varphi y_{Ht}^2 - 2(1+\varphi)a_{Ht}y_{Ht} + \frac{\varepsilon}{\kappa}\pi_{Ht}^2\right].$$

We can then replace the demand for consumption and intermediate goods from (42) and (43)

$$\begin{aligned}\mathcal{L}_t \;=\; & -\frac{N^{1+\varphi}}{2}\left\{\left[s_c + \frac{s_m}{(1-\eta)^2}\right]\tau_t^2 + 2\left[s_c y_t^* + \frac{s_m}{(1-\eta)^2}(a_{Ot} + p_{Ot}^*)\right]\tau_t \right.\\ & \left. +\varphi y_{Ht}^2 - 2(1+\varphi)a_{Ht}y_{Ht} + \frac{\varepsilon}{\kappa}\pi_{Ht}^2\right\} + t.i.p.\end{aligned}$$

From expression (29), we note that

$$s_c y_t^* + \frac{s_m}{(1-\eta)^2}(a_{Ot} + p_{Ot}^*) = -\varphi\gamma_\tau y_{Ht}^e - \left[s_c + \frac{s_m}{(1-\eta)^2}\right]\tau_t^e + (1+\varphi)\gamma_\tau a_{Ht}.$$

We use this result in the loss function above to obtain

$$\begin{aligned}\mathcal{L}_t &= -\frac{N^{1+\varphi}}{2}\left\{\left[s_c+\frac{s_m}{(1-\eta)^2}\right](\tau_t^2-2\tau_t^e\tau_t)+\varphi y_{Ht}^2-2(1+\varphi)a_{Ht}(y_{Ht}-\gamma_\tau\tau_t)\right.\\ &\quad \left. -2\varphi\gamma_\tau y_{Ht}^e\tau_t+\frac{\varepsilon}{\kappa}\pi_{Ht}^2\right\}+t.i.p.\end{aligned}$$

From the resource constraint, we can see that $y_{Ht}-\gamma_\tau\tau_t$ is independent of policy and that

$$\gamma_\tau y_{Ht}^e\tau_t = y_{Ht}^e(y_{Ht}-t.i.p.).$$

Therefore, we can finally rewrite the per-period loss function as

$$\mathcal{L}_t = -\frac{N^{1+\varphi}}{2}\left\{\left[s_c+\frac{s_m}{(1-\eta)^2}\right](\tau_t^2-2\tau_t^e\tau_t)+\varphi(y_{Ht}^2-2y_{Ht}^e y_{Ht})+\frac{\varepsilon}{\kappa}\pi_{Ht}^2\right\}+t.i.p.$$

or

$$\mathcal{L}_t = -\frac{N^{1+\varphi}}{2}\left[\lambda_\tau(\tau_t-\tau_t^e)^2+\varphi(y_{Ht}-y_{Ht}^e)^2+\frac{\varepsilon}{\kappa}\pi_{Ht}^2\right]+t.i.p.,$$

where

$$\lambda_\tau \equiv s_c+\frac{s_m}{(1-\eta)^2}.$$

The per-period loss function penalizes departures of output, the terms of trade and inflation from their efficient equilibrium counterparts. Note, however, that the efficient output gap and terms of trade gap are proportional to each other

$$y_{Ht}-y_{Ht}^e = \gamma_\tau(\tau_t-\tau_t^e).$$

Consequently, the per-period loss function can be written more compactly in terms of efficient output gap and inflation only as

$$\mathcal{L}_t = -\frac{\Omega}{2}(\pi_{Ht}^2+\lambda_x x_{Ht}^2), \tag{44}$$

where $x_{Ht}\equiv y_{Ht}-y_{Ht}^e$ is the efficient output gap and the parameters in the loss function are

$$\begin{aligned}\Omega &\equiv \frac{(1-\alpha)\varepsilon}{\kappa\gamma_\tau},\\ \lambda_x &\equiv \frac{\kappa}{\varepsilon}\left(\frac{\lambda_\tau}{\gamma_\tau^2}+\varphi\right).\end{aligned}$$

The form of the loss function in our model coincides with the one in GM. The relative weight on the efficient output gap, however, is different. The text discusses how the relative weight changes depending on the size of the oil sector.

F The Phillips Curve

Marginal costs are given as

$$mc_t = w_t - p_{Ht} - a_{Ht} = \varphi\left(y_{Ht} - a_{Ht}\right) + y_t^* + \tau_t - a_{Ht}, \tag{45}$$

where the second equality uses labor supply and the risk sharing condition. With flexible prices, firms keep marginal costs fixed so that in deviations from the efficient steady state we have

$$0 = \varphi y_{Ht}^n + \tau_t^n - (1+\varphi)a_{Ht} + y_t^*.$$

This corresponds to (31) in the text. Independently of price setting, we can use the risk sharing and demand relations to write the resource constraint as

$$y_{Ht} = \gamma_\tau \tau_t + s_c y_t^* + \frac{s_m}{1-\eta}\left(a_{Ot} + p_{Ot}^*\right) + (1 - s_c - s_m)g_t, \tag{46}$$

from which it follows that

$$y_{Ht} - y_{Ht}^n = \gamma_\tau\left(\tau_t - \tau_t^n\right).$$

Combining (31) with (46) gives (30) in text. Taking the difference between (45) and (31) now yields

$$mc_t = \varphi\left(y_{Ht} - y_{Ht}^n\right) + \left(\tau_t - \tau_t^n\right) = \left(\gamma_\tau^{-1} + \varphi\right)\left(y_{Ht} - y_{Ht}^n\right). \tag{47}$$

Inserting (47) in (33) and rearranging gives (35) with

$$u_t = \xi(y_{Ht}^e - y_{Ht}^n) = \kappa[\varphi y_{Ht}^e + \tau_t^e - (1+\varphi)a_{Ht} + y_t^*]$$

where the second equality follows, after some manipulations, from the relation between the solutions for the efficient and natural level of output.

G Alternative Monetary Policy Rules

This appendix presents impulse responses to key mainland variables for a set of alternative simple rules index by j within the class of inertial Taylor-type rules

$$i_t = \rho_j i_{t-1} + (1-\rho_j)\left(\phi_{\pi j}\pi_{jt}^T + \phi_{yj} y_{jt}^T\right)$$

where π_{jt}^T is the inflation target and y_{jt}^T is the output target for rule j.

Each rule is characterized in Table 3, where we define output growth as $\Delta y_{Ht} \equiv y_{Ht} - y_{Ht-1}$. The first four rules are versions of the CITR and DITR in the text extended with a term in output, either in levels of mainland output (CITRL), in terms of growth (CITRG), or specified as the efficient output gap (CITRE and DITRE). The next four are inertial versions of these rules further augmented with an interest rate smoothing term. The subsequent four are optimized versions of the inertial rules, where the parameters have been set through a grid search to minimize the expected period welfare loss. The last three rules are strict targeting rules, respectively fully stabilizing domestic inflation (SDIT), CPI inflation (SCIT), and the growth of mainland nominal GDP (SNGDPGT).

Table 3: Alternative Taylor rule specifications

j	π_{jt}^T	y_{jt}^T	ρ_j	$\phi_{\pi j}$	ϕ_{yj}
CITRL	π_t	y_{Ht}	0.00	1.50	1.125
CITRG	π_t	Δy_{Ht}	0.00	1.50	1.125
CITRE	π_t	x_t	0.00	1.50	1.125
DITRE	π_{Ht}	x_t	0.00	1.50	1.125
ICITRL	π_t	y_{Ht}	0.75	1.50	1.125
ICITRG	π_t	Δy_{Ht}	0.75	1.50	1.125
ICITRE	π_t	x_t	0.75	1.50	1.125
IDITRE	π_{Ht}	x_t	0.75	1.50	1.125
OICITRL	π_t	y_{Ht}	0.00	1.00	0.09
OICITRG	π_t	Δy_{Ht}	0.00	30.77	49.40
OICITRE	π_t	x_t	0.00	1.02	0.07
OIDITRE	π_{Ht}	x_t	0.00	1090	10.00
SDIT	π_{Ht}	–	0.00	∞	0.00
SCIT	π_t	–	0.00	∞	0.00
SNGDPGT	–	$(\pi_{Ht} + \Delta y_{Ht})$	0.00	0.00	∞

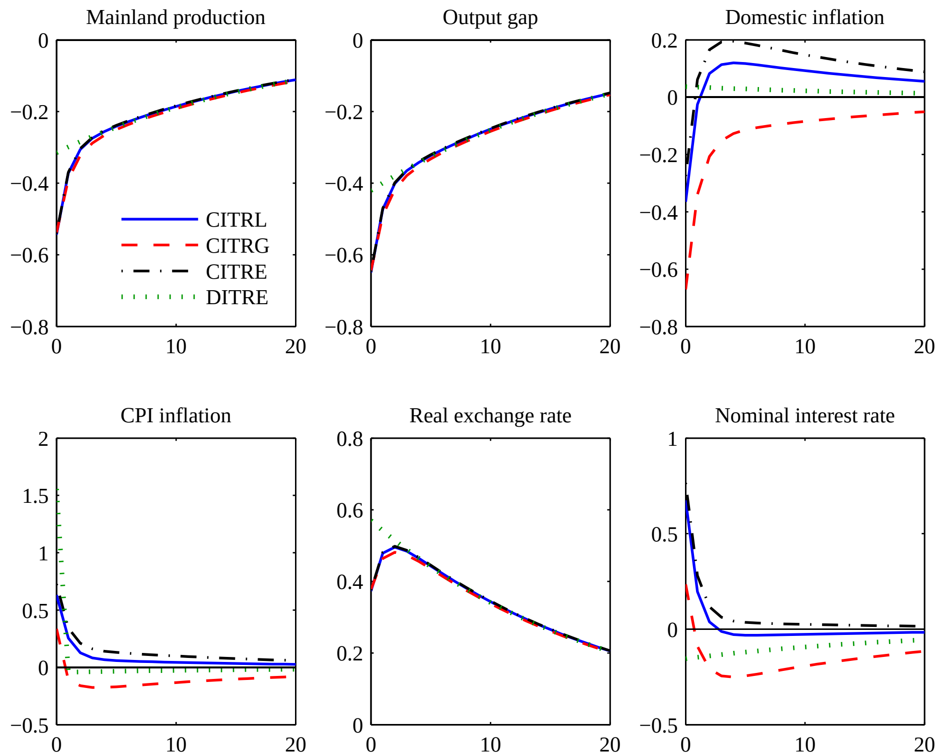

Figure 6: Impulse responses to a 10% negative shock to the USD oil price in the mainland economy with alternative monetary policy rules.

Figure 6 shows impulse responses in the mainland under four Taylor rules with a positive term on output: a *CPI inflation-based Taylor rule with output in levels* (CITRL; solid lines), a *CPI inflation-based Taylor rule with output growth* (CITRG; dashed lines), a *CPI inflation-based Taylor rule with the efficient output gap* (CITRE; dashed-dotted lines), and a *domestic inflation-based Taylor rule with the efficient output gap* (DITRE; dotted lines). The rules lead to similar output gap dynamics. Adding the output level or the output gap to the CITR leads to a larger increase in the interest rates. Adding the output gap to the DITR allows the simple rule to reproduce the feature of optimal policy that domestic inflation increases while the output gap falls.

Figure 7 shows responses to inertial versions of the rules shown in Figure 6. With interest rate smoothing, monetary policy becomes less activistic with smaller movements in the interest rate. By implication, the responses of output and inflation are somewhat stronger than under non-inertial rules.

Figure 8 shows responses to optimized versions of the rules shown in Figure 7, where parameter values have been set to minimize the expected period welfare loss subject to the specification of the rule. The optimized rules have no inertia and lead to similar dynamics as the stylized rules in 6 with the exception of the optimized rule with output growth. The *optimized inertial CPI inflation-based Taylor rule with with output growth* (OICITRG) induces dynamics similar to optimal policy.

Figure 9 show responses under strict targeting regimes. Under *strict domestic inflation targeting*

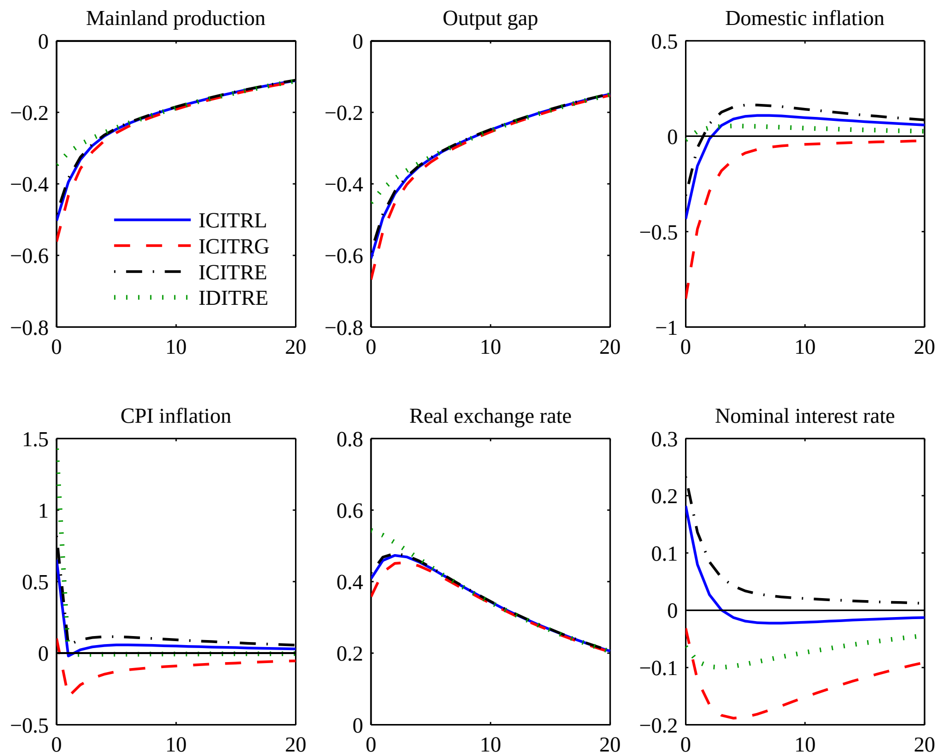

Figure 7: Impulse responses to a 10% negative shock to the USD oil price in the mainland economy with alternative monetary policy rules.

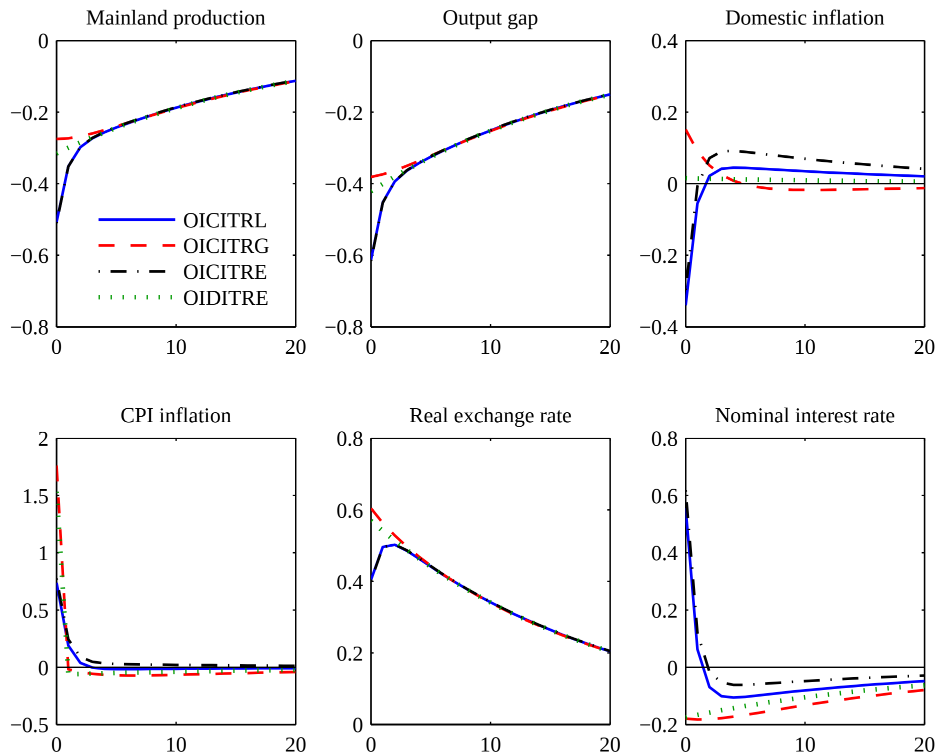

Figure 8: Impulse responses to a 10% negative shock to the USD oil price in the mainland economy with alternative monetary policy rules.

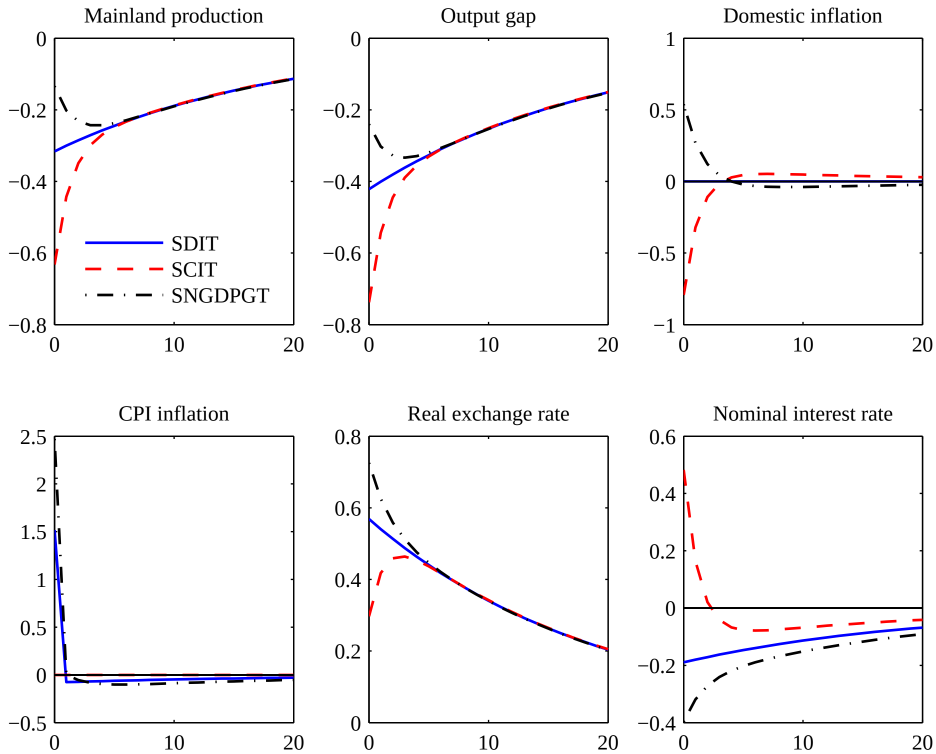

Figure 9: Impulse responses to a 10% negative shock to the USD oil price in the mainland economy with alternative monetary policy rules.

(SDIT; solid lines), monetary policy successfully stabilizes domestic price inflation at all times, while under *strict CPI inflation targeting* (SCIT) CPI inflation is fully stabilized. With SDIT, monetary policy stimulates the economy enough to keep domestic prices constant going some way towards the prescriptions of optimal policy. In contrast, a SCIT calls for a larger increase in the interest rate than the CITR. With *nominal GDP growth targeting* (SNGDPGT), monetary policy keeps mainland nominal GDP growth constant. This rule induces dynamics similar to optimal policy.

Standard deviations and relative welfare loss contributions are given in Table 4. Adding an output term to the simple rules generally reduces the relative loss, while interest rate smoothing increases it. SDIT, DITRE and IDTRE all come close to delivering the minimum loss under optimal policy. The optimized rule with CPI inflation and output growth also performs well. The term in growth introduces a backward-looking component to monetary policy as under optimal policy. Similarly, SNGDPGT trades off domestic inflation with output growth according to a targeting rule. While closer to the optimal rule, it contains output growth itself rather than the growth of the output gap, and the slope of the targeting rule is arbitrarily set at one. This makes SNGDPGT stabilize mainland output slightly too much at the expense of too much domestic inflation variability.

Table 4: Fluctuations and welfare conditional on oil price shocks

	Standard deviations (in %)						Losses	
Policy regime	i_t	π_{Ht}	π_t	Δe_t	y_{Ht}	x_t	Dev.	Rel.
CITRL	1.07	0.84	1.09	0.84	1.68	2.19	0.09	1.38
CITRG	1.39	1.32	1.07	0.78	1.73	2.23	0.17	1.69
CITRE	1.25	1.12	1.43	0.89	1.67	2.17	0.12	1.50
DITRE	0.74	0.18	2.34	1.45	1.51	2.02	0.01	1.04
ICITRL	0.33	0.92	1.03	0.86	1.69	2.20	0.11	1.43
ICITRG	1.06	1.59	0.88	0.63	1.80	2.30	0.24	1.96
ICITRE	0.46	1.04	1.39	0.95	1.66	2.17	0.11	1.44
IDITRE	0.55	0.30	2.14	1.36	1.53	2.04	0.02	1.07
OICITRL	1.01	0.57	1.15	0.92	1.65	2.16	0.06	1.25
OICITRG	1.02	0.30	2.68	1.59	1.47	1.98	0.00	1.01
OICITRE	1.00	0.66	1.24	0.93	1.65	2.15	0.07	1.27
OIDITRE	0.84	0.07	2.32	1.44	1.52	2.02	0.00	1.03
SDIT	0.91	0.00	2.30	1.44	1.52	2.03	0.01	1.03
SCIT	0.88	1.33	0.00	0.50	1.83	2.32	0.20	1.80
SNGDPGT	1.37	0.95	3.75	2.04	1.38	1.88	0.03	1.11

Note: Welfare losses are unconditional expectations of one-period welfare losses in deviation from the minimum loss achieved under optimal policy in terms of percentages of steady-state consumption (dev.) and in relative terms (rel.).