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The Informational Benefit of Price Disccrimination

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Abstract

We consider a monopoly supplying a homogeneous good to two separate markets with different demands. In one of the markets, some buyers do not know the quality of the good, but learn about it from observing prices. Under noisy demand, third-degree price discrimination is shown to alter the informational content of the price-signals received by the uninformed buyers. Specifically, discriminatory pricing have informational benefits over uniform pricing, i.e., the posterior beliefs of the uninformed buyers have a smaller bias and a lower variance.

Keywords: Market segmentation, Monopoly, Quality of information, Signaling, Third-degree price discrimination.

JEL Classifications: D82, D83, L12, L15.

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1 Introduction

As long as there is some easily observable characteristic (e.g., age, income, or geographic location) by which a firm can group buyers and arbitrage can be prevented, it is possible for the firm to segment markets and engage in (third-degree) price discrimination. An important question is whether market segmentation is beneficial for society. The welfare analysis on market segmentation has generally been undertaken under the assumption of complete information on the part of consumers.[1] In this case, the welfare effects are ambiguous. One has to weight the losses of consumers in low-elasticity markets against the gains of those in high-elasticity markets and of the producer. Moreover, the elimination of discriminatory pricing may lead to the closure of some markets. Yet, little is known about the effect of market segmentation on welfare and especially on consumers' well-being in the case of incomplete information. This is relevant since the differences among the segmented groups might concern not only tastes, but also information regarding the quality of the good. For instance, consider the case of a prescription drug readily available in the US which is introduced in a developing country. In addition of being able to pay less for the drug, consumers in the developing country might be less informed about the effectiveness of the prescription drug.[2]

The introduction of asymmetric information among buyers in markets leads naturally to the issue of the informative role of prices. Indeed, prices have been shown to be instrumental in disseminating information to market participants (Grossman, 1989).[3] It is the purpose of this paper to study

[1] See Armstrong (2006) for a survey on price discrimination.

[2] The implementation of drug information centers is a primary concern in many developing countries (Flores Vidotti, 2004). Proper sources of information on drugs are not easily accessible in developing countries. There are several reasons for the absence of information: inadequate translation in local languages, prohibitive cost to acquire information, and even customers' unawareness on how to obtain information.

[3] Several studies have provided conditions under which privately-held information by firms becomes public through prices, beginning with perfectly competitive markets (Kihlstrom and Mirman, 1975; Grossman, 1976, 1978; Grossman and Stiglitz, 1980) and continuing with imperfectly competitive markets (Wolinsky, 1983; Riordan, 1986; Bagwell and Riordan, 1991; Judd and Riordan, 1994; Daughety and Reinganum, 1995, 2005, 2007, 2008a,b; Janssen and Roy, 2010; Daher et al., 2012).

the effect of market segmentation on the informational content of prices. Specifically, does discriminatory pricing provide less or more information to the uninformed buyers? If it were not for the endogeneity of prices, it could be argued that an increase in the number of price-signals from one to two (due to market segmentation) yields more information to the uninformed buyers (i.e., more precise posterior beliefs). However, since the firm sets prices, the distribution of the price-signals (i.e., the informational content) does depend on whether the firm uses discriminatory pricing. There is thus a trade-off. Price discrimination generates more price-signals, but each of these signals might be less precise.

To study the effect of discriminatory pricing on the dissemination of information via market prices, we consider the simplest model of third-degree price discrimination of a monopoly selling a homogeneous good to two separate markets. In one of the markets, some buyers do not know the quality of the good. Yet, the presence of informed buyers makes it possible for prices to disseminate information. Under noisy demand, we show that market segmentation alters the informational content of price-signals received by the uninformed buyers. Specifically, discriminatory pricing have informational benefits over uniform pricing, i.e., the posterior beliefs of the uninformed buyers have a smaller bias and a lower variance.

There is a small literature on signaling in a stochastic setting beginning with Matthews and Mirman (1983) in a limit pricing environment. Judd and Riordan (1994) and Mirman et al. (2012) study noisy signaling in the monopoly case.[4] In this paper, we contribute to the literature on noisy signaling by studying the informational role of prices in the presence of market segmentation. The noisy environment enables us to study thoroughly the effect that market segmentation has on the informational content of prices. In a noiseless environment, the firm reacts to the informational externality, but has limited control over the flow of information. In other words, in equilibrium, either the unknown quality is not revealed and learning buyers revert

[4]Recent experimental work suggests that the stochastic environment in signaling maps better into experimental subject behavior (de Haan et al., 2011; Jeitschko and Norman, 2012).

to their prior beliefs, or it is fully revealed. Hence, under noiseless demand, whether the firm uses discriminatory pricing has no particularly meaningful effect on the posterior beliefs.

The remainder of the paper is organized as follows. Section 2 presents the model of third-degree price discrimination and characterizes the noisy signaling equilibrium. Section 3 studies the effect of market segmentation on the quality of information received by the uninformed buyers. Section 4 concludes the paper.

2 Model and Equilibrium

In this section, we present the model and characterize the noisy signaling equilibrium under both uniform and discriminatory pricing. In the next section, we study the effect of discriminatory pricing on the dissemination of information.

Consider a monopolist that sells a good of quality $\mu > 0$ in markets A and B. In market A, the buyers are informed, i.e., they know μ. Their demand is $q_A^i = \mu - P_A$ where quality μ is also the reservation price and P_A is the price in market A.[5] Aggregate demand in market A is given by $Q_A(P_A, \mu, \eta_A) = q_A^i + \eta_A$ or

$$Q_A(P_A, \mu, \eta_A) = \mu - P_A + \eta_A \tag{1}$$

where η_A is a demand shock that is unobserved by the buyers. In market B, the buyers have a lower demand. Specifically, conditional on μ, the reservation price of the good in market B is $\gamma\mu$, where $\gamma \in (0, 1]$ reflects the difference in demand between the two markets. Unlike market A, market B is composed of both informed and uninformed buyers. The informed buyers know μ and have demand $q_B^i = \gamma\mu - P_B$ where P_B is the price in market B. The uninformed buyers do not know μ, and have prior beliefs with the corresponding p.d.f. $\xi(\cdot)$. Using Bayes' rule to update beliefs, the uninformed buyers extract information about μ from observing the prices.

[5] The superscript i stands for *informed*.

Let $\hat{\xi}(\cdot|P_A, P_B)$ be the posterior p.d.f. of $\tilde{\mu}$ upon observing P_A and P_B.[6] The demand of the uninformed buyers is $q_B^u = \gamma\mathbb{E}[\tilde{\mu}|P_A, P_B] - P_B$ where $\mathbb{E}[\tilde{\mu}|P_A, P_B] \equiv \int_{\mathbb{R}} x\hat{\xi}(x|P_A, P_B)\mathrm{d}x$ is the posterior mean for quality upon observing P_A and P_B.[7] Normalizing the mass of buyers to one and letting $\lambda \in [0, 1]$ be the fraction of the informed buyers, aggregate demand in market B is $Q_B(P_B, \mu, \hat{\xi}(\cdot|P_A, P_B), \eta_B) = \lambda q_B^i + (1-\lambda)q_B^u + \eta_B$ or

$$Q_B(P_B, \mu, \hat{\xi}(\cdot|P_A, P_B), \eta_B) = \lambda(\gamma\mu - P_B) + (1-\lambda)(\gamma\mathbb{E}[\tilde{\mu}|P_A, P_B] - P_B) + \eta_B \tag{2}$$

where η_B is a demand shock that is unobserved by the buyers.

Next, we describe the firm's maximization problem. In addition of knowing the quality μ, the firm has complete information about demand, i.e., both η_A ans η_B are known to the firm. This reflects the idea that the firm knows more about demand than the buyers do. Hence, using (1) and (2), the firm's maximization problem with price discrimination is[8]

$$\max_{P_A, P_B} \left\{ P_A \cdot Q_A(P_A, \mu, \eta_A) + P_B \cdot Q_B(P_B, \mu, \hat{\xi}(\cdot|P_A, P_B), \eta_B) \right\}. \tag{3}$$

To study the effect of discriminatory pricing on the dissemination of information, we need to consider the benchmark model of no price discrimination. When markets are not segmented, the firm sets one price, i.e., $P \equiv P_A = P_B$, and the uninformed buyers receive only one signal. Using (1) and (2), the firm's maximization problem without price discrimination is

$$\max_{P} \left\{ P \cdot \Big(Q_A(P, \mu, \eta_A) + Q_B(P, \mu, \hat{\xi}(\cdot|P), \eta_B) \Big) \right\}. \tag{4}$$

Having described the agents and the markets, we now define the noisy signaling equilibrium when the firm price discriminates. The equilibrium consists of the firm's price strategies, $\{P_A^{**}(\mu, \eta_A, \eta_B), P_B^{**}(\mu, \eta_B, \eta_A)\}$; the distribution of the price-signals conditional on any quality x, $\phi^{**}(P_A, P_B|x)$; and the uninformed buyers' posterior beliefs about the quality upon observing

[6] A tilde sign distinguishes a random variable from the true quality.
[7] The superscript u stands for *uninformed*.
[8] For simplicity, cost is assumed to be zero.

any prices $\{P_A, P_B\}$, $\hat{\xi}^{**}(x|P_A, P_B)$.[9] In equilibrium, the posterior beliefs are consistent with Bayes' rule and the equilibrium distribution of prices.

Definition 2.1. *The tuple* $\left\{P_A^{**}(\mu, \eta_A, \eta_B), P_B^{**}(\mu, \eta_B, \eta_A), \phi^{**}(P_A, P_B|\cdot), \hat{\xi}^{**}(\cdot|P_A, P_B)\right\}$ *is a noisy signaling equilibrium with discriminatory pricing if, for all* $\mu > 0$*,*

1. *Given* $\hat{\xi}^{**}(\cdot|P_A, P_B)$*, and for any* η_A *and* η_B*, the firm's price strategies are*

$$\begin{aligned}\{P_A^{**}(\mu, \eta_A, \eta_B), P_B^{**}(\mu, \eta_B, \eta_A)\} = \arg\max_{P_A, P_B} \Big\{ & P_A \cdot Q_A(P_A, \mu, \eta_A) \\ & + P_B \cdot Q_B(P_B, \mu, \hat{\xi}^{**}(\cdot|P_A, P_B), \eta_B)\Big\}.\end{aligned} \tag{5}$$

2. *Given the distribution of* $\{\tilde{\eta}_A, \tilde{\eta}_B\}$*,* $\phi^{**}(P_A, P_B|x)$ *is the p.d.f. of the random price-signals* $\{P_A^{**}(x, \tilde{\eta}_A, \tilde{\eta}_B), P_B^{**}(x, \tilde{\eta}_B, \tilde{\eta}_A)\}$ *conditional on any quality* x*.*

3. *Given* $\phi^{**}(P_A, P_B|\cdot)$ *and prior beliefs* $\xi(\cdot)$*, the uninformed buyers' posterior beliefs about quality upon observing* P_A *and* P_B *is* $\tilde{\mu}^{**}|P_A, P_B$ *with the p.d.f.*

$$\hat{\xi}^{**}(x|P_A, P_B) = \frac{\xi(x)\phi^{**}(P_A, P_B|x)}{\int_{x' \in \mathbb{R}} \xi(x')\phi^{**}(P_A, P_B|x')\mathrm{d}x'}, \tag{6}$$

$x \in \mathbb{R}$.

Before proceeding with the characterization of the equilibrium, we discuss the distributional assumption for prior beliefs and the random demand shocks. We assume that the demand shocks are known to the firm, but unobserved by the buyers, which implies that the prices cannot fully reveal quality since they also depend on unobserved demand shocks. We rely on the fact that the family of normal distributions with an unknown mean is a

[9] The variable μ refers to the true quality whereas x is used as a dummy variable for quality.

conjugate family for samples from a normal distribution.[10] With the normality assumption, we obtain a unique linear equilibrium, i.e., an equilibrium in which the uninformed buyers' updating rule is linear in the price-signals. Although negative demand shocks can yield a negative price or a negative posterior mean, the values of the parameters of the model can be restricted to ensure that the probability of such events be arbitrarily close to zero. Moreover, it turns out that, for any parameters, equilibrium values for mean prices are always positive.

Assumption 2.2. *Prior beliefs are $\tilde{\mu} \sim N(\rho, \sigma_\mu^2)$, with $\rho > 0$. Distributions of demand shocks are $\tilde{\eta}_A \sim N(0, \sigma_\eta^2), \tilde{\eta}_B \sim N(0, \sigma_\eta^2)$ such that $\mathbb{E}[\tilde{\eta}_A \tilde{\eta}_B] = 0$.*

Using Definition 2.1, Proposition 2.3 characterizes the noisy signaling equilibrium when the firm price discriminates. Specifically, the price strategies and the posterior beliefs as a function of the signals are provided. The joint distribution of the price-signals is immediate from Assumption 2.2, (7), and (8).

Proposition 2.3. *Suppose that markets are segmented. For $\lambda \in [0, 1)$, there exists a noisy signaling equilibrium with discriminatory pricing.*[11] *In equilibrium,*

1. *Given quality μ and demand shocks $\{\eta_A, \eta_B\}$, the firm sets prices*

$$P_A^{**}(\mu, \eta_A, \eta_B) = \frac{\delta_0^{**}\delta_1^{**}\gamma^2(1-\lambda)^2 + (2 - 2\delta_2^{**}\gamma(1-\lambda) + \delta_1^{**}\gamma^2\lambda(1-\lambda))\mu}{4 - \delta_1^{**2}\gamma^2(1-\lambda)^2 - 4\delta_2^{**}\gamma(1-\lambda)} + \frac{(2 - 2\delta_2^{**}\gamma(1-\lambda))\eta_A + \delta_1^{**}\gamma(1-\lambda)\eta_B}{4 - \delta_1^{**2}\gamma^2(1-\lambda)^2 - 4\delta_2^{**}\gamma(1-\lambda)} \tag{7}$$

[10] Normal assumption combined with linear demand yields closed-form equilibrium values and makes the analysis tractable by focusing on the mean and variance of price and posterior beliefs. See Grossman and Stiglitz (1980), Kyle (1985), Judd and Riordan (1994), and Mirman et al. (2012) for the use of normal distributions to study the informational role of prices in single-agent problems (without market segmentation).

[11] When $\lambda = 1$, all buyers are informed and no updating rule needs to be specify. In this case, there exists an equilibrium with equilibrium prices $P_A^{**}(\mu, \eta_A, \eta_B) = (\mu + \eta_A)/2$ and $P_B^{**}(\mu, \eta_B, \eta_A) = (\gamma\mu + \eta_B)/2$, which are equal to (7) and (8) evaluated at $\lambda = 1$, respectively.

and

$$P_B^{**}(\mu,\eta_B,\eta_A)=\frac{2\delta_0^{**}\gamma(1-\lambda)+(\delta_1^{**}\gamma(1-\lambda)+2\gamma\lambda)\mu+\delta_1^{**}\gamma(1-\lambda)\eta_A+2\eta_B}{4-\delta_1^{**2}\gamma^2(1-\lambda)^2-4\delta_2^{**}\gamma(1-\lambda)}. \tag{8}$$

2. *Given any observation $\{P_A,P_B\}$, the uninformed buyers' posterior beliefs are*

$$\tilde{\mu}^{**}|P_A,P_B\sim N\left(\delta_0^{**}+\delta_1^{**}P_A+\delta_2^{**}P_B,\frac{\sigma_\eta^2\sigma_\mu^2}{\sigma_\eta^2+(1+\gamma^2\lambda^2)\sigma_\mu^2}\right). \tag{9}$$

Here,

$$\delta_0^{**}=\frac{\rho\sigma_\eta^2}{\sigma_\eta^2+\sigma_\mu^2(1+\gamma^2\lambda)}, \tag{10}$$

$$\delta_1^{**}=\frac{2\sigma_\mu^2}{\sigma_\eta^2+\sigma_\mu^2(1+\gamma^2\lambda)}, \tag{11}$$

$$\delta_2^{**}=\frac{2\gamma(\lambda\sigma_\mu^2(\sigma_\eta^2+2\sigma_\mu^2)-\sigma_\mu^4(1-\gamma^2\lambda^2))}{(\sigma_\eta^2+\sigma_\mu^2(1+\gamma^2\lambda))(\sigma_\eta^2+\sigma_\mu^2(1+\gamma^2\lambda(2-\lambda)))}. \tag{12}$$

Proof. See Appendix A. □

Next, we define and characterize the noisy signaling equilibrium for the benchmark model of uniform pricing. Definition 2.4 provides the noisy signaling equilibrium for the benchmark case in which the firm does not price discriminate.

Definition 2.4. *The tuple $\left\{P^*(\mu,\eta_A,\eta_B),\phi^*(P|\cdot),\hat{\xi}^*(\cdot|P)\right\}$ is a noisy signaling equilibrium with uniform pricing if, for all $\mu>0$,*

1. *Given $\hat{\xi}^*(\cdot|P)$, and for any η_A and η_B, the firm's price strategy is*

$$P^*(\mu,\eta_A,\eta_B)=\arg\max_P\left\{P\cdot\Big(Q_A(P,\mu,\eta_A)+Q_B(P,\mu,\hat{\xi}^*(\cdot|P),\eta_B)\Big)\right\}. \tag{13}$$

2. *Given the distribution of $\{\tilde{\eta}_A,\tilde{\eta}_B\}$, $\phi^*(P|x)$ is the p.d.f. of the random price-signal $P^*(x,\tilde{\eta}_A,\tilde{\eta}_B)$ conditional on any quality x.*

3. *Given $\phi^*(P|\cdot)$ and prior beliefs $\xi(\cdot)$, the uninformed buyers' posterior beliefs upon observing any P is $\tilde{\mu}^*|P$ with p.d.f.*

$$\hat{\xi}^*(x|P) = \frac{\xi(x)\phi^*(P|x)}{\int_{x' \in \mathbb{R}} \xi(x')\phi^*(P|x')\mathrm{d}x'}, \tag{14}$$

$x \in \mathbb{R}$.

Proposition 2.5 characterizes the noisy signaling equilibrium when the firm does not price discriminate. The distribution of the price-signal is immediate from Assumption 2.2 and (15).

Proposition 2.5. *Suppose that markets are not segmented. For $\lambda \in [0, 1)$, there exists a noisy signaling equilibrium with uniform pricing.*[12] *In equilibrium,*

1. *Given quality μ and demand shocks $\{\eta_A, \eta_B\}$, the firm sets the price*

$$P^*(\mu, \eta_A, \eta_B) = \frac{\beta_0^*\gamma(1-\lambda) + (1+\gamma\lambda)\mu + \eta_A + \eta_B}{4 - 2\beta_1^*\gamma(1-\lambda)} \tag{15}$$

in markets A and B.

2. *Given any observation P, the uninformed buyers' posterior beliefs are*

$$\tilde{\mu}^*|P \sim N\left(\beta_0^* + \beta_1^* P, \frac{2\sigma_\eta^2\sigma_\mu^2}{2\sigma_\eta^2 + (1+\lambda\gamma)^2\sigma_\mu^2}\right). \tag{16}$$

Here,

$$\beta_0^* = \frac{2\rho\sigma_\eta^2}{2\sigma_\eta^2 + \sigma_\mu^2(1+\gamma+\gamma\lambda+\gamma^2\lambda)}, \tag{17}$$

$$\beta_1^* = \frac{4(1+\gamma\lambda)\sigma_\mu^2}{2\sigma_\eta^2 + \sigma_\mu^2(1+2\gamma+2\gamma^2\lambda-\gamma^2\lambda^2)}. \tag{18}$$

Proof. See Appendix A. □

[12] When $\lambda = 1$, all buyers are informed and no updating rule needs to be specify. In this case, there exists an equilibrium with $P^*(\mu, \eta_A, \eta_B) = ((1+\gamma)\mu + \eta_A + \eta_B)/4$ which is equal to (15) evaluated at $\lambda = 1$.

3 Analysis

In this section, we study the effect of discriminatory pricing on the dissemination of information. Specifically, we consider two aspects for the quality of information: the bias of the posterior mean and the posterior variance.

We first show that price discrimination reduces the bias of the (unconditional) posterior mean. From Proposition 2.3, the expected bias with discriminatory prices is the absolute value of the difference between the unconditional posterior mean for quality and the true quality μ, i.e.,

$$\mathcal{B}^{**} \equiv \int_{P_A,P_B} \mathbb{E}[\tilde{\mu}^{**}|P_A,P_B]\phi^{**}(P_A,P_B|\mu)\mathrm{d}P_A\mathrm{d}P_B - \mu \tag{19}$$

whereas, from Proposition 2.5, the expected bias with uniform prices is

$$\mathcal{B}^{*} \equiv \int_{P} \mathbb{E}[\tilde{\mu}^{*}|P]\phi^{*}(P|\mu)\mathrm{d}P - \mu. \tag{20}$$

Proposition 3.1 states that the direction of the effect depends only on the bias of the prior mean. The posterior mean of quality is on average unbiased for unbiased prior mean belief, i.e., for $\rho = \mu$. However, if the prior mean is biased, the posterior mean is biased on average, but price discrimination amplifies the reduction in the bias of the posterior. In other words, the posterior bias is always smaller under price discrimination than under uniform pricing.

Proposition 3.1. *From* (19) *and* (20),

1. *For* $\rho = \mu$, $|\mathcal{B}^{**}| = |\mathcal{B}^{*}| = 0$.

2. *For* $\rho \neq \mu$, $|\mathcal{B}^{**}| < |\mathcal{B}^{*}|$.

Proof. From (7), (8), and (9),

$$\int_{P_A,P_B} \mathbb{E}[\tilde{\mu}^{**}|P_A,P_B]\phi^{**}(P_A,P_B|\mu)\mathrm{d}P_A\mathrm{d}P_B = \frac{\rho\sigma_\eta^2 + \mu(1+\gamma^2\lambda^2)\sigma_\mu^2}{\sigma_\eta^2 + (1+\gamma^2\lambda^2)\sigma_\mu^2}. \tag{21}$$

From (15) and (16),

$$\int_P \mathbb{E}[\tilde{\mu}^*|P]\phi^*(P|\mu)\mathrm{d}P = \frac{2\rho\sigma_\eta^2 + \mu(1+\gamma\lambda)^2\sigma_\mu^2}{2\sigma_\eta^2 + (1+\gamma\lambda)^2\sigma_\mu^2}. \tag{22}$$

Plugging (21) into (19) and (22) into (20) yields the results stated in Proposition 3.1. □

Next, we consider the effect of discriminatory pricing on the posterior variance. Proposition 3.2 states that the posterior variance for quality is always greatest with non-discriminatory prices. Hence, price discrimination provides more information to the uninformed buyers, i.e., the posterior beliefs for quality are more precise. Equation (23) characterizes the variance differential. Notice from (23), that the presence of both demand uncertainty and prior uncertainty are necessary for market segmentation to have an effect on the informational content of prices. If there is no prior uncertainty, then there is no reason to learn from observing prices. Moreover, if there is no demand shock, then observing more prices does not provide more information to the uninformed buyers. Indeed, in this case, the uninformed buyers can back out the true quality by observing a unique price-signal.

Proposition 3.2. *From* (9) *and* (16),

$$\mathbb{V}[\tilde{\mu}^*] - \mathbb{V}[\tilde{\mu}^{**}] = \frac{(1-\gamma\lambda)^2\sigma_\eta^2\sigma_\mu^4}{(\sigma_\eta^2 + (1+\gamma^2\lambda^2)\sigma_\mu^2)(2\sigma_\eta^2 + (1+\gamma\lambda)^2\sigma_\mu^2)} \geq 0. \tag{23}$$

Before discussing Proposition 3.2, it is worth considering three special cases of (23). Consider first the benchmark case of full information with two identical markets, i.e., $\gamma = \lambda = 1$. Then, for $\eta_A = \eta_B$, the firm sets the same price in both markets. Hence, for an uninformed *outsider*, market segmentation yields no gain in precision of the posterior beliefs. Next, consider two special cases for which discriminatory prices provide more precise posterior beliefs. If $\lambda = 1$ and $\gamma \in [0, 1)$, then discriminatory prices provide better information to an uninformed *outsider*. The fact that two signals about two fully informed markets be available makes the posterior beliefs more precise.

In other words, the market price is more informative to outsiders.[13] If $\gamma = 1$ and $\lambda \in (0, 1)$, then preferences over the good are the same between the two markets, but some buyers in market B are uninformed. In the presence of uninformed buyers, segmenting a market into two identical markets provides more precise information to the uninformed buyers.[14]

We now discuss Proposition 3.2. The gain in precision due to discriminatory prices holds in spite of changes in the variances of the price-signals. On the one hand, discriminatory pricing (compared to uniform pricing) implies that the buyers receive two signals instead of one. Hence, holding everything else constant, price discrimination provides more signals and increases the precision of the posterior beliefs. On the other hand, since the firm sets prices, the variance of the price-signals are endogenous. In particular, it is possible for the variance of the price-signals to increase as a consequence of market segmentation. Hence, there is a trade-off. Price discrimination offers more signals but each of these signals might be less precise. To show this, we provide some numerical evidence regarding the precision of the signals. Specifically, Figure 1 shows that the variance of the price-signals under market segmentation can be higher than the variance of the price-signal under no segmentation. Although there is a trade-off, it turns out that third-degree price discrimination always reduces the variance of the posterior mean for quality even when each price-signal becomes less precise.

We conclude this section with a comparative analysis on (23). Specifically, the parameters of the model can mitigate or reinforce the positive effect of price discrimination on the variance of the posterior beliefs. Remark 3.3 presents the effect of noise on the variance differential. An increase in the variance of the prior beliefs always increases the variance differential. Specifically, from (24), if the prior beliefs are very precise, then the differential in the posterior variance stemming from the observation of two price-signals instead of one is relatively small. Since the uninformed buyers are quite certain that the true quality lies within a constraint interval, the firm's signaling

[13] Given $\lambda = 1$, from (9) and (16) we have $\mathbb{V}[\tilde{\mu}^{**}] = \sigma_\eta^2\sigma_\mu^2/(\sigma_\eta^2 + (1+\gamma^2)\sigma_\mu^2)$ and $\mathbb{V}[\tilde{\mu}^{*}] = 2\sigma_\eta^2\sigma_\mu^2/(2\sigma_\eta^2 + (1+\gamma)^2\sigma_\mu^2)$ respectively.

[14] Given $\gamma = 1$ and $\lambda \in (0,1)$, from (9) and (16), we have $\mathbb{V}[\tilde{\mu}^{**}] = \sigma_\eta^2\sigma_\mu^2/(\sigma_\eta^2 + (1+\lambda^2)\sigma_\mu^2)$ and $\mathbb{V}[\tilde{\mu}^{*}] = 2\sigma_\eta^2\sigma_\mu^2/(2\sigma_\eta^2 + (1+\lambda)^2\sigma_\mu^2)$, respectively.

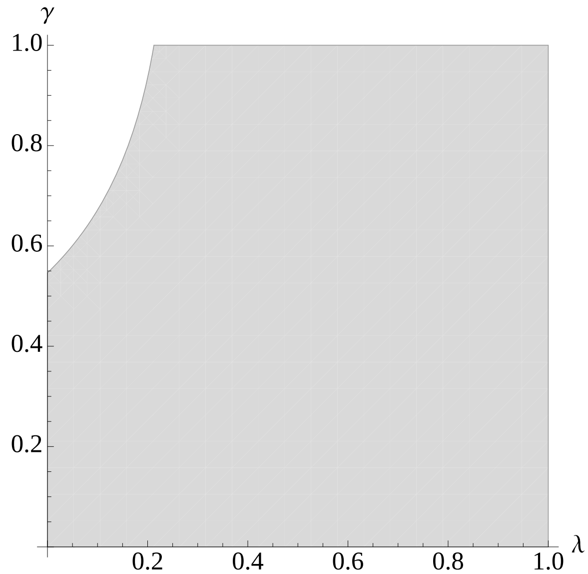

Figure 1: Region where $\mathbb{V}(P^*) < \min\{\mathbb{V}(P_A^{**}), \mathbb{V}(P_B^{**})\}$ with $\sigma_\eta^2 = 1$ and $\sigma_\mu^2 = 1$

activity does not play a prominent role as the informational reaction to the price-signals is small, i.e., β_1^* or δ_1^{**} and δ_2^{**} are small. On the other hand, if the prior beliefs are very diffuse, the firm's signaling activity matters and the differential in information that two price-signals convey instead of one is significantly more important.

Remark 3.3. *From* (23)*,*

$$\frac{\partial(\mathbb{V}[\tilde{\mu}^*] - \mathbb{V}[\tilde{\mu}^{**}])}{\partial \sigma_\mu} \geq 0. \tag{24}$$

Moreover,

$$\frac{\partial(\mathbb{V}[\tilde{\mu}^*] - \mathbb{V}[\tilde{\mu}^{**}])}{\partial \sigma_\eta} \geq 0 \tag{25}$$

if and only if

$$\frac{\sigma_\eta^2}{\sigma_\mu^2} \leq (1+\gamma\lambda)\sqrt{\frac{1+\gamma^2\lambda^2}{2}}. \tag{26}$$

Next, consider the effect of the proportion of informed buyers and the differential in demand on (23). Remark 3.4 states that the larger is the proportion of informed buyers and the lesser is the differential in buyers' valuation, then the smaller is the variance differential coming from market segmentation.

Remark 3.4. *From* (23),

$$\frac{\partial \mathbb{V}[\tilde{\mu}^*] - \mathbb{V}[\tilde{\mu}^{**}]}{\partial \lambda} \leq 0 \tag{27}$$

and

$$\frac{\partial \mathbb{V}[\tilde{\mu}^*] - \mathbb{V}[\tilde{\mu}^{**}]}{\partial \gamma} \leq 0. \tag{28}$$

From (27), as λ increases, under both uniform pricing and third-price discrimination, the posterior beliefs become less volatile as the price-signals incorporate more information from the mere fact that a larger proportion of buyers knows the true quality μ. However, the posterior variance decreases more rapidly when the uninformed buyers observe a single price-signal rather than two price-signals. This means that the benefit on the flow of information arising from a signal of a better quality is subject to some form of decreasing return. From (9) and (16), $\beta_1^* > \delta_1^{**} + \delta_2^{**}$. Hence, the uninformed buyers are always more sensitive to an improvement in the quality of a price-signal (due to more informed buyers) under uniform pricing than under discriminatory pricing. This higher sensibility translates into a higher decay rate of the posterior variance.

Finally, we investigate the effect of the parameter γ on the variance differential in (23). From (28), an increase in γ reduces the benefit from observing two price-signals rather than one. Recall that γ is an indicator of how elastic is market B relative to market A, i.e., as $\gamma \to 1$, the two market segments are more similar to each other. Hence an increase in γ, by homogenizing the two markets, implies that P_A and P_B incorporate and convey a more similar content to the uninformed buyers.[15] Therefore, as $\gamma \to 1$, the uninformed buyers gain less from observing a second price-signal as it contains little supplementary informative content.

[15] Using the criterion of mutual information $MI(\tilde{P}_A, \tilde{P}_B) = -log(1-\rho^2)/2$ where ρ is the correlation coefficient between $\tilde{P}_A$ and $\tilde{P}_B$, then we have $\partial MI(\tilde{P}_A, \tilde{P}_B)/\partial\gamma > 0$ such that the mutual information of $\tilde{P}_A$ and $\tilde{P}_B$ increases with γ.

4 Final Remarks

In this paper, we study the effect of market segmentation on the informational content of prices. We find that market segmentation improves the informational content of the price-signals, which benefits the uninformed buyers by yielding more precise posterior beliefs. Since the introduction of noise precludes complete learning, the uninformed buyers continue to face uncertainty about the good's quality. In future work, it would be interesting to study the effect of risk-aversion under incomplete learning.

It is important to note that our analysis implicitly assumes that both markets are served whether pricing is discriminatory or not. In general, price discrimination makes it profitable to serve markets that would otherwise not be served with uniform pricing. In order words, discriminatory pricing may lead to the opening of new markets. In the presence of uninformed buyers, uniform pricing might make it more likely to exclude the buyers of one of the markets. The reason is that the informational externality generally leads to an increase in the mean prices. Hence, the benefit of market segmentation (in terms of accessibility of the good) is enhanced by the presence of uninformed buyers. See Appendix B.

A Proofs

Let $\mathbb{E}$ and $\mathbb{V}$ be the *expectation* and *variance* operators, respectively.

Proof of Proposition 2.3. Using Definition 2.1, we proceed in three steps. First, given the uninformed buyers' updating rule, we solve for the firm's optimal price strategies. Second, we derive the distribution of the posterior beliefs that follows from the firm's price strategies and the prior beliefs. Finally, we check that the uninformed buyers's updating rule and the distribution of the price-signals are mutually consistent.

1. Given (9), $\mathbb{E}[\tilde{\mu}^{**}|P_A, P_B] = \delta_0^{**} + \delta_1^{**} P_A + \delta_2^{**} P_B$. Plugging (1), (2), and $\mathbb{E}[\tilde{\mu}^{**}|P_A, P_B] = \delta_0^{**} + \delta_1^{**} P_A + \delta_2^{**} P_B$ into (5) yields

$$\begin{aligned} \max_{P_A, P_B} \; & \{P_A \cdot (\mu - P_A + \eta_A) \\ & + P_B \cdot (\lambda(\gamma\mu - P_B) + (1-\lambda)(\gamma(\delta_0^{**} + \delta_1^{**} P_A + \delta_2^{**} P_B) - P_B) + \eta_B)\} . \end{aligned} \tag{29}$$

Taking the first-order conditions with respect to prices yields

$$P_A : \mu - 2P_A + \eta_A + (1-\lambda)\gamma\delta_1^{**} P_B = 0, \tag{30}$$

$$P_B : \lambda(\gamma\mu - 2P_B) + (1-\lambda)(\gamma(\delta_0^{**} + \delta_1^{**} P_A + 2\delta_2^{**} P_B) - 2P_B) + \eta_B = 0. \tag{31}$$

Given (11) and (12), the Hessian matrix is negative definite. Solving (30) and (31) for the price strategies yields (7) and (8).

2. Next, given the firm's price strategies, we solve for the buyers' posterior beliefs. Specifically, using (7) and (8), let

$$\begin{aligned} \tilde{z}_A \equiv\; & 2\left(P_A^{**}(\mu, \tilde{\eta}_A, \tilde{\eta}_B) - \frac{\delta_0^{**}\delta_1^{**}\gamma^2(1-\lambda)^2}{D_0}\right) \\ & \cdot \left(\frac{D_0}{D_0 + \delta_1^{**2}\gamma^2(1-\lambda)^2 + 2\delta_1^{**}\gamma^2\lambda(1-\lambda)}\right), \end{aligned} \tag{32}$$

$$= \mu + \left(\frac{2 - 2\delta_2^{**}\gamma(1-\lambda)}{D_1}\right)\tilde{\eta}_A + \left(\frac{\delta_1^{**}\gamma(1-\lambda)}{D_1}\right)\tilde{\eta}_B, \tag{33}$$

and

$$\tilde{z}_B \equiv \left(P_B^{**}(\mu, \tilde{\eta}_B, \tilde{\eta}_A) - \frac{2\delta_0^{**}\gamma(1-\lambda)}{D_0}\right)\left(\frac{D_0}{\delta_1^{**}\gamma(1-\lambda)+2\gamma\lambda}\right), \quad (34)$$

$$= \mu + \left(\frac{\delta_1^{**}\gamma(1-\lambda)}{\delta_1^{**}\gamma(1-\lambda)+2\gamma\lambda}\right)\tilde{\eta}_A + \left(\frac{2}{\delta_1^{**}\gamma(1-\lambda)+2\gamma\lambda}\right)\tilde{\eta}_B, \quad (35)$$

where

$$D_0 \equiv 4 - 4\delta_2^{**}\gamma(1-\lambda) - \delta_1^{**2}\gamma^2(1-\lambda)^2, \quad (36)$$

$$D_1 \equiv 2 - 2\delta_2^{**}\gamma(1-\lambda) + \delta_1^{**}\gamma^2\lambda(1-\lambda). \quad (37)$$

From (33) and (35), $\tilde{\mathbf{z}}|\mu \equiv [\tilde{z}_A, \tilde{z}_B]'|\mu$ is jointly normally distributed. Hence, given the prior distribution $\tilde{\mu} \sim N(\rho, \sigma_\mu^2)$, the posterior distribution of the quality μ upon observing $\mathbf{z}$ (i.e., upon observing $\{P_A, P_B\}$) is

$$\tilde{\mu}^{**}|\mathbf{z} \sim N(\rho + \sigma_\mu^2 \mathbb{1}\mathbf{\Sigma}^{-1}(\mathbf{z} - \rho\mathbb{1}'), \sigma_\mu^2 - \sigma_\mu^4 \mathbb{1}\mathbf{\Sigma}^{-1}\mathbb{1}') \quad (38)$$

where $\mathbb{1}$ is a 1×2 vector of ones and

$$\mathbf{\Sigma} \equiv \begin{bmatrix} \sigma_\mu^2 + \sigma_\eta^2 \frac{4(1+\delta_2\gamma(\lambda-1))^2+\delta_1^2\gamma^2(1-\lambda)^2}{D_1^2} & \sigma_\mu^2 + \sigma_\eta^2 \frac{2\delta_1\gamma(1-\lambda)(2+\delta_2\gamma(\lambda-1))}{D_1(\delta_1\gamma(1-\lambda)+2\gamma\lambda)} \\ \sigma_\mu^2 + \sigma_\eta^2 \frac{2\delta_1\gamma(1-\lambda)(2+\delta_2\gamma(\lambda-1))}{D_1(\delta_1\gamma(1-\lambda)+2\gamma\lambda)} & \sigma_\mu^2 + \sigma_\eta^2 \left(\frac{\delta_1^2\gamma^2(1-\lambda)^2+4}{(\delta_1\gamma(1-\lambda)+2\gamma\lambda)^2}\right) \end{bmatrix}. \quad (39)$$

Hence, using (38), given P_A and P_B, the posterior mean and variance are

$$\mathbb{E}[\tilde{\mu}^{**}|P_A, P_B] = \frac{\rho\sigma_\eta^2 - \delta_0^{**}\gamma^2\lambda(1-\lambda)\sigma_\mu^2 + (2 - \delta_1^{**}\gamma^2\lambda(1-\lambda))\sigma_\mu^2 P_A}{\sigma_\eta^2 + (1+\gamma^2\lambda^2)\sigma_\mu^2} + \frac{(2\gamma\lambda(1-\delta_2^{**}\gamma(1-\lambda)) - \delta_1^{**}\gamma(1-\lambda))\sigma_\mu^2 P_B}{\sigma_\eta^2 + (1+\gamma^2\lambda^2)\sigma_\mu^2}, \quad (40)$$

and

$$\mathbb{V}[\tilde{\mu}^{**}|P_A, P_B] = \frac{\sigma_\eta^2\sigma_\mu^2}{\sigma_\eta^2 + (1+\gamma^2\lambda^2)\sigma_\mu^2}, \quad (41)$$

respectively.

3. Setting (40) equal to $\delta_0^{**} + \delta_1^{**} P_A + \delta_2^{**} P_B$ and solving for δ_0^{**}, δ_1^{**} and δ_2^{**} yields (10), (11), and (12). Since δ_0^{**}, δ_1^{**} and δ_2^{**} uniquely exist, the posterior beliefs are normally distributed as defined by (9) and are consistent with (40) and (41). Moreover, from (7) and (8), the price-signals are jointly normally distributed.

Proof of Proposition 2.5. The proof of Proposition 2.5 follows the same steps of the proof of Proposition 2.3. Using Definition 2.4, we proceed as follows.

1. Given (16), $\mathbb{E}[\tilde{\mu}^*|P] = \beta_0^* + \beta_1^* P$. Plugging (1), (2), and $\mathbb{E}[\tilde{\mu}^*|P] = \beta_0^* + \beta_1^* P$ into (13) yields

$$\max_P \left\{ P \cdot ((\mu - P + \eta_A) + \lambda(\gamma\mu - P) + (1-\lambda)(\gamma(\beta_0^* + \beta_1^* P) - P) + \eta_B) \right\}. \tag{42}$$

Taking the first-order condition with respect to price yields

$$(1+\gamma\lambda)\mu + \eta_A + \eta_B + \beta_0^*\gamma(1-\lambda) - 2P(2 - \beta_1^*\gamma(1-\lambda)) = 0. \tag{43}$$

Given (18), the second-order condition holds, i.e., $-2\left(2 - \beta_1^*\gamma(1-\lambda)\right) < 0$. Solving (43) for the price strategy yields (15).

2. Next, given the firm's price strategy, we solve for the buyers' posterior beliefs. Specifically, using (15), let

$$\tilde{z} \equiv \frac{2(2 - \beta_1^*\gamma(1-\lambda))P^*(\mu, \tilde{\eta}_A, \tilde{\eta}_B) - \beta_0^*\gamma(1-\lambda)}{1+\gamma\lambda} \tag{44}$$

$$= \mu + \frac{\tilde{\eta}_A + \tilde{\eta}_B}{1+\gamma\lambda} \tag{45}$$

such that $\tilde{z}|\mu$ is normally distributed with mean μ and variance $\sigma_z^2 \equiv 2\sigma_\eta^2/(1+\gamma\lambda)^2$. Given the prior distribution, $\tilde{\mu} \sim N(\rho, \sigma_\mu^2)$, the posterior belief upon observing z (i.e., upon observing P) is

$$\tilde{\mu}^*|z \sim N\left(\frac{\rho\sigma_z^2 + z\sigma_\mu^2}{\sigma_z^2 + \sigma_\mu^2}, \frac{1}{1/\sigma_z^2 + 1/\sigma_\mu^2}\right). \tag{46}$$

Hence, using (46), given P, the posterior mean and variance are

$$\mathbb{E}[\tilde{\mu}^*|P] = \frac{2\rho\sigma_\eta^2 + ((4 - 2\beta_1^*\gamma(1-\lambda))P - \beta_0^*\gamma(1-\lambda))(1+\gamma\lambda)\sigma_\mu^2}{2\sigma_\eta^2 + (1+\lambda\gamma)^2\sigma_\mu^2} \tag{47}$$

and

$$\mathbb{V}[\tilde{\mu}^*|P] = \frac{2\sigma_\eta^2\sigma_\mu^2}{2\sigma_\eta^2 + (1+\lambda\gamma)^2\sigma_\mu^2}, \tag{48}$$

respectively.

3. Setting (47) equal to $\beta_0^* + \beta_1^* P$ and solving for β_0^* and β_1^* yields (17) and (18). Since β_0^* and β_1^* exist, the posterior beliefs are normally distributed as defined by (16) and are consistent with (47) and (48). Finally, from (15), the price-signal is normally distributed.

B Probability of Exclusion

In this section, we study whether the presence of uninformed buyers (or the informational externality) decreases or increases the probability of excluding market B under nondiscriminatory pricing. In market B, the informed buyers and the uninformed buyers with unbiased prior beliefs do not buy the good if the price is above the reservation price, i.e., $P > \gamma\mu$. Using (15), we compare the probability of such an event under the two scenarios of complete and incomplete information, i.e., $P^*(\mu, \tilde{\eta}_A, \tilde{\eta}_B)|_{\lambda=1}$ and $P^*(\mu, \tilde{\eta}_A, \tilde{\eta}_B)|_{\lambda\in(0,1)}$. In Figure 2, the shaded area encompasses the points $\{\gamma, \lambda\}$ for which the presence of uninformed buyers increases the probability of exclusion, i.e., $\mathbb{P}[\gamma\mu < P^*(\mu, \tilde{\eta}_A, \tilde{\eta}_B)|_{\lambda=1}] < \mathbb{P}[\gamma\mu < P^*(\mu, \tilde{\eta}_A, \tilde{\eta}_B)|_{\lambda\in(0,1)}]$.[16] An increase in the variance of the demand shock increases the size of the shaded area.

[16] To generate Figure 2, we set $\mu = 1$ and $\sigma_\mu^2 = 1$.

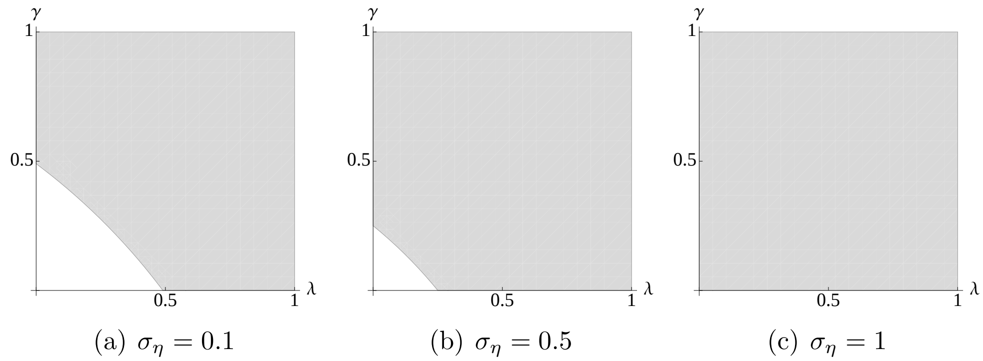

Figure 2: $\mathbb{P}[\gamma\mu < P^*(\mu, \tilde{\eta}_A, \tilde{\eta}_B)|_{\lambda=1}]$ vs. $\mathbb{P}[\gamma\mu < P^*(\mu, \tilde{\eta}_A, \tilde{\eta}_B)|_{\lambda\in(0,1)}]$.

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