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Noisy Signaling in Monopoly*

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Abstract

We study the informational role of prices in a stochastic environment. We provide a closed-form solution of the monopoly problem when the price imperfectly signals quality to the uninformed buyers. We then study the effect of noise on output, market price, information flows, and expected profits. The presence of noise may reduce the informational externality due to asymmetric information, which increases the firm's expected profits.

Keywords: Asymmetric information, Learning, Monopoly, Noise, Quality, Rational Expectations, Signaling.
JEL Classifications: D21, D42, D82, D83, D84, L12, L15.

1 Introduction

Market prices are instrumental not only in allocating resources in an economy, but also in disseminating information to market participants (Grossman, 1989). In particular, in the consumer problem, buyers face uncertainty about many aspects of the market, e.g., the characteristics or the quality of the goods that they consider purchasing. While signaling models have been developed to study asymmetric information, they all rely on a deterministic setting (Bagwell and Riordan, 1991; Daughety and Reinganum, 1995, 2005, 2007, 2008a,b; Janssen and Roy, 2010; Mirman and Santugini, 2013). That is, except for the unknown quality, every other aspect of the market is known to the buyers, e.g., demand and cost of the firm are known. However, there are many circumstances that reduce the buyers' ability to extract information from observable outcomes. In other words, uncertainty plays a crucial role in revealing information. This uncertainty in turn affects the ability of the firm to influence the learning process. For example, in addition to being uncertain about the quality, buyers lack knowledge about the demand faced by the firm, i.e., the environment is *noisy*. This affects not only the decision of the firm, but also the informativeness of the price regarding the quality of the good.

The informational role of prices in a noisy environment influences the behavior of all the agents in the model. In particular, the market outcomes depend on both the firm sending out a *signal* that is contained in prices, as well as the uninformed buyers receiving and interpreting the information contained in the price signal. Studying the informational role of prices in a noisy environment would deepen our understanding of information flows in a more complex and realistic environment. Indeed, in a noiseless environment, firms can only react to the informational externality due to asymmetric information. However, in a noisy environment, while still facing an informational externality, firms are able to take advantage of the noise by manipulating the beliefs of the uninformed buyers.

There is a small literature beginning with Matthews and Mirman (1983) on signaling in a stochastic setting. Although done in a limit pricing environment this idea is central to all models with asymmetric information. Matthews and Mirman (1983) shows, among other things, that noise may remove the necessity to define out-of-equilibrium beliefs. Yet, although the use of noise removes difficulties in characterizing the equilibrium, little is known about the effect of learning through prices when prices convey partial and incomplete information to the buyers. In light of recent experimental work that suggests that the stochastic environment actually maps better into experimental subject behavior (Jeitschko and Norman, 2012), it is important that this avenue of research be further pursued.

In this paper, we study the informational role of prices in a stochastic environment. To that end, we consider a single market in which a static monopolist supplies a good to price-taking buyers.[1] The quality of the good is known to the monopolist, but only to some buyers. The remaining buyers are uninformed, but anticipate that the market price conveys some information about quality. In other words, the uninformed buyers anticipate learning, and, thus, form expectations about the informativeness of the price that are consistent with their knowledge of the structure of the economy. In addition of not knowing quality, the buyers have partial knowledge of the market, i.e., the environment is noisy. This, in turn, implies that the market price conveys only partial information about quality.

We first provide a closed-form solution of the monopoly problem when the price imperfectly signals quality to the uninformed buyers, as well as expressions for the effects of noise on output, price, and information flows. We show that equilibrium output can be decomposed into two components. The first component is related to the effect of the informational externality in a noiseless environment. The second component is due to the presence of noise in demand.

Second, we study the effect of noise on the equilibrium. We begin by recalling that in a noiseless environment the effect of the informational externality is to decrease output. As noise increases, the posterior mean for quality depends less on the new information (from the price-signal) and more

[1] Judd and Riordan (1994) studies the behavior of a noisy monopoly in a dynamic context in which buyers learn from observing the price as well as from experience.

on the prior beliefs. Hence, the effect of the informational externality identified in the noiseless case is reduced by the presence of noise, which leads to an increase in output. An alternative explanation is as follows. Note that expected demand depends positively on the uninformed buyers' posterior mean about quality. Moreover, the posterior mean increases with the prior mean. Hence, the equilibrium quantity increases with the prior mean quality. We show that the equilibrium output in the noiseless case is equivalent to the equilibrium quantity with noise when the prior quality mean is zero. Since equilibrium quantity is minimal when the prior mean is zero, the introduction of noise necessarily increases output. We then show that the introduction of noise may increase or decrease the mean price depending on the bias of the prior relative to the true quality. In particular, if the prior mean is much greater than the true quality, then noise increases the mean price. But, if prior beliefs are unbiased or downward-biased, then the mean price decreases with noise. Finally, we note that noise may reduce the informational externality due to asymmetric information, which increases the firm's expected profits.

The paper is organized as follows. Section 2 presents the model and characterizes the noisy REE. Section 3 discusses the benchmark case of a noiseless environment. Section 4 studies the effect of noise on the equilibrium. Section 5 concludes and suggests possible extensions for the informational role of prices in a noisy environment.

2 Model and Equilibrium

In this section, we present a model of noisy information flows. Specifically, we consider a single market in which a monopolist supplies a good to price-taking buyers. The quality of the good is known to the monopolist, but only to some buyers. The remaining buyers are uninformed, but anticipate that the market price conveys some information about quality. In other words, the uninformed buyers anticipate learning, and, thus, form expectations about the informativeness of the price that are consistent with their knowledge of the structure of the economy. In addition of not knowing quality, the buyers have partial knowledge of the market, i.e., the environment is noisy. This, in turn, implies that the market price conveys only partial information about quality. We first describe the agents and the market in which they operate. We then define and characterize the noisy rational expectations equilibrium (REE). In the next section, we study the effect of noise on the equilibrium.

Consider a market for a homogeneous good of quality θ sold at price p. There are both *informed* and *uninformed* price-taking buyers. The informed buyers know θ and have demand $q_I^d = \theta - p$. The uninformed buyers do not know θ, and have prior beliefs $\tilde{\theta} \sim N\left(\mu_\theta, \sigma_\theta^2\right), \mu_\theta > 0$ with the corresponding p.d.f. $\xi(\theta)$. The uninformed buyers extract information about θ from observing the price, using Bayes' rule to update beliefs. Hence, for the uninformed buyers, the price plays an informative role about quality along with the usual role of a parameter defining the feasible set of purchases. Let $\hat{\xi}(\theta|p)$ be the posterior p.d.f. of $\tilde{\theta}$ given p. The posterior beliefs depends only on the price because the quantity supplied is not observable. Since the only difference between informed and uninformed buyers concerns information, the demand of the uninformed buyers is $q_U^d = \hat{\mu}_\theta(p) - p$ where $\hat{\mu}_\theta(p) = \int_{\mathbb{R}} x\hat{\xi}(x|p)\mathrm{d}x$ is the posterior mean for quality. Normalizing the mass of buyers to one and letting $\lambda \in [0,1]$ be the fraction of informed buyers, aggregate demand is

$$D(p, \theta, \hat{\xi}(\cdot|p), \eta) = \lambda(\theta - p) + (1-\lambda)\left(\int_{\mathbb{R}} x\hat{\xi}(x|p)\mathrm{d}x - p\right) + \eta, \qquad (1)$$

where η is a demand shock unknown to all buyers. Moreover, it is assumed that η is a realization of $\tilde{\eta} \sim N(0, \sigma_\eta^2)$.

Next, we describe the firm's maximization problem. The monopolist knows the quality θ, but faces uncertainty in demand due to the demand shock η. The monopolist supplies q units of the good to maximize expected profits $\mathbb{E}_{\tilde{\eta}} P(\theta, q, \tilde{\eta}, \hat{\xi}(\cdot))q$ where $\mathbb{E}_{\tilde{\eta}}$ is the expectation operator for the random variable $\tilde{\eta}$ and $P(\theta, q, \tilde{\eta}, \hat{\xi}(\cdot))$ is the inverse random demand function corresponding to expression (1). Information flows, working through the posterior mean, influence profits and constitute an informational externality to the monopolist.

Before proceeding with the definition and the characterization of the equilibrium, we comment on the distributional assumption. In order to study noisy signaling in monopoly, we rely on the fact that the family of normal distributions with an unknown mean is a conjugate family for samples from a normal distribution. Normal assumption combined with linear demand yields closed-form equilibrium values and makes the analysis tractable by focusing on the mean and variance of price and posterior beliefs.[2] Hence, the normality assumption allows us to gain insight on information flows in a noisy environment. Although equilibrium price and posterior mean quality can admit negative values, restrictions on parameter values ensures that the probability of a negative price or a negative posterior mean be arbitrarily close to zero. Moreover, it turns out that the equilibrium value for output is always positive.

Having described the agents and the market in which they operate, we now define the noisy REE. Equilibrium consists of the monopolist's strategy $q^*(\theta)$, the distribution of the price signal $\phi^*(p|\theta)$ for the random price $\tilde{p}^*(\theta)$, and the uninformed buyers' posterior beliefs about the quality upon observing the price, $\hat{\xi}^*(\cdot)$. In equilibrium, posterior beliefs are restricted by the uninformed buyers' partial knowledge of the structure of the economy (i.e., η is unknown to buyers). In other words, using Bayes' rule, posterior beliefs are consistent with demand and the firm's strategy. Specifically, given equilibrium output and posterior beliefs, for any quality θ and demand shock η, a realization p of the random variable $\tilde{p}^*(\theta)$ satisfies

$$\lambda(\theta - p) + (1-\lambda)\left(\int_{\mathbb{R}} x\hat{\xi}^*(x|p)\mathrm{d}x - p\right) + \eta = q^*(\theta). \tag{2}$$

Definition 2.1. *The tuple* $\left\{q^*(\theta), \phi^*(p|\theta), \hat{\xi}^*(\cdot)\right\}$ *is a noisy REE if, for all* $\theta \geq 0$,

1. *Given* $\hat{\xi}^*(\cdot)$,
$$q^*(\theta) = \arg\max_{q\geq 0} \mathbb{E}_{\tilde{\eta}} P(\theta, q, \tilde{\eta}, \hat{\xi}^*(\cdot))q. \tag{3}$$

[2] See Grossman and Stiglitz (1980), Kyle (1985), Judd and Riordan (1994) for the use of normal distributions to study the informational role of prices in single-agent problems.

2. *Given $q^*(\theta)$ and $\hat{\xi}^*(\cdot)$,*

$$\tilde{p}^*(\theta) = P(\theta, q^*(\theta), \tilde{\eta}, \hat{\xi}^*(\cdot)) \tag{4}$$

is the random price signal with the corresponding p.d.f. $\phi^*(p|\theta)$.

3. *Given $\phi^*(p|\theta)$ and prior beliefs, the uninformed buyers' posterior beliefs is*

$$\hat{\xi}^*(\theta|p) = \frac{\xi(\theta)\phi^*(p|\theta)}{\int_{\mathbb{R}} \xi(x)\phi^*(p|x)\mathrm{d}x}. \tag{5}$$

Proposition 2.2 states that there exists a unique noisy REE in which the price retains the normal distribution. In equilibrium, the updating rule is a linear function of the price signal.

Proposition 2.2. *There exists a unique REE with a linear updating rule. In equilibrium, the monopolist sells*

$$q^*(\theta) = \frac{\lambda\theta}{2} + \frac{2(1-\lambda)\sigma_\eta^2\mu_\theta}{4\sigma_\eta^2 + \lambda\sigma_\theta^2}. \tag{6}$$

The distribution of the price signal is $\tilde{p}^*(\theta) \sim N(\mu_p^*, \sigma_p^{*2})$ where

$$\mu_p^* = \frac{4\sigma_\eta^2 + (2-\lambda)\lambda\sigma_\theta^2}{4\sigma_\eta^2 + \lambda^2\sigma_\theta^2} q^*(\theta), \tag{7}$$

$$\sigma_p^{*2} = \frac{(4\sigma_\eta^2 + (2-\lambda)\lambda\sigma_\theta^2)^2\sigma_\eta^2}{(4\sigma_\eta^2 + \lambda^2\sigma_\theta^2)^2}. \tag{8}$$

The uninformed buyers' posterior beliefs are $\tilde{\theta}|p \sim N(\hat{\mu}_\theta^*(p), \hat{\sigma}_\theta^{*2})$ where

$$\hat{\mu}_\theta^*(p) = \frac{4\sigma_\eta^2\mu_\theta}{4\sigma_\eta^2 + \lambda\sigma_\theta^2} + \frac{2\lambda\sigma_\theta^2 p}{4\sigma_\eta^2 + (2-\lambda)\lambda\sigma_\theta^2}, \tag{9}$$

$$\hat{\sigma}_\theta^{*2} = \frac{4\sigma_\eta^2\sigma_\theta^2}{4\sigma_\eta^2 + \lambda^2\sigma_\theta^2}. \tag{10}$$

Proof. Given (9), plugging $\hat{\mu}_\theta^*(p) = \int_{\mathbb{R}} x\hat{\xi}^*(x|p)\mathrm{d}x = a^* + b^*p$ into (1) and solving for the price as a function of q yields the *revised* random inverse demand

$$P(\theta, q, \tilde{\eta}, \hat{\xi}^*(\cdot)) = \frac{\lambda\theta + (1-\lambda)a^* + \tilde{\eta} - q}{1-(1-\lambda)b^*} \tag{11}$$

where

$$a^* = \frac{4\sigma_\eta^2 \mu_\theta}{4\sigma_\eta^2 + \lambda\sigma_\theta^2}, \tag{12}$$

$$b^* = \frac{2\lambda\sigma_\theta^2}{4\sigma_\eta^2 + (2-\lambda)\lambda\sigma_\theta^2}. \tag{13}$$

Given (11), the first-order condition corresponding to $\max_{q\geq 0} \mathbb{E}_{\tilde{\eta}} P(\theta, q, \tilde{\eta}, \hat{\xi}^*(\cdot))q$ is $(\lambda\theta + (1-\lambda)a^* - 2q)/(1-(1-\lambda)b^*) = 0$, which yields (6).

Plugging (6), (12), and (13) into (11) yields the price signal

$$\begin{aligned} \tilde{p}^*(\theta) &= \frac{\lambda\theta + (1-\lambda)a^* + \tilde{\eta} - q^*(\theta)}{1-(1-\lambda)b^*}, && (14) \\ &= \frac{4\sigma_\eta^2 + (2-\lambda)\lambda\sigma_\theta^2}{4\sigma_\eta^2 + \lambda^2\sigma_\theta^2}(q^*(\theta) + \tilde{\eta}). && (15) \end{aligned}$$

Since $\tilde{\eta} \sim N(0, \sigma_\eta^2)$, the distribution of the price signal is normal with mean and variance (7) and (8), respectively.

Given the normal distribution of the price with mean and variance (7) and (8), respectively, and prior beliefs $\tilde{\theta} \sim N(\mu_\theta, \sigma_\theta^2)$, the uninformed buyers' posterior beliefs is $\tilde{\theta}|p \sim N(\hat{\mu}_\theta^*(p), \hat{\sigma}_\theta^{*2})$, where $\hat{\mu}_\theta^*(p)$ and $\hat{\sigma}_\theta^{*2}$ are defined by (9) and (10), respectively.[3] □

The parameter λ is the fraction of informed buyers in the market and provides information about the intensity of the informational externality faced

[3] Note that expression (15) can be rewritten as $\tilde{p}^*(\theta) = \frac{\lambda\theta/2 + (1-\lambda)a^*/2 + \tilde{\eta}}{1-(1-\lambda)b^*}$. Let $\tilde{z} \equiv 2(1-(1-\lambda)b^*)\tilde{p}^*(\theta)/\lambda - (1-\lambda)a^*/\lambda = \tilde{\theta} + 2\tilde{\eta}/\lambda$, so that $\tilde{z}|\tilde{\theta} \sim N(\tilde{\theta}, 4\sigma_\eta^2/\lambda^2)$. Given that $\tilde{\theta} \sim N(\mu_\theta, \sigma_\theta^2)$, the posterior distribution upon observing $z = 2(1-(1-\lambda)b^*)p/\lambda - (1-\lambda)a^*/\lambda$ is

$$\tilde{\theta}|z \sim N\left(\frac{\frac{4\sigma_\eta^2}{\lambda^2}\mu_\theta + \sigma_\theta^2(2(1-(1-\lambda)b^*)p/\lambda - (1-\lambda)a^*/\lambda)}{\frac{4\sigma_\eta^2}{\lambda^2} + \sigma_\theta^2}, \frac{1}{\frac{1}{\frac{4\sigma_\eta^2}{\lambda^2}} + \frac{1}{\sigma_\theta^2}}\right). \tag{16}$$

Equating the posterior mean defined by (16) to $a^* + b^*p$ and solving for a^* and b^* confirms (12) and (13).

by the firm. The case of $\lambda = 1$ refers to the benchmark case of full information in which there is no informational externality. If $\lambda \in [0, 1)$, then there are uninformed buyers who have rational expectations about the relationship between quality and the price. The parameter σ_η^2 provides information about the level of noise in the market. When $\sigma_\eta^2 = 0$, the environment is assumed noiseless as done generally in the signaling literature. When $\sigma_\eta^2 > 0$, the environment is noisy. The expression for output stated in (6) shows the influence of these parameters on behavior.

Equilibrium output is the sum of two components. The first component of (6) depends on the fraction of informed buyers, but not on noise, which thus refers to the equilibrium level of output set by the firm in a noiseless environment with information flows. The second component depends on both the fraction of informed buyers and noise and represents the effect of noise on the firm's behavior in a noisy REE.

Having characterized the REE, we use Proposition 2.2 to study the effect of noise on the REE. We proceed in two steps. We first discuss the benchmark noiseless REE (when $\sigma_\eta^2 = 0$). We then study how an increase in the variance of the noise alters the equilibrium by comparing the noisy and noiseless outcomes for quantity, price distribution and posterior beliefs.

3 The Noiseless Case

In the noiseless case (i.e., $\sigma_\eta^2 = 0$), there is complete learning. Indeed, from (12) and (13), $\hat{\mu}_\theta^*(p)|_{\sigma_\eta^2=0} = a^* + b^* p = 2p/(2 - \lambda)$, so that, from Proposition 2.2,

$$\mathbb{E}_{\tilde{p}^*(\theta)} \hat{\mu}_\theta^*(\tilde{p}^*(\theta))|_{\sigma_\eta^2=0,\lambda>0} = \theta \tag{17}$$

and $\sigma_\theta^{*2}|_{\sigma_\eta^2=0,\lambda>0} = 0$. Although there is complete learning, the presence of uninformed buyers has an effect on behavior. In other words, the full information solution is not a noiseless REE. To see why, suppose to the contrary that the full information solution is a noiseless REE. Then, from (6) and (7), given $q^*(\theta)|_{\sigma_\eta^2=0,\lambda=1} = \mu_p^*|_{\sigma_\eta^2=0,\lambda=1} = \theta/2$, the uninformed buyers' posterior mean is $\hat{\mu}_\theta^*(p) = 2p$. However, this is inconsistent with the firm's optimal behavior. Indeed, given $\hat{\mu}_\theta^*(p) = 2p$, the firm's optimal output decision is not $q^*(\theta)|_{\sigma_\eta^2=0,\lambda=1} = \theta/2$, which destabilizes the equilibrium.

As the firm maximizes expected profit, it takes account of the effect of output on the price directly through the market as well as indirectly through the posterior beliefs. Hence, the presence of uninformed buyers has an effect on the firm's behavior. Specifically, from (6), $\partial q^*(\theta)|_{\sigma_\eta^2=0}/\partial\lambda = \theta/2 > 0$ so that a decrease in the fraction of informed buyers reduces the quantity produced by the firm, i.e., $q^*(\theta)|_{\sigma_\eta^2=0,\lambda=1} = \theta/2 > q^*(\theta)|_{\sigma_\eta^2=0,\lambda\in[0,1)} = \lambda\theta/2$. This in turn increases the mean price. There is thus a loss in efficiency because expected profits are reduced and buyers pay on average a higher price. This loss of efficiency decreases with the fraction of informed buyers, tending to the full information case as $\lambda \to 1$.

4 The Effect of Noise

Having discussed the noiseless case, we now proceed with the effect of noise on behavior. Let

$$\psi_q = q^*(\theta)|_{\sigma_\eta^2>0} - q^*(\theta)|_{\sigma_\eta^2=0}\,, \tag{18}$$

$$\psi_{\mu_p} = \mu_p^*\big|_{\sigma_\eta^2>0} - \mu_p^*\big|_{\sigma_\eta^2=0}\,, \tag{19}$$

$$\psi_{\sigma_p^2} = \sigma_p^{*2}\big|_{\sigma_\eta^2>0} - \sigma_p^{*2}\big|_{\sigma_\eta^2=0}\,, \tag{20}$$

$$\psi_{\hat{\mu}_\theta} = \int_{\mathbb{R}} \hat{\mu}_\theta^*(p)\phi^*(p)\mathrm{d}p\bigg|_{\sigma_\eta^2>0} - \int_{\mathbb{R}} \hat{\mu}_\theta^*(p)\phi^*(p)\mathrm{d}p\bigg|_{\sigma_\eta^2=0}\,, \tag{21}$$

$$\psi_{\hat{\sigma}_\theta^2} = \hat{\sigma}_\theta^{*2}\big|_{\sigma_\eta^2>0} - \hat{\sigma}_\theta^{*2}\big|_{\sigma_\eta^2=0} \tag{22}$$

be the effects of noise on output, the mean and variance of the equilibrium price, and the posterior mean and variance of quality (evaluated at the equilibrium mean price), respectively.

We begin with the effect of noise on posterior beliefs, which is the basis for understanding the effect of noise on the other variables. Proposition 4.1 states the effect of noise on posterior beliefs. The posterior mean evaluated at the equilibrium mean price may increase or decrease with noise depending on the bias of the prior mean. For instance, if prior beliefs are biased upward, i.e., $\mu_\theta > \theta$, then posterior beliefs remain biased upward, although the bias is reduced. The reduction in the bias depends on the composition of buyers, the variance of the demand shock, and the variance of the prior beliefs. The posterior variance necessarily increases with noise since there is complete learning in the noiseless REE.

Proposition 4.1. *From Proposition 2.2, and using* (21) *and* (22)*,*

$$\psi_{\hat{\mu}_\theta} = \frac{4\sigma_\eta^2(\mu_\theta - \theta)}{4\sigma_\eta^2 + \lambda^2\sigma_\theta^2} > 0, \tag{23}$$

if and only if $\mu_\theta > \theta$*. Moreover,*

$$\psi_{\hat{\sigma}_\theta^2} = \frac{4\sigma_\eta^2\sigma_\theta^2}{4\sigma_\eta^2 + \lambda^2\sigma_\theta^2} > 0. \tag{24}$$

Proposition 4.2 provides the effects of noise on output and the price distribution. On the one hand, the effect of noise is to unambiguously increase output. On the other hand, noise may increase or decrease the mean price depending on the bias of the prior relative to the true quality. In particular, if the prior mean is much greater than the true quality, then noise increases the mean price. But, if prior beliefs are unbiased or downward-biased, i.e., $\mu_\theta \leq \theta$, then the mean price decreases with noise.

Proposition 4.2. *From Proposition 2.2, and using* (18)*,* (19)*, and* (20)*,*

$$\psi_q = \frac{2(1-\lambda)\sigma_\eta^2\mu_\theta}{4\sigma_\eta^2 + \lambda\sigma_\theta^2} > 0. \tag{25}$$

Moreover,

$$\psi_{\mu_p} = \frac{4\sigma_\eta^2 + (2-\lambda)\lambda\sigma_\theta^2}{4\sigma_\eta^2 + \lambda^2\sigma_\theta^2}\left(\frac{\lambda\theta}{2} + \frac{2(1-\lambda)\sigma_\eta^2\mu_\theta}{4\sigma_\eta^2 + \lambda\sigma_\theta^2}\right) - \frac{(2-\lambda)\theta}{2} > 0, \tag{26}$$

if and only if $\mu_\theta > \frac{2\sigma_\eta^2+\lambda\sigma_\theta^2/2}{\sigma_\eta^2+(2-\lambda)\lambda\sigma_\theta^2/4}\theta > \theta$, *and*

$$\psi_{\sigma_p^2} = \frac{\left(4\sigma_\eta^2 + (2-\lambda)\lambda\sigma_\theta^2\right)^2}{\left(4\sigma_\eta^2 + \lambda^2\sigma_\theta^2\right)^2 \sigma_\eta^2} > 0. \tag{27}$$

We now discuss Proposition 4.2 by focusing on the effect of noise on equilibrium output and mean price defined by (6) and (7). We begin with equilibrium output. In the absence of noise, the presence of uninformed buyers induces the firm to reduce output, which increases the mean price. Hence, the learning activity of the uninformed buyers has a negative effect on output. As the level of noise increases, this negative effect is mitigated. Indeed, an increase in σ_η^2 makes the price less informative, and thus the signal has less influence on the posterior mean for quality. In other words, the uninformed buyers learn less from the price and posterior beliefs depend more on prior beliefs. From (6), an increase in σ_η^2 reduces the negative effect of the informational externality and thus output increases toward the level of output in which the uninformed buyers use only their prior beliefs. That is, $\lim_{\sigma_\eta^2\to\infty} q^*(\theta) = (\lambda\theta + (1-\lambda)\mu_\theta)/2$. In the limit, equilibrium output depends directly on the prior beliefs, as no learning occurs. Equilibrium output tends to a solution with the perceived quality being the weighted mean of the beliefs of both types of buyers. The limiting case is thus analogous to the full information case as all buyers use their prior information whether it is the truth or prior beliefs. In particular, the limiting output is equal to the full information solution when prior beliefs are unbiased, i.e., $\mu_\theta = \theta$.

To see this from another point of view, output can be written as a function of the posterior mean conditional on any price p, $\hat{\mu}_\theta^*(p) = a^* + b^*p$. From (6), $q^*(\theta) = \lambda(\theta/2) + (1-\lambda)(a^*/2)$ where a^* is defined by (12). Without noise, prior beliefs are irrelevant for posterior beliefs, i.e., $a^* = 0$. As $\sigma_\eta^2 \to \infty$,

$a^* \to \mu_\theta$. Formally, from (6),

$$\begin{aligned}\frac{\partial q^*(\theta)}{\partial \sigma_\eta^2} &= \frac{\partial q^*(\theta)}{\partial a^*}\frac{\partial a^*}{\partial \sigma_\eta^2} \\ &= \frac{2(1-\lambda)\lambda\sigma_\theta^2\mu_\theta}{(4\sigma_\eta^2+\lambda\sigma_\theta^2)^2} > 0 \end{aligned} \tag{28}$$

where

$$\frac{\partial a^*}{\partial \sigma_\eta^2} = \frac{4\lambda\sigma_\theta^2\mu_\theta}{(4\sigma_\theta^2+\lambda\sigma_\eta^2)^2} > 0. \tag{29}$$

Next, we study the effect of noise on the price. The effect of noise on the market price is two-fold. Indeed, an increase in σ_η^2 changes the posterior mean via expressions a^* and b^* as well as induces the firm to increase output. The increase in output unambiguously decreases the price. However, the effect of noise on the parameters for the posterior beliefs depends on the prior beliefs. If the prior beliefs are downward biased, then so are the posterior beliefs. Hence, the effect of noise on output and demand go in the same direction of reducing expected price. However, when the prior beliefs about quality are upward biased, posterior beliefs are also upward biased. Here, the two effects pull in opposite direction. Specifically, noise induces the firm to increase output, which decrease expected price whereas upward biased posterior beliefs increases demand, which leads to higher expected price. Hence, an increase in noise lead to a higher or a lower expected price depending on the values of the parameters of the model. Formally, from (7),

$$\frac{\partial \mu_p^*}{\partial \sigma_\eta^2} = -\frac{8\sigma_\theta^2(1-\lambda)\lambda}{\left(4\sigma_\eta^2+\lambda^2\sigma_\theta^2\right)^2}q^*(\theta) + \frac{4\sigma_\eta^2+(2-\lambda)\lambda\sigma_\theta^2}{4\sigma_\eta^2+\lambda^2\sigma_\theta^2}\frac{\partial q^*(\theta)}{\partial \sigma_\eta^2} \tag{30}$$

where, from (28), $\frac{\partial q^*(\theta)}{\partial \sigma_\eta^2} > 0$. As noted, the effect of noise on the mean price is ambiguous.

Finally, we study the effect of noise on expected profits. In our model, the introduction of noise has an ambiguous effect on expected profits. From (6) and (7), $\pi^*(\theta) = \mu_p^* q^*(\theta)$ or

$$\pi^*(\theta) = \frac{4\sigma_\eta^2 + (2-\lambda)\lambda\sigma_\theta^2}{4\sigma_\eta^2 + \lambda^2\sigma_\theta^2}\left(\frac{\lambda\theta}{2} + \frac{2(1-\lambda)\sigma_\eta^2\mu_\theta}{4\sigma_\eta^2 + \lambda\sigma_\theta^2}\right)^2. \tag{31}$$

Recall that in a noiseless environment, signaling (or the presence of uninformed buyers) reduces the firm's expected profits compared to full information. Specifically, using (31), expected profits under no noise are

$$\pi^*(\theta)|_{\sigma_\eta^2=0,\lambda\in[0,1)} = \frac{(2-\lambda)\lambda\theta^2}{4}, \tag{32}$$

which are smaller than expected profits under full information. That is, $\pi^*(\theta)|_{\sigma_\eta^2=0,\lambda=1} = \frac{\theta^2}{4} > \pi^*(\theta)|_{\sigma_\eta^2=0,\lambda\in[0,1)} = \frac{(2-\lambda)\lambda\theta^2}{4}$.

However, in a noisy environment, signaling might increase expected profits. To see this, we compare expected profits under full information with the limiting expected profits when the variance of the demand shock tends to infinity. If expected profits under full information is smaller than the limiting expected profits, then, unlike the noiseless case, signaling can increase expected profits for a finite variance of the demand shock. In the limit, expected profits when there are uninformed buyers are

$$\lim_{\sigma_\eta^2\to\infty} \pi^*(\theta)|_{\lambda\in[0,1)} = (\lambda\theta + (1-\lambda)\mu_\theta)^2/4. \tag{33}$$

It follows that (33) is smaller, equal or greater than full-information expected profits if and only if μ_θ is smaller, equal or greater than θ . Hence, from (32) and (33), $\pi^*(\theta)|_{\sigma_\eta^2=0,\lambda\in[0,1)} < \lim_{\sigma_\eta^2\to\infty} \pi^*(\theta)|_{\lambda\in[0,1)}$ if and only if

$$(1-\lambda)^2\mu_\theta^2 + 2\lambda(1-\lambda)\theta(\mu_\theta - \theta) > 0. \tag{34}$$

In particular, suppose that prior beliefs are unbiased, i.e., $\mu_\theta = \theta$. Then, (34) is unambiguously positive. Hence, noise may reduce the informational externality, which increases expected profits.[4]

5 Final Remarks

In this paper, we study the effect of noise on the informational role of prices in a static monopoly. We show that the introduction of noise has an effect on both the behavior of the firm and the distribution of the market price. It is important to continue studying information in a noisy environment in order to understand the effect of information flows in various economic environments. Two important extensions come to mind. First, in this model, the source of noise is assumed to be in demand. However, buyers know even less about the structure of the economy, especially they have little information about the cost structure of the firm. For instance, a cost parameter is known to the monopolist, but unknown to the buyers, which makes the price a noisy signal of quality. It would be interesting to study the effect of different sources of noise on equilibrium outcomes. Second, in this paper, we have ignored the link between uncertainty and risk aversion. However, risk aversion has a profound influence on the behavior of buyers and thus their demand. In future work, we plan to study the effect of risk aversion on information flows in a noisy environment.

[4] In a Cournot model without noise, Daher et al. (2012) shows that signaling mitigates the negative effect of the market externality inherent in the Cournot equilibrium on the profits of the firms.

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