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Cahier de recherche n° IEA-09-04
Version révisée
Novembre 2012

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ISSN : 0825-8643

Step-by-Step Explanation of Hendricks and Kovenock (1989)'s Model of Social Learning

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November 7, 2012

Abstract

We explain the Hendricks and Kovenock (1989) framework by studying the behavior of two strategic firms under an informational externality. The informational externality arises when each firm of a social network is endowed with private information regarding the profitability of the investment. In such situations, the past decisions of the firms are informative and, thus, are used as partially revealing signals of private information. Asymmetric information and the observability of actions render the firm's problem dynamic and strategic because the investment decision of one firm affects the other firms' future payoffs through the learning process. We describe the model and we show that there exists a unique symmetric Bayesian Nash equilibrium. The informational externality increases the likelihood for a firm to refrain from investing immediately in order to make a more informed decision in the future.

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1 Introduction/Motivation

Firms generally make investment decisions under uncertainty. There are two main sources of uncertainty. The first one is uncertainty in outcomes through the standard inclusion of random shocks in the profit function. The second one is structural uncertainty, i.e., the values of parameters in the profit function are unknown to the firms, and, thus, are treated as random variables from their point of view. For instance, firms are usually unaware of the profitability of their investment, and, thus, have prior beliefs about it.

Unlike uncertainty in outcomes, structural uncertainty evolves through learning. Indeed, in order to acquire more information, firms engage in econometric activities by gathering and analyzing observations, which reduces structural uncertainty. While firms may learn from exogenous signals or market variables such as the price, firms might also extract information from observing the investment decisions of the other firms in their social network. In other words, firms might engage in social learning.

Social learning occurs when the firms are initially endowed with reliable private information regarding a good. In such situations, firms are induced to observe each other's past investment decisions because these actions convey and reflect partially their private information. In addition, in a small social network, the presence of social learning leads to strategic and dynamic interactions among firms through the informational externality.

In this pedagogical note, we study a dynamic model in which two firms engage in social learning about the profitability of the investment. Specifically, each firm lives two periods and is initially endowed with reliable private information about the profitability of the investment. In the first period, each firm decides whether to invest based on his private information, or to wait in order to infer (imperfectly) the other firm's private information through observing his previous investment decision. The cost of waiting comes from discounting, while the benefit of waiting comes from acquiring more information in the second period by observing what the other firm has done.

Hendricks and Kovenock (1989) is the first paper to introduce social learning with strategic interaction. Specifically, they consider a two-period model, in which each firm decides when and whether to drill a common pool. In the first period, each firm receives a private and informative signal about the value of the area to drill. If a firm chooses to drill today, it collects its profits and does not drill tomorrow. If the firm chooses to wait in the first period, it can drill tomorrow. A firm's anticipation of social learning in the second period affects his behavior in the first period. In fact, the anticipation of social learning increases the likelihood for a firm to wait in order to make a more informed decision in the second period.

The Hendricks and Kovenock (1989) framework includes two important features of learning from observing each other's actions. First, the flow of information is endogenous and partially revealing, as it depends on optimal behavior.[1] Second, the learning process is embedded directly into a dynamic program, implying that the anticipation of acquiring information results in a game, because this information comes from the action of the other agent. In other words, there are strategic and dynamic interactions among agents through the informational externality.

While we explain the Hendricks and Kovenock (1989) framework to the firm's problem, we also relax the assumption that all the information is revealed if one of the agents undertakes a decision. In our case, a firm always extracts partially revealing information regardless of the other firm's past decision.

Before proceeding with the model and the characterization of the equilibrium, we briefly review the subsequent literature on social learning and its relation to Hendricks and Kovenock (1989). The majority of the social learning literature generally abstracts from one of the two important features of the Hendricks and Kovenock (1989) framework.[2] Some papers abstract from the endogeneity of the flow of information. In McFadden and Train (1996), firms are forward-looking as they anticipate information from other firms,

[1] In the original Hendricks-Kovenock framework, the flow of information is endogenous when neither agent undertakes the action, but it is assumed to be revealed once an agent undertakes the action. We will relax this assumption.

[2] Another related literature considers how agents learn from communicating information to each other. For instance, Ellison and Fudenberg (1993, 1995) examine word-of-mouth communication under exogenously specified rules for behavior and social learning, while Vettas (1997) study how agents communicate information about product quality.

but the flow of information is always exogenous, i.e., exogenous signals that are independent of other agents' actions. Other models consider how strategic behavior affects the timing of information revelation. Specifically, undertaking an action triggers revelation exogenously, rather than being used as a signal to update beliefs via Bayesian methods. See Alexander-Cook et al. (1998), Caplin and Leahy (1994), Chamley and Gale (1994), and Zhang (1997) for models of irreversible investment decision, and Caplin and Leahy (1998, 2000) for search models. In other words, once an agent decides to invest, the information is revealed (perfectly or partially), but the strategy of this agent has no effect on the quality of information.

The herding literature abstract from the dynamic aspect of the firm's problem because agents are assumed to live one period. See Banerjee (1992), Bikhchandani et al. (1992), and Smith and Sorensen (2000). The herding literature focuses more on the effect of learning on optimal behavior, rather than the effect of the anticipation of learning on present strategies. In a herding model, each agent is endowed with private information and makes a single and irreversible decision in a predetermined order.[3] Hence, there are no strategic and dynamic interactions because each agent simply reacts to what his predecessors have done.

The pedagogical note is organized as follows. Section 2 introduces the model. Section 3 presents the symmetric Bayesian Nash equilibrium, while Section 4 provides a discussion of the equilibrium.

2 Model

2.1 Environment

Consider a social network composed of two firms living for two periods. For $i = A, B$, firm i decides investing today or exercising the outside option. He may invest only once but may exercise the outside option in either periods. That is, if a firm invests in the first period, he automatically exercises the

[3]While Gale and Kariv (2003) substitute sequential for simultaneous moves, the firm's problem remains static.

outside option in the second period. Firm i's net profit from deciding to invest is $\hat{\mu}$. The value of not investing, i.e., exercising the outside option, is normalized to zero in either period.

2.2 Learning

Suppose firm i does not know $\hat{\mu}$, although he knows it was initially drawn from the p.d.f. $\xi(\mu)$, $\mu \in \mathbb{R}$. Each period, firm i receives an informative signal, and uses Bayesian methods to update his beliefs about $\hat{\mu}$.

In period 1, firm i receives a private and exogenous signal $\hat{s}_i$, a realization of $\tilde{s}_i$ with the p.d.f. $\phi(s_i|\hat{\mu})$, $s_i \in \mathbb{R}$, and updates his beliefs to $\xi(\mu|\hat{s}_i) \propto \phi(\hat{s}_i|\mu)\xi(\mu)$, $\mu \in \mathbb{R}$, according to Bayes' theorem.[4] Thus, the expected net value of buying the good in period 1 is $v_1(\hat{s}_i) = \int_{\mu\in\mathbb{R}} \mu\xi(\mu|\hat{s}_i)\mathrm{d}\mu$.

In period 2, each firm observes whether the other firm has invested or not in period 1. While firms receive an exogenous signal in period 1, the signal in period 2 is endogenous and depends on firms' strategies. For the remaining of the paper, the problem is studied from the perspective of firm A. Firm B's maximization problem is identical. Firm B's decision in period 1 is informative to firm A about $\hat{\mu}$ because it depends on firm B's private signal $\hat{s}_B$, which, in turn, is a realization of $\tilde{s}_B$ with the p.d.f. $\phi(s_B|\hat{\mu})$. Formally, let firm B's strategy be the set of private signals $\hat{s}_B$ corresponding to investing in period 1, i.e., $S_B = \{\hat{s}_B : \chi_B(\hat{s}_B) = 1\}$, and $\chi_B : \mathbb{R} \to \{0, 1\}$, where $\chi_B(\hat{s}_B) = 1$ means that firm B invests, and $\chi_B(\hat{s}_B) = 0$ means that he does not invest, i.e., he exercises the outside option in period 1.

If firm B invests in period 1, firm A updates his beliefs to $\xi(\mu|\hat{s}_A, \tilde{s}_B \in S_B) \propto \Phi(\tilde{s}_B \in S_B|\mu)\xi(\mu|\hat{s}_A)$, where $\Phi(\tilde{s}_B \in S_B|\mu) = \int_{\hat{s}_B\in S_B} \phi(\hat{s}_B|\mu)\mathrm{d}\hat{s}_B$ is the probability that, given the strategy S_B, firm B invests in period 1 conditional on μ. His corresponding expected value of investing in period 2 is

$$v_2(\hat{s}_A, \tilde{s}_B \in S_B) = \int_{\mu\in\mathbb{R}} \mu\xi(\mu|\hat{s}_A, \tilde{s}_B \in S_B)\mathrm{d}\mu, \tag{1}$$

if he did not invest previously, and zero if firm A has already invested in

[4] For $Z \subset \mathbb{R}$, firm$i$'s probability that $\hat{\mu} \in Z$ is $\int_{\mu\in Z} \xi(\mu|\hat{s}_i)\mathrm{d}\mu$ in period 1.

period 1.[5]

If firm B does not invest in period 1,[6] firm A updates his beliefs to $\xi(\mu|\hat{s}_A, \tilde{s}_B \notin S_B) \propto \Phi(\tilde{s}_B \notin S_B|\mu)\xi(\mu|\hat{s}_A)$, where $\Phi(\tilde{s}_B \notin S_B|\mu) = \int_{\hat{s}_B \notin S_B} \phi(\hat{s}_B|\mu)\mathrm{d}\hat{s}_B$ is the probability that, given the strategy S_B, firm B exercises the outside option conditional on μ. His corresponding expected value of investing in period 2 is

$$v_2(\hat{s}_A, \tilde{s}_B \notin S_B) = \int_{\mu \in \mathbb{R}} \mu \xi(\mu|\hat{s}_A, \tilde{s}_B \notin S_B)\mathrm{d}\mu, \tag{2}$$

if he did not invest previously, and zero if firm A has already invested in period 1.

The change in the expected value of investing from $v_1(\hat{s}_A)$ to either (1) or (2), depending on firm B's decision, characterizes social learning, which possibly leads to a change in behavior in period 2.

2.3 Anticipation of Social Learning

While social learning affects firm A's behavior in period 2, the anticipation of social learning affects his behavior in period 1. In other words, the possibility to learn from each other renders the firm's problem strategic as well as dynamic.[7]

In period 1, firm A's decision depends on his private signal $\hat{s}_A$, as well as the anticipation of acquiring information in period 2, which depends on firm B's strategy S_B. Firm A chooses between two alternatives. The first is investing in period 1 and exercising the outside option in period 2 yielding an expected profit of

$$v_1(\hat{s}_A) + 0. \tag{3}$$

The second is exercising the outside option in period 1 and having the opportunity to invest in period 2 for the first time, with (possibly) more information

[5] When firm A has invested in period 1, learning is useless in period 2.

[6] This is an important departure from the original framework. We learn not only when people do things (e.g., invest), but also when they do nothing (e.g., not invest).

[7] In a full-information environment, firm i is static and nonstrategic, i.e., he invests if and only if $\hat{\mu} > 0$.

about $\hat{\mu}$,[8] yielding an expected profit of

$$0+\beta\Pr[\tilde{s}_B\in S_B|\hat{s}_A]\max\{0,v_2(\hat{s}_A,\tilde{s}_B\in S_B)\}+\beta\Pr[\tilde{s}_B\notin S_B|\hat{s}_A]\max\{0,v_2(\hat{s}_A,\tilde{s}_B\notin S_B)\},\tag{4}$$

where $\beta\in(0,1)$ is the discount factor. Since firm A does not observe firm B's private signal $\hat{s}_B$, firm A faces uncertainty about the information he will acquire in period 2, which, in turn, affects his future profit. Firm A's anticipation of social learning depends on firm B's strategy S_B. Thus, not only is there anticipation of social learning, but the information firm A anticipates receiving in period 2 is endogenous to the game.

From (4), when firm A does not invest in period 1, there are two possible pieces of information firm A anticipates receiving given firm B's strategy S_B. The first one is that firm B invests in period 1. Given $\hat{s}_A$ and firm B's strategy S_B, this event occurs with probability

$$\Pr[\tilde{s}_B\in S_B|\hat{s}_A]=\int_{m\in\mathbb{R}}\Phi(\tilde{s}_B\in S_B|m)\xi(m|\hat{s}_A)\mathrm{d}m,\tag{5}$$

and the resulting expected profit is $\max\{0,v_2(\hat{s}_A,\tilde{s}_B\in S_B)\}$, i.e., firm A invests in period 2 if and only if $v_2(\hat{s}_A,\tilde{s}_B\in S_B)>0$. The second is that firm B exercises the outside option in period 1. Given $\hat{s}_A$ and firm B's strategy S_B, this event occurs with probability

$$\Pr[\tilde{s}_B\notin S_B|\hat{s}_A]=\int_{m\in\mathbb{R}}\Phi(\tilde{s}_B\notin S_B|m)\xi(m|\hat{s}_A)\mathrm{d}m,\tag{6}$$

and the resulting expected profit is $\max\{0,v_2(\hat{s}_A,\tilde{s}_B\notin S_B\}$, i.e., firm A invests in period 2 if and only if $v_2(\hat{s}_A,\tilde{s}_B\notin S_B)>0$.

From (3) and (4), firm A's value function in period 1 is

$$V_A(\hat{s}_A)=\max\{v_1(\hat{s}_A),\beta W(\hat{s}_A,S_B)\},\tag{7}$$

[8] I say possibly because if firm B's strategy is to invest in period 1 regardless of the value of his own signal, then there is no information emanating from firm B's decision.

where

$$W(\hat{s}_A, S_B) = \Pr[\tilde{s}_B \in S_B|\hat{s}_A] \max\{0, v_2(\hat{s}_A, \tilde{s}_B \in S_B)\}$$
$$+ \beta \Pr[\tilde{s}_B \notin S_B|\hat{s}_A] \max\{0, v_2(\hat{s}_A, \tilde{s}_B \notin S_B)\}. \quad (8)$$

Note that from (7), if firm A does not discount the future expected profit, i.e., $\beta = 1$, then the informational externality induces firm A to exercise the outside option in period 1, regardless of firm B's strategy. Indeed, by Jensen's inequality, $v_1(\hat{s}_A) \leq W(\hat{s}_A, S_B)$ for all $\hat{s}_A$ since

$$v_1(\hat{s}_A) = \Pr[\tilde{s}_B \in S_B|\hat{s}_A]v_2(\hat{s}_A, \tilde{s}_B \in S_B) + \Pr[\tilde{s}_B \notin S_B|\hat{s}_A]v_2(\hat{s}_A, \tilde{s}_B \notin S_B). \quad (9)$$

3 Symmetric Bayesian Nash Equilibrium

Having described the Hendricks-Kovenock model in details, we now characterize the unique symmetric Bayesian Nash equilibrium. The proofs are relegated to the appendix. We make the following distributional assumption.

Assumption 3.1. *The signals $\tilde{s}_A$ and $\tilde{s}_B$ are continuous and independent random variables with the strict monotone likelihood ratio property.*[9]

Assumption 3.1 implies that the posterior beliefs $\xi(\mu|\hat{s}_i)$ possess the strict monotone likelihood ratio property, and $v_1(\hat{s}_i)$ is continuous and strictly increasing in $\hat{s}_i$, for $i = A, B$. Moreover, since $\xi(\mu|\hat{s}_i)$ possesses the strict monotone likelihood ratio property, so do $\xi(\mu|\hat{s}_i, \tilde{s}_k \in S_k)$ and $\xi(\mu|\hat{s}_i, \tilde{s}_k \notin S_k)$, implying that $v_2(\hat{s}_i, \tilde{s}_k \in S_k)$ and $v_2(\hat{s}_i, \tilde{s}_k \notin S_k)$ are strictly increasing in $\hat{s}_i$, for $i, k = A, B, i \neq k$. Finally, $v_2(\hat{s}_i, \tilde{s}_k \in S_k)$ and $v_2(\hat{s}_i, \tilde{s}_k \notin S_k)$ are also continuous in $\hat{s}_i$, for $i, k = A, B, i \neq k$.

To discard uninteresting cases, we make the following assumptions.

[9] Formally, for $s > s'$,

$$\frac{\xi(x'|s')}{\xi(x'|s)} > \frac{\xi(x|s')}{\xi(x|s)}. \quad (10)$$

if and only if $x > x'$.

Assumption 3.2. *There exists $t_0 \in \mathbb{R}$ such that $v_1(t_0) = 0$.*

Assumption 3.3. *There exists $t_1, t_2 \in \mathbb{R}$ such that $v_2(t_1, \tilde{s}_i \in S_i) = 0$ and $v_2(t_2, \tilde{s}_i \notin S_i) = 0$ for $i = A, B$.*

Assumption 3.2 discards cases in which firm i never invests in period 1, regardless of his private signal, i.e., we allow $v_1(\hat{s}_i) > 0$ for $\hat{s}_i > t_0, i = A, B$. Assumption 3.3 discards cases in which a firm never invests in period 2 because of the other firm's decision in period 1 and regardless of his private signal. That is, we allow $v_2(\hat{s}_i, \tilde{s}_k \in S_k) > 0$ for $\hat{s}_i > t_1$ and $v_2(\hat{s}_i, \tilde{s}_k \notin S_k) > 0$ for $\hat{s}_i > t_2, i, k = A, B, i \neq k$.

Proposition 3.4 shows that a unique response function $R_A(\hat{s}_A, S_B) = 1_{[\hat{s}_A \geq c_B]}$ to firm B's strategy S_B exists. The same result holds for firm B's reaction function. Formally,

Proposition 3.4. *From (7), $R_A(\hat{s}_A, S_B) = 1_{[\hat{s}_A \geq c_A]}$ where $c_A \in (t_0, \max\{t_1, t_2\})$ is implicitly defined by $v_1(c_A) = \beta W(c_A, S_B)$.*

The Bayesian Nash equilibrium in period 1 is characterized by a collection of sets $\{S_A^*, S_B^*\}$, where $S_A^* = \{\hat{s}_A : \chi_A^*(\hat{s}_A) = 1\}$ and $S_B^* = \{\hat{s}_B : \chi_B^*(\hat{s}_B) = 1\}$, such that $\hat{s}_A \in S_A^*$ if and only if $v_1(\hat{s}_A) \geq \beta W(\hat{s}_A, S_B^*)$ and $\hat{s}_B \in S_B^*$ if and only if $v_1(\hat{s}_B) \geq \beta W(\hat{s}_B, S_A^*)$. Proposition 3.4 implies that the Bayesian Nash equilibrium is completely defined by two threshold values c_A^* and c_B^* such that $\chi_A^*(\hat{s}_A) = 1_{[\hat{s} \geq c_A^*]}$ and $\chi_B^*(\hat{s}_B) = 1_{[\hat{s} \geq c_B^*]}$. Proposition 3.5 shows that there is a unique symmetric Bayesian Nash equilibrium in period 1. Formally,

Proposition 3.5. *There is a unique symmetric Bayesian Nash equilibrium in period 1, $\{S^*, S^*\}$, $S^* = \{\hat{s} : \chi^*(\hat{s}) = 1_{[\hat{s} \geq c^*]}\}$, where $c^* \in (t_0, \max\{t_1, t_2\})$ is implicitly defined by $v_1(c^*) = \beta W(c^*, S^*)$.*

4 Discussion

The presence of an informational externality induces firms to delay investing in order to acquire more information in period 2. If there is no informational externality, firm A invests in period 1 if and only if $v_1(\hat{s}_A) > 0$, or $\hat{s}_A > t_0$.

Since $c^* \in (t_0, \max\{t_1, t_2\})$, the informational externality reduces the a priori probability to invest in period 1, i.e., $\Pr[\tilde{s}_A > c^*] < \Pr[\tilde{s}_A > t_0]$.

Proposition 3.5 is similar to Hendricks and Kovenock (1989). However, the a priori probability to invest in period 1 is weaker in our case. Indeed, the Hendricks and Kovenock (1989) original framework assumes that the unknown parameter $\hat{\mu}$ is automatically revealed in period 2, when one of the firms invests in period 1. We relax this assumption as each firm extracts partially revealing information from the optimal behavior of the other firm, whether or not investment takes place in the first period. Hence, the benefit of waiting is reduced. To see this, compare the expected profit of investing when there is never full revelation, i.e., equation (3), with the expected profit of waiting when there is full revelation once the investment is made,

$$\begin{aligned}\beta W(\hat{s}_A, S_B) =& \beta \Pr[\tilde{s}_B \in S_B | \hat{s}_A] \int_{\mu \in \mathbb{R}} \max\{0, \mu\} \xi(\mu | \hat{s}_A, \tilde{s}_B \in S_B) \mathrm{d}\mu \\ &+ \beta \Pr[\tilde{s}_B \notin S_B | \hat{s}_A] \max\{0, v_2(\hat{s}_A, \tilde{s}_B \notin S_B)\}, \end{aligned} \tag{11}$$

Since $\max\{0, \mu\}$ is convex in μ, by Jensen's inequality, (3) is smaller than (11). Therefore, the a priori probability to invest in period 1 is higher in our case than in the case of Hendricks and Kovenock (1989), because it is not as valuable to wait.

A Proofs

The proofs are in the spirit of Hendricks and Kovenock (1989).

Proof of Proposition 3.4. From (7),

$$\Delta(\hat{s}_A, S_B) = v_1(\hat{s}_A) - \beta W(\hat{s}_A, S_B), \tag{12}$$

$$= \int_{\mu\in\mathbb{R}} \mu\xi(\mu|\hat{s}_A)\mathrm{d}\mu - \beta \max\left\{0, \int_{\mu\in\mathbb{R}} \mu\Phi(\tilde{s}_B \in S_B|\mu)\xi(\mu|\hat{s}_A)\mathrm{d}\mu\right\}$$

$$- \beta \max\left\{0, \int_{\mu\in\mathbb{R}} \mu\Phi(\tilde{s}_B \notin S_B|\mu)\xi(\mu|\hat{s}_A)\mathrm{d}\mu\right\}, \tag{13}$$

$$= \int_{\mu\in\mathbb{R}} \mu \left[1 - 1_{[\hat{s}_A \geq t_1]}\beta\Phi(\tilde{s}_B \in S_B|\mu) - 1_{[\hat{s}_A \geq t_2]}\beta\Phi(\tilde{s}_B \notin S_B|\mu)\right] \xi(\mu|\hat{s}_A)\mathrm{d}\mu, \tag{14}$$

$$= \int_{\mu\in\mathbb{R}} \mu \left[1 - 1_{[\hat{s}_A > t_1]}\beta\Phi(\tilde{s}_B \in S_B|\mu) - 1_{[\hat{s}_A > t_2]}\beta\Phi(\tilde{s}_B \notin S_B|\mu)\right] \xi(\mu|\hat{s}_A)\mathrm{d}\mu \tag{15}$$

be the difference between the value of investing in period 1, and the value of exercising the outside option in period 1. Note that $\Delta(\hat{s}_A, S_B) < 0$ for $\hat{s}_A \leq t_0$, and $\Delta(\hat{s}_A, S_B) > 0$ for $\hat{s}_A \geq \max\{t_1, t_2\}$. Since $\Delta(\hat{s}_A, S_B)$ is continuous in $\hat{s}_A$, there is at least one $c_A \in (t_0, \max\{t_1, t_2\})$, such that $\Delta(c_A, S_B) = 0$. It remains to show the uniqueness of $c_A \in (t_0, \max\{t_1, t_2\})$, such that $\Delta(c_A, S_B) = 0$.

1. For $\hat{s}_A > c_A$. Let

$$f(\mu, \hat{s}_A) = \mu \left[1 - 1_{[\hat{s}_A \geq t_1]}\beta\Phi(\tilde{s}_B \in S_B|\mu) - 1_{[\hat{s}_A \geq t_2]}\beta\Phi(\tilde{s}_B \notin S_B|\mu)\right] \tag{16}$$

and $\rho = \xi(0|c_A)/\xi(0|\hat{s}_A)$ when $\mu = 0$. Then,

$$\rho\Delta(\hat{s}_A, S_B) = \rho \int_{\mu\in\mathbb{R}} f(\mu, \hat{s}_A)\xi(\mu|\hat{s}_A)\mathrm{d}\mu, \tag{17}$$

$$= \rho \int_{\mu\in\mathbb{R}} f(\mu, \hat{s}_A)\xi(\mu|\hat{s}_A)\mathrm{d}\mu - \int_{\mu\in\mathbb{R}} f(\mu, c_A)\xi(\mu|c_A)\mathrm{d}\mu, \tag{18}$$

$$= \int_{\mu\in\mathbb{R}} f(\mu, \hat{s}_A) \left(\rho - \frac{f(\mu, c_A)}{f(\mu, \hat{s}_A)} \frac{\xi(\mu|c_A)}{\xi(\mu|\hat{s}_A)}\right) \xi(\mu|\hat{s}_A)\mathrm{d}\mu. \tag{19}$$

Given the strict monotone likelihood ratio property, $\rho > \xi(\mu|c_A)/\xi(\mu|\hat{s}_A)$

if and only if $\mu > 0$. Since $f(\mu, \hat{s}_A)$ is right continuous in $\hat{s}_A$, there is $\varepsilon > 0$ such that

$$\rho - \frac{f(\mu, c_A)}{f(\mu, \hat{s}_A)} \frac{\xi(\mu|c_A)}{\xi(\mu|\hat{s}_A)} > 0 \tag{20}$$

for $\hat{s}_A \in (c_A, c_A + \varepsilon)$ if and only if $\mu > 0$. This, combined with the fact that $f(\mu, \hat{s}_A) > 0$ if and only if $\mu > 0$, implies that $\rho\Delta(\hat{s}_A, S_B) > 0$ for $\hat{s}_A \in (c_A, c_A + \varepsilon)$. Since $\rho > 0$, $\Delta(\hat{s}_A, S_B) > 0$ for $\hat{s}_A \in (c_A, c_A + \varepsilon)$.

2. For $\hat{s}_A < c_A$. Use

$$g(\mu, \hat{s}_A) = \mu \left[1 - 1_{[\hat{s}_A > t_1]} \beta\Phi(\tilde{s}_B \in S_B|\mu) - 1_{[\hat{s}_A > t_2]} \beta\Phi(\tilde{s}_B \notin S_B|\mu)\right] \tag{21}$$

instead of $f(\mu, \hat{s}_A)$, because g is left continuous in $\hat{s}_A$. The steps are identical to 1. Since $g(\mu, \hat{s}_A)$ is left continuous in $\hat{s}_A$, there is $\varepsilon > 0$ such that $\Delta(\hat{s}_A, S_B) < 0$ for $\hat{s}_A \in (c_A, c_A - \varepsilon)$.

Combining the facts that there is a $c_A \in (t_0, \max\{t_1, t_2\})$, such that $\Delta(c_A, S_B) = 0$, $\Delta(\hat{s}_A, S_B)$ is continuous in $\hat{s}_A$, $\Delta(\hat{s}_A, S_B) < 0$ for $\hat{s}_A \leq t_0$, $\Delta(\hat{s}_A, S_B) > 0$ for $\hat{s}_A \geq \max\{t_1, t_2\}$, $\Delta(\hat{s}_A, S_B) > 0$ for $\hat{s}_A \in (c_A, c_A + \varepsilon)$, and $\Delta(\hat{s}_A, S_B) < 0$ for $\hat{s}_A \in (c_A, c_A - \varepsilon)$ implies that $c_A \in (t_0, \max\{t_1, t_2\})$ is unique. Thus, a unique response function $R_A(\hat{s}_A, S_B) = 1_{[\hat{s}_A \geq c_A]}$ to firm B's strategy S_B exists.

Proof of Proposition 3.5. Given Proposition 3.4, firms A and B's strategies in a symmetric Bayesian Nash equilibrium are $S^* = \left\{\hat{s} : \chi(\hat{s}) = 1_{[\hat{s} \geq c^*]}\right\}$, where c^* is defined by $v_1(c^*) = \beta W(c^*, S^*)$. From (7), let

$$\begin{aligned}
\Gamma(c) =& v_1(c) - \beta W(c, S), && (22)\\
=& \int_{\mu\in\mathbb{R}} \mu\xi(\mu|c)\mathrm{d}\mu - \beta \max\left\{0, \int_{\mu\in\mathbb{R}} \mu\Phi(\tilde{s}_B \geq c|\mu)\xi(\mu|c)\mathrm{d}\mu\right\}\\
&- \beta \max\left\{0, \int_{\mu\in\mathbb{R}} \mu\Phi(\tilde{s}_B < c|\mu)\xi(\mu|c)\mathrm{d}\mu\right\}, && (23)\\
=& \int_{\mu\in\mathbb{R}} \mu\left[1 - 1_{[c\geq t_1]}\beta\Phi(\tilde{s}_B \geq c|\mu) - 1_{[c\geq t_2]}\beta\Phi(\tilde{s}_B < c|\mu)\right]\xi(\mu|c)\mathrm{d}\mu, && (24)\\
=& \int_{\mu\in\mathbb{R}} \mu\left[1 - 1_{[c>t_1]}\beta\Phi(\tilde{s}_B \geq c|\mu) - 1_{[c>t_2]}\beta\Phi(\tilde{s}_B < c|\mu)\right]\xi(\mu|c)\mathrm{d}\mu, && (25)
\end{aligned}$$

where $S = \{\hat{s} : \chi(\hat{s}) = 1_{[\hat{s}\geq c]}\}$. Note that $\Gamma(c) < 0$ for $c \leq t_0$, because $v_1(t_0) = 0 < v_2(t_0, s_B \geq t_0)$, and $\Gamma(c) > 0$ for $c \geq \max\{t_1, t_2\}$. Since $\Gamma(c)$ is continuous in c, there is at least one $c^* \in (t_0, \max\{t_1, t_2\})$, such that $\Gamma(c^*) = 0$. It remains to show the uniqueness of $c^* \in (t_0, \max\{t_1, t_2\})$, such that $\Gamma(c^*) = 0$.

1. For $c > c^*$. Let $f(\mu, c) = \mu\left[1 - 1_{[c\geq t_1]}\beta\Phi(\tilde{s}_B \geq c|\mu) - 1_{[c\geq t_2]}\beta\Phi(\tilde{s}_B < c|\mu)\right]$ and $\rho = \xi(0|c^*)/\xi(0|c)$ when $\mu = 0$. Then,

$$\begin{aligned}
\rho\Gamma(c) =& \rho\int_{\mu\in\mathbb{R}} f(\mu, c)\xi(\mu|c)\mathrm{d}\mu = \rho\int_{\mu\in\mathbb{R}} f(\mu, c)\xi(\mu|c)\mathrm{d}\mu - \int_{\mu\in\mathbb{R}} f(\mu, c^*)\xi(\mu|c^*)\mathrm{d}\mu, && (26)\\
=& \rho\int_{\mu\in\mathbb{R}} f(\mu, c)\left(\rho - \frac{f(\mu, c^*)}{f(\mu, c)}\frac{\xi(\mu|c^*)}{\xi(\mu|c)}\right)\xi(\mu|c)\mathrm{d}\mu. && (27)
\end{aligned}$$

   Given the strict monotone likelihood ratio property, $\rho > \xi(\mu|c^*)/\xi(\mu|c)$ if and only if $\mu > 0$. Since $f(\mu, c)$ is right continuous in c, there is $\varepsilon > 0$ such that

$$\rho - \frac{f(\mu, c^*)}{f(\mu, c)}\frac{\xi(\mu|c^*)}{\xi(\mu|c)} > 0 \qquad (28)$$

   for $c \in (c^*, c^* + \varepsilon)$ if and only if $\mu > 0$. This, combined with the fact that $f(\mu, c) > 0$ if and only if $\mu > 0$, implies that $\rho\Gamma(c) > 0$ for

$c \in (c^*, c^* + \varepsilon)$. Since $\rho > 0$, $\Gamma(c) > 0$ for $c \in (c^*, c^* + \varepsilon)$.

2. For $c < c^*$. Let $g(\mu, c) = \mu \left[1 - 1_{[c>t_1]}\beta\Phi(\tilde{s}_B \geq c|\mu) - 1_{[c>t_2]}\beta\Phi(\tilde{s}_B < c|\mu)\right]$ instead of $f(\mu, c)$, because g is left continuous in c. The steps are identical to 1. Since $g(\mu, c)$ is left continuous in c, there is $\varepsilon > 0$ such that $\Gamma(c) > 0$ for $c \in (c^*, c^* - \varepsilon)$.

Combining the facts that there is a $c^* \in (t_0, \max\{t_1, t_2\})$, such that $\Gamma(c^*) = 0$, $\Gamma(c)$ is continuous in c, $\Gamma(c) < 0$ for $c \leq t_0$, $\Gamma(c) > 0$ for $c \geq \max\{t_1, t_2\}$, $\Gamma(c) > 0$ for $c \in (c^*, c^* + \varepsilon)$, and $\Gamma(c) < 0$ for $c \in (c^*, c^* - \varepsilon)$ implies that $c^* \in (t_0, \max\{t_1, t_2\})$ is unique.

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