

**Integrated Assessment of Climate Change
Using an OLG Model**

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Integrated Assessment of Climate Change Using an OLG Model

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Abstract

This paper presents an economy and climate model of 60 overlapping generations of finite lived agents and competitive firms interacting with a physical environment. Use of energy in production results in the release of carbon to the atmosphere which can affect global climate, and thus productivity. The model is calibrated to global economic activity over the 30 years ended in 1995. The model is solved using an Euler equation approach, and simulated for three climate change scenarios, capturing optimistic, median, and pessimistic predictions on the rate and severity of climate change in response to CO₂ emissions.

JEL classification: Q21; H23; L51

Keywords: Climate Change; Environmental Taxes; Environmental Regulation.

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1 Introduction

Global climate change induced by atmospheric accumulation of CO₂ and other greenhouse gases (GHG) poses a potentially grave threat to our wellbeing as a society. The economics literature has seen the development of Integrated Assessment Models (IAM) for the evaluation and optimization of climate change mitigating policies. This paper addresses three key areas: the effect of the introduction of finite lived agents, the sensitivity of model results to assumptions on both the mechanism of climate change and the commitment period for the policy, and the ability of the model to match global aggregate economic and climate data. The contribution of this paper to the economic modelling of climate change is to allow for an overlapping generations structure while maintaining important advancements in the modelling literature: finite resource stocks and an annual time interval. The results of this paper clearly show the importance of considering finite lived agents, as agents' preference over policies depends strongly on their age at the time the policy is implemented. As might be intuitive, older agents are shown to prefer the status quo, while agents not yet born are shown to prefer the most stringent regulation. The results are quantitatively sensitive to assumptions on the parameters of the climate model and the policy timeframe. The model is able to match trends in aggregate labour supply, capital stock, output, emissions and temperature data over the 1965-2000 time period.

Manne and Richels (1992) and Nordhaus and Boyer (2000) are benchmark models in the assessment of climate change mitigation policies. These models, like much of the literature, rely on the optimal growth models proposed in Ramsey (1925), Cass (1965) or Koopmans (1965). The features of climate change are most compatible with the economic problems addressed by versions of the Overlapping Generations Model, first proposed by Diamond (1965). Previously, this model has been successfully applied to problems of international debt (Diamond (1965)), social security (Cooley and Soares (1999), Andolfatto and Gervais (1999)), life-cycle learning (Heckman (1976), Andolfatto et al. (2000)), and dynamic fiscal policy (Auerbach and Kotlikoff (1987), Rios-Rull (1997), Erosa and Gervais (2002)). In a problem closely related to that presented here, Olson and Knapp (1997) examine the extraction of finite resources in an overlapping generations economy. Climate change mitigation requires a sustained effort over several generations, with costs being felt by agents who are likely not to accrue many of the benefits. As such, the Diamond (1965) model is well suited for the integrated assessment of climate change in this paper. There has been

limited attention in the literature to combining the features of climate and economic models with the environment proposed by Diamond, and generally it has been accomplished at the expense of other important aspects. In a recent paper, Howarth (1998) presents a model of overlapping generations of two period lived agents and climate change. Most recently, papers by Rasmussen (2002) and Kavuncu and Knabb (2002) have also looked at the intergenerational aspect of climate change mitigation. Section 2 presents a more thorough review of the relevant literature in environmental science, economics and political economy.

The economic model of 60 overlapping generations of finite lived agents and competitive firms is presented in Section 3. The climate and economy characterization is analogous to Nordhaus and Boyer (2000), such that use of energy in production results in the release of carbon to the atmosphere which can affect global climate. In turn, climate state affects productivity over a long time horizon. The model is calibrated to match global economic activity over the 30 years ending in 1995, and captures the evolution of the economy from 1965 starting values to a balanced growth path on which emissions tend to zero and the climate reverts to a pre-industrial state. Calibrated parameter and starting values are discussed in Section 7. The model is solved using an Euler equation approach outlined in Section 6. The results of the model are verified for external consistency over the period of 1965-2000 using economic data from the Penn World Tables and climate data from the IPCC and CDIAC.

Generally, policies aimed at mitigating climate change focus on the imposition of carbon quotas or taxes. The model is modified to admit carbon taxes of \$50 and \$10 per ton of carbon extraction and emissions quotas of 5.735GtC annually. The policies are then simulated and compared to a status quo simulation. Each policy is evaluated in terms of effects on climate, production, and cohort welfare. The results show a clear welfare preference for the status quo for the cohorts of agents alive at the time policies are imposed in 2003. Depending on assumptions on the severity of climate change, the tax policies are preferred to the status quo by some younger and not yet born agents. Production is most affected by the quota policies, and the net loss in production is positively related to the carbon tax level. Climate change mitigation is accomplished most effectively under the quota system, and varies positively with the tax level. The simulations demonstrate the key difficulty with regulating climate change mitigation: benefits are felt on a much longer time horizon than costs, and thus agents are less likely to support stringent mitigation policies at any point in time.

Consideration of uncertainty is of paramount importance, and the model is simulated for baseline values and subjected to sensitivity analysis in Section 8. Much of the climate change debate centers on two parameters of interest in modelling the relationship between atmospheric CO₂ and other GHG and climate change. The science literature is divided on the level of temperature change for a doubling of atmospheric CO₂ (see Wigley et. al. (1998)) and the damages occurring in proportion to changes in temperature (See Nordhaus and Boyer (2000)). To address this uncertainty, the model is solved and simulated for three climate change scenarios, capturing the optimistic, median, and pessimistic predictions on the rate and severity of climate change. An additional sensitivity analysis is performed on the important assumption that, once implemented, policies can be sustained ad infinitum. In order to examine the impact of this assumption, the results are replicated for limited commitment policies.

2 Modelling Climate Change

IAMs are most easily classified based on the level of optimization and policy evaluation in the models. First, macro-based models focus on the dynamics of the economy given a stream of investment, production, emissions, and labour supply. Agent maximization models, generally based on the Ramsey model, endogenously generate these variables over time, using agent and firm optimization at the microeconomic level. Either model type can focus on either the choice of an optimal policy or the evaluation of proposed policy schemes. *Policy evaluation* models use the IAM to provide estimates of effects of a proposed policy or a series of potential scenarios. In these models, economic variables such as tax or permit levels, or control ratios are set exogenously. *Policy Optimization* models look at the optimal policy choice within a set of policy instruments, allowing a government to endogenously determine policy according to the maximization of a social welfare function. Much of the early work on climate change economics is summarized by Weyant (1993). Kolstad and Kelly (1999a) provide a more up to date review of IAMs in the economics and environmental science literature, beginning with the work of Manne and Richels (1992) and Nordhaus (1994). Table 1 summarizes the key features of the most recent contributions to the IAM literature. Taxes and permit based policies are grouped under price mechanisms. The model proposed in this paper addresses each of these areas with the exception of alternative energy sources.

Manne and Richels (1992) use the combination of a computable general equilibrium

Table 1: Features of Selected Integrated Assessment Papers

Author(s)	Policy Opt./Eval	Agent Opt.	Price Mech.	Command+ Control	Finite Res.	Alt. Energy	Finite Lives
Manne and Richels(1992)	eval.	✓	✓	✓		✓	
Nordhaus and Boyer(2000)	opt.+ eval.	✓	✓		✓	✓	
Pizer(1996)	opt.	✓		✓			
Chakravorty et al.	opt.		✓		✓	✓	
Howarth (1998)	opt	✓					✓
Kavuncu and Knabb (2002)	eval	✓	✓				✓
Rasmussen (2002)	opt	✓	✓				✓
This Paper	eval.	✓	✓	✓	✓		✓

model and a global macroeconomic model to examine the costs of limiting CO₂ emissions. Their book only fits loosely into the IAM literature, since it does not possess the climate feedback into welfare. Their model has a very complete treatment of the energy production sector, and explicitly models the transition to cleaner energy sources. The work in this paper is updated in Manne, Mendelsohn and Richels (1995) to include a more extensive climate sub-model and damage assessment model.

Pizer (1996) examines the effect of uncertainty in a Nordhaus (1993) framework. Pizer extends upon Nordhaus' contributions in several ways. First, the role of uncertainty in parameter values is examined, showing significant differences in optimal policies and welfare effects. Much of the effect of uncertainty is isolated on the rate of time preference. Pizer's results suggest that there is a distribution of discount rates among agents. Second, the use of annual intervals is shown to have a significant effect on conclusions. Finally, a comparison is offered between tax and quantity regulation, the former being shown to have more positive welfare consequences.

Howarth (1998) extends the same framework to account for finite lived agents. Using

this framework, the paper is able to examine the potential for inter-generational transfers of wealth and climate change abatement strategies to generate pareto improvements over the status quo.

Nordhaus and Boyer (2000) present a comprehensive model of climate change which allows for cross-country equilibrium prices of emissions permits. Their framework is a dynamic, deterministic model in which governments seek to maximize discounted social welfare in their country, in Nash equilibrium with other countries. Countries are able to trade emissions permits up to certain constraints. Within this framework, Nordhaus and Boyer are able to impose the constraints of the Kyoto protocol through adjusting the endowment of emissions permits in each country and restricting trade. Also important in this iteration of the model is the introduction of finite resources. Nordhaus and Boyer introduce a firm which extracts resources and provides the input to final production at extraction cost. The finite nature of the resource is captured in rising extraction costs.

Nordhaus and Boyer (2000) do not explicitly account for the substitution between energy resources over time. Rather they abstract from this using a type of composite fuel which has an emissions level corresponding to a weighted average of the emissions from all currently used sources of energy, and finite resource levels calculated in the same way. Chakravorty et al. (1997) look at the effect of endogenous substitution on future energy use profiles, and subsequently climate change. The key results of their paper are the time to complete conversion to solar energy (370 years baseline), with carbon emissions peaking in 2175. The resulting 6 degree increase in global temperature is persistent through 2275. Policy analysis conducted on the effect of taxes and research and development subsidies is done only in partial equilibrium, so it is not possible to address the growth consequences of the taxes in the model proposed. Of additional interest is the effect of the different simulations on the exhaustion rates of resources. Under the baseline simulations, all resources are extracted before the conversion to solar energy takes place. Conversely, under policy simulations which attach a tax to the use of carbon, only 8% of the remaining coal stock is exhausted.

Rasmussen (2002) presents a model with 55 overlapping generations of agents. This model concentrates more on the costs of taxation to the economy than on the direct climate change impacts, examining the changes to steady state growth rates arising from the imposition of a carbon tax. The results of this paper show that the consumption effects of a carbon tax are highly unequally distributed across generations, with the tax leading to decreased economic growth and long term costs to generations born far into the future.

There is no direct effect of climate on the economy however.

Kavuncu and Knabb (2002) present a model where agents live for two periods. The paper imposes a policy analogous to the Kyoto protocol and simulates the economy for approximately 350 years. It is worth noting that since a period in the model is 35 years, this only accounts for 10 model time periods. Their results are intuitive given the assumptions, showing that benefits arise only over a long time horizon and that these benefits arise sooner if more detrimental affects of climate change are assumed.

Integrated assessment models have become an important tool in the evaluation and development of environmental policy. One of the key difficulties with the application of these models has been the computational burden and complexity resulting from the large state space required to simultaneously model climate and economic variables. Kelly and Kolstad (1999b) present a background paper on the computational burdens in IAMs and present a formalized structure and methodology for solution and estimation of these models. Section 6 details an Euler equation approach to the computing of equilibria in overlapping generations models of climate change.

3 The Model

This section introduces an IAM of climate and economy made up of $L = 60$ overlapping generations of agents. Agents work for the first 45 periods of life, and accumulate assets to finance 15 periods of retirement. The production technology has energy as a factor which is used at an increasing rate of emissions efficiency. Use of carbon in production results in the release of carbon to the atmosphere which can affect global climate. In turn, climate state affects productivity over a long time horizon.

3.1 The Structure of the Model

The IAM proposed below follows a three sector setup with a physical sector, an economic sector and an emissions sector. The structure of the model is denoted by mapping functions which characterize the evolution of each of the sectors of the model:

$$\begin{aligned}
 S_{P,t+1} &= f_P(S_{P,t}, S_{M,t}, C) \\
 S_{M,t} &= f_E(S_{E,t}, C) \\
 S_{E,t+1} &= f_E(S_{E,t}, S_{P,t}, C)
 \end{aligned} \tag{1}$$

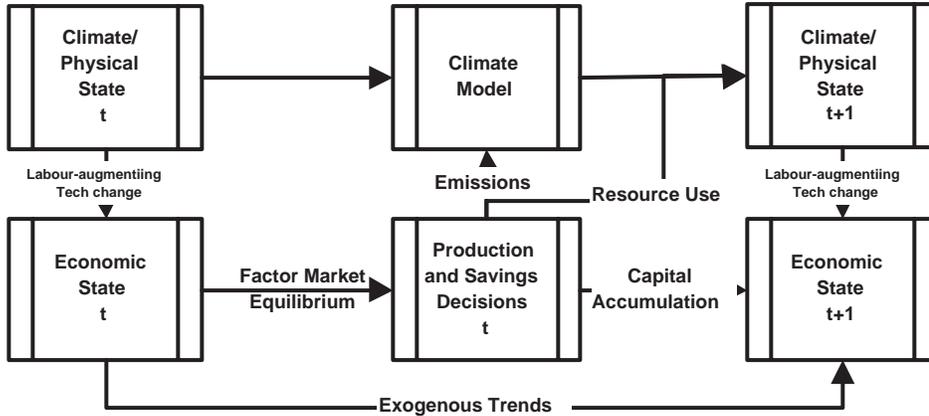


Figure 1: Schematic Diagram of the Integrated Assessment Model

The interrelationships in the model are shown in the schematic diagram in Figure 1. The sectors are denoted P(physical), M(emissions), and E(economic). The model is deterministic. Choice variables are denoted by C. The laws of motion f_i capture the relationships between sectors. In summary, next period’s physical state is affected by this period’s climate, resource stocks, emissions and the choice of resource extraction. Current emissions are affected by economic state and choice variables. Next period’s economic outcome is determined by current economic and physical state, and choice variables. It is important to note that choice variables are assumed not to be able to affect climate directly, ruling out the ‘miracle solution’ where we will be able to apply new technology to change climate directly. Choice variables do affect resource extraction from the physical climate. The remainder of this section lays out in more specific detail the relationships in each section.

3.2 The Economic Environment

An initial population is spread over L generations of agents identical in all aspects except age. In each period, a large number of identical agents is born. The population grows at a convergent rate, such that population growth eventually limits to zero. In the limit therefore, there is a constant age distribution of agents. Agents in the model act to maximize their discounted stream of utility through consumption and savings paths. Agents choose consumption of a final good which is produced by competitive firms. Final good production uses carbon resources, capital and labour for which the firm pays competitive prices.

3.2.1 Agents

Agents in each generation face the same optimization problem, since they begin with the same asset holdings, have the same certain lifetimes, and face the same income. Agents choose consumption and savings to maximize their expected lifetime utility, which is given by:

$$\sum_{i=1}^L \beta^{i-1} U(c_{i,t+i-1}). \quad (2)$$

where β gives the agent's discount factor $\beta \in (0, 1)$, $c_{i,t+i-1}$ is consumption by an age i agent at time $t+i-1$. Utility has constant relative risk aversion form, with risk aversion parameter σ as:

$$U(c_{i,t+i-1}) = \frac{c_{i,t+i-1}^{1-\sigma}}{1-\sigma}. \quad (3)$$

Agents accumulate assets in the form of claims on physical capital ($a_{i,t}$), and use asset holdings to smooth consumption over time according to the following individual budget constraint shown in (4).

$$a_{i,t+1} = (1 + \iota_t - \delta_k) a_{i,t} + y_{i,t} - c_{i,t} \quad (4)$$

where y is period labour income, c is consumption, and ι is the rate of return on assets from the previous period. Agents also bear the cost of depreciation δ_k . There is no asset bequest in the model, so agents will set $a_{M+1,t} = 0 \forall t$. There is also a borrowing constraint in the model such that asset holdings must be $a_{M+1,t} \geq 0 \forall t$. Agents derive no utility from the bequest of environmental quality. The lack of a bequest motive is not inconsistent with the literature. In Howarth (1998), there is no consumption or environmental bequest, while in Nordhaus and Boyer (2000), an annual discount factor of .96% leads to almost no consideration outside of the time horizon of an average lifespan.¹

At any point in time, an agent's resources are the combination of asset holdings $a_{i,t}$ and a labour endowment e_i which is age but not time dependent. The structure of the human capital profile is discussed further in Section 7. The problem of each individual in a generation is identical, so we can generalize by looking at a representative age i agent's problem. The maximization problem for an agent is summarized by the value function given below, with the sequence of budget constraints substituted for consumption:

$$V_i(a_{i,t}, t) \equiv \max_{a_{i+1,t+1}} \frac{(A_t^* w_{it} e_i + (1 + \iota_t - \delta_k) a_{i,t} - a_{i+1,t+1})^{1-\sigma}}{1-\sigma} + \beta V_{i+1}(a_{i+1,t+1}, t+1). \quad (5)$$

¹For a complete treatment of the role of the discount factor, see Manne (1995).

The Euler equations for this problem are:

$$a_{i+1,t+1} \quad : \quad c_{i,t}^{-\sigma} = \beta V_1(a_{i+1,t+1}, t+1). \quad (6)$$

and substituting from the envelope condition in (7) yields, for each age group, an Euler equation(8).

$$V_1(a_{t+1}, t+1) = U_1(c_{t+1}) \quad (7)$$

$$c_{i,t}^{-\sigma} = \beta(1 + \iota_{t+1} - \delta_k)c_{i+1,t+1}^{-\sigma} \quad (8)$$

Finally, substituting the budget constraint back into (8) yields a difference equation which describes the asset holdings through time subject to a sequence of prices:

$$\begin{aligned} & (A_t^* w_t e_i + (1 + \iota_t - \delta_k)a_{i,t} - a_{i+1,t+1})^{-\sigma} \\ & = \beta(1 + \iota_{t+1} - \delta_k) (A_{t+1}^* w_{t+1} e_{i+1} + (1 + \iota_{t+1} - \delta_k)a_{i+1,t+1} - a_{i+2,t+2})^{-\sigma}. \end{aligned} \quad (9)$$

Equation (9) describes consumer behaviour as it relates to physical capital accumulation and life cycle savings decisions. Feasibility requires that aggregate consumption and net investment be less than or equal to total output in all time periods.

3.2.2 Production

Production in the economy is Cobb-Douglas with three inputs; capital, labour and resources. The representative firm faces competitive prices w_t for an efficiency unit of labour, ι_t for a unit of physical capital, and γ_t for a ton of carbon-based fuel. Technology in production is given by three parameters, overall production scale Ω , labour augmenting technical change A^* and energy efficiency ϕ . Denoting by N the aggregate labour supplied by all agents, the firm's problem is therefore summarized as:

$$\max_{K_t, N_t, R_t} Y_t = F(\Omega_t, K_t, N_t, R_t) - w_t A_t^* N_t - \iota_t K_t - (\gamma_t) R_t \quad (10)$$

$$= \Omega_t K_t^\alpha [A_t^* N_t]^{1-\alpha-\theta} (\phi_t R_t)^\theta - w_t A_t^* N_t - \iota_t K_t - (\gamma_t) R_t. \quad (11)$$

Since the firms are solving a one-period maximization problem, the solution to is described by three first order conditions:

$$F_{A^*N} = w_t \quad (12)$$

$$F_R = \gamma_t \quad (13)$$

$$F_K = \iota_t. \quad (14)$$

Resource supply is treated as in Nordhaus and Boyer (2000) by assuming that a firm provides resources competitively each period, setting price equal to marginal extraction cost. The marginal cost of extraction increases in the cumulative extraction of carbon. This is consistent with finite, scarce resources.

$$\gamma_t = \xi_1 + \xi_2 \left[\frac{X_t + R_t}{X^*} \right]^{\xi_3} \quad (15)$$

The link between climate, emissions, and productivity occurs through the labour augmenting technical change parameter A_t^* :

$$A_t^* = (1 + (D_0/9) \cdot G_t^2)^{\frac{-1}{1-\alpha}} A_t. \quad (16)$$

The intuition here is that the change in climate will lead to a lowering in our ability to use labour effectively. Examples of this change occur in the learning required for agriculture to adapt to new climatic conditions and the labour and resources used in cleanup from increased environmental disasters. G_t is the 30 year normal average surface temperature, D_0 describes damages in terms of a percentage expected reduction in GDP from a 3 degree increase in G relative to preindustrial levels. Damages are quadratic in temperature which is consistent with the relationship to random climatic events, proportional to the variance in temperature. The use of a feedback through labour augmenting technical change as opposed to Hicks neutral technical change is analogous to Pizer(1996). A_t is an exogenous trend in labour augmenting technical change, such that the exogenous trend for A is given by:

$$A_t = A_0 \exp \left(\frac{\gamma_a}{\delta_a} (1 - e^{-\delta_a t}) \right) \quad (17)$$

This law of motion is identical to that used in Pizer (1996). The exogenous trend for labour supply N_t is given by the rule for the size of the new generation born each period and retirement age R:

$$N_{0t} = N_{00} \exp \left(\frac{\gamma_n}{\delta_n} (1 - e^{-\delta_n t}) \right) \quad (18)$$

$$N_t = \sum_{i=0}^{L-1-R} N_{it}. \quad (19)$$

3.2.3 Aggregate Laws of Motion

The two endogenous stocks in the economy are physical and resource capital, which evolve according to the actions of agents and firms. The cumulative extraction of the resource

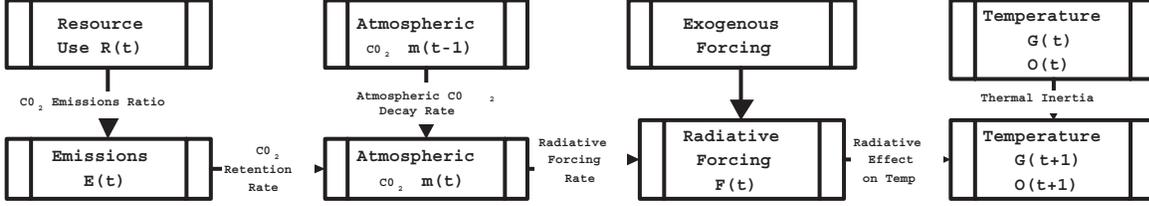


Figure 2: Schematic Diagram of the Climate Model

stock evolves according to use as:

$$X_{t+1} = X_t + R_t \quad (20)$$

and physical capital evolves according to investment and depreciation as:

$$K_{t+1} = \sum_{i=1}^L a_{i,t+1} * N_{it} \quad (21)$$

3.3 The Climate and Emissions Model

The role of the climate change model is to provide a law of motion for climatic state as a function of the emission of greenhouse gases as a result of production. Other IAMs have used multi-stage processes to define this motion, at the expense of a large number of (largely irrelevant) state variables. The model proposed below uses a stylized two-stage system, where atmospheric carbon stock evolves as a function of emissions and current stock. The level of atmospheric carbon alters the level of radiative forcing (heat retention) which leads to changes in surface and ocean temperature. Damages are incurred as a result of changes in temperature relative to pre-industrial normals as described in (16).

3.3.1 The Carbon Cycle

The model presented in this paper provides a simplified characterization of the global carbon cycle. Emissions E_t into the atmosphere are governed by the exogenous emissions release rate parameter ϕ_t . This parameter determines the relationship between fuel burned and emissions released into the atmosphere. R_t denotes the resource use in period t, while emissions are denoted E_t .

$$E_t = \phi_t R_t \quad (22)$$

Denoting atmospheric concentration of CO₂ as m_t , retention rates (δ_e) for new emissions E_t and (δ_a) for current stock m_t net of pre-industrial levels (m_b) are factors in the law of motion for atmospheric carbon as:

$$m_t = m_b + \delta_e E_{t-1} + \delta_a (m_{t-1} - m_b). \quad (23)$$

This assumes, as in Pizer (1996), that the evolution of atmospheric carbon can be approximated by the decay of carbon above zero emissions levels.² Over the transition path, what is important is providing a realistic characterization of the atmospheric half-life of carbon, which this model is able to provide. This characterization differs from Nordhaus and Boyer (2000) in the fact that a no-emissions steady state for atmospheric carbon exists at preindustrial levels m_b . The reason for incorporating different sink rates for emissions and existing atmospheric carbon is that emissions are released throughout the year, and carbon sink absorption is continuous. It is natural to assume that the discrete process will imply lower average absorption for emitted carbon than atmospheric carbon.

3.3.2 Radiative Forcing and Temperature

Greenhouse gases provide a change in radiative forcing, increasing heat retention relative to the baseline for the atmosphere. Radiative forcing, F_t , is modelled as a function of atmospheric carbon as follows:

$$F_t = \left\{ \frac{\log \frac{m_t}{m_b}}{\log[2]} \right\} + \bar{F}_t \quad (24)$$

where the units are the relative increase in radiative forcing from pre-industrial times, and \bar{F}_t is the exogenous forcing levels of other greenhouse gases. The characterization of forcing in terms of a doubling of atmospheric CO₂ is standard in both the scientific and economic literature on climate change.(see Pizer (1996) or Wigley et al. (1998))

The evolution of temperature, and thus climate change, occurs through a slow warming of the world's oceans and atmosphere, which is prevented in the short run by thermal inertia, and is modelled as a two-stage process:

$$G_t = \lambda_1 G_{t-1} + \eta(F_t) + \omega O_{t-1} \quad (25)$$

$$O_t = \lambda_2 O_{t-1} + (1 - \lambda_2) G_{t-1} \quad (26)$$

²See calibration information in Section 7 for more detail.

The state variable G is surface temperature in $^{\circ}C$ relative to the pre-industrial global average for 30 year normals, while O represents the change in temperature in the world's upper oceans, globally and seasonally averaged, since 1900. The parameter η is measured in $^{\circ}C$ and denotes the potential temperature, or long run warming from a doubling in CO_2 . Parameters $\lambda_1 \in (0, 1)$ and $\lambda_2 \in (0, 1)$ are the coefficients of autoregression in the surface and ocean temperature relationships respectively. This characterization is identical to that used in Pizer (1996) and Nordhaus and Boyer (2000). The value of G_t feeds back through (16) to generate period labour-augmenting technological change.

4 Dynamics

The model presented in this paper seeks to characterize the transition path of the economy as agents use the scarce carbon resource allocated to them. The intuition here is that the economy begins on a balanced growth path using an arbitrarily small stock of carbon energy (wood as fuel), which we can denote \bar{R} . The discovery of a large useful stock of exhaustible carbon resources is a shock to the economy which leads the state to follow a transition path over a long time horizon until it eventually returns to the balanced growth path. While it is important to demonstrate the existence and convergence properties of such a balanced growth path, it is the transitional path of the model which is of particular interest to this paper.

4.1 Balanced Growth

Balanced growth in this class of economy is characterized by constant rates of change for physical and economic state variables and prices in per capita terms over time. The following balanced growth path characterization relies on a growth rate of A given by γ_a that converges to a constant. This is shown in Proposition 1 and the exogenous trend for A_t given in 17. While this paper does not model an explicit stock level, there is a level of extraction after which the use of the stock of carbon resources in production will decline asymptotically to zero, leading to a resource input to production of \bar{R} . It is also important to note the population and age distribution properties of the model, which converge to a constant population and age distribution over a long time horizon. As long as population and technology level limit to a constant rate of change, and resource use above the backstop \bar{R} limits to zero, the existence of an equilibrium can be shown as in Auerbach and Kotlikoff

(1987) or any other standard overlapping generations model of consumption and savings.

The long run behaviour of the climate sector and thus the endogenous portion of the A^* state variable is dependent on the emissions path over time. The resource stock is used up along the transition path, so the amount used above the backstop level in each period tends to 0. With resource use tending to 0, all that is required for convergence in the climate sector is to show that emissions tend to zero, the inertia process will revert to preindustrial state at some point in the future. As resource use tends to 0, emissions also tend to zero.³

Proposition 1 *The convergence of the environmental variables to preindustrial levels implies a unique balanced growth path with environmental variables satisfying:*

$$E \xrightarrow{\infty} 0 \tag{27}$$

$$m \xrightarrow{\infty} 590 = m_b \tag{28}$$

$$G \xrightarrow{\infty} 0 \tag{29}$$

$$O \xrightarrow{\infty} 0 \tag{30}$$

Proof: Immediate

4.2 Transitional Dynamics

The balanced growth path for the economy is characterized above, but relies on the eventual return of climatic state variables to pre-industrial levels, in response to the asymptotic tendency of emissions to zero. The algorithm proposed in this paper allows us to analyze the effects of policies over the transition path, from an arbitrary set of initial conditions to the balanced growth path described above, and to comment on the welfare implications for agents over these transitions. In order to define the transition paths, dynamic equilibrium must be defined for the economy in transition to a balanced growth path.

Definition 4.2.1 *Equilibrium along the transition path is defined by a sequence of prices $(w_t, \iota_t, \gamma_t)_{t=1}^T$ for labour, capital and resources and time horizon T , given initial capital stock, population, and physical state. Along the transition path, the price sequence must be such that:*

1. *Agents satisfy their Euler equation given in (9) and supply labour inelastically.*

³It is important to note here that all climate normals are in terms that are relative to pre-industrial times, and thus take into account the previous use of the backstop technology \bar{R} . Intuitively, the 590GtC concentration can be seen as the steady state level which arises in the climate sector from a long run per-period emissions of \bar{R} .

2. *Factors are paid their marginal products*
3. *Factor markets for capital, labour, and resources clear.*
4. *Carbon resources are supplied according to their marginal private cost*

5 Policy

The model admits carbon taxes and emissions quotas as policy parameters. Sequences of taxes and emissions quota levels comprise the policy space of the economy. In terms of the structure of the model, the policy sector can be seen as a fourth sector in (1) which maps policies into economic and emissions states.

5.0.1 Carbon Taxes

Carbon taxes are admitted in the model as an increment to the price charged by the carbon extraction firm for the use of resources in final production. The carbon pricing function is therefore modified to:

$$\gamma_t = \tau_t + \xi_1 + \xi_2 \left[\frac{X_t + R_t}{X^*} \right]^{\xi_3}. \quad (31)$$

where τ_t is the dollar/ton carbon tax rate. The model admits policies which charge the carbon tax only above a threshold level. Tax revenues are assumed to be distributed lump-sum to agents in the model, such that each agent receives an equal share of the carbon tax revenue. Denoting as $\tilde{\tau}_t$ the per capita carbon tax revenue, and v the proportion of tax revenue returned to the agent, the agent's value function and budget constraint is therefore modified to:

$$\begin{aligned} V_i(a_{i,t}, t) &\equiv \max_{a_{i+1,t+1}} \frac{(A_i^* w_t + (1 + \iota_t - \delta_k) a_{i,t} + v \tilde{\tau}_t - a_{i+1,t+1})^{1-\sigma}}{1 - \sigma} \\ &+ \beta V_{i+1}(a_{i+1,t+1}, t + 1). \end{aligned} \quad (32)$$

which then modifies the Euler equation to account for the transfer of tax income.

5.0.2 Emissions Quotas

Because of the nature of the model as a global characterization, the imposition of market-based mechanisms is per se uninformative. Quotas are introduced in the model to capture the scenario where firms are not able to pay any amount in order to increase emissions where the quota levels are binding. This differs from the tax policy introduced in the model

in that resource use is directly regulated. The supply of resources is therefore constrained by the sequence of quota levels $\{Q\}_{t=0}^T$ as:

$$R_s = \begin{cases} R_s & \text{if } \phi_t R_d(\gamma_t) < Q_t \\ \frac{Q_t}{\phi_t} & \text{otherwise.} \end{cases}$$

6 Computation

In order to solve and simulate the model, an Euler equation approach is used. In each period, agents' Euler equations and budget constraints and firms' first order conditions must be satisfied. Prices for labour (w_t), physical capital (ι_t) and resource capital (γ_t) determine demand and supply of each commodity. The computational algorithm uses a sequence of prices to evaluate a corresponding sequence of excess demand levels, and uses a fixed point algorithm to converge to the vector of prices which satisfies the equilibrium conditions over the entire vector from time 1..T. Given a convergence criterion, the transition path of the economy from starting values is established by the converged system of prices as follows:

Algorithm 1

Objective: Solve transition path for the climate/economy system given starting values for state variables and climate normals.

Algorithm Preliminaries: Choose a convergence criterion ϵ_c and an adjustment parameter λ_c . Choose T , the time horizon, and initial guess for price sequence $\mathbf{P} \equiv (w_t, \iota_t, \gamma_t)_{t=1}^T$. Set prices beyond T to evolve according to balanced growth conditions.

Step 1: Solve the system of Euler equations given vector of prices for all time periods.

Step 2: For each time period t , complete the following sequence of steps:

- *Step 2a : Given the guess (w_t, ι_t, γ_t) , compute factor supply and demand values for resources, labour and capital in time period t . Label supply and demand vectors as (L_t^s, K_t^s, R_t^s) and (L_t^d, K_t^d, R_t^d) . Supply of labour and resources are inelastic.*
- *Step 2b : Given the factor supplies, calculate implied emissions and carbon extraction. Given this, update cumulative carbon extraction and temperature values using the climate and emissions module.*
- *Step 2c : Given new climate state variables, update labour augmenting technological change for the next period.*

Step 3: Compute the vector of percentage excess demands given by the difference between each element of the time period factor supply and demand values

Step 4: Compute a new guess of prices according to the following adjustment formula for each of the prices for each $t=1..T$:

$$\begin{aligned} w_t^{i+1} &= w_t^i + \lambda_w \left(\frac{L_t^d - L_t^s}{L_t^d} \right) \\ \iota_t^{i+1} &= \iota_t^i + \lambda_\iota \left(\frac{K_t^d - K_t^s}{K_t^d} \right) \\ \gamma_t^{i+1} &= \gamma_t^i + \lambda_\gamma \left(\frac{R_t^d - R_t^s}{R_t^d} \right) \end{aligned}$$

and return to step 1 if the maximum excess demand value exceeds the criterion ϵ_c .

Step 5: Having reached convergence criteria ϵ_c , define the model as an excess demand system as:

$$E \equiv (L_t^d - L_t^s, K_t^d - K_t^s, R_t^d - R_t^s)_{t=1}^T$$

as a function of:

$$\mathbf{P} = (w_t, \iota_t, \gamma_t)_{t=1}^T$$

which is a system in $3T+T*L$ unknowns and $3T+T*L$ simultaneous equations, comprising the excess demand functions and the Euler equations of each age agent at each time period. Use a Newton routine to compute a zero of $E(\mathbf{P})$.

The final step of the algorithm provides gains only in processing time. Since the Tatonnement process converges linearly and does not make use of gradient information, the Newton-Raphson method will provide substantial acceleration gains.(see Judd (1998))

7 Calibration

In the calibration of the IAM proposed above, it is important to account for the sensitivity of simulation results to certain parameters in the climate sector. Of particular interest are the parameters linking the relationship between atmospheric CO₂ levels and temperature changes, and the link between temperature changes and productivity decline. The parameters of the economic model are relatively standardized in the literature. Papers such as Pizer (1996) estimate parameters of the economic model and the elements of the climate model that can be identified with variance in the data. In response to the uncertainty over the parameters governing the effects of climate change, three scenarios are considered in

the calibration of the model. Scenarios are described as optimistic, median and pessimistic, and are calibrated to correspond with the prevalence of estimates in the literature.

7.1 Economic Sector

The economy is calibrated to a global production function as outlined in Pizer (1996). Baseline values for the model are 1965 economic aggregates derived from the Penn World Tables (2002). Parameters of the economic model are standard, with rates being given in annual equivalents in Table 3. Economic sector calibrations are summarized in Tables 2 and 3.

An earnings distribution is used to calibrate e_i , the vector of human capital in the model, as in Huggett (1996). Data are obtained from the Canadian Labour Force Survey, 1998.⁴ Median wages for agents in 5 year age groups were used to construct the earnings profile, and the remaining wages were interpolated using a cubic spline. The values are adjusted so that the mean labour contribution is 1 unit. The resulting age-earnings profile is shown in Figure 4. Earnings peak from ages 45-48, which is consistent with models such as Huggett(1996) which uses U.S. data to calibrate the earnings profile.⁵

Growth rates are parameterized as in Nordhaus and Boyer (2000) and Howarth (1998). The calibration of population growth is accomplished by comparing the model population to patterns of population from the United Nations(UN) Population Division (2001) data and projections on world population. Population growth is initially at 4.5% in 1960, which is consistent with the fastest growth in the history of the human race taking place between 1950 and 2000. Global population limits toward a long run constant population of slightly over 10 billion people, which coincides with UN estimates of future population growth; estimates which take into account the limiting factors of density, climate, and technology. Figure 3 shows the population of agents in the model compared with UN global population estimates for people over the age of 15. Labour supply matches Penn World Tables data for the period of 1965-2000 by assuming agents retire after 45 periods. This retirement age is assumed to remain constant through time. Productivity growth in the model is parameterized in a similar way, with initial growth rates of 1.1%, converging to a constant level of technology of 153% higher than levels in the year 2000. This corresponds with the

⁴The Canadian data are used to generate an earnings profiles. Wage rates are determined in equilibrium, but the difference in productivity levels by age is calibrated to the Canadian earnings profile.

⁵Agents enter the model at 15 years of age, and supply labour for 45 periods, retiring at the age of 60. The modelling assumptions therefore set labour endowment for agents above 45 periods to zero.

assumptions on the trend variables in Nordhaus and Boyer (2000) and Pizer (1996).

Emissions and extraction in the model are calibrated such that emissions match global emissions data from the CDIAC (2002). This is accomplished by fixing the initial rate of energy efficiency to match 1965 emissions data, and calibrating the growth rate of efficiency to match emissions data over the time period in question. Energy efficiency is assumed to continue improving at a 1% per year rate through the simulation period of the model. Figure 5 shows the performance of the model in terms of predicting actual global carbon emissions over the 1965-2000 period.

7.2 Climate and Emissions Sectors

The fixed values in the climate sector are taken from a synthesis of five sources: Marland et al. (2002), Nordhaus and Boyer (2000), Pizer (1996), Wigley et al. (1998) and IPCC (2001). Climate sector calibrations for each of the three scenarios considered are summarized in Table 4. The key calibrated value are the retention rates for atmospheric CO₂ and emissions, which are derived using a first order autoregression on data from the CDIAC (2002). The model used a single parameter to represent global damages from climate change, a process which clearly has regional dimensions. This parameter is meant to capture the average decline in global productivity brought on by climate change.

8 Policy Evaluation

Models of climate change can take on two policy related roles: policy evaluation or policy optimization. This model seeks to provide a cardinal measure of the productivity and welfare costs of the transitional paths produced as a result of different implementations of mitigation strategies.

The Kyoto protocol places a target for an emissions reduction by member nations based on 1990 baseline levels to be implemented between 2008 and 2012, and the maintenance of those standards over time. This paper seeks to evaluate three policies which have been proposed as means to implement these types of CO₂ emissions reduction. The policies considered are the reduction of emissions to the estimated global protocol levels using a command and control quota regime, or the imposition of carbon taxes at levels which have been suggested as potential global carbon prices under Kyoto (see Government of Canada (2000,2002)). Two stages of evaluation are provided. First, the three policy proposals are

compared to the status quo under median assumptions about climate change. Following this, the sensitivity of each policy's effects to climate change assumptions is evaluated. The first policies are assumed to be continued for the entire transition path. The second set of simulations analyzes the effect of policies with a finite commitment period of fifty years after the imposition of the policies.

Policy evaluation in the model looks at three key measures. First, the overlapping generations structure allows us to look at the welfare levels for each cohort. Welfare is defined as the discounted sum of lifetime utility from consumption. The effect on the economy is measured through total output and labour productivity. Total output of the economy shows the slowdown effects of policies implemented to mitigate climate change. Aggregate productivity decline is measured through the percentage reduction in labour augmenting technical change incurred as a result of climate change. The predicted surface temperature deviations are a measure of the level of actual climate change projected by the model. Additionally, the level of cumulative carbon extraction and resulting levels of atmospheric CO₂ mass are secondary indicators of the success of each policy. In both cases, the economy is simulated for 380 time periods(years) with a new cohort of agents born each period ($L = 60$).⁶

8.1 Infinite Commitment

The first set of figures, labelled Figure 6 - Figure 10 show the effects of policies which remain in effect over the entire transition path under median climate change assumptions. The model predictions on welfare demonstrate one of the key issues with climate change mitigation. Figure 6 shows welfare levels for agents born in each time period of the model, under each of the policies. For agents born before 2118, the status quo is preferred to any policy. Agents born after 2118 prefer the \$10 tax, but never prefer the \$50 tax or the quota system.

The projected levels of final production output are shown in Figure 7. The slowdown effects of each of the policies are clearly visible in this figure. The largest single period drop in production occurs as a result of the \$50 tax, with incrementally smaller changes caused by the imposition of the quota system, and the \$10 tax.

⁶The model was simulated for a 1000 time period test to ensure that there was no perceptible difference in the behaviour of agents on the portion of the transition path we are examining. Looking at the behaviour of the economy from the imposition of policies in 2008, there is no perceptible difference in the transition path to 2075 in a 1000 period simulation.

Productivity effects of each of the policies are shown in Figure 8. The effects are shown as percentage deviations of A^* from the trend variable A . Deviations are highest under the status quo of no regulation, and next highest under the tax schemes. Productivity deviation is increasing throughout the domain of the simulation.

In terms of actual changes in 30 year normal surface temperature relative to the normals for the 30 years previous to 1990, the most dramatic changes are predicted in the status quo, with temperature increase estimates of $2.42971^\circ C$ over the 50 years following the imposition of policies in 2003. The evolution of surface temperatures under the status quo and each policy option are shown in Figure 9. In comparison to the status quo, the tax policies of \$10/ton and \$50/ton produce temperature change over the same period of $2.3596^\circ C$ and $2.15441^\circ C$ respectively. The quota policy is most successful in terms of climate change mitigation, showing a 50 year warming of $2.04843^\circ C$.

Figure 10 shows equilibrium in atmospheric CO_2 mass. Carbon use rises linearly under the quota policy, but continues to increase at an increasing rate in each time period of interest under both the tax policies and the status quo simulation. As a result, atmospheric carbon levels begin to stabilize under the quota scheme at around 920GtC, or roughly 56% above preindustrial levels. Under the status quo and tax policies, we see some evidence of convergence in the levels of atmospheric CO_2 as well.⁷

8.2 Sensitivity to Parameter Values

In order to test the sensitivity of the predictions to assumptions on the mechanism of climate change, the infinite commitment policy simulations are repeated for optimistic and pessimistic climate change assumptions, as outlined in Table 4. The ordering of the policy evaluation projections reached above are not subject to sensitivity, however the age at which agents' preferences switch to different policies depends intuitively on the severity of climate change. In Figures 11-14, the model projections of utility, final production, productivity and temperature normals are shown under an optimistic set of climate change and damage parameters. In Figures 15-18, the same projections are reported for the more pessimistic set of parameters.

Agents' discounted streams of utility are greatly affected by assumptions on the severity of climate change. In Figure 11, the policy preferences of agents when climate change has

⁷Simulating 1000 periods ahead, atmospheric CO_2 levels peak and begin to drop back toward preindustrial levels under the tax policies and the status quo.

minimal effect is shown. In this simulation, welfare is never improved under the imposition of any climate change policy. Conversely, in Figure 15 we see that agents prefer more stringent regulation earlier. Agents born before 2033 prefer no regulation, while agents born between 2033 and 2056 prefer the \$10/ton tax. The \$50/ton tax is preferred by agents born after 2056 in the pessimistic case.

Figures 12 and 16 show the dynamics of final production under alternate climate change assumptions, while Figures 13 and 17 show the dynamics of technical change. The results here are intuitive, since we see more magnified slowdown effects as the economy is also slowed by climate change in the pessimistic scenario, with the opposite being true under the optimistic assumptions.

The most sensitivity to assumptions on climate change occurs in the projected changes in 30 year normal surface temperature. Under optimistic assumptions, temperature increase estimates of $1.42434^{\circ}C$ over the 50 years following the imposition of policies in 2003 are recorded, compared to $3.18171^{\circ}C$ under pessimistic assumptions. The effect of the policy options are shown in Figures 14 and 18. The quota policy leads to a 50 year temperature increases of $1.17751^{\circ}C$ and $2.72745^{\circ}C$ respectively. The tax policies are projected to lead to temperature changes of $1.38025^{\circ}C$ and $1.25099^{\circ}C$ for the optimistic assumptions and $3.0957^{\circ}C$ and $2.84423^{\circ}C$ for the pessimistic case.

8.3 Sensitivity to Finite Commitment

The policy simulations are repeated in this Section, however it is assumed that the commitment period is 50 years, after which time the economy reverts to the status quo. Predictably, the effects of the policies on both welfare and climate are reduced by the lack of a long run change in behaviour. Figures 19-22 show the results of the finite commitment simulations. 50 years is chosen somewhat ad hoc, although the simulations are meant to frame the discussion of coalition stability and the Kyoto protocol. If we consider that the regulations imposed in the Kyoto protocol might survive for 50 years, it is interesting to consider how long it will take for the economy and the climate to revert to pre-policy transition paths. This serves to examine the questions of whether it is better to undertake a less stable policy which may not last, since we feel it is important to implement some regulation.

The welfare implications of each of the policies are radically altered under the limited commitment period policies. Intuitively, agents born after the end of the commitment period prefer the most stringent policies to have been imposed previously, since these endow them

with the most productive economy. As such, agents born before the policies are implemented and until 2044 prefer that the status quo be maintained. Beyond 2045, we see that there is a rapid transition of preferences such that agents born after 2051, two years before the end of the commitment period, prefer the imposition of the most stringent quota policy.

The effects of the policies on the economy are intuitive as well. Over the commitment period, the effects are identical to those under the infinite policies. After the policies are relaxed, the economy is able to benefit from the increased productivity of a more favorable climate, and agents respond with higher levels of growth, until the economy returns to the original transition path within approximately 100 years of the end of the commitment period.

The mitigation effects are small, with the maximum temperature effects achieved at the end of the commitment period, and totalling less than $0.5^{\circ}C$, and disappearing all but completely within 100 years of the end of the policy. This is important since, for 50 years of large welfare effects, we have bought very little in terms of climate change mitigation. This confirms the intuition that, where we cannot be reasonably sure that the policy will be sustained for several generations, and robust to changes within the coalition, we may be better with less stringent, longer-lived policies that ensure the stability required.

9 Discussion

The question of how we measure the costs and benefits of climate change mitigation policies is a powerful one. The model and simulations presented here provide some important context to this question. First, we see clearly that, where price based mechanisms are preferred to other policies, it is important that they be set to optimal dynamic levels, not fixed over time in order to achieve maximum effectiveness and political feasibility. The simulations here show evidence of the fact that, while policies allow us to pass on a cleaner, more productive environment to future generations, these effects are tempered by the growth constraints placed on the economy by the policies. Where policies are continued indefinitely, it is only under the severest of climate change assumptions that the economy reaches higher consumption levels on the transition path than it would have under the status quo. When the policies are imposed for a finite period of time, the results are different. The reduction of emissions allows the economy to grow faster in the future, leading to higher levels of future utility. Intuitively, agents alive in the future prefer policies implemented before they

are born which improve the economic environment, but do not constrain their behaviour.

The simulations shown above constitute preliminary evidence of the nature of the potential gains and losses to welfare under the implementation of climate change mitigating policies. In particular, the gains must be discussed in terms of welfare and productivity terms, as well as in terms of the net effect on climate. The key source of uncertainty not addressed in these measures is the cost of implementation of these measures, as the model allows the quota system to be imposed with no cost, and the tax system to fully refund all revenue to agents. Each of these also have large fixed costs to implementation which must be compared with the gains in productivity and welfare in order to determine the feasibility of a particular policy scheme. Intuitively, the majority of costs would be incurred at implementation with either the tax or quota scheme, although the quota scheme has higher enforcement costs which would be absorbed over time. The interpretation of this in the model would be a further shift outward in the preference for the status quo.

The political ramifications of these findings are clear. In order to develop a feasible climate change policy that will be accepted by today's decision makers, accounting for future generations at some discounted rate will not be enough to induce effective mitigation policy. Some system of consumption transfers (ie. national debt) and climate policies will be required. The use of an overlapping generations model structure proves important for several reasons. First, the model is consistent with the problem of climate change. It is evident that the agents who bear a large portion of the cost of climate change policies do not directly reap any of the benefits, and this is captured in the results of the policy simulations. A representative agent model with infinitely lived agents cannot capture this feature of the problem. It can be inferred that policies with negative net benefit to agents alive at the time are not likely to be successfully imposed and continued.

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Table 2: Initial Period Values

Variable	Description	Calibrated Value
K_0	Capital Stock	$4.328 * 10^{12}$
N_0	Labour Supply	$1.299 * 10^9$
A_0	Productivity	2.5
m_0	Atmospheric CO ₂ levels	690
G_0	Surface temperature change	0
O_0	Ocean temperature change	0
X_0	Aggregate Carbon Extraction	94 GtC
\bar{F}	Forcings from other greenhouse gases	$1.42/\eta \forall t$

Table 3: Calibrated Values in the Economic Sector

Variable	Description	Calibrated Value
σ	Coefficient of Relative Risk Aversion	.97
β	Discount Rate	.96125
δ_k	Capital Depreciation Rate	.045
α	Production Share of Capital	.3764
θ	Production Share of Resources	.05
ξ_1	Minimum extraction cost of carbon(\$)	113
ξ_2	Linear rate in extraction cost of carbon	700
ξ_3	Exponent in extraction cost of carbon	4
γ_a	Growth rate of labour augmenting tech. change	.028
δ_a	Decay rate on γ_a	.028
γ_n	Growth rate of population	0.045
δ_n	Decay rate on γ_n	.049
ϕ	Emissions Ratio from Carbon Fuel	.27
γ_ϕ	Growth rate of ρ	1.005
Ω	Global Technology Scaling	$100 \forall t$
v	Proportion of Carbon Tax Paid Out	1

Table 4: Calibrated Values in the Climate Sector

Variable	Description	Sc.1	Sc.2	Sc.3
m_b	Preindustrial concentration of CO ₂		590	
δ_a	atmpspheric retention of CO ₂		.9846	
δ_e	atmpspheric retention of emissions		.9301	
λ_1	AR(1) parameter on temperature deviations		.9112	
λ_2	AR(1) parameter on ocean temperature deviations		.998	
η	Temperature sensitivity to CO ₂ doubling	1.5000	2.980	4.500
D_0	Damage from a 3°C increase in temp.	.01	.0150	.0250

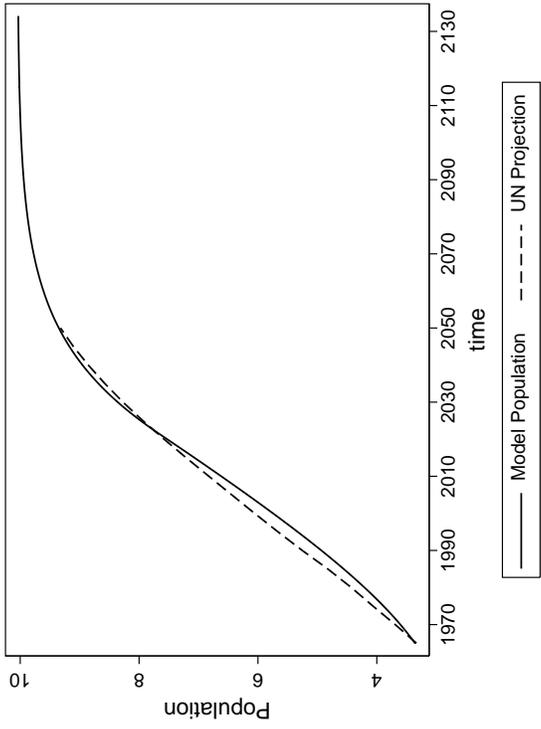


Figure 3: Population: Model and UN Projection

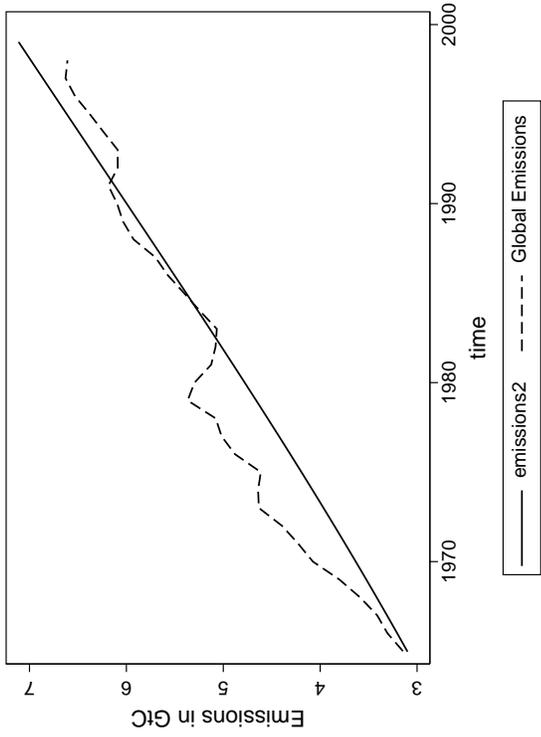


Figure 5: Predicted and Actual Emissions

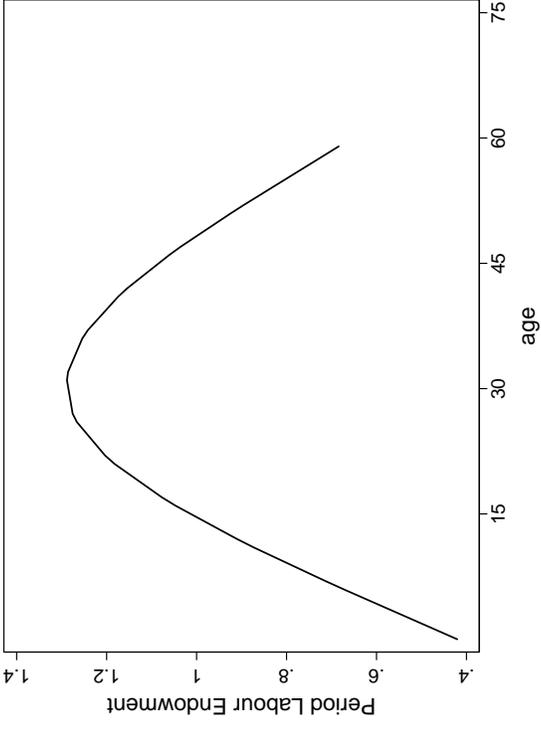


Figure 4: Earnings Profile (Productivity ratio to mean=1)

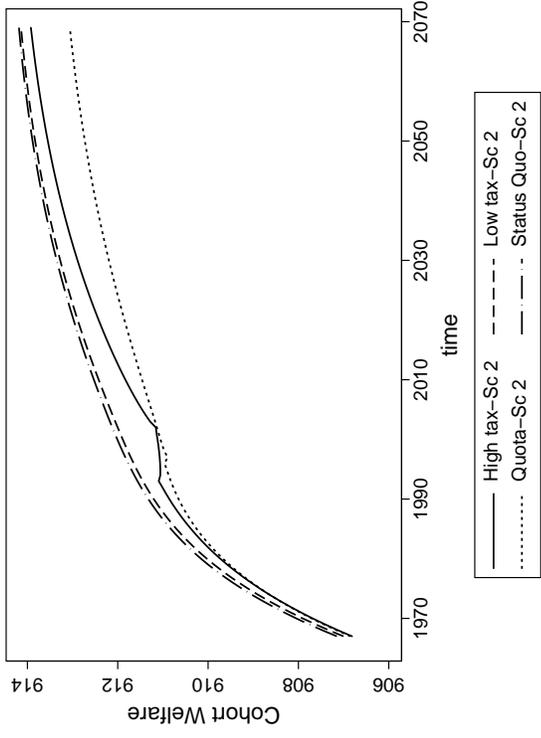


Figure 6: Welfare Levels by cohort

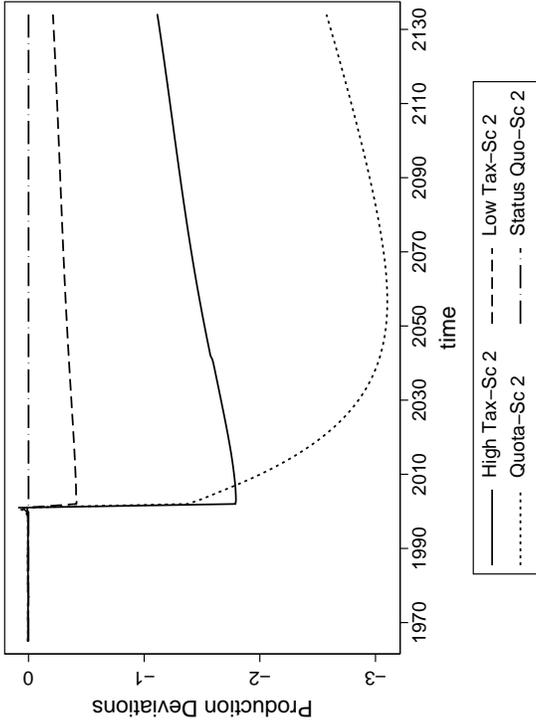


Figure 7: Percentage Deviations in Production

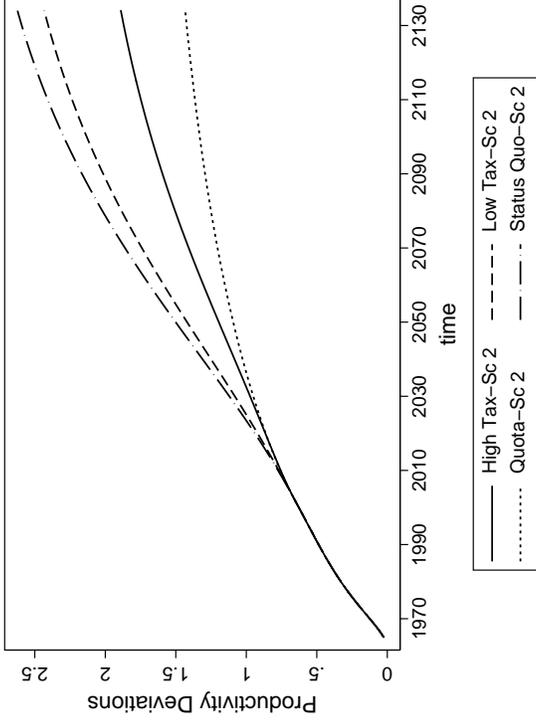


Figure 8: Productivity Decreases

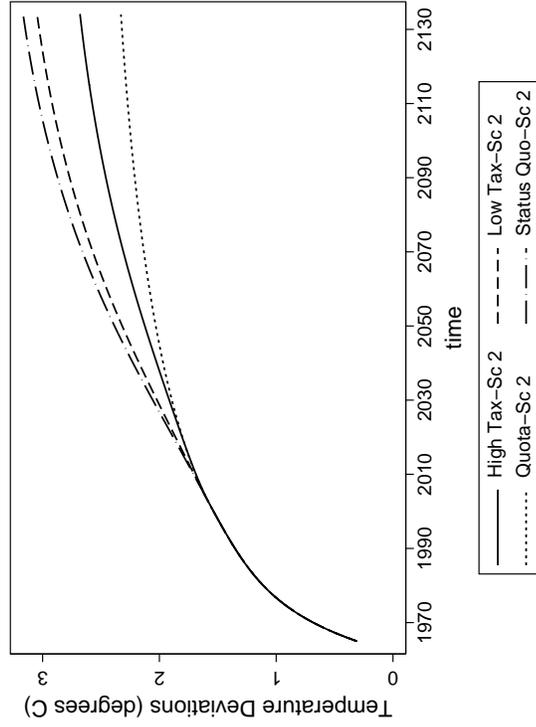


Figure 9: Median Temperature Deviations

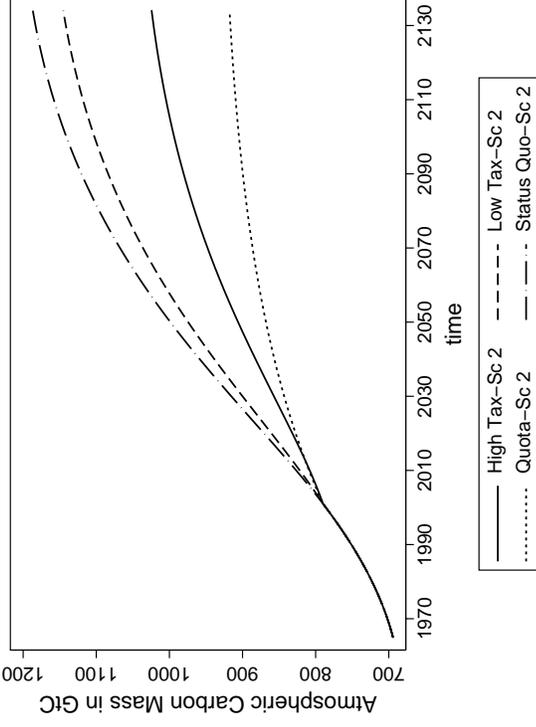


Figure 10: Atmospheric Carbon Levels

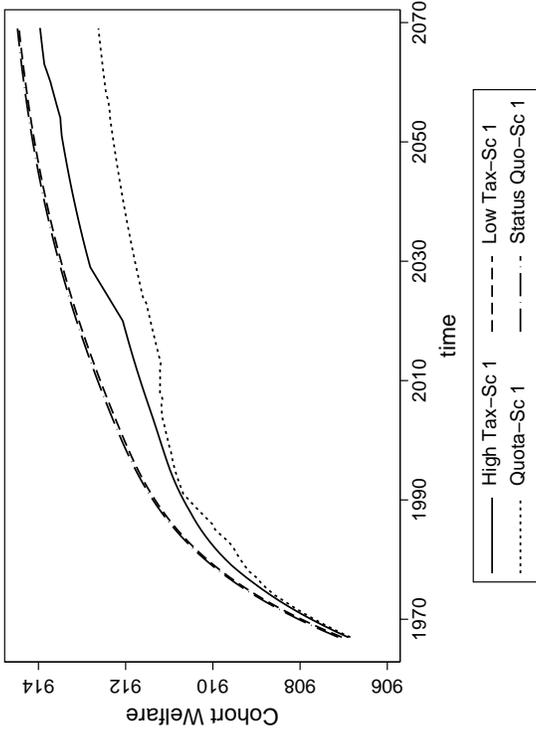


Figure 11: Welfare Levels by cohort

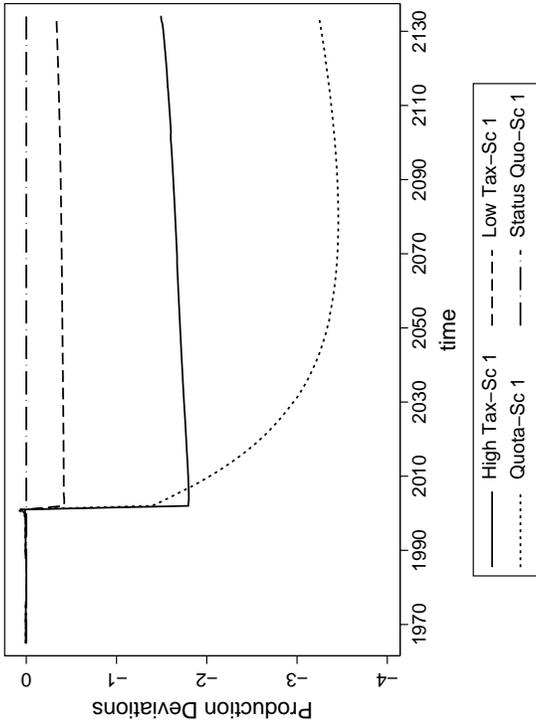


Figure 12: Percentage Deviations in Production

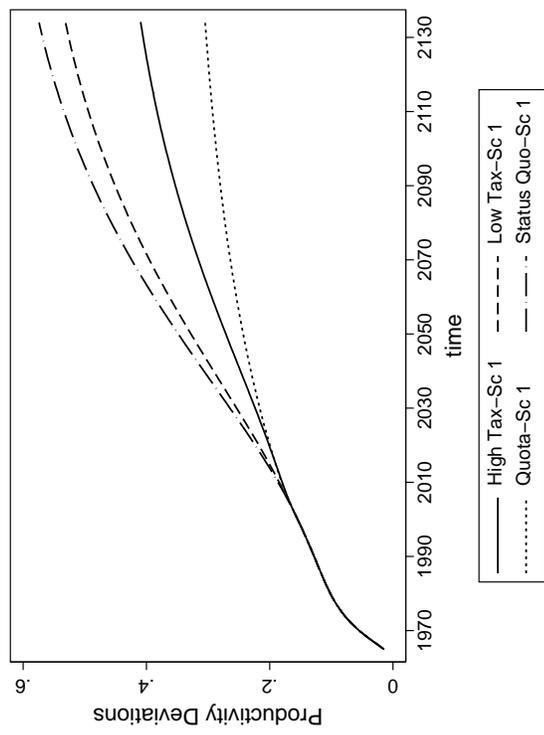


Figure 13: Productivity Decreases

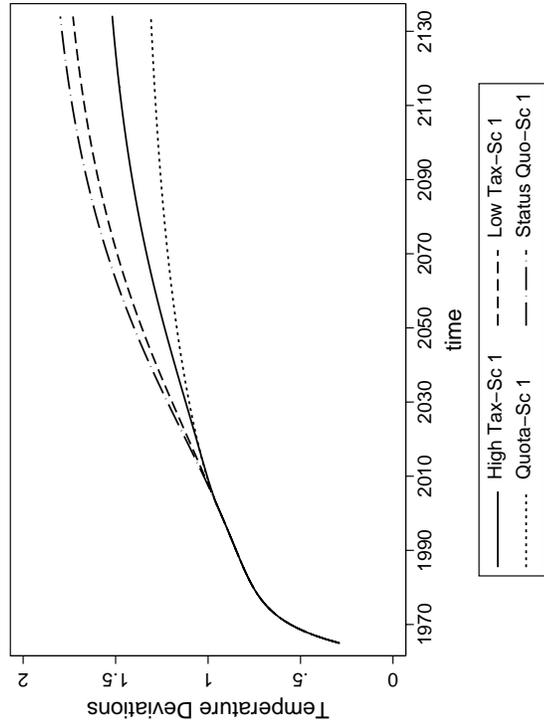


Figure 14: Median Temperature Deviations

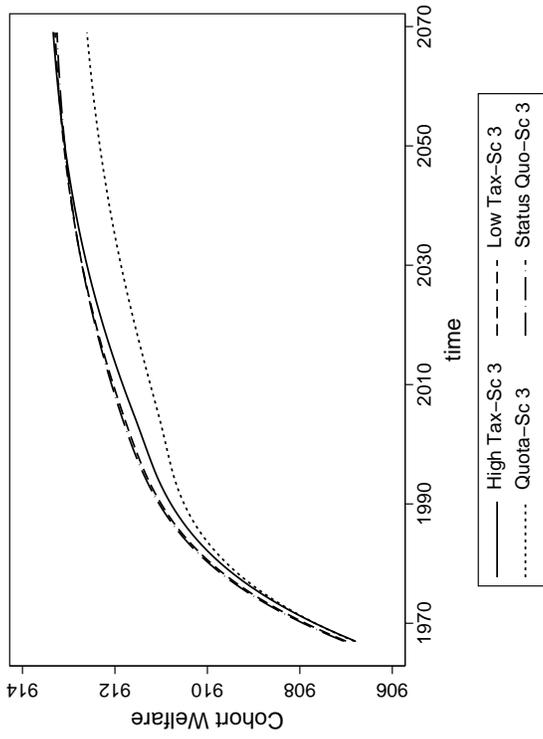


Figure 15: Welfare Levels by cohort

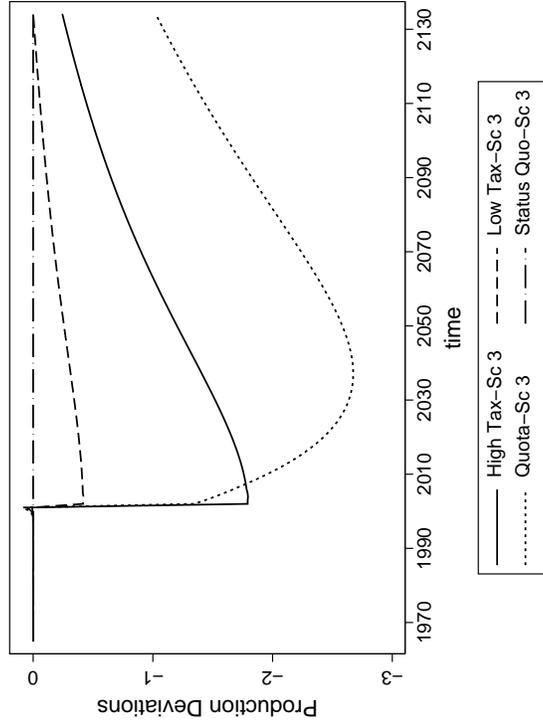


Figure 16: Percentage Deviations in Production

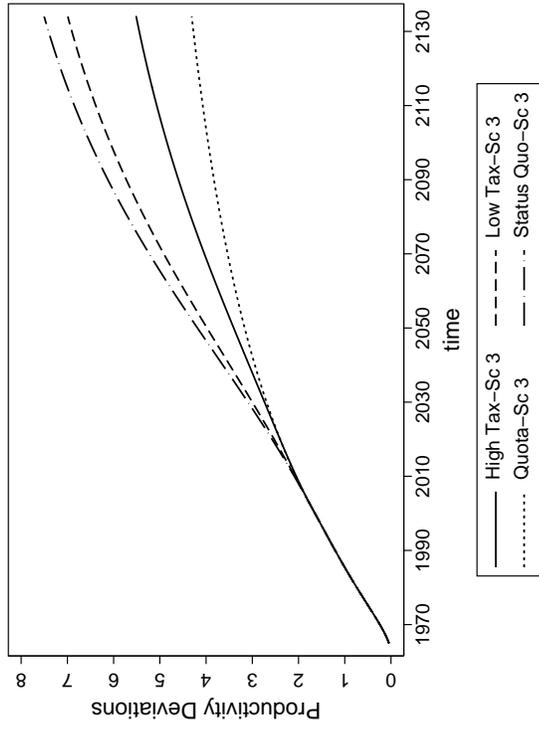


Figure 17: Productivity Decreases

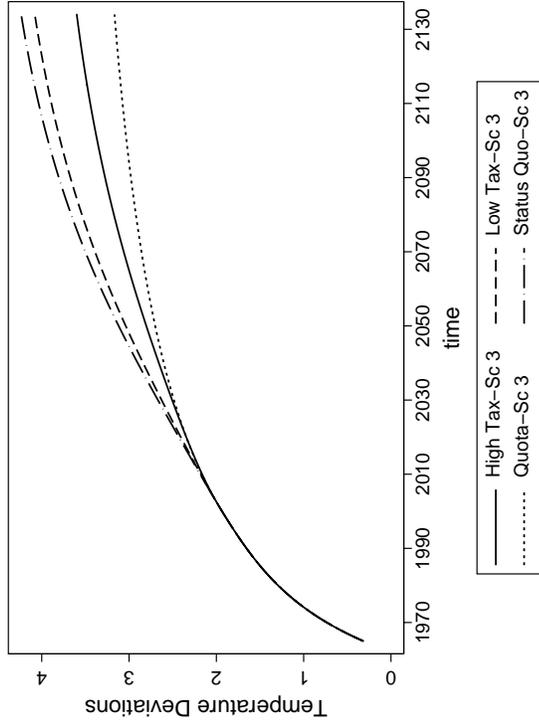


Figure 18: Median Temperature Deviations

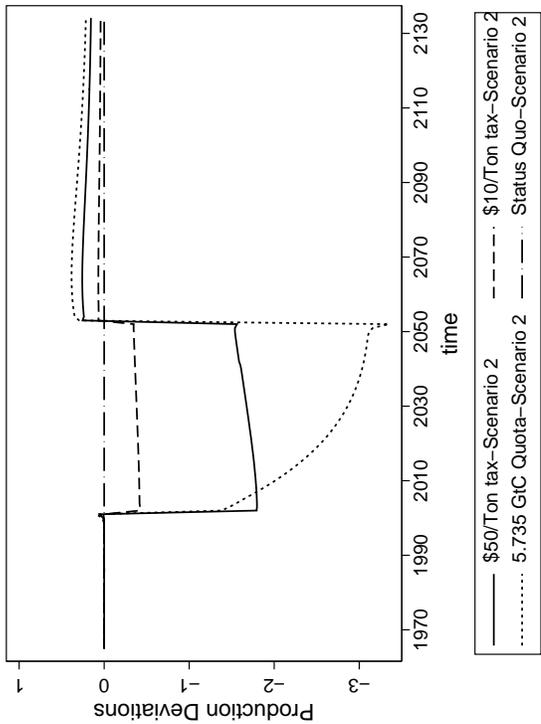


Figure 19: Welfare Levels by cohort-Finite commitment

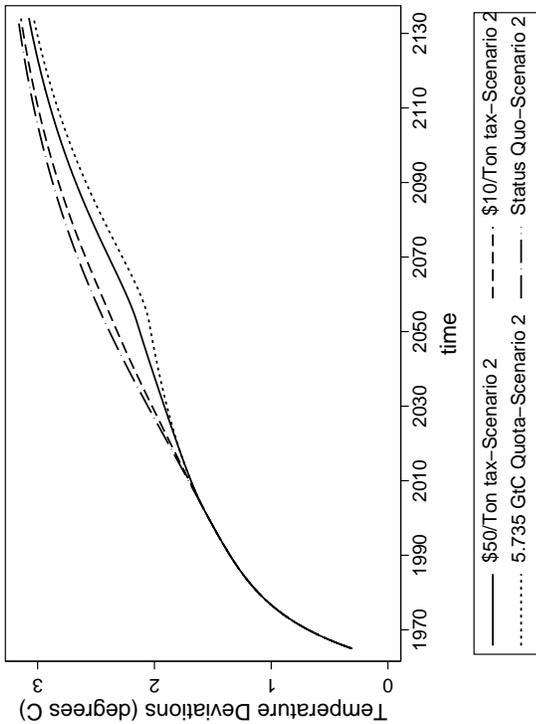


Figure 20: Percentage Deviations in Production-Finite commitment

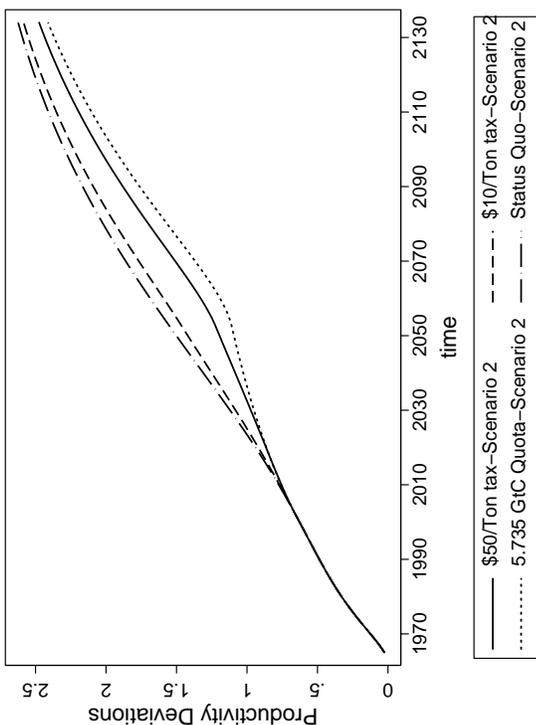


Figure 21: Simulated Median Temperature Deviations-Finite commitment

Figure 19: Cohort Welfare by cohort-Finite commitment- Figure 22: Simulated Median Temperature Deviations-Finite commitment