The Valuation and Information Content of Options on Crude-Oil Futures Contracts

J. Glenn Andrews

and

Ehud I. Ronn
Department of Finance
University of Texas at Austin

Feb. 2009
Revised: May 26, 2009

Communications
Author: Ehud I. Ronn

Address: Department of Finance
McCombs School of Business
1 University Station, B6600
University of Texas at Austin
Austin, TX. 78712-1179

Tel.: (512) 471-5853
FAX: (512) 471-5073
E-mail: eronn@mail.utexas.edu

Abstract

Using market prices for crude-oil futures options and the prices of their underlying futures contracts, we estimate the volatility skew in two ways. As a benchmark for our theoretical model, on each date we first estimate a cross-sectional polynomial structure for each maturity to demonstrate the strength and weaknesses of a purely-mechanical model. We then apply to the empirical data a Merton-style jump-diffusion model, with a rich structure of cross-sectional constraints on the parameters. Both models are tested with respect to their mark-to-market accuracy over time, as well as their efficacy in hedging intertemporal option price changes. The postulated Merton-style model is shown to yield useful parameters from which market prices can be computed, option prices can be market-to-market and (imperfectly) hedged, as well as an informationally-rich structure covering the time period of the turbulent past six months.

Key Words: Crude-oil futures and options

The Valuation and Information Content of Options on Crude-Oil Futures Contracts

1 Introduction

The market for crude-oil futures and option contracts, traded on the New York Mercantile Exchange, is one of the deepest and certainly one of the most important futures and options markets in the world. In this paper, we seek to extend our knowledge and understanding of the pricing of futures options the presence of the volatility "skew" but as importantly we seek to infer what market prices are "telling us about the future," what might be termed the "Message from Markets."

The literature on the pricing of futures and futures options, even when restricted to the crude-oil contracts is extremely rich. The literature includes one- through four-factor models which address the movements of prices and volatilities over time. The range of relevant literature which pertains to the crude-oil futures literature includes equilibrium models that simultaneously address all futures prices (and implicitly, options), production/extraction and investment decisions, and Heath-Jarrow-Morton (HJM) models.

Our purpose in this paper is to address the valuation, calibration and understanding of the volatility "skew" through the use of both a model, and a model-free, approach, the latter constituting a benchmark for the specific model we will examine. The modeling approach we postulate takes each futures price as given, and does not attempt to embed that futures contract in a more-general limited-number-of-factors model. Rather, our approach is in the spirit of the HJM models, in which the term structure of prices is allowed to be totally general.

The literature is rich with equilibrium-based models which strive to explain the cross-sectional term structure of Gibson and Schwartz (1990), Brennan (1991), Schwartz (1997), Hilliard and Reis (1998), Schwartz and Smith (2000), Richter and Sørensen (2002), Nielsen and Schwartz (2004) and Casassus and Collin-Dufresne (2005) and more recently Trolle and Schwartz (2008). These models attempt to represent the set of futures contracts and their accompanying options with a small set of factors, in some cases including an additional factor representing *volatility.* In the current model, we take the futures prices as exogenously specified, in the spirit of the HJM model, and we impose a cross-sectional constraint on the parameters of the futures contract via the imposition and calibration of a Merton (1976) jump-diffusion model.

Another important strand of the literature is that of estimation of stochastic processes. With respect to the modeling of the implied-vol "skew," the two primary approaches have been stochastic volatility, jump-diffusion and their variants and combinations. The relevant literature begins with Merton (1976), continues with the Ball-Torous (1983) estimation of such processes in stocks, Bates (1996)

We begin our approach by utilizing a "model-free" approach, in which the maturity-specific cross-sectional of volatilities is estimated by estimating a polynomial structure to each maturity. This model-free approach is complemented by demonstrating its strengths and weaknesses with respect to a measure of intertemporal mark-to-market accuracy, as well as its effectiveness in yielding a prescription for the hedging of intertemporal option price changes. We then turn to applying the empirical data to a Merton-style jump-diffusion model, endowed with a structure of cross-sectional constraints on the parameters. This model, too, is tested with respect to mark-to-market accuracy and hedging efficacy. The postulated Merton-style model is shown to yield useful parameters from which market prices can be computed, option prices can be marked-to-market and (imperfectly) hedged, as well as an informationally-rich structure covering the time period of the turbulent past six months.

This paper is now organized as follows. Section 2 applies what we term a curve-fitting approach to the cross-sectional of futures options, and investigates the model's properties as well as its intertemporal properties in marking-to-market and hedging. Whereas Section 3 briefly describes the data, Section 4 considers the Merton (1976) model, implements the cross-sectional parameter constraint approach, considers the model's mark-to-market and hedging properties — and then proceeds to explore the model's informational content over the turbulent period of 2008. Section 5 considers several tests across the curve-fitting and Merton-(1976) methodologies. Section 6 concludes.

2 An Atheoretic Curve-Fitting Approach

As a benchmark to the Merton (1976) jump-diffusion model, alternatively using quadratic or cubic polynomials, we initially estimate a purely-technical curve-fitting exercise cross-sectionally at each point in time. This will establish some empirical regularities of the volatility skew, to be subsequently contrasted with the Merton model.

2.1 Curve-Fitting Model

In this approach, we estimate two smooth polynomial curves, one of the quadratic and cubic forms, to the observed Term Structure of Volatility (TSOV) and the volatility skews.

Thus, we observe the following data:

1. For alternate maturities T, the options' market prices yield the *cross-sectional* implied vols, $\sigma_T(K)$, which are observable and are a function of the strike-price K

2. In this case, we have date-T volatility skews, and we seek to estimate *for each maturity* T the quadratic and cubic fits

$$\sigma_T(K) = \alpha_T + \beta_T(K - F_T) + \gamma_T(K - F_T)^2 \quad (1)$$

$$\sigma_T(K) = \alpha_T + \beta_T(K - F_T) + \gamma_T(K - F_T)^2 + \delta_T(K - F_T)^3 \quad (2)$$

   In each case, setting $K = F_T$ produces α_T as the ATM implied vol for maturity T

2.2 Empirics

On each trading day over the period 6/1/08 – 12/31/08, we observe the cross-strike implied vols $\sigma_T(K)$.[1] Accordingly, we can perform the vega-weighted minimum-absolute value analysis:

$$\min_{\{\{\alpha_i,\, \beta_i,\, \gamma_i,\, \text{all } i\},\, j=2,\,3\}} \sum_T \sum_K \text{vega}(K, T) \left| \sigma_T(K) - \text{polynom}_j(T, K) \right| \quad (3)$$

where $n(x)$ is the standard normal density and

$$\begin{aligned}
\text{vega}(K, T) &\equiv \exp\{-rT\} F_T\, n(d_1)\sqrt{T} \\
d_1 &\equiv \frac{\ln(F_T/X) + \sigma_T^2(K)\, T/2}{\sigma_T(K)\sqrt{T}} \\
\text{polynom}_2(T, K) &\equiv \alpha_T - \beta_T(K - F_T) - \gamma_T(K - F_T)^2 \\
\text{polynom}_3(T, K) &\equiv \alpha_T - \beta_T(K - F_T) - \gamma_T(K - F_T)^2 - \delta_T(K - F_T)^3 \\
c_T(K) &\geq \max\{.05,\ F_T - K + .05\}
\end{aligned}$$

Here we report the following sets of results: Based on data from June 13, 2008,

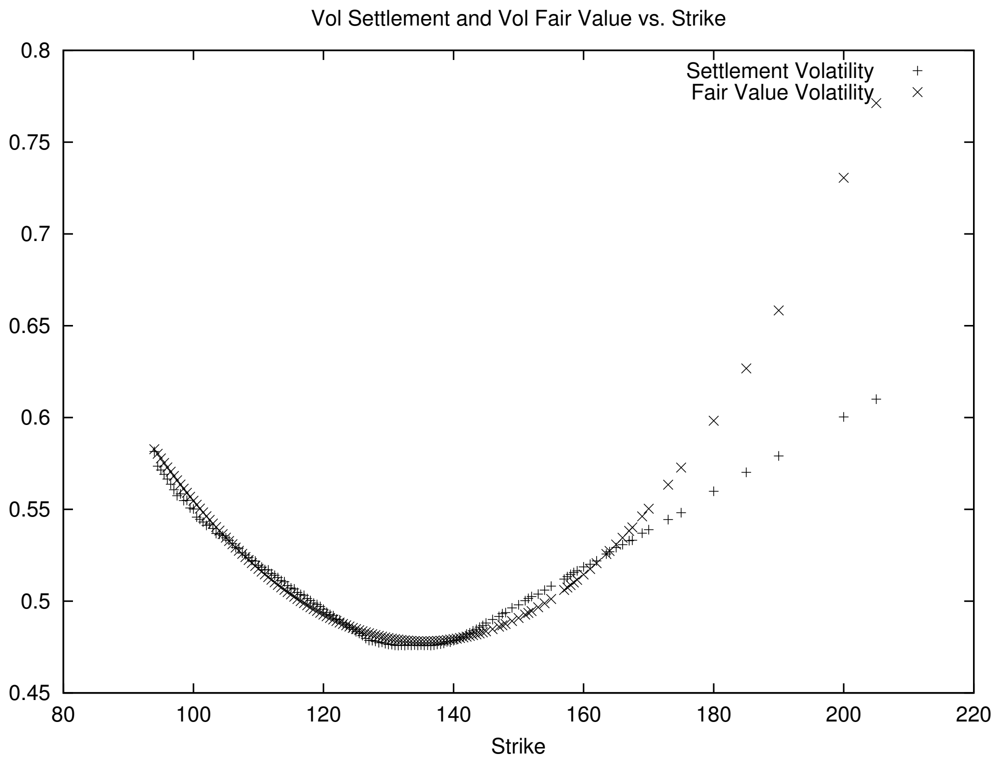
1. Figures 1 and 2 display the cross-sectional observed market implied vols contrasted with those fitted from quadratic and cubic polynomials to maturities of 34 and 1614 days.

[1] Traded NYMEX options are American in style. Accordingly, we attempted to apply the Barone-Adesi/Whaley (BAW) algorithm to extract implied vols. This exercise was not useful due to data limitations:

1. Deep in-the-money options have low time values, rendering the extraction of implied vols difficult

2. For the ATM options, the difference between European- and American-style implied vols was minimal

Figure 1: The 7/17/2008 Volatility Skew, Quadratic Curve-Fitting, 34 Days to Expiration



2. Figures 3 and 4 plot the set of α_T's for that day using the quadratic and cubic. The purpose of these two graphs is to demonstrate the quadratic and cubic functional forms provide essentially the same TSOV.

3. Based on a time-series run across the sample period 6/1/08 – 12/31/08, Figure 3 plots α_T for the shortest maturity exceeding 28 days, as well as the longest maturity closest to 1600 days

   Figure 2 clearly displays the implied TSOV volatility impacts of surging prices in the Summer of 2008: A substantial increase in the short-term vol $\alpha_{T \cong 34 \text{ days}}$ accompanied by a virtually stable long-term vol $\alpha_{T \cong 1600 \text{ days}}$.

2.3 Mark-to-Market Analyses

1. For each option, we compute the fair-value $v_T(K)$ by substituting α_T, β_T, γ_T, δ_T into Black (1976). For date t, we thus have market prices $c_{tT}(K)$ and fair-values $v_{tT}(K)$

Figure 2: Term Structure of Volatility, Quadratic Curve-Fitting, June 13, 2008

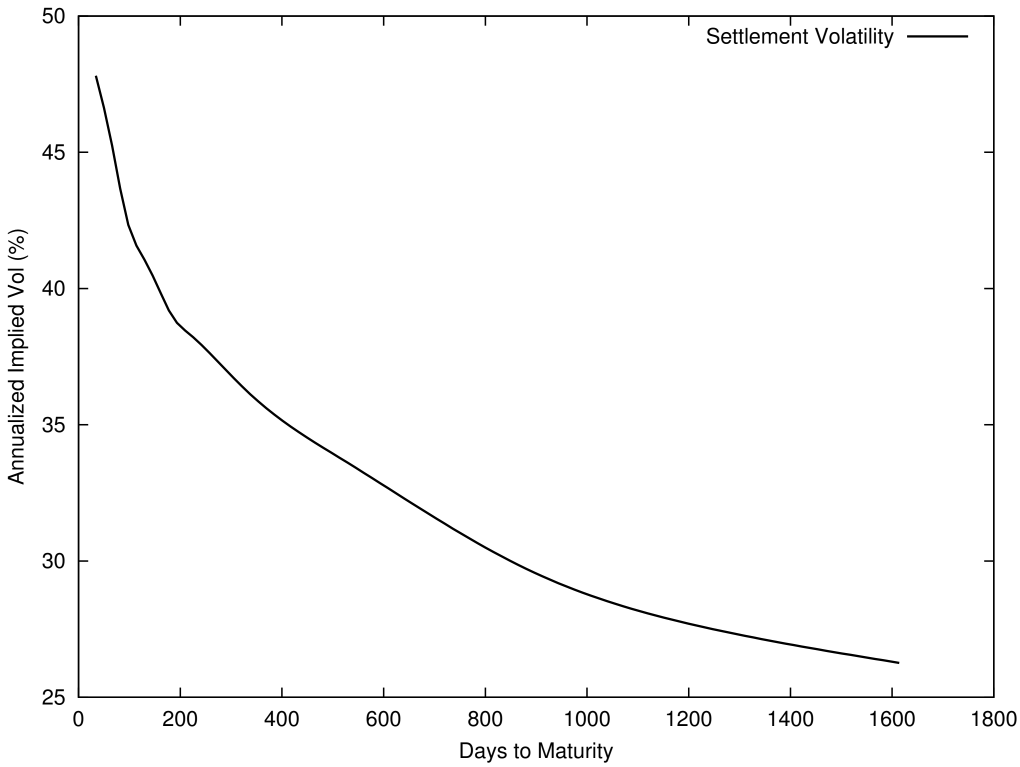


Figure 3: Term Structure of Volatility, Cubic Curve-Fitting, June 13, 2008

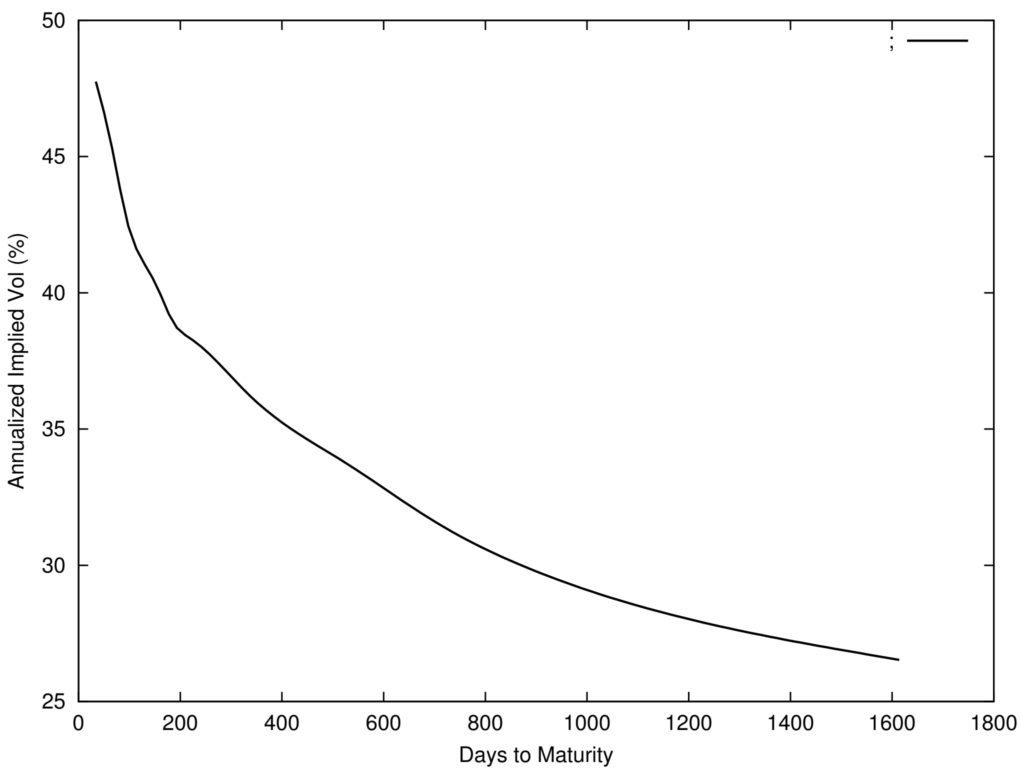
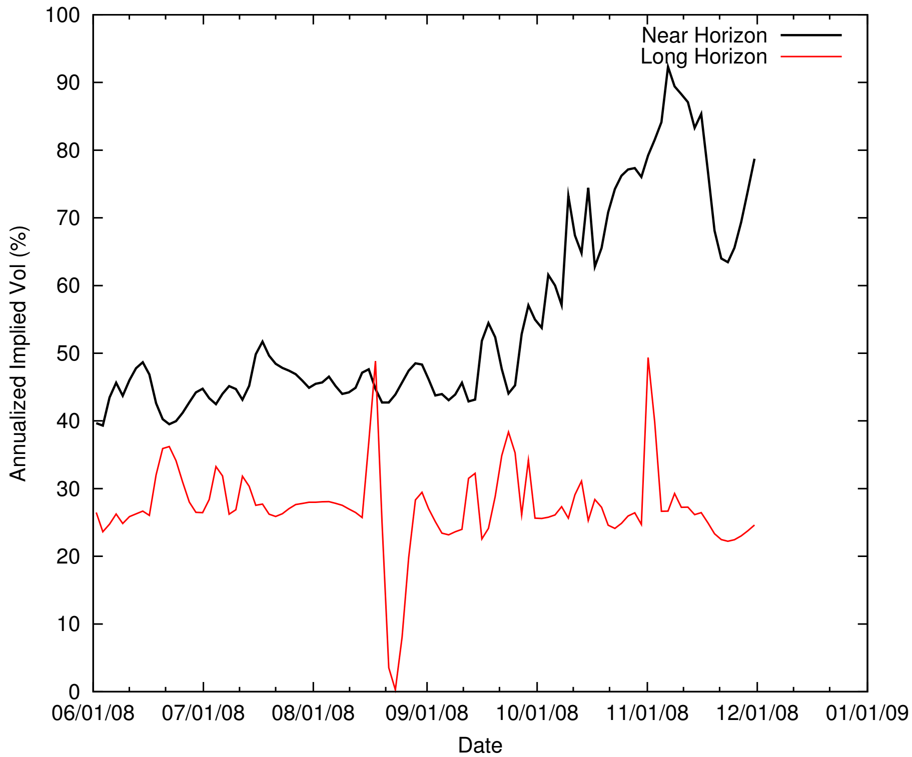


Figure 4: Time-Series Estimates of Short ($T = 34$ days) and Long-Term α_T's ($T = 1600$ days)



2. The identification of rich and cheap options is then straightforward:

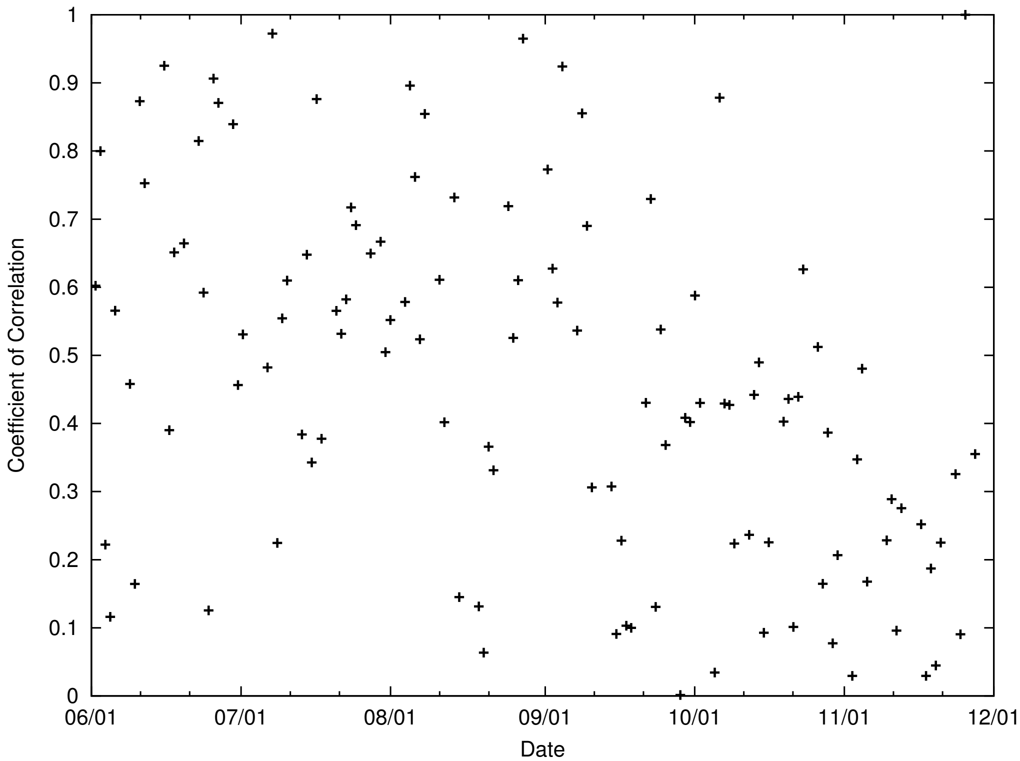
$$\begin{array}{llcll} \text{If} & c_{tT}(K) & > & v_{tT}(K), & \text{RICH} \\ \text{If} & c_{tT}(K) & < & v_{tT}(K), & \text{CHEAP} \end{array}$$

3. With these results in hand, we compute

$$\text{Corr}\left[c_{tT}(K) - v_{tT}(K),\, c_{t+1,T}(K) - v_{t+1,T}(K) \right]. \tag{4}$$

Figure 4 displays these correlations across our data period 6/1/08 – 12/31/08.

Figure 5: Time-Series of Daily Correlations, Curve-Fitting Model, Quadratic



4. Further, we now run the regression

$$c_{t+1,T}(K) - v_{t+1,T}(K) = A + B\left[c_{tT}(K) - v_{tT}(K)\right] \tag{5}$$

whose coefficients A, B and their associated t-statistics are reported in Table 1.

2.4 Hedging Analysis

The hedging analysis here is conducted according to the following algorithm:

1. For a given maturity T and strike price K, compute the fair-value implied vol $\sigma_T(K)$ in accordance with the estimated values of α_T, β_T, γ_T and (for the cubic) δ_T. This fair-value implied vol should correspond to the implied volatility of the previously calculated $v_T(K)$

2. If the option is RICH (CHEAP), at time t sell the option and purchase (sell) a Δ-hedged portfolio of F_T, where the Δ is computed from the fair-value implied vol (*not* from the implied vol of the option price)

3. At the succeeding date $t+1$, compute the trading gain/loss by closing out the option position at its market price, and closing out the Δ-hedged position using the futures

contract $F_{t+1,T}$ at date $t+1$. Using the computed hedge ratios, the tracking gain/loss Π for an option designated RICH is given by:

$$\Pi \equiv c_{tT}(K) - c_{t+1,T}(K) - \Delta\left(F_{tT} - F_{t+1,T}\right).$$

If the option is CHEAP, report $-\Pi$.

3 Data

For the period 6/1/2008 – 12/31/2008, on each trading day we observe NYMEX data for crude-oil futures contracts and option prices:

1. Futures prices for maturities $T : F_T$
2. Call option prices for maturity T, $c_T(K)$, or implied vols $\sigma_T(K)$ which can be converted into option prices thru Black (1976)'s

$$c_T(K) = e^{-rT}\left[F_T N(d) - KN\left(d - \sigma_T(K)\sqrt{T}\right)\right], \tag{6}$$

where

$$d \equiv \frac{\log\left(F_T/K\right)}{\sigma_T(K)\sqrt{T}} + \tfrac{1}{2}\sigma_T(K)\sqrt{T}$$
$$\sigma_T(K) = \text{annualized futures volatility}$$

4 The Merton (1976) Jump-Diffusion Model

4.1 Analytical Model

Recall that Merton (1976) option pricing model is given by:

$$v_T(K_T) = \sum_{n=0}^{\infty} \frac{e^{-\lambda' T}\left(\lambda' T\right)^n}{n!} c_n\left(F_T, X, T, r_n, q, \sigma_n\right) \tag{7}$$

where

$$v_T(K_T) = \text{European call option}$$
$$\lambda' = \lambda\left(1 + \overline{k}\right)$$
$$T = \text{option expiration}$$

$c_n\left(F_T,\, X,\, T,\, r_n,\, q,\, \sigma_n\right)$ = Black-Scholes [*not* Black (1976)] call option value with parameters $\{F_T,\, X,\, T,\, r_n,\, q,\, \sigma_n\}$, where q is the *dividend yield*

$$\begin{aligned} c_n\left(F_T,\, X,\, T,\, r_n,\, q,\, \sigma_n\right) &= F_T\, e^{-qT} N(d) - K e^{-r_n T} N(d - \sigma_n \sqrt{T}) \\ d &\equiv \frac{\ln\left(F_T / K\right) + \left(r_n - q\right) T}{\sigma_n \sqrt{T}} + \tfrac{1}{2}\, \sigma_n \sqrt{T} \end{aligned}$$

$\sigma_n^2 = \sigma^2 + n\delta^2 / T$

$r_n = r - \lambda \overline{k} + n \ln\left(1 + \overline{k}\right) / T$

$q = r$

Notes:

1. Although in principle (7) requires a summation over an infinite number of terms, in practice the option value converges after a summation over the first ten terms.

2. The parameters of the jump process are:

   λ = Intensity of the jump process

   $\overline{k}$ = Average amplitude of the jump process

   δ^2 = Variance of the jump process amplitude

   σ^2 = Variance of the diffusion process

3. $q = r$ in this case, since the F_T's are futures contracts

4.2 Modeling λ, $\overline{k}$, δ_T^2 and σ_T^2

The modeling of these four sets of parameters is guided by the following intuition:

1. When a crude-oil price shock occurs, it occurs at all maturities

2. Such a shock is more profoundly felt at the lower maturities, and "dies off" for succeedingly longer maturities. This is modeled as an exponentially-declining function, where the three parameters are the initial value, the asymptotic value and the rate of decline

Accordingly, we impose the following time-declining functional forms on λ, $\overline{k}_T$, δ_T^2 and σ_T^2 :

$$\begin{aligned} \lambda &= \text{const. for all } T && (8) \\ \overline{k}_T &= \left(b_k - a_k\right) \exp\left\{-c_k T\right\} + a_k && (9) \\ \delta_T^2 &= \left(b_\delta - a_\delta\right) \exp\left\{-c_\delta T\right\} + a_\delta && (10) \\ \sigma_T^2 &= \left(b_\sigma - a_\sigma\right) \exp\left\{-c_\sigma T\right\} + a_\sigma && (11) \end{aligned}$$

As previously noted, the unitary λ across all T reflects the attribute that when a price shock occurs, it occurs at all maturities.

Across the three sets of parameters $\overline{k}_T$, δ_T^2 and σ_T^2, this modeling has the characteristics that:

1. For $T \to 0$,
$$\begin{bmatrix} \overline{k}_0 \\ \delta_0^2 \\ \sigma_0^2 \end{bmatrix} = \begin{bmatrix} b_k \\ b_\delta \\ b_\sigma \end{bmatrix},$$
where b_k can be of either sign.

2. For $T \to \infty$,
$$\begin{bmatrix} \overline{k}_\infty \\ \delta_\infty^2 \\ \sigma_\infty^2 \end{bmatrix} = \begin{bmatrix} a_k \\ a_\delta \\ a_\sigma \end{bmatrix}.$$

3. The rates of decline for each parameter are given by the respective magnitudes of c_k, c_δ and c_σ.[2]

4. There are several parameter constraints imposed on the set of parameters $\{\lambda, b_k, a_k, c_k, b_\delta, a_\delta, c_\delta, b_\sigma, a_\sigma, c_\sigma\}$, most of which were empirically non-binding:

Table 2 — Set of Parameter Constraints

#	Constraint	Motivation
1.	$b_k \geq -0.95$	b_k is the expected magnitude of the jump at the $T = 0$ end of the futures curve cannot be lower than -100%
2.	$\text{abs}(b_k) \geq \text{abs}(a_k)$	Recall a_k is the average magnitude of the jump size at the $T \to \infty$ end of the futures curve. Irrespective of the sign of b_k, the absolute magnitude of a_k should be no greater than that of b_k
3.	$b_\delta \geq a_\delta \geq 0$, $b_\sigma \geq a_\sigma \geq 0$	The variance rates of the jump (δ) and diffusion (σ) processes should be greater at $T = 0$ than at $T \to \infty$, and should always be non-negative

[2]In alternate tests, we have permitted σ_T^2 to be unconstrained/unmodeled, with results essentially similar to those obtained when the exponential model was in fact imposed.

4.3 Calibration and Statistical Significance

We now consider the values of the estimated parameters and derive the appropriate tests for their statistical significance:

1. With observed prices given by $c_{tT}(K_T)$, and their theoretical (7) counterparts given by $v_T(K_T)$, the objective function is:

$$\min_{\{\lambda,\, b_k,\, a_k,\, c_k,\, b_\delta,\, a_\delta,\, c_\delta,\, b_\sigma,\, a_\sigma,\, c_\sigma\}} \sum_T \sum_K \text{vega}(T,\, K)\, |\, c_T(K) - v_T(K)\, | \tag{12}$$

   Here, using all options satisfying

$$c_T(K) \geq \max\{.05,\; F_T - K + .05\}$$

   observable at a given point in time t,[3] we seek to find the set of $\{b_k,\, a_k,\, c_k,\, b_\delta,\, a_\delta,\, c_\delta,\, b_\sigma,\, a_\sigma,\, c_\sigma\}$ parameters, as well as the single jump-intensity parameter λ, that best explain the cross-section of option prices.

   Non-linear estimation of necessity requires a set of initial parameter estimates. To provide such a set, the estimation procedure utilized a three-step process:

   (a) Initially, set $a_k = b_k = c_k = 0$. This eliminates the optimization over these three parameters and the constraints that pertain to a_k and b_k : $b_k \geq -.95$ and $\text{abs}(b_k) \geq \text{abs}(a_k)$.

   (b) Using the final values of the previous stage, restrict a_k to equal 0, and reintroduce the constraint $b_k \geq -.95$

   (c) Using the final values of the previous run as initial values, now optimize with respect to all variables *including* b_k and c_k

2. We ran several tests to determine the level of statistical significance of the estimated parameters. Specifically, using standard non-linear analysis — see e.g., Judge, Griffiths, Hill and Lee (1980) — for a value of $\boldsymbol{\beta^*}$ which minimizes the objective function, the

[3] The constraint $c_T(K) \geq \max\{.05,\; F_T - K + .05\}$ is imposed as we observed numerous instances of low-time value and -open interest options whose prices appear substantially "out of line" with the remaining same-date T options.

relevant variance-covariance matrix is $\mathbf{\Sigma^2}$:

$$
\begin{aligned}
\boldsymbol{y}(\boldsymbol{\beta}) &= \boldsymbol{f}(\boldsymbol{X},\, \boldsymbol{\beta}) + \boldsymbol{e} \\
\boldsymbol{e} &\sim N\left(\mathbf{0},\, \sigma^2 \boldsymbol{I}_T\right) \\
\boldsymbol{f}(\boldsymbol{X},\, \boldsymbol{\beta}) &\cong \boldsymbol{f}(\boldsymbol{X},\, \boldsymbol{\beta}^*) + \left[\left. \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{\beta}'} \right|_{\boldsymbol{\beta}^*} \right] (\boldsymbol{\beta} - \boldsymbol{\beta}^*) \\
S(\boldsymbol{\beta}) &\equiv [\boldsymbol{y} - \boldsymbol{f}(\boldsymbol{X},\, \boldsymbol{\beta})]' [\boldsymbol{y} - \boldsymbol{f}(\boldsymbol{X},\, \boldsymbol{\beta})] \\
\sigma^2 &= \frac{S(\boldsymbol{\beta}^*)}{T - K} \\
\mathbf{\Sigma^2} &= \sigma^2 \left[\left. \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{\beta}} \right|_{\boldsymbol{\beta}^*} \left. \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{\beta}'} \right|_{\boldsymbol{\beta}^*} \right]^{-1},
\end{aligned}
\tag{13}
$$

where $\left. \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{\beta}'} \right|_{\boldsymbol{\beta}^*}$ is a matrix of first-order derivatives evaluated at the optimal point $\boldsymbol{\beta}^*$.

The statistical tests took two alternate forms:

(a) To obtain a smoothly-differentiable objective function, we replaced the objective function $\sum_T \sum_K \text{vega}(T,\, K)\, |\, c_T(K) - v_T(K)\, |$ with its sum-of-squares analogue $\sum_T \sum_K \text{vega}(T,\, K)\, [\, c_T(K) - v_T(K)\,]^2$.

(b) In testing for the rank of the ten-parameter matrix $\boldsymbol{f}(\boldsymbol{X},\, \boldsymbol{\beta})$, we found it to have eight significant eigenvalues. Accordingly, we reduced the estimation to eight parameters by setting the $T \to \infty$ values of the average jump amplitude and its variance to zero: $a_k = a_\delta = 0$.

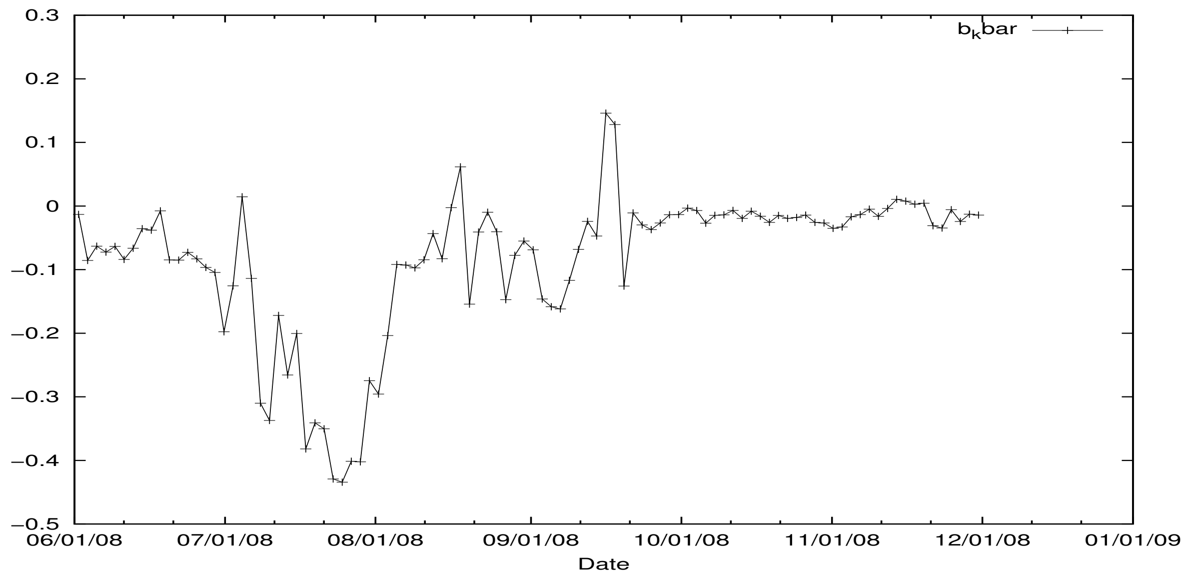
Table 1: Coefficient Estimates of Min-Absolute-Value Optimization

Stage	λ	a_σ	b_σ	c_σ	a_k	b_k	c_k	a_δ	b_δ	c_δ
1	0.035	0.000	0.183	0.610	0.000	0.000	0.000	0.000	2.282	-0.499
2	0.032	0.014	0.186	0.686	0.000	-0.063	-0.394	0.000	2.951	-0.452
3	0.032	0.014	0.185	0.689	0.045	-0.045	-0.337	0.000	2.955	-0.457

4.4 Time-Series Analyses of $\{b_k,\, a_k,\, c_k,\, b_\delta,\, a_\delta,\, c_\delta,\, b_\sigma,\, a_\sigma,\, c_\sigma\}$

Having fit these parameter structures to multiple dates, it is of interest to examine graphs of $\{b_k,\, a_k,\, c_k,\, b_\delta,\, a_\delta,\, c_\delta,\, b_\sigma,\, a_\sigma,\, c_\sigma\}$ four key parameters b_k, b_δ, b_σ and a_σ :

Figure 6: Time-Series of b_k, $T = 0$ Average Jump Amplitude, 6/1/08 – 12/31/08



The key conclusion to be extracted from Figure 3 follows from the relationship between b_k and crude-oil prices over the 2008 summer months: As crude prices reached their peak in early July, the skew was indicating an ever-more significant average jump *decline.* Subsequently, as prices indeed fell, the average jump amplitude reverted to near zero value.

Figure 4 mirrors some of the effects of Figure 3. That is, summer-2008 peak prices were accompanied by high-variance estimates, which subsided as prices retreated from their lofty levels.

Figure 5 also reflects the "volatility of the times": As was shown in the atheoretic curve-fitting approach, $T = 0$ diffusion-volatility also increased during summer 2008 peak prices. On the other hand, Figure 6 reveals unsurprisingly the estimates of $T \to \infty$ diffusion volatility is not estimated with the same precision.

4.5 Mark-to-Market Analyses

1. Analogous to the MTM analysis of the previous section, now let $v_{tT}(K)$ be the fitted value from (12), using the estimated parameter values $\{b_k, a_k, c_k, b_\delta, a_\delta, c_\delta, b_\sigma, a_\sigma, c_\sigma\}$ estimated at date t.

2. Identify the rich and cheap options:

$$\begin{array}{llcll} \text{If} & c_{tT}(K) & > & v_{tT}(K), & \text{RICH} \\ \text{If} & c_{tT}(K) & < & v_{tT}(K), & \text{CHEAP} \end{array}$$

Figure 7: Time-Series of $\sqrt{b_\delta}$, $T = 0$ Jump Amplitude Volatility, over 6/1/08 – 12/31/08

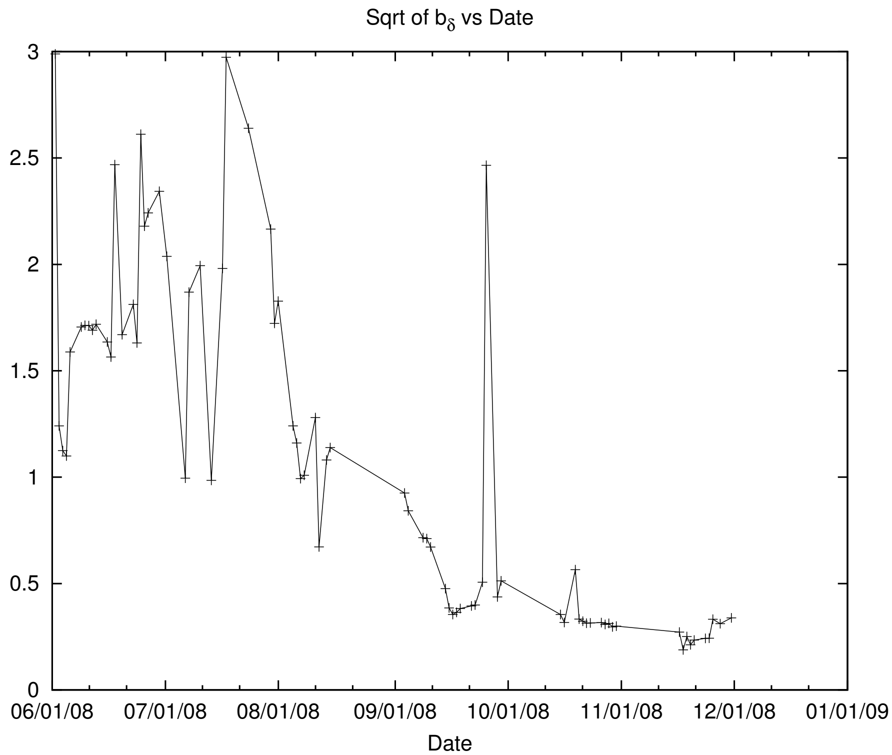


Figure 8: Time-Series of $\sqrt{b_\sigma}$, $T = 0$ Jump Amplitude Volatility, over 6/1/08 – 12/31/08

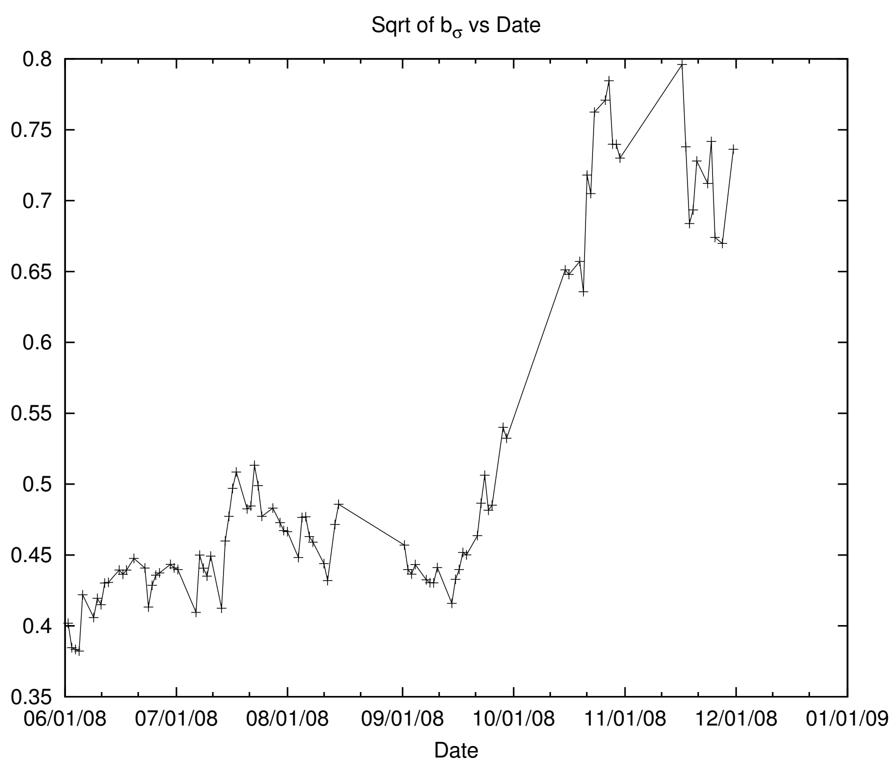
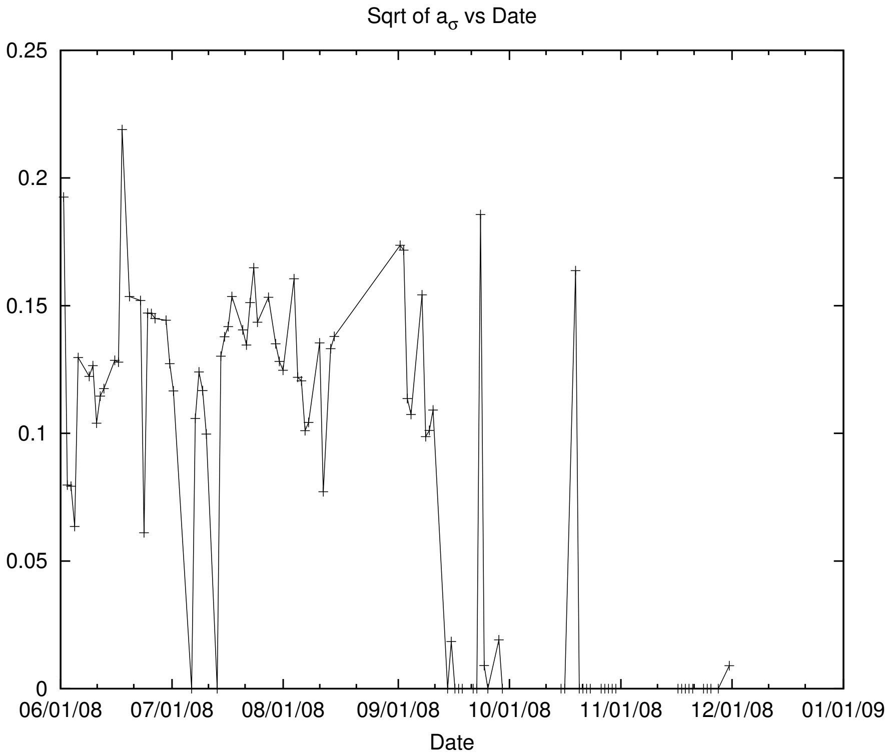


Figure 9: Time-Series of $\sqrt{a_\sigma}$, $T \to \infty$ Jump Amplitude Volatility, over 6/1/08 – 12/31/08



3. Compute

$$\text{Corr}\left[\, v_{tT}\,(K) - c_{tT}\,(K)\,,\; v_{t+1,T}\,(K) - c_{t+1,T}\,(K)\,\right] \tag{14}$$

4. Run the regression

$$v_{t+1,T}\,(K) - c_{t+1,T}\,(K) = A + B\left[v_{tT}\,(K) - c_{tT}\,(K)\right] \tag{15}$$

and report the coefficients A, B as well as their associated t-statistics

The results of (14) and (15) should be reported in a *table* across the time period for which we have data: Date 1, Date 2, Corr, A, B, t-stat A, $t-$stat B.

4.6 Hedging Analysis

1. Assume for given day t we have calibrated the maturity-T specific values of σ_T, λ, $\overline{k}_T$ and δ_T. For this test, assume that the date-T values of σ_T, $\overline{k}_T$ and δ_T have been obtained through the calibration of the three negative-exponential forms for $\{a_\sigma,\, b_\sigma,\, c_\sigma,\, a_k,\, b_k,\, c_k,\, a_\delta,\, b_\delta,\, c_\delta\}$

2. For each maturity T, compute the theoretical fair-valued ATM option value as v_T^{ATM}

3. For all options of maturity T (including the ATM one), compute two partial derivatives:

$$\frac{\partial v_T}{\partial F_T} \tag{16}$$

$$\text{vega}\,(K,\,T) \;\equiv\; \frac{\partial v_T}{\partial \Sigma_{KT}}, \tag{17}$$

where Σ_{KT} is the specific option's fitted implied vol

4. For each option with maturity T, let the hedge portfolio be obtained from the equation

$$v_T = \alpha_1 v_T^{\text{ATM}} + \alpha_2 F_T. \tag{18}$$

Thus, solve the two-equation system

$$\frac{\partial v_T}{\partial F_T} \;=\; \alpha_1 \frac{\partial v_T^{\text{ATM}}}{\partial F_T} + \alpha_2 \tag{19}$$

$$\frac{\partial v_T}{\partial \Sigma_{KT}} \;=\; \alpha_1 \frac{\partial v_T^{\text{ATM}}}{\partial \Sigma_{KT}^{\text{ATM}}} \tag{20}$$

You can solve for α_1 from (19) and substitute that result into (20) to solve for α_2.

5. Now, using the computed hedge ratios $\widehat{\alpha}_1$, $\widehat{\alpha}_2$ from (19) and (20), the tracking gain/loss Π for an option designated RICH is given by[4]:

$$\Pi \equiv c_{tT}\,(K) - \alpha_1 v_{tT}^{\text{ATM}} - c_{t+1,T}\,(K) + \alpha_1 v_{t+1,T}^{\text{ATM}} - \alpha_2\,(F_{tT} - F_{t+1,T})\,. \tag{21}$$

If the option is CHEAP, report $-\Pi$.

5 Analyses across Curve-Fitting and Merton Analyses

Consider the contemporaneous richness/cheapness analyses conducted under the two primary estimation methods, the Curve-Fitting and Merton (1976) Analyses. At date t, the relevant definitions in both cases are:[5]

$$\begin{array}{lccl} \text{If} & c_T\,(K) > v_T\,(K)\,, & & \text{RICH} \\ \text{If} & c_T\,(K) < v_T\,(K)\,, & & \text{CHEAP} \end{array}$$

[4] Although the date $t+1$ ATM option will likely have a different strike price, for purpose of the tracking error computation, it is important to *retain* the previously-defined date-t ATM.

[5] The subscript t is omitted, since these are correlations computed at a given point in time, not across time.

Letting "CV" stand for curve-fitting and "M" for Merton, now *super*script both c_T and v_T with their respective methods: c_T^{CV}, c_T^{M}, v_T^{CV}, v_T^{M} and compute the following two statistics:

$$\text{Corr}\left[v_T^{\text{CV}}(K) - c_T^{\text{CV}}(K), v_T^{\text{M}}(K) - c_T^{\text{M}}(K)\right] \tag{22}$$

$$v_T^{\text{CV}}(K) - c_T^{\text{CV}}(K) = A + B\left[v_T^{\text{M}}(K) - c_T^{\text{M}}(K)\right] \tag{23}$$

and report the coefficients A, B as well as their associated t-statistics

The results of (22) and (23) should be reported in a *table* across the time period for which we have data: Date 1, Date 2, Corr, A, B, t-stat A, $t-$stat B.

6 Conclusions

This paper set out with two primary objectives in mind:

1. To determine the degree to which a purely technical, atheoretical curve-fitting model could capture the cross-sectional and intertemporal regularities in the important market for crude-oil futures and options

2. To examine whether a jump-diffusion model could outperform the curve-fitting model, and whether it had the ability to convey the "message from markets" as these latter were reacting to crude-oil price changes through the important calendar year of 2008

While the work of summarizing and presenting this model is on-going, we can already declare at this juncture that the theoretical Merton-based model conveyed economically important and interesting information regarding the sign and magnitude of a price crash during the peak oil prices of summer 2008, a price decline that did in fact subsequently materialize. Subsequent to the price decline in the Fall of 2008, the absolute value of the average magnitude of the jump change exhibited a market decrease towards zero.

References

1. Barone-Adesi, G. and R. Whaley (1987): "Efficient analytic approximation of american option values," Journal of Finance, 42:301320.

2. Bates, D. S. (1996): "Jumps and stochastic volatility: Exchange rate processes implicit in Deutsche Mark options," Review of Financial Studies, 9:69107. (2000): "Post-87 crash fears in the S&P 500 futures option market," Journal of Econometrics, 94:181238.

3. Bessembinder, H., J. F. Coughenour, P. J. Seguin, and M. M. Smoller (1995): "Mean-reversion in equilibrium asset prices: Evidence from the futures term structure," Journal of Finance, 50:361375. 53

4. Black, F. (1976): "The pricing of commodity contracts," Journal of Financial Economics, 3:167179.

5. Brennan, M. J. (1991): "The price of convenience and the valuation of commodity contingent claims," in, Diderik Lund, and Bernt ksendal, eds.: Stochastic Models and Option Values, North-Holland, Amsterdam.

6. Broadie, M., M. Chernov, and M. Johannes (2004): "Model specification and risk premiums: Evidence from futures options," Working paper, Columbia University, forthcoming in Journal of Finance.

7. Carlson, M., Z. Khoker, and S. Titman (2006): "Equilibrium exhaustible resource price dynamics," Working paper, NBER # 12000.

   Clewlow, L. and C. Strickland (1999): "A multi-factor model for energy derivatives," Working paper, University of Technology, Sydney.

8. Cortazar, G. and E. S. Schwartz (1994): "The valuation of commodity contingent claims," Journal of Derivatives, pages 2739.

   Deaton, A. and G. Laroque (1992): "On the behaviour of commodity prices," Review of Economic Studies, 59:123. (1996): "Competitive storage and commodity price dynamics," Journal of Political Economy, 104:896923.

9. Duan, J.-C. and J.-G. Simonato (1999): "Estimating and testing eksponential-affine term structure models by Kalman filter," Review of Quantitative Finance and Accounting, 13:111135.

10. Eraker, B. (2004): "Do Stock Prices and Volatility Jump? Reconciling Evidence from Spot and Option Prices," Journal of Finance, 59:13671403.

11. Eydeland, A. and H. Geman (1998): "Pricing power derivatives," Risk, September.

12. Gibson, R. and E. S. Schwartz (1990): "Stochastic convenience yield and the pricing of oil contingent claims," Journal of Finance, 45:959976.

13. Heath, D., R. Jarrow, and A. Morton (1992): "Bond pricing and the term structure of interest rates: A new methodology for contingent claims valuation," Econometrica, 60:77105.

14. Heston, S. (1993): "A closed form solution for options with stochastic volatility," Review of Financial Studies, 6:327343.

15. Hilliard, J. E. and J. Reis (1998): "Valuation of commodity futures and options under stochastic convenience yields, interest rates and jump diffusions in the spot," Journal of Financial and Quantitative Analysis, 33:6186.

16. Judge, George G., William E. Griffiths, R. Carter Hill and Tsoung-Chao Lee, **The Theory and Practice of Econometrics,** 1980, John Wiley & Sons, New York.

17. Litzenberger, R. H. and N. Rabinowitz (1995): "Backwardation in oil futures markets: Theory and empirical evidence," Journal of Finance, 50:15171545.

18. Longstaff, F. and E. Schwartz (2001): "Valuing American options by simulation: A simple least-square approach," Review of Financial Studies, 14:113147.

19. Lund, J. (1997): "Econometric analysis of continuous-time arbitrage-free models of the term structure of interest rates," Working paper, Aarhus School of Business.

20. Miltersen, K. (2003): "Commodity price modelling that matches current observables: A new approach," Quantitative Finance, 3:5158.

21. Miltersen, K. and E. S. Schwartz (1998): "Pricing of options on commodity futures with stochastic term structures of convenience yields and interest rates," Journal of Financial and Quantitative Analysis, 33:3359.

22. Ng, V. K. and C. Pirrong (1994): "Fundamentals and volatility: Storage, spreads and the dynamics of metal prices," Journal of Business, 67:203230.

23. Nielsen, M. J. and E. S. Schwartz (2004): "Theory of Storage and the Pricing of Commodity Claims," Review of Derivatives Research, 7:524.

24. Richter, M. and C. Srensen (2002): "Stochastic volatility and seasonality in commodity futures and options: The case of soybeans," Working paper, Copenhagen Business School.

25. Routledge, B. R., D. J. Seppi, and C. S. Spatt (2000): "Equilibrium forward curves for commodities," Journal of Finance, 55:12971338.

26. Schwartz, E. S. (1997): "The stochastic behavior of commodity prices: Implications for valuation and hedging," Journal of Finance, 52:923973.

27. Schwartz, E. S. and J. E. Smith (2000): "Short-term variations and long-term dynamics of commodity prices," Management Science, 46:893911. 57

28. Trolle, A. B. and E. S. Schwartz (2006): "A general stochastic volatility model for the pricing of interest rate derivatives," Working paper, UCLA, forthcoming in Review of Financial Studies. 58