# Life-Cycle Portfolio Allocation for Disappointment Averse Agents

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**Abstract**

I solve the life-cycle portfolio allocation problem of a disappointment averse (DA) agent with labor income risk. DA preferences overweight disappointing outcomes and are consistent with behavior highlighted by the Allais paradox. I show that unlike constant relative risk aversion (CRRA) investors, DA investors drastically cut their allocation to stocks when they retire. This result is consistent with empirical evidence on portfolio shares and with the allocation rules of target-date retirement funds. I also show that sufficiently disappointment averse agents abstain from stocks after retirement, which is consistent with the observed low rates of stock market participation among retirees. I further show that when crashes are possible, agents with low levels of wealth invest little (or nothing) in the stock market.

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# 1 Introduction

The aversion to risky assets such as stocks affects the asset income available to the elderly and their ability to support themselves (Hurd (2002)). A recent survey documents that the asset income for those aged 65 and above was less than 13% of their total income.[1] The low asset income reflects the lower stock market participation rates. The fraction of households participating in the stock market decreases from the highs of 60% (for ages 45 to 54) to about 40% (for ages 65 to 74) around retirement.[2] The proportion of financial assets invested in equity drops by about half between the ages of 60 to 70.[3] Studies using proprietary 401(K) data also find that the stock share of the portfolio drops in old age.[4] Life-cycle or target-date funds also suggest a decline in asset holdings around retirement. They are one of the mandated default investment options[5] in retirement accounts. They exhibit increasing conservativeness by reducing the risky asset holdings[6] around retirement (Figure 1.1). These funds represented holdings of $256 billion as of 2009.[7]

Despite the evidence in the data and the advice of financial planners, the decline in risky asset holdings around retirement is puzzling from the perspective of a standard life-cycle model with constant relative risk aversion (CRRA) or Epstein and Zin (EZ) preferences. The theoretical models based on CRRA preferences and idiosyncratic non-tradable labor income generate post-retirement portfolio shares in stocks close to that before the retirement (Cocco, Gomes, and Maenhout (2005)). The uncertainty in human capital decreases as the agent approaches retirement and reduces to a sure non-tradable bond (the pension income) at retirement. The drop in uncertainty balanced by the drop in the size of the human capital sets the optimal risky investment close to that before the retirement. The presence of correlation between labor income and risky returns, however, creates an even more puzzling outcome. The positive correlation between labor income and stock market returns implies that the human capital is more stock-like before than after retirement.[8] Thus agents should optimally invest more in the stock market after they retire, contrary to the empirical evidence.

I use disappointment aversion (DA) preferences to explain the puzzling portfolio allocations around retirement and in old age. DA preferences, proposed by Gul (1991), incorporate expected utility as a special case and are consistent with Allais paradox-type non-expected

[1]See Purcell (2009). Asset income is primarily interest and dividends but also includes rents and royalties.

[2]See Bucks, Kennickell, Mach, and Moore (2009).

[3]See Ameriks and Zeldes (2002) for estimates based on the Survey of Consumer Finances, and Guiso, Haliassos, and Jappelli (2002) for international evidence.

[4]Agnew, Balduzzi, and Sunden (2003) and Holden, VanDerhei, and Quick (2000).

[5]Department of Labor website lists the default investment alternatives.

[6]The stock allocation drops by almost 1.5% per year after age 40, and accelerates to 2% per year at age 60 and 4% at age 65. These calculations are based on asset allocations for vanguard target-date funds in Viceira (2009).

[7]Investment Company Institute 2010 Fact Book (http://www.icifactbook.org/).

[8]See Gomes and Michaelides (2005) for an analysis based on Epstein-Zin preferences.

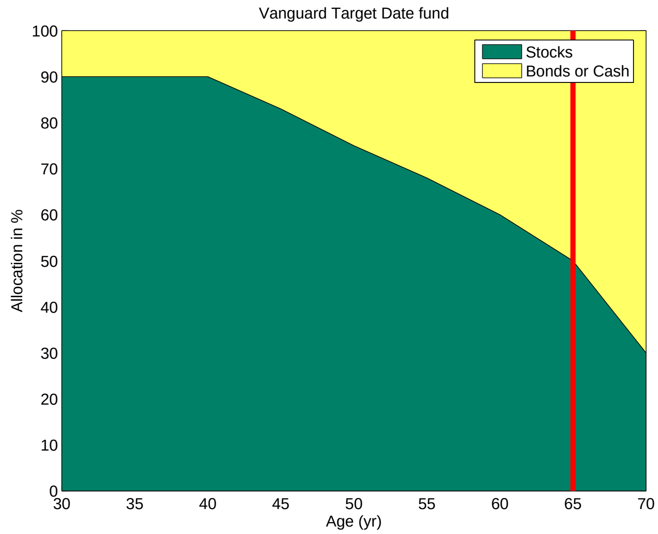


Figure 1.1: The asset allocation is based on data for the Vanguard target date fund presented in Table 2a in Viceira (2009).

utility behavior. DA preferences are a single-parameter extension of expected utility theory. To understand why the Allais paradox is related to the decline in risky asset holdings, consider the following example adapted from Gul (1991). Agents are first offered a choice between Lottery 1A, which pays a guaranteed \$200, and Lottery 1B which offers an 80% chance of winning \$300 and a 20% chance of winning \$0. When faced with this choice, most people prefer Lottery 1A over Lottery 1B. Agents are then offered a second choice between Lottery 2A, a 50% chance of winning \$200 and a 50% chance of winning \$0, and Lottery 2B, a 40% chance of winning \$300 and a 60% chance of winning \$0. When faced with this choice, most people prefer Lottery 2B. Formally,

$$\text{Choice 1} \quad \Rightarrow \quad u(\$200) > 0.8u(\$300) + 0.2u(\$0) \tag{1.1}$$

$$\text{Choice 2} \quad \Rightarrow \quad 0.5u(\$200) + 0.5u(\$0) < 0.4u(\$300) + 0.6u(\$0) \tag{1.2}$$

If agents' preferences are from the expected utility class, choosing Lottery 1A over Lottery 1B is inconsistent with choosing Lottery 2B over Lottery 2A. These choices imply violation of the independence axiom.[9] The choices reflect an affinity toward a sure outcome if it is one

[9]The independence axiom states that if an agent is indifferent between lottery A and lottery B, then the agent also is indifferent between a convex combination of A and C and the same combination of B and C; that is if, $A \sim B$, then $\alpha A + (1-\alpha)C \sim \alpha B + (1-\alpha)C \;\forall\; \alpha \in [0,1]$. The same applies for weakly ($\succsim$) and strictly preferred relations($\succ$). For DA preferences, the ordering after the convex combination still holds if the disappointing and the elating outcomes of lotteries A and B do not cross over after they are combined with lottery C. To demonstrate why agents' choices violate the independence axiom, the following steps reconstruct

of the alternatives and a choice for the risky but attractive gamble if the alternate choice is also risky.

This example mirrors the situation households face around retirement. The agents choose to allocate savings between the risk-free asset and the risky asset, along with making a consumption decision. The background risk limits the alternative choices available to the household. The non-tradable certain retirement income results in a choice akin to the first choice in the Allais paradox (Eq. 1.1). The affinity toward the sure alternative reduces the agent's appetite for risk after retirement. This increased risk aversion results in a drop of savings in risky assets and possible non-participation in the stock market. The risky labor income before retirement leaves the agent with a risky gamble even if he were not to invest any of the savings in stocks. The choice the agent faces in this scenario is similar to the second choice in the Allais paradox (Eq. 1.2). The agent faced with a risky alternative gamble prefers to take advantage of the risky but attractive stock returns. This results in higher share of savings invested in stocks before the retirement. Hence DA preferences naturally lead to a decline in the share of savings in risky assets around retirement when uncertainty about background risk gets resolved.

The life-cycle model with DA preferences has four interesting implications. First, I show that disappointment aversion provides a preference based explanation for the drop in the risky share of savings after retirement. As the Allais paradox example highlights, the drop in background risk around retirement generates greater aversion to risk and hence a drop in portfolio weights. Second, I show that sufficiently disappointment averse agents withdraw from the stock market altogether after retirement. I further show that the excess return distribution approximately characterizes the critical threshold of disappointment aversion, above which agents do not invest in the stock market at all. The discrete and positive risk price for the first unit of risk under DA preferences drives this non-participation. Third, I show that the perceived correlation between the labor income and the stock returns is higher than that under the data-generating distribution. The perceived increase in correlation raises the hedging motive.

Fourth, I show that the DA agent saves more for the retirement. He anticipates the need for higher savings to support consumption after retirement. The higher savings rate is a reflection of the conservative investment strategy in retirement that, on average, produces lower asset income. The agent driven by the consumption-smoothing motive thus saves more

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the second set of gambles from the first one under the assumption that the independence axiom holds:

$$\begin{aligned}
\text{Choice 1:}\quad u(\$200) &> 0.8u(\$300) + 0.2u(\$0)\\
0.5u(\$200) &> 0.4u(\$300) + 0.1u(\$0)\\
\Rightarrow\quad 0.5u(\$200) + 0.5u(\$0) &> 0.4u(\$300) + 0.6u(\$0),
\end{aligned}$$

where $u(\cdot)$ can be any of the utility functions from the expected utility theories. These steps show that if agents choose Lottery 1A in the first gamble, they should always choose Lottery 2A in the second gamble.

before retirement. The consumption stream is also less smooth due to a disparate attitude toward risky investment before and after retirement. The agent draws down on the wealth faster, as he does not take advantage of the attractive premium on risky returns. This lowers consumption after retirement, unlike the consumption pattern for an agent with the CRRA preferences.

In light of the 2007-8 financial crisis, considering how stock market crashes affect agents' optimal allocations is also interesting. As Rietz (1988), Barro (2006), and Gabaix (2008) among others, emphasize, such crashes have potentially important implications for asset prices. I study the implications of crashes in the life-cycle model for an agent with DA preferences. I show that when crashes are possible, DA agents with low levels of wealth stay out of the stock market. The optimal investment rules before retirement generate stock-allocation profiles that are increasing in wealth.

The paper closest to mine is Ang, Bekaert, and Liu (2005). They show that stock market nonparticipation is a possible optimal outcome when agents are sufficiently disappointment averse. In contrast to Ang, Bekaert, and Liu (2005), who consider the terminal utility problem and focus on the participation decision, I study the life-cycle asset allocation decisions of DA agents who earn non-tradeable labor income and derive utility from intermediate consumption.

The thesis is organized as follows. Section 2 discusses the background to the study. Section 3 contrasts the risk-reward attitude under DA and CRRA preferences. Section 4 describes the life-cycle model and the solution method. Section 5 details the model parameters used in calibrations. Section 6 examines optimal allocation decisions in the benchmark case with uncorrelated background risk. Section 7 includes various extensions of the model. Section 8 concludes.

# 2 Background

DA preferences have also been used in an equilibrium consumption asset pricing framework. Epstein and Zin (1991) and Epstein and Zin (2001) embed DA preferences among other alternative preferences in a recursive infinite horizon model. Bekaert, Hodrick, and Marshall (1997) find a better match for asset-return predictability in a consumption-based asset pricing model with DA preferences. Campanale, Castro, and Clementi (2010) match unconditional asset returns in a general equilibrium production economy with DA preferences and convex adjustment costs. Routledge and Zin (2010) propose generalized disappointment aversion (GDA), which is a one parameter extension of DA preferences. They obtain a large equity premium and a low-volatility risk-free rate that is pro-cyclical in an endowment economy.

I obtain conservative investment strategies as a result of the drop in labor uncertainty. The agents may also exhibit aversion to risky investments after retirement due to the inflexibility in labor supply. Bodie, Merton, and Samuelson (1992) study the flexibility in labor supply and find additional flexibility to choose labor supply induces the agent to take on greater financial risk.[10] Farhi and Panageas (2007) explore the effects of irreversible retirement choice along with the option to retire early. The irreversibility generates the inflexibility in labor supply after retirement. They show that the retirement optionality in general increases the investment in stocks prior to retirement and especially for those with high levels of wealth.

Yogo (2009) and Pang and Warshawsky (2010), among others, numerically solve for the effects of health risks on portfolio choice. They find that health risk lowers risk taking. The empirical evidence, however, is mixed. Love and Smith (2009) find little or no causal effect for health on portfolio choice after accounting for the unobserved heterogeneity (2%-3% at most for married households). Edwards (2008) finds that the households' adjustment of their portfolios in response to self-perceived health risk may explain only 20% of the age-related decline in financial risk taking after retirement. Berkowitz and Qiu (2006) find that health shocks indirectly affect portfolio choice by lowering financial wealth. They find that the relation between health and portfolio choice disappears after controlling for differences in the financial wealth of the sick and the healthy households.

The other appealing model for preferences is habits. Gomes and Michaelides (2003) explore ratio and additive habit models. Polkovnichenko (2007) studies the additive habit model. The additive habit model results in a drop in the portfolio allocation as the agents age, because the old maintain their consumption levels and hence their habit levels as they decumulate the assets. The agent at old age follows a conservative investment strategy to ensure his habit level is maintained. The consumption pattern, however, does not yield a sharp drop-off as observed with DA preferences. Polkovnichenko (2007) finds that the consumption pattern is not substantially different from that under the CRRA preference model, although in some cases, consumption increases slightly after retirement. He also notes that the model implies a high level of wealth accumulation. Gomes and Michaelides (2003) study the ratio habit model in detail. The authors find that this preference model generates more conservative portfolios than the CRRA model. However, these preferences also generate speedy wealth accumulation. They note that this feature impedes the model's fit for generating the low market participation in the presence of fixed costs. The age effects on the portfolio shares are similar to those for the CRRA model.

The literature on portfolio allocation is extensive. Campbell, Cocco, Gomes, and Maenhout (1999), Cocco, Gomes, and Maenhout (2005), Bertaut and Haliassos (1997), and Heaton and Lucas (1997, 2000b), among others, explore the effects of background risk for explaining the heterogeneity in portfolio allocation. The low correlation of labor income with stock returns implies that despite the riskiness of the labor income, the capitalized labor mimics a risk-free asset more than a risky stock index. This feature of labor income yields optimal stock investments that are large at a young age and drop with age as the capitalized value of labor drops. Benzoni, Collin-Dufresne, and Goldstein (2007) obtain low stock investments at a young age in an environment in which labor income is co-integrated with the market dividends.

[10]See Gomes, Kotlikoff, and Viceira (2008) for a calibrated life-cycle model.

Gomes and Michaelides (2005) introduce one-time entry costs proportional to the permanent income of the agent in an Epstein-Zin preference model to explain the low stock market participation of the young agents. Yao and Zhang (2005), Flavin and Yamashita (2002), and Cocco (2005) model portfolio choice in the presence of housing. They find that the price risk associated with housing crowds out stock holdings for the homeowners.

Disappointment Aversion and CRRA Preferences

Disappointment aversion is an axiomatic model of preferences by Gul (1991) that accommodates Allais paradox-type behavior. These preferences generate proportionately greater aversion to small gambles. The preference model achieves this by overweighting the outcomes below the certainty equivalent of the gamble. For example, consider the simple static gamble on wealth $W$. The certainty equivalent of this gamble $\mu_{da}$ is given by Eq. 2.1:

$$\mu_{da}^{1-\gamma} = E\left[W^{1-\gamma}\right] - \theta E\left[\left(\mu_{da}^{1-\gamma} - W^{1-\gamma}\right) I(W < \mu_{da})\right] \tag{2.1}$$

$$\mu_{crra}^{1-\gamma} = E\left[W^{1-\gamma}\right]. \tag{2.2}$$

The parameter $\theta$ controls disappointment aversion ($I(\cdot)$ is an indicator function). If $\theta$ is set to zero, the preference model reduces to that of constant relative risk aversion (CRRA, with relative risk aversion $\gamma$). The positive values for $\theta$ imply overweighting of disappointing outcomes. The disappointing outcomes are, however, the ones below the certainty equivalent of the gamble itself. Thus the outcomes that are overweighted and the certainty equivalent $\mu_{da}$ are implicitly determined and dependent on the nature of the gamble. I also define the disappointment probability, $\Phi(W < \mu_{da}) = E\left[I(W < \mu_{da})\right]$, as the probability that the gamble outcomes will be disappointing.

The log-normal gamble illustrates the changes in the risk-reward attitude of an agent with the DA preferences as the size of the gamble changes. The gamble on wealth $W(\epsilon)$ is given by $\ln(W(\epsilon)) = k + \sigma\epsilon$, where $\epsilon$ is a standard normal random variable. I compare the risk premia for the DA and the CRRA preferences. CRRA preferences exhibit only second-order risk aversion. The risk premium is proportional to the variance of the gamble $RP^{crra} = \ln(E(W)/\mu_{crra}) = \gamma\sigma^2/2$. The incremental premium or the incremental reward per unit rise in the risk is thus proportional to the size or the standard deviation of the gamble $d(RP^{crra})/d\sigma = \gamma\sigma$. Thus the incremental reward the agent demands is negligible for small gambles. In addition to second-order aversion to risk, DA preferences also exhibit first-order risk aversion. The risk premium includes terms proportional to both the standard deviation and the variance of the gamble. Thus the incremental premium for small-size gambles can be substantial depending on the value for parameter $\theta$.[11] I plot examples in Figures 2.1 and 2.2 for DA ($\theta = 1$ and $\gamma = 5$) and CRRA preferences ($\gamma = 5$).

[11] The limit value of incremental premium is $\theta/(\theta + 2)$ for an equally probable two outcome gamble $\{1 - \sigma, 1 + \sigma\}$ as $\sigma \to 0^+$.

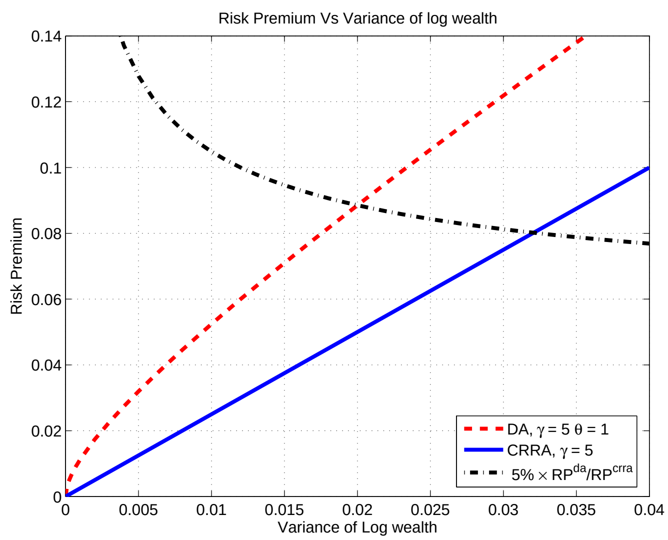


Figure 2.1: The risk premium for log-normal gamble as a function of the variance of the gamble for (1) DA preferences and (2) CRRA preferences. The plot also includes the ratio of risk premia for the two preferences.

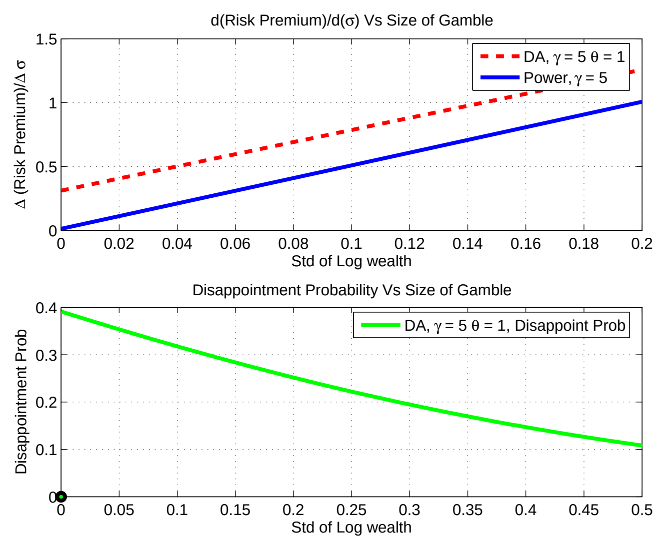


Figure 2.2: The characteristics of a log-normal gamble. Top panel: The incremental risk premium per unit additional risk versus the size of the gamble. Bottom panel: The disappointment probability as the size of the gamble increases.

The risk premium is linearly increasing in variance for the CRRA preference model, reflecting second-order risk aversion (Fig. 2.1). However, the risk premium for the DA preference increases at more than linear trend in variance. The incremental risk premium for the DA preference (Fig. 2.2) is not negligible for small gambles and in fact has a positive intercept of about a third for $\theta = 1$. This non-negligible incremental premium for small gambles generates greater aversion to small size gambles and thus a plausible model consistent with the first choice in the Allais paradox (Eq. 1.1). The first choice is between a sure payoff and a well rewarded gamble. The large incremental premium for the first unit of risk, however, implies that the agent demands a significant reward to accept even a small size risky choice. The incremental premia for the two preferences (with same $\gamma$) asymptote to each other as the size of the gamble gets larger. This helps explain the second choice in the Allais paradox, which is suggestive of lower aversion to risk unlike the first choice.

For low levels of background risk, DA preferences also generate increasing preference for gambles that pay a premium as the background risk increases.[12] This increasing relationship is due to the endogenous dependence of the certainty equivalent that determines the disappointing and elating outcomes. I illustrate this phenomenon in a one period terminal consumption problem. The investor has savings worth $A_t$ dollars that he optimally splits between a risky asset and a risk-free asset at time $t$. Further, the investor also receives a non-tradable positive labor income $Y_{t+1}$ at time $t+1$. The investor consumes all of the terminal wealth $W_{t+1}$ at time $t+1$ and has no date $t$ consumption:

$$
\begin{aligned}
W_{t+1} &= A_t R_{p,t+1} + Y_{t+1} \\
R_{p,t+1} &= (R_{t+1} - R_f)x_t + R_f \\
\ln(R_{t+1}) &= \ln(\overline{R}) + \epsilon_{r,t+1} \\
\epsilon_{r,t+1} &\sim N(-\sigma_r^2/2, \sigma_r^2).
\end{aligned}
$$

The risky return $R_{t+1}$ has a log-normal distribution with mean $\overline{R}$. The risk-free rate is $R_f$. The portfolio weights are constrained such that, $0 \leq x_t \leq 1$. I consider a mean-preserving spread on the background risk. I set the expected value of labor income at $\overline{Y}$ and vary the standard deviation, $\sigma_y$, of log-labor income. I set the correlation between the stock returns and the labor income to zero:

$$
\begin{aligned}
Y_{t+1} &= \overline{Y} \exp(\epsilon_{y,t+1}) \\
\epsilon_{y,t+1} &\sim N(-\sigma_y^2/2,\ \sigma_y^2).
\end{aligned}
$$

[12] Safra and Segal (2008) show that calibration result of Rabin (2000) may not hold for non-expected utilities under some conditions in the presence of background risk. Chapman and Polkovnichenko (2008), however, obtain a set of reasonable conditions under which an individual may reject a small bet and take on a large bet even in the presence of background risk for non-expected rank-dependent utility.

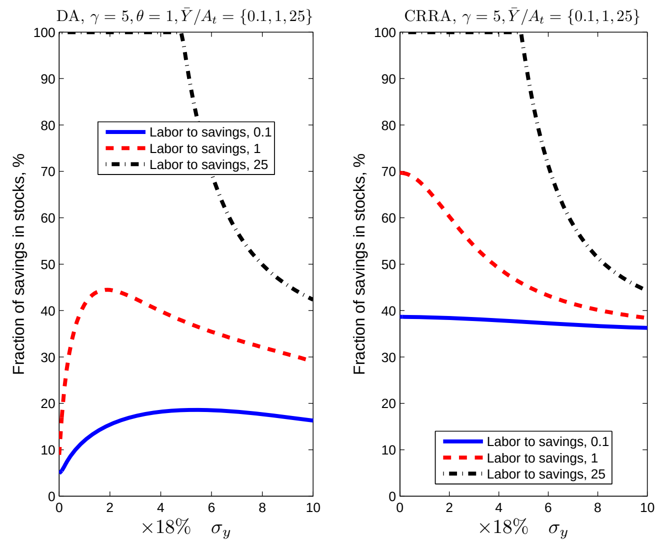


Figure 2.3: Optimal portfolio weights as the standard deviation of labor income increases. The expected value of the labor income is unchanged. The parameters are (1) equity premium at 6%, (2) risk-free rate at 2%, (3) standard deviation of stock returns at 18%, and (4) the correlation between returns and labor income is zero. The left panel is for DA preference with $\gamma = 5$ and $\theta = 1$. The right panel is for CRRA preference with $\gamma = 5$. The graphs in each panel are for different ratios of the expected labor income to savings. The three values for the ratio are, 0.1, 1, and 25.

The agent's objective is to maximize the certainty equivalent (CE) by choosing the optimal portfolio weights. I compute the optimal portfolio weight $x_t^*$ as a function of $\sigma_y$. I repeat the certainty equivalent for CRRA ($\mu_{crra,t}$) and DA ($\mu_{da,t}$) preferences below:

$$\begin{aligned}
\mu_{da,t}^{1-\gamma} &= E_t\left[W_{t+1}^{1-\gamma}\right] - \theta E_t\left[\left(\mu_{da,t}^{1-\gamma} - W_{t+1}^{1-\gamma}\right) I(W_{t+1} < \mu_{t,da})\right] \\
\mu_{crra,t}^{1-\gamma} &= E_t\left[W_{t+1}^{1-\gamma}\right].
\end{aligned}$$

I plot the optimal stock share of savings against the standard deviation of the log-labor income in Figure 2.3. The optimal stock share of savings is increasing for DA preferences as the background risk increases. However, the investment strategy turns conservative once the non-tradable income is sufficiently risky. The portfolio weight, on the contrary, is decreasing over the entire range for the CRRA preferences[13].

[13]This result is true under most conditions for power utility and preferences of HARA class (Campbell and Viceira (2002)). One sufficient condition is that the relative risk aversion be greater than the reciprocal of the elasticity of consumption (terminal wealth, $W_{t+1}$) with respect to financial wealth.

The source of the non-monotonic investment strategy under DA preferences is the perceived premium of the risky asset. The certainty equivalent under DA preferences can be interpreted in terms of that under the CRRA preferences, where the gamble outcomes are drawn from a distorted probability distribution (Eqs. 2.3 and 2.4). This distorted probability distribution ($\hat{p}(W_{t+1})$) increases the likelihood of outcomes below the certainty equivalent of the gamble:

$$\mu_{da,t}^{1-\gamma} = \frac{E\left[W_{t+1}^{1-\gamma}\left(1+\theta I(W_{t+1} < \mu_{da,t})\right)\right]}{1+\theta\Phi(W_{t+1} < \mu_{da,t})} \tag{2.3}$$

$$\mu_{da,t}^{1-\gamma} = \sum_{W_{t+1}} \hat{p}(W_{t+1}) W_{t+1}^{1-\gamma} \tag{2.4}$$

$$\hat{p}(W_{t+1}) = \frac{1+\theta I(W_{t+1} < \mu_{da,t})}{1+\theta\sum_{W_{t+1}} \hat{p}(W_{t+1}) I(W_{t+1} < \mu_{da,t})} \times p(W_{t+1}).$$

I obtain an approximate formula for the portfolio weights in Eq. 2.5 using log-linearization. The derivation is in Appendix H. The $\sigma$ represents the standard deviation and the hat ( $\hat{}$ ) represents the quantities under the distorted distribution. $\hat{\rho}$ represents the elasticity of terminal wealth with respect to the financial wealth under the distorted distribution.

$$x_t^* \approx \frac{1}{\hat{\rho}}\left(\frac{\widehat{\overline{r_{t+1} - r_f}} + \hat{\sigma}_{r,t}^2/2}{\gamma\hat{\sigma}_{r,t}^2}\right) + \left(1 - \frac{1}{\hat{\rho}}\right)\frac{\hat{\sigma}_{y,r,t}}{\hat{\sigma}_{r,t}^2}, \quad \text{where } \hat{\rho} < 1 \tag{2.5}$$

The log-linearized formula for the optimal portfolio weight is similar to the one for CRRA preferences except that all quantities are computed under the distorted distribution. The increasing elasticity of terminal consumption with respect to financial wealth reduces the portfolio weights for CRRA preferences (unless the correlation between the risky return and the labor income is negative). This effect is also present under the DA preferences. However, overweighting disappointing outcomes generates perceived risky return premium ( $\widehat{\overline{r_{t+1} - r_f}} + \hat{\sigma}_{r,t}^2/2$ ) that varies with the background risk.

Figure 2.4 plots the two quantities. The elasticity of terminal consumption with respect to financial wealth ($\hat{\rho}$) is monotonically increasing, which effectively increases the riskiness of stock investments. However, the increase in perceived premium with the increasing background risk makes the risky investment attractive. The perceived premium does degrade beyond a level of riskiness of the non-tradable income. These two opposing forces yield an initial affinity toward the risky asset and an aversion to the same once the background risk is sufficiently high. The variation in disappointment probability has a pattern that is a mirror image of the perceived premium.

The low perceived premium for small background risk generates investment behavior akin to the first choice in the Allais paradox. The small perceived premium implies aversion to risky

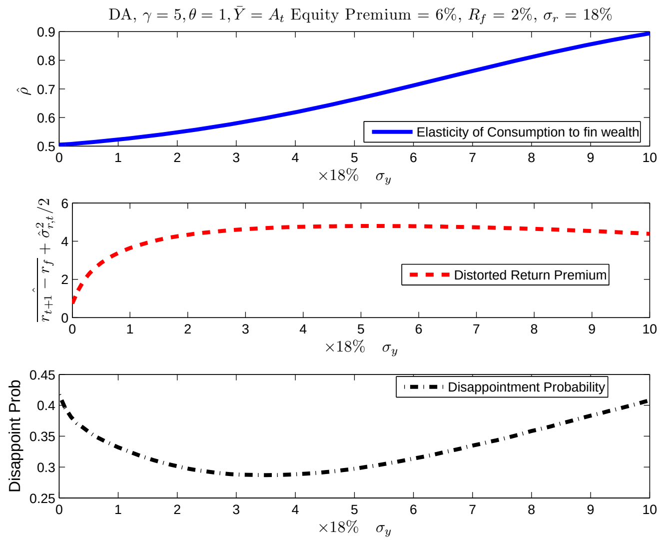


Figure 2.4: Bottom panel: disappointment probability, Middle panel: perceived return premium, and Top panel: the elasticity of terminal wealth with respect to the financial wealth, as the background risk increases. These graphs are for the case in which the expected labor income and the savings are equal.

assets and the agent prefers a conservative portfolio. The increase in background risk lowers the aversion to risk, and the agent is willing to take gambles that are rewarded in expectation, a behavior similar to the second choice in the Allais paradox. However, increased elasticity of terminal consumption with respect to financial wealth and lower perceived premium together reduce the longing for risky investments as the background riskiness continues to increase. This behavior is similar to that of an agent with CRRA preferences.

The effects of changing background risk on the perceived standard deviation of the risky returns are negligible. The perceived correlation between labor income and returns is negligibly positive, although no correlation exists under the data generating distribution.

# 3 Model

## 3.1 Wealth and Asset Returns

The savings $A_t$ are transformed into future tradable wealth according to Eq. 3.1. The benefit of saving is the portfolio return $R_{p,t+1}$, which depends on the chosen portfolio weight $x_t$ and the stochastic excess return $R_{e,t+1}$ on the risky asset, and a known risk-free return $R_f$. The term $W_t$ that I refer to as the tradable wealth is also known as cash-on-hand in the life-cycle portfolio allocation literature (following Deaton (1991)). Also, I refer to savings $A_t = (W_t - C_t)$

as the financial wealth, as it is the wealth available for investing in financial instruments. $C_t$ is the consumption at time $t$. In addition to returns on savings, the agent also receives exogenous non-tradable labor income $Y_{t+1}$ in every period:

$$\begin{aligned} W_{t+1} &= A_t R_{p,t+1} + Y_{t+1}, \text{ Where, } A_t = W_t - C_t \\ R_{p,t+1} &= R_{e,t+1} x_t + R_f \\ R_{e,t+1} &= R_{t+1} - R_f. \end{aligned} \tag{3.1}$$

The agent is constrained from borrowing or selling the risky assets short. I place two constraints on the risky asset weight. The first constraint of positive weight on the risky asset ensures that the agent does not short the risky asset and invest the proceeds at the risk-free rate. The other constraint that the weight be less than one ensures that the agent does not borrow at the risk-free rate to invest in the risky asset:

$$C_t < W_t, \quad x_t \geq 0, \quad x_t \leq 1. \tag{3.2}$$

The risky asset at the agent's disposal is a value-weighted market index (henceforth also referred to as stock). The raw returns on the market index follow a log-normal distribution with a standard deviation of $\sigma_r$ and an expected rate of return $\overline{R}$ per period (Eq. 3.3). The risk-free asset (henceforth also referred to as bond) return is equivalent to a treasury bill return. I assume that the investor faces a constant investment opportunity set:

$$\ln(R_t) = \ln(\overline{R}) + \eta_t \quad \text{Where } \eta_t \sim N(-\sigma_r^2/2, \sigma_r^2). \tag{3.3}$$

I consider the cases in which labor income may or may not be correlated with the stock returns.

## 3.2 Labor Income

The labor income has a deterministic component $l_t \equiv l(t,\ Z_t)$, dependent on the age and other personal characteristics $Z_t$. Eq. 3.4 describes the labor income process until retirement.

$$\begin{aligned} Y_t &= \exp(l_t + \nu_t + \epsilon_t) \qquad \forall\, t \leq K \\ \nu_t &= \nu_{t-1} + u_t \quad \text{Where } u_t \sim N(0, \sigma_u^2) \quad \epsilon_t \sim N(0, \sigma_\epsilon^2). \end{aligned} \tag{3.4}$$

In addition to the deterministic trend $l_t$, labor income is also determined by a permanent component $\nu_t$ driven by the shocks $u_t$ and an idiosyncratic component $\epsilon_t$. The two shocks distributed as $N(0, \sigma_u^2)$ and $N(0, \sigma_\epsilon^2)$ are uncorrelated. The permanent component $\nu_t$ is modeled as a unit root process following Carroll (1997) and Gourinchas and Parker (2002). The transitory shock $\epsilon_t$ when combined across the agents (say a cohort) averages to zero in the cross-section, as it is uncorrelated across the agents. Further, the transitory shock is also uncorrelated with the random component of the stock returns $\eta_t$. However, I decompose the permanent shock $u_t$ into aggregate component $\zeta_t$ and an idiosyncratic shock $\omega_t$ uncorrelated across agents, $u_t = \zeta_t + \omega_t$. I consider the possibility of correlation between the permanent income shock $u_t$ and the unexpected return shock $\eta_t$ via the correlation between $\zeta_t$ and $\eta_t$:

$$u_t = \zeta_t + \omega_t. \tag{3.5}$$

The above decomposition implies that the unexpected component of aggregate labor income is a random walk and conforms to the assumptions in Jagannathan and Wang (1996) and Fama and Schwert (1977).

The retirement income is a fixed fraction of the permanent income in the last working period before retirement. This specification avoids an additional state variable and simplifies the solution to the model. I do not model the flexibility in labor supply.[14]

$$\begin{aligned} \ln(Y_t) &= \ln(\lambda) + l_K + \nu_K; \quad \forall K+1 \le t \le T \\ \Rightarrow Y_t &= \lambda \exp(l_K + \nu_K), \quad K = \text{the last period agent works} \end{aligned} \tag{3.6}$$

## 3.3 Preferences

The preference specification follows Epstein and Zin (2001). I set the reciprocal of the second-order risk aversion parameter[15] equal to the elasticity of intertemporal substitution. The preference over the temporal and intertemporal gambles is summarized by the value function $J$, which depends on time $t$, agent's wealth $W_t$, and the level of his/her permanent income $\nu_t$:

$$\frac{J_t(W_t,\nu_t)^{1-\gamma}}{1-\gamma} = \max_{C_t,x_t} \frac{C_t^{1-\gamma}}{1-\gamma} + p_t\beta \frac{\mu_t(J_{t+1}(W_{t+1},\nu_{t+1}))^{1-\gamma}}{1-\gamma} \tag{3.7a}$$

$$\begin{aligned} \mu_t^{1-\gamma} &= E_t\left[J_{t+1}(W_{t+1},\nu_{t+1})^{1-\gamma}\right] \\ &\quad -\theta \underset{J_{t+1}<\mu_t}{E_t} \left[\mu_t^{1-\gamma} - J_{t+1}(W_{t+1},\nu_{t+1})^{1-\gamma}\right]. \end{aligned} \tag{3.7b}$$

At time $t$, the investor faces the trade-off between current consumption $C_t$ and the utility gain from savings $A_t$ as measured by the future value function $J_{t+1}$. DA preferences summarize the investor's attitude towards uncertainty in $J_{t+1}$ via the implicit formula Eq. 3.7b. The

[14] See Bodie, Merton, and Samuelson (1992), Gomes, Kotlikoff, and Viceira (2008), Chan and Viceira (2000), and Farhi and Panageas (2007) and references therein for models with flexible labor supply:

[15] In the absence of disappointment aversion, $\gamma$ produces only second-order risk aversion.

certainty equivalent (CE), $\mu_t$, of the future value function gamble incorporates overweighting[16] the outcomes that are below the certainty equivalent, yielding an implicit equation. $\beta$ captures the time rate of preference. $\gamma$ captures the reciprocal of the elasticity of intertemporal substitution as well as most of the risk attitudes due to second-order risk aversion for atemporal gambles. $p_t$ represents the probability of surviving to the next period, $t+1$, conditional on having survived up to period $t$. $p_t$ captures the mortality risk by changing the time rate of preference and thus effectively changes the planning horizon.

## 3.4 Solution Technique

The household's problem, summarized in Eqn. 3.7a, contains three state variables, the time $t$, wealth $W_t$, permanent income $\nu_t$, and two control variables $C_t$ and $x_t$. I scale the variables by the permanent income and define

$$W_t^\nu = W_t e^{-\nu_t},\ C_t^\nu = C_t e^{-\nu_t},\ Y_t^\nu = Y_t e^{-\nu_t} = e^{g_t+\epsilon_t}.$$

The scaling reduces the number of state variables to two, $W_t^\nu$ and $t$, and the control variables are reduced to a function of only these two variables; that is, $C_t^\nu \equiv C_t^\nu(W_t^\nu, t)$ and $x_t \equiv x_t(W_t^\nu, t)$. The budget equation with these scaled variables is

$$W_{t+1}^\nu = (W_t^\nu - C_t^\nu)\left[R_{e,t+1}x_t + R_f\right]\exp(-u_{t+1}) + Y_{t+1}^\nu.$$

The value function with the scaled variables is[17]

$$\frac{J_t^\nu (W_t^\nu)^{1-\gamma}}{1-\gamma} = \max_{C_t^\nu, x_t} \frac{(C_t^\nu)^{1-\gamma}}{1-\gamma} + p_t\beta \frac{\mu_t(\exp(u_{t+1})J_{t+1}^\nu(W_{t+1}^\nu))^{1-\gamma}}{1-\gamma}.$$

The constraints are also rescaled to $C_t^\nu \leq W_t^\nu$ and $1 \geq x_t^\nu \geq 0$. The rescaling significantly reduces the computational time. I recover the original unscaled variables in the simulations by keeping track of the permanent level and multiplying the scaled variables by $\exp(\nu_t)$.

The first-order conditions for the household's problem are similar to those in the CRRA case aside from the distribution used for computing expectations. The expectation in the case of DA preferences is computed under the distorted data-generating process. The distortion overweights disappointing outcomes, that is, the outcomes below the certainty equivalent.

Eqn. 3.8 characterizes the optimal portfolio weight $x_t^*$ at time $t$. The term multiplying

[16] I only consider cases with $\theta \geq 0$. DA agents are risk averse in the sense of weakly not preferring the mean-preserving spreads iff $\theta \geq 0$ and $\gamma > 0$ (Gul (1991)).

[17] The certainty equivalent under the DA preferences is scalable.

$R^*_{e,t+1}$ is proportional to the marginal utility of savings for the chosen portfolio weight.[18]

$$
\begin{aligned}
0 &= E_t\left[\left[e^{u_{t+1}}C^{\nu,*}_{t+1}\right]^{-\gamma}\left[R_{e,t+1}\right]\left(1+\theta I(e^{u_{t+1}}J^{\nu}_{t+1}<\mu^*_t)\right)\right] \\
0 &= \hat{E}_t\left[\left[e^{u_{t+1}}C^{\nu,*}_{t+1}\right]^{-\gamma}R^*_{e,t+1}\right]
\end{aligned}
\tag{3.8}
$$

The optimal portfolio weight sets the marginal valuation orthogonal to the excess returns, thus leaving no room for welfare improvement by simply reallocating the stock portion with risk-free holdings or vice versa:

$$
\begin{aligned}
(C^{\nu,*}_t)^{-\gamma} &= p_t\beta\frac{E_t\left[\left[e^{u_{t+1}}C^{\nu,*}_{t+1}\right]^{-\gamma}R^*_{p,t+1}\left(1+\theta I(e^{u_{t+1}}J^{\nu}_{t+1}<\mu^*_t)\right)\right]}{\left[1+\theta E_t\left[I(e^{u_{t+1}}J^{\nu}_{t+1}<\mu_t)\right]\right]} \\
(C^{\nu,*}_t)^{-\gamma} &= p_t\beta\times\hat{E}_t\left[\left[e^{u_{t+1}}C^{\nu,*}_{t+1}\right]^{-\gamma}R^*_{p,t+1}\right].
\end{aligned}
\tag{3.9}
$$

Eqn. 3.9 states the first-order condition for the optimal consumption. In case the borrowing constraint is active and the agent does not save for the future, the consumption equals wealth, that is, $C^{\nu}_t = W^{\nu}_t$. I derive the first order conditions in Appendix B.

The optimization program ends at age 100. The agent consumes all the remaining wealth at age 100 and has no savings. I numerically solve for the optimal policy rules for all other periods. I use the endogenous grid method of Carroll (2006), which involves using a grid on savings instead of a grid on wealth. I compute the integrals in the form of expectations using draws from the distribution of returns and the labor income. I use the equi-distributed Sobol sequences (Judd (1998)) to generate these draws[19]. Appendix A further details the numerical method.

# 4 Calibration

## 4.1 Preference Parameters and Mortality

I set the value of DA parameter $\theta$ to 1 and $\gamma$ to 5. Ang, Bekaert, and Liu (2005), in a portfolio-allocation model in a multiperiod setting without intermediate consumption or non-tradable income, consider $\theta$ ranging from 0.18 to 2 and $\gamma$ from 2 to 5. Choi, Fisman, Gale,

[18]The envelope condition helps substitute the marginal utility of savings with the future consumption. The following may help explain the way portfolio weight affects the expectation: for every choice of portfolio weight $x_t$, the budget equation yields a distribution of wealth at time $t+1$ that maps to the optimal consumption at time $t+1$ and a certainty equivalent $\mu^*_t$ and thus obtains all quantities in the expectation.

[19]This is a quasi-Monte Carlo method (Judd (1998)) and I based the choice on the convergence merits of the quasi-Monte Carlo methods. I also computed the policy rules using the pseudo-random number generator and did not find any significant difference.

and Kariv (2007) find experimental evidence for DA preferences in a laboratory setting, but estimate parameters based on static gamble payoffs only. Their estimates for $\theta$ range from 0 to 1.8. [20]

The CRRA preference model is nested within the DA model, which allows me to compare the optimal consumption and portfolio allocation plans generated by the two preference models. For comparison purposes, I set the relative risk aversion $\gamma$ for the CRRA preferences at 9. These two preferences are not equivalent, as the CRRA model implies little risk aversion over small gambles unless $\gamma$ is extremely high. However, a high $\gamma$ implies unrealistic risk aversion over large gambles. I compare the risk premia implied by DA and CRRA preferences for a simple log-normal gamble. I choose the parameter $\gamma$ for the CRRA preference so that the risk premia the two preferences imply (the parameters for DA preference are set at $\theta = 1$ and $\gamma = 5$) for simple log-normal gamble match approximately. I base this match on the size of the gamble the agent faces early in the young adult life.[21] See Appendix C for details.

The agent faces mortality risk from age 66 till age 100 and dies with probability 1 at age 100. I obtain the estimates for $p_t$ from National Center for Health Statistics, (2003)[22]. The adult age starts at 20 for agents without college degrees and at age 22 for those with college degrees. The agents retire at age 65 irrespective of their educational attainment.

## 4.2 Labor Income and Asset Returns

The estimates for the labor income process in Eqn. 3.4 are from Cocco, Gomes, and Maenhout (2005). They use PSID data to obtain these estimates. I treat the labor income as exogenous. The labor income besides the reported income also includes unemployment compensation, workers compensation, social security, supplemental social security, other welfare, child support, and total transfers. The labor income combines these items for both the head of the household and the spouse. Cocco, Gomes, and Maenhout (2005) provide estimates[23] for three groups according to their educational attainment (Figure 4.1): (1) no high-school (2) high-school graduates (but no college degree), and (3) college degree. The labor income

[20] These values are the $5^{th}$ and the $95^{th}$ percentiles.

[21] I first compute the actual volatility of the log of the value function gambles under DA preferences from simulation. I use this volatility to choose the $\gamma$ parameter for CRRA preference such that the agent demands the same risk premium under the two preferences for a simple log-normal gamble with the same volatility. I further constrain myself to choosing a whole number for the $\gamma$ parameter.

[22] Link: ftp://ftp.cdc.gov/pub/Health_Statistics/NCHS/Publications/NVSR/54_14/. $p_t$ is the probability of surviving to period $t+1$, conditional on having survived up to period $t$. The link above provides the mortality probabilities $1 - p_t$.

[23] Their estimation controls for fixed household effect, marital status, and household size. The estimation, however, lacks controls for occupation, as the unemployed and those not participating in the labor force are categorized together in PSID dataset (1975 onwards). Thus, including occupation controls will have an undesirable consequence of modeling unemployment, a significant source of risk, as a switch in occupation.

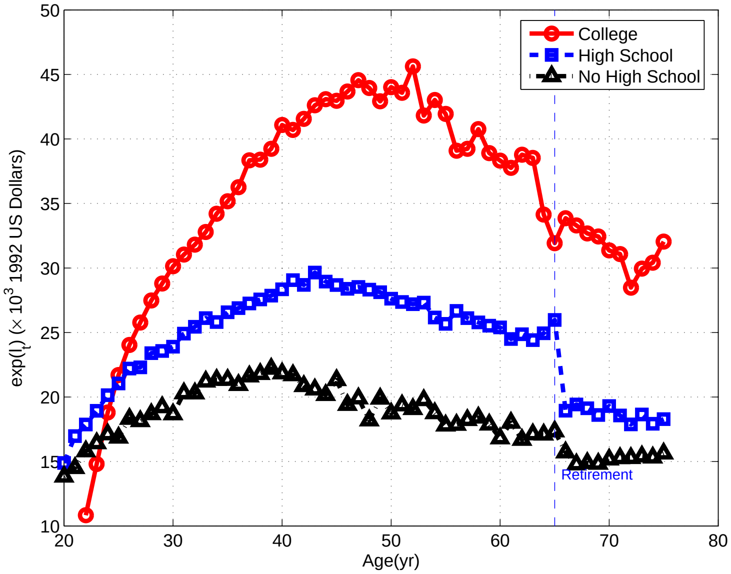


Figure 4.1: Labor income implied by the profile $l_t$ over the life for different education groups. Source: Cocco, Gomes, and Maenhout (2005).

profiles are different across these education groups.[24] I fit a third order polynomial to the estimated profile for numerical implementation.

The estimates for the replacement ratio $\lambda$ (ratio of retirement income to permanent income before retirement)[25] and the idiosyncratic and the permanent shocks[26] for the three groups are in Table 4.1. Purcell (2009) notes that social security and pension income comprised 60% of the income for the elderly above the age of 65 in 2008. The contribution of social security and pension income was as high as 93% for the bottom quartile of the elderly. The estimates for the variance of idiosyncratic shocks vary widely across studies, the lowest being 0.15 for college graduates and 0.18 for non-college graduates.[27] I compute the optimal portfolio rules and the consumption policies using both the low estimates and the estimates in Table 4.1. The differences in estimates do not change any of the conclusions.

Campbell, Cocco, Gomes, and Maenhout (1999) estimate the correlation between the per-

[24]The difference in income-age profiles by educational attainment is consistent with the evidence in Attanasio (1995) and Hubbard, Skinner, and Zeldes (1995).

[25]$\lambda$ is estimated as the ratio of the average labor income for retirees in an education group to the average labor income in the last working period before retirement.

[26]The procedure is similar to variance decomposition method in Carroll and Samwick (1997).

[27]Kolusheva (2009) combines the high-school and non-high-school graduates. The estimates are based on data from PSID for the period 1977 to 2005. She also obtains similar age-labor income profile over the life and similar replacement ratios.

| | No High School | High School | College |
|---|---|---|---|
| $\sigma_\epsilon$ | 0.3250 | 0.2717 | 0.2417 |
| $\sigma_u$ | 0.1025 | 0.1030 | 0.1300 |
| $\lambda$ | 0.8898 | 0.6821 | 0.9388 |

Table 4.1: The standard deviation of idiosyncratic and permanent shocks for the labor income process from the variance estimates in Cocco, Gomes, and Maenhout (2005). The authors note that the variance estimates are statistically significant at a less than 1% significance level. The last row includes the replacement ratio for the three education groups.

manent component of labor income and the returns.[28] They set the correlation between the idiosyncratic component and the returns to zero. Their estimates imply a correlation of 0.15 between the permanent component and the returns for the high-school graduates. Heaton and Lucas (2000a) and Davis and Willen (2002) do not separate the permanent and idiosyncratic components. Their estimates range from -0.07 to 0.14 and -0.25 to 0.30, respectively. Viceira (2001) explores the correlation between the permanent component and the returns over the range of 0 to 0.25. I consider the cases with and without the correlation between the permanent component and the returns.

Barro and Ursua (2008) estimate the asset returns based on data going as far back as 1870. They estimate the yearly real returns on U.S. stocks at 8.27%, with a standard deviation of 18.66% and the real t-bill returns at 1.99%. I set the real rate of return for the risk-free asset at 2% and equity premium at 6%. I set the standard deviation of stock returns at 18%.[29]

# 5 Benchmark Case

I analyze the case for high-school graduates and keep the correlation between stock returns and either of the labor income shocks at zero. Table 5.1 lists parameters for this case.

## 5.1 Policy rules and simulated portfolio shares

The optimal policy rules for the share of savings to invest in risky assets around retirement are in Figure 5.1. The rules are a function of the scaled tradable wealth or cash-on-hand (scaled by

[28] Campbell, Cocco, Gomes, and Maenhout (1999) estimate the correlation between the aggregate component part of the permanent shocks and the returns ($\rho_{\zeta\eta}$, in Eqn. 3.5). The values are between 0.33 to 0.52, with the higher value for the higher-education group of college graduates. These values suggest a correlation of 0.15 between the permanent shock and the returns.

[29] The real returns on the assets are lower after considering holding costs and taxes. Jagannathan, McGrattan, and Scherbina (2000) argue that the equity premium is much lower than 6% due to diversification costs, taxes, and liquidity premium for bills.

| | |
|---|---|
| $\gamma$ | 5 or 9 |
| $\theta$ | 1 and 0 |
| $\beta$ | 0.96 |
| Age at the last non-retirement income, $K$ | 65 |
| | |
| Correlation of labor income shocks with stock returns, | 0 |
| Std. deviation of transitory labor shocks, $\sigma_\epsilon$ | 0.27 |
| Std. deviation of permanent labor shocks, $\sigma_\nu$ | 0.10 |
| Std. deviation of log stock returns, $\sigma_r$ | 0.18 |
| Average excess returns, $(\overline{R} - R_f)$ | 6% |
| Risk free rate, $(R_f - 1)$ | 2% |

Table 5.1: Parameter values for the benchmark case.

permanent income). I obtain a pattern of decreasing allocation to stocks with the increasing cash-on-hand. The decrease in the risky allocation is due to the decrease in the capitalized non-tradable labor income or the retirement income (post-retirement) relative to the tradable wealth. Retirement income is certain, resulting in a substitute non-tradable risk-free asset in the agent's portfolio. Agents with both DA and CRRA preferences offset this implicit risk-free position by investing more in stocks. However, the borrowing constraints are binding at low enough levels of tradable wealth. The constraints prevent the agent from investing more than his savings in the risky assets, which limits the portfolio share to at most 100%. The increase in the tradable wealth lowers the relative share of the implicit non-tradable risk-free asset, resulting in an increasingly conservative portfolio. The conservativness with the decreasing share of the implicit risk-free asset gives rise to the declining pattern in Figure 5.1. The portfolio rules asymptote to the all-tradable-wealth case as the tradable wealth increases.

Figure 5.1 shows the investment policy rules turn increasingly conservative for both preferences as the agent gets older. The conservativeness is attributable to the drop in the capitalized value of the future retirement income as the investor ages. The drop in capitalized retirement income results in a decreasing share of this implicit risk-free asset in the total wealth for a given level of the tradable wealth. Thus the optimal portfolio for a given tradable wealth grows conservative with age.

The most dramatic difference between the policy rules for the two preferences is around retirement, when the policy rule for DA preferences becomes incredibly conservative whereas the policy rule under CRRA preferences changes very little. The source of this disparity is in the resolution of uncertainty about the post-retirement income and the overweighting of disappointing states associated with the DA preferences. Age 65 is the last period the agent spends in the labor market and he enters this period with the realization of the permanent income for the period, which also sets his retirement income (retirement income is a fraction

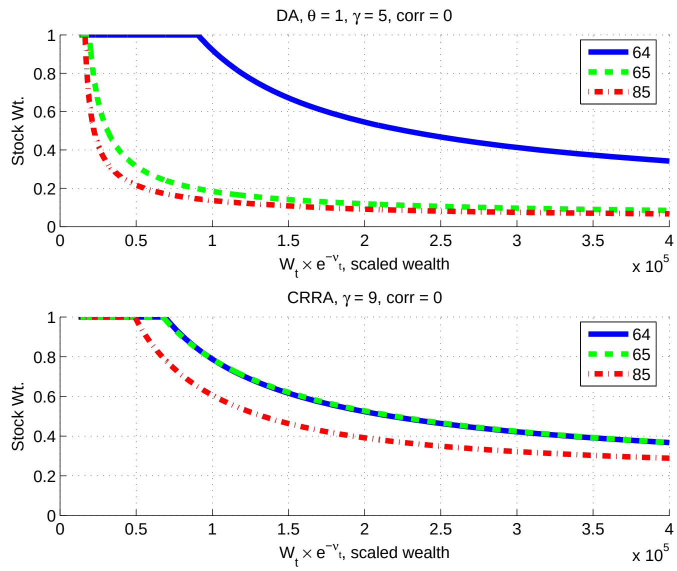


Figure 5.1: The optimal portfolio policy rules as a function of wealth scaled by the permanent income around retirement and late old age for DA and CRRA preferences.

of the permanent income in the last working period before retirement). Thus the portfolio allocation from age 65 and onwards is predicated on no remaining risk in the non-tradable income going forward. This drop in uncertainty is sizeable, as it resolves the uncertainty on the entire stream of retirement income, which has a large present value. The future value function without any uncertainty (setting stock investments are zero), a sure alternative, is also one of the choices the agent faces while choosing his optimal risky investment. The decision problem the agent faces is akin to the first choice in the Allais paradox, where the agent prefers the sure alternative over the risky gamble. The drop in non-tradable risk exogenously sets the alternative choices available to the agent. The agent demands a larger premium for holding the same level of risk, just as in the first choice of the Allais paradox. Since the risk premium on the risky asset is fixed whereas the agent's tolerance for risk has fallen, he optimally lowers his portfolio risk by reducing the fraction of his wealth invested in the risky asset.

Large relative size (or the total wealth share) of the implicit risk-free capitalized retirement income mitigates the drive to lower stock allocation. The agent offsets this large implicit risk-free position by investing more in stocks. Thus although the exogenous drop in non-tradable income uncertainty drives the dramatic drop in the risky allocation, the varying importance of non-tradable capitalized retirement income compared to the tradable wealth gives the downward-sloping shape to the portfolio rule (Figure 5.1). I address the lingering question of whether the DA preference model can drive down the risky allocation to zero in Section 6.2.

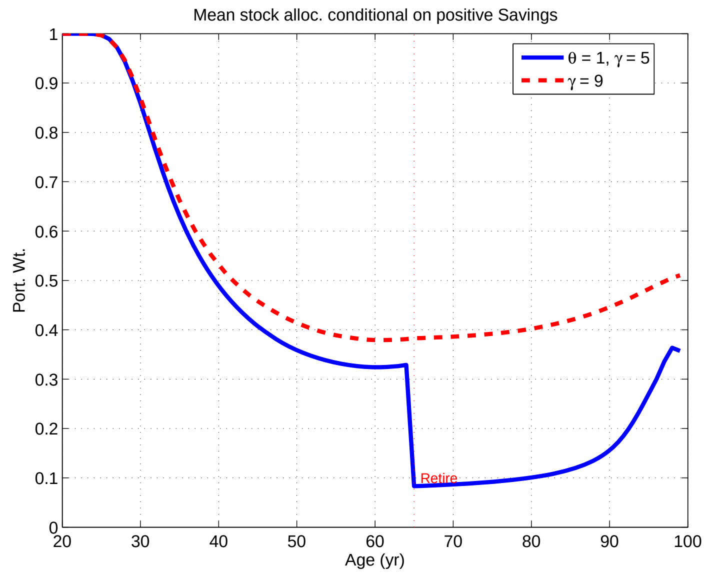


Figure 5.2: The cross-sectional average of the fraction of savings invested in risky assets at all ages over the life of the agents with CRRA and DA preferences.

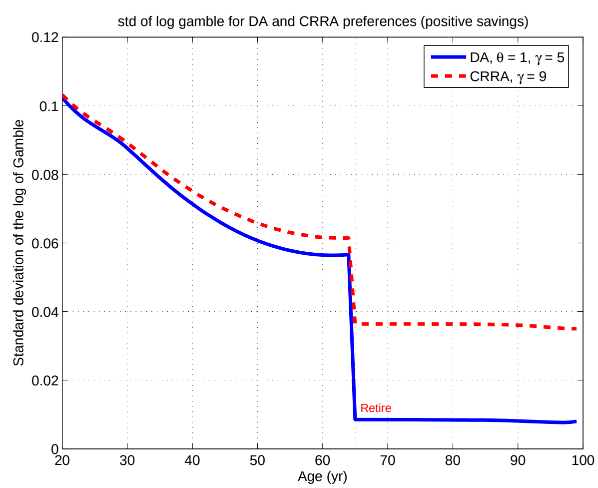


Figure 5.3: The future gamble outcomes at time $t$ with optimal portfolio weight $x^*_{t+1}$ and consumption $C^{*,\nu}_{t+1}$ are given by $e^{u_{t+1}} J^{\nu}_{t+1}(W^{*,\nu}_{t+1})$. The plot is the cross-sectional average of the standard deviation of the log of this future gamble the investor faces. Further, the average is conditional on positive savings.

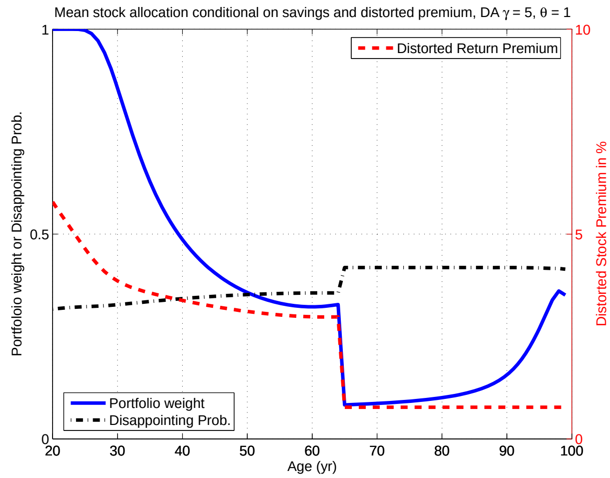


Figure 5.4: The cross-sectional averages of the portfolio weights, disappointment probability and the distorted stock return premium at all ages through life for DA agent.

I plot the mean stock allocation at every age as the fraction of savings (hence conditional on positive savings) invested in stock over the life of the agent in Figure 5.2. I obtain these values via simulation of 10,000 identical individuals that start off their lives at age 20.[30] The overly conservative approach toward stock investing around retirement is noticeable in the form of a significant drop in the mean portfolio allocation at age 65. The average portfolio allocation to stocks for the two preferences track each other closely until early adult life. This match in portfolio allocation is reflective of the choice of $\gamma$ parameter for the CRRA preference such that the demanded risk premia match given the uncertainty at young adult age. The volatility of the value function gambles the investor faces under the two preferences are the same over the young adult age (Figure 5.3). The drop in the capitalized value of the future labor income drives the drop in the stock allocation as the agents age. This conservative approach with the increasing age and the lower residual risk due to fewer remaining uncertain labor flows reduce the volatility of the log of the gambles. The drop in the uncertainty of the scaled gambles drives the two preferences apart as the agent navigates mid-adult life. The substantial drop in uncertainty around retirement amplifies this wedge in the risk attitudes and results in a drop in stock allocation for the DA agent.

The simplified representation of retirement with an abrupt drop in uncertainty causes the sharp decline in stock investments around retirement. The differences in retirement age across the agents and progressive decline in income uncertainty can help match the observed gradual

[30]The agents begin with wealth, $W_{20}$ set at $\exp(l_{20})$ and the permanent level of income at zero; that is, $\nu_t = 0$. The conclusions are unchanged and the profiles hardly change if I instead draw the initial wealth from the distribution of the labor income at age 20; that is, $W_{20} = e^{l_{20}+\nu_{20}+\epsilon_{20}}$, where $\nu_{20} \sim N(0, \sigma_u^2)$ and $\epsilon_{20} \sim N(0, \sigma_\epsilon^2)$.

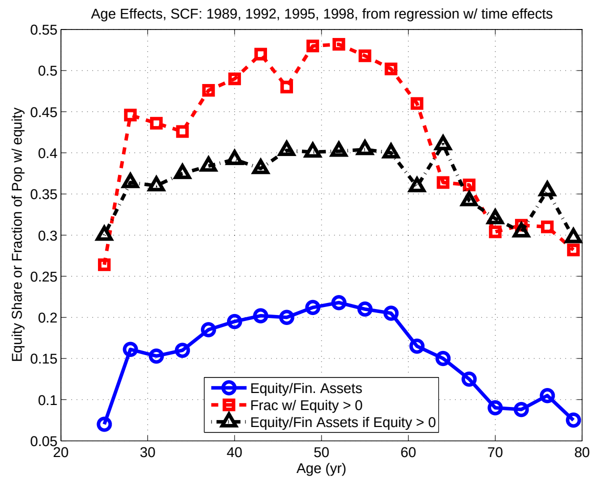


Figure 5.5: Three different measures of age effects on stock investments over life from Ameriks and Zeldes (2002). The three measures are (1) equity as a fraction of savings, (2) fraction of population with positive equity holdings, and (3) equity as a fraction of savings conditional on positive equity holdings. The plots are for the age dummies from a probit regression that includes time dummies (but excludes cohort dummies). The data are from 1989, 1992, 1995, and 1998 SCFs.

drop in the portfolio share of stocks in the data (Figure 5.5). I pursue the case of declining uncertainty prior to retirement in Section 6.3.

The increase in the stock investment in old age as the agent nears age 100 is characteristic of a majority of life-cycle models without bequest or some other form of risk (e.g. health risk) in old age. Two opposing effects determine the stock investment in old age. The first effect is mortality. The agent, anticipating certain mortality at age 100 and increased impatience due to uncertain mortality, decumulates savings. The drop in savings, the tradable wealth, increases the relative share of non-tradable but risk-free retirement income in the total wealth. This effect encourages the agent to invest in stocks. The second effect is the drop in capitalized labor income due to fewer remaining periods (age 100 is the last period) as the agent transits through old age. This second effect discourages stock investments. The first effect dominates and props up the stock share of savings for most of the remaining life. The second effect dominates for a few short periods before age 100.

The drive to decumulate the savings is also seen in the consumption policy rules in old age (Figure 5.6). The consumption policy rules are concave in tradable wealth and the pattern indicates the agent aggressively decumulates savings in retirement. The agent faces increasing mortality risk as the conditional probability of survival to the next period $p_t$ continues to drop. This drop in survival probability makes the agent increasingly impatient. Thus the consumption level for a given amount of tradable wealth rises as the agent approaches age

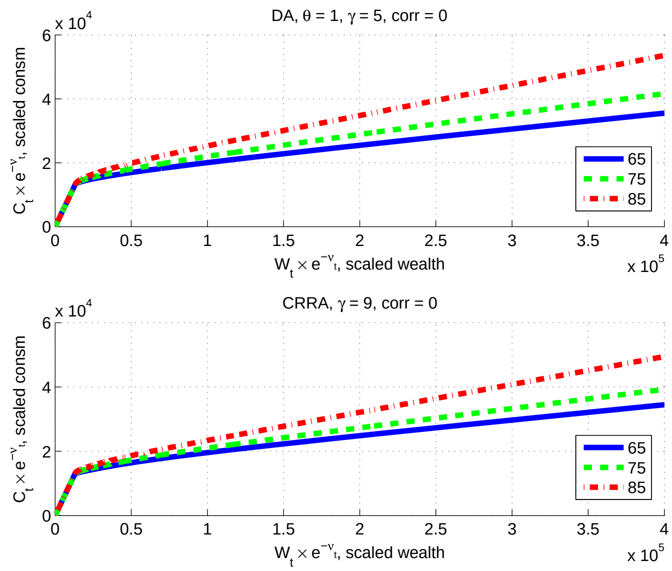


Figure 5.6: The optimal consumption policy rules as a function of wealth scaled by the permanent income around retirement and late old age for DA and CRRA preferences. Consumption on y-axis is scaled by the permanent income.

100, which we see in the fanning out pattern of the consumption policy rules in old age. The consumption policy rules also suggest the DA agent decumulates savings faster than the agent with the CRRA preference. This is due to the lower expected benefit from savings as the optimal stock allocation for the DA agent is lower, thereby leading to a greater incentive to decumulate rather than save.

The consumption rules for both the CRRA and DA preferences move out in early adult life as the permanent income rises with age and peaks in the mid- to late thirties due to the hump shape in earnings.[31] I include the consumption policy rules in early and mid-adult life in Figure I.2 in the appendix. Cocco, Gomes, and Maenhout (2005) find a small drop in the stock allocation in early young age for CRRA preferences. The portfolio policy rules reveal an analogous pattern. The policy rules are conservative in early young age and turn aggressive until the mid- to late thirties. The authors note that this pattern is due to the hump shape in the earnings profile (Figure 4.1). The capitalized value of the labor income or the human capital keeps rising in early young age and peaks around the mid- to late thirties.[32] The portfolio rules in early and mid-adult life are in Figure I.1 in the appendix.

[31]The consumption rules are almost identical over the downward-sloping part of the earning profile. The earning profile drops but the attractive risk premium also encourages the agent to save. The effective outcome in late mid-adult life is dependent on the parameter choice.

[32]I recover the slight hump-shaped pattern in Cocco, Gomes, and Maenhout (2005), with the parameters therein for the CRRA preferences.

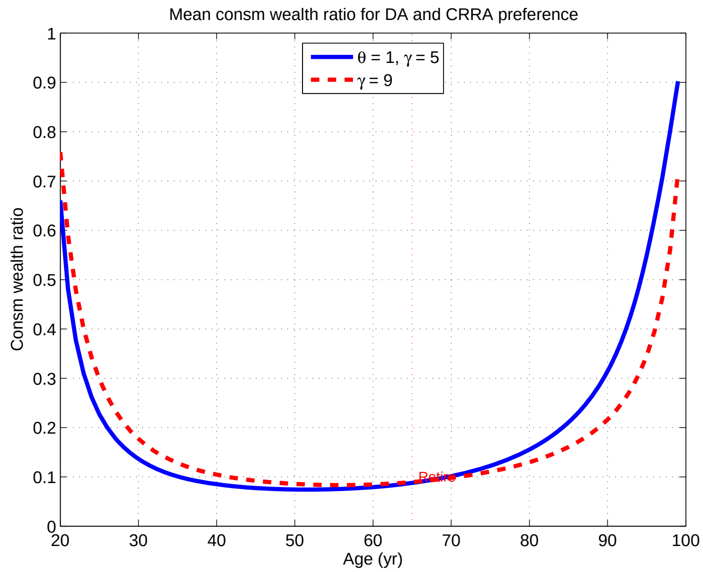


Figure 5.7: The cross-sectional mean of consumption-wealth ratio at every age over the life of the agents with CRRA and DA preferences.

## 5.2 Savings and consumption from simulations

I compare the consumption and savings decisions for the two preferences using consumption-wealth ratios. The cross-sectional averages of the ratios at every age over the life are in Figure 5.7. The consumption-wealth ratio refers to ratio of consumption to tradable wealth throughout the article.

The agent with DA preference saves more until retirement but decumulates wealth faster after retirement. The DA agent expects lower earnings from investments in old age due to lower stock allocations after retirement. Thus he must save more for retirement and hence accumulates greater wealth until retirement. The lower stock investments until retirement also drive the greater accumulation. The DA agent's conservative investing strategy implies that he must save more in order to smooth consumption. This greater savings rate produces greater accumulated wealth for the DA agent prior to retirement. The decumulation in retirement is due to mortality risk as noted above. However, the faster rate for the DA agent is the result of the agent's low stock allocation in retirement. The lower expected income from savings encourages the DA agent to dissave faster. The plot of the cross-sectional averages of savings at every age over life (Figure 5.8) reflects the pattern observed in the consumption-wealth ratio.

I plot the cross-sectional average consumption at all ages in Figure 5.9. The consumption profile for the DA agent is uneven compared to the agent with the CRRA preference. This uneven pattern is another manifestation of the DA agent's drive to use sav-

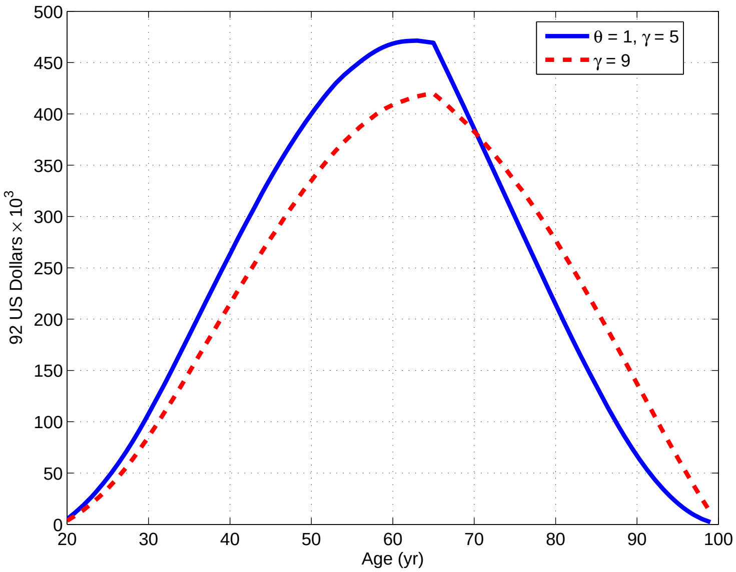


Figure 5.8: The cross-sectional mean savings at every age over the life of the agents with CRRA and DA preferences.

ings and risk-free asset rather than the attractive stock investment to smooth consumption. The peak in consumption at retirement is consistent with the observed drop in consumption after retirement in the data. Ameriks, Caplin, and Leahy (2007), Attanasio (1999), Bernheim, Skinner, and Weinberg (2001), Hurd and Rohwedder (2005) among others consider agents' aparent inability to use savings to smooth out the effects of the predictable drop in income around retirement to be a puzzle. Since reducing stock allocation and investing more in the risk-free asset is a less efficient (though optimal for the DA agent) way to smooth consumption, the DA agent requires greater savings to achieve any given level of consumption smoothing. This higher "cost" of smoothing and aversion to risky investments post retirement leads to the more uneven pattern we see for the DA agent compared with the CRRA agent.

I also compare the risk attitudes under the two preferences by comparing the constant consumption stream the two agents will willingly accept in place of the labor income and the stock return gamble. The DA agent's greater aversion to risk due to inherent extra weight on disappointing states suggests he would be indifferent at a low level of constant consumption stream. In fact, the DA agent will trade away the gamble for a constant consumption stream that is 15% below that for the CRRA agent (Table 5.2). I detail the method used to compute the constant consumption equivalent in Appendix G.

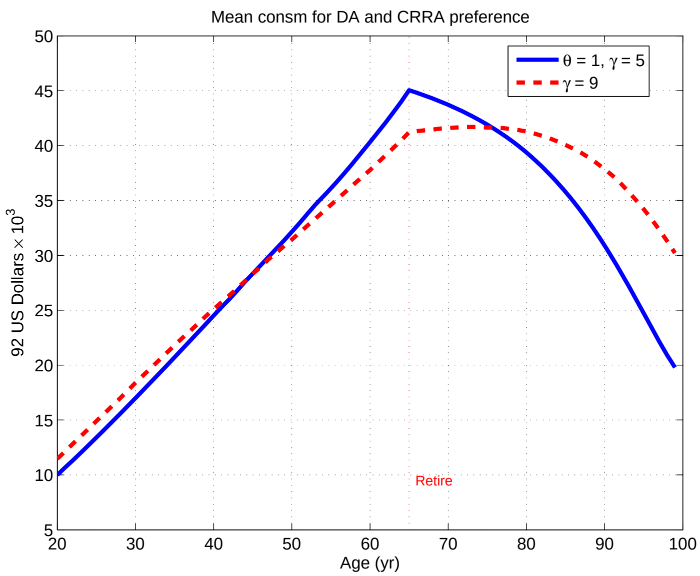


Figure 5.9: The cross-sectional mean consumption at every age over the life of the agents with CRRA and DA preferences.

| | |
|---|---|
| ΔPeak Accumulated Savings | 14% |
| ΔEquivalent Constant Consumption | -15% |

Table 5.2: Comparison of savings and welfare for the CRRA and DA preferences. The change in savings and the equivalent constant consumption stream is relative to the CRRA preference.

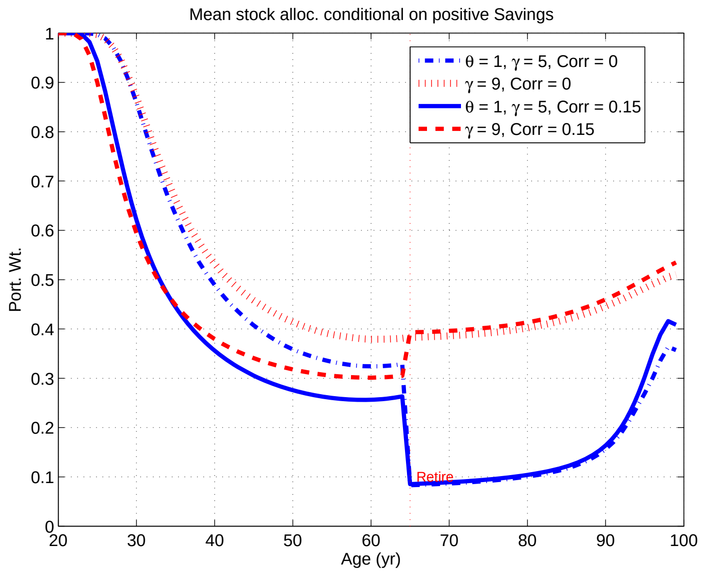


Figure 6.1: The cross-sectional average of the fraction of savings invested in risky assets at all ages over the life of the agents with CRRA and DA preferences. The plots are for the correlation between the permanent income and the returns set at 0.15 and zero.

# 6 Model Extensions

## 6.1 Labor Income Correlated with Stock Market Returns

Cocco, Gomes, and Maenhout (2005) find that correlated labor income helps explain lower stock investments in early young life. They also find that the differences in the correlation of income generate significant heterogeneity in stock allocation. Benzoni, Collin-Dufresne, and Goldstein (2007) explore a setup in which the aggregate component of labor income is co-integrated with the dividend on the market index, and find that the young agents do not participate in the stock market. I consider the effect of correlation between the labor income and returns on the optimal stock allocation strategy for the DA preferences. I follow Campbell, Cocco, Gomes, and Maenhout (1999) and set the correlation between the permanent component of labor income and the stock returns at 0.15. The idiosyncratic shocks to income are uncorrelated with the returns.

Figure 6.1 plots the cross-sectional averages of stock allocation conditional on positive savings at all ages throughout life. The correlation reduces the resemblance of human capital to a risk-free asset. Increased hedging demands reduce the portfolio allocation for both the DA and the CRRA agent. The drop in the uncertainty around retirement, however, has the same effect for the DA agent. The stock allocation is substantially curtailed once the uncertainty surrounding the non-tradable income is resolved. The agent with the CRRA preferences, however, finds the stocks attractive once labor income correlated with the stocks reduces to a non-tradable bond. The hedging demands cease to exist after retirement.

The average stock allocation after retirement is higher than in the case with no-correlation for both the DA and CRRA preferences. This is due to lower savings when the income is correlated with stock returns. The correlated labor income lowers the benefit to savings, and thus the accumulated saving at retirement is lower. However, the lower level of tradable wealth translates into higher share of the capitalized retirement income (which is risk-free after retirement) in the total wealth. This rise in the relative share of the implicit risk-free asset in the total wealth drives the investor more towards stocks. This effect is present both for the DA and the CRRA preferences. Still, the DA agent substantially reduces the stock investments once the uncertainty in the background risk vanishes.

The incentive to increase stock share of savings after retirement also exists in other models of life-cycle portfolio allocation. Gomes and Michaelides (2005) explore a life-cycle model with entry costs, bequests, and Epstein-Zin preferences in the presence of correlated income. Gomes and Michaelides (2003) present results for correlated income with ratio habit preferences. The drop in hedging demands and the lower savings for the case of correlated labor income result in the increase of stock share of savings around retirement in these settings as well.

## 6.2 Heterogeneity in Disappointment Aversion

The households exhibit considerable heterogeneity in their portfolio allocation (Curcuru, Heaton, Lucas, and Moore (2004)). The most notable variation is their choice to participate in the stock market. The 2007 Survey of Consumer Finances finds that either directly or indirectly, only 51% of the U.S. population participates in the stock market. The heterogeneity is either sourced through the differences in environment or the preferences. I examine the effects of variation in disappointment aversion on the optimal portfolio allocation.

I consider the benchmark case of zero correlation between the labor income and stock returns and examine the effects of changing the degree of disappointment aversion ($\theta$). I consider three cases, $\theta = \{0.5,\ 1,\ 1.5\}$, which are within the range of estimates observed in experiments (Choi, Fisman, Gale, and Kariv (2007)). The cross-sectional averages of the optimal stock allocations are in Figure 6.2.

An increase in $\theta$ increases the weight on the disappointing states and thus lowers the risky asset allocation. Table 6.1 summarizes the drop in stock allocation and increasing aversion to risk. The increase in $\theta$ from 0.5 to 1.5 drops the stock allocation by about half during mid-adult life and until retirement. The stock investments are, however, substantial at a young age although somewhat lower for higher $\theta$. The driver behind the stock investments at young age is the large outstanding labor stock that resembles a risk-free asset more than a stock index. This large stock along with the attractive risk premium, continues to encourage high stock investment at young age.

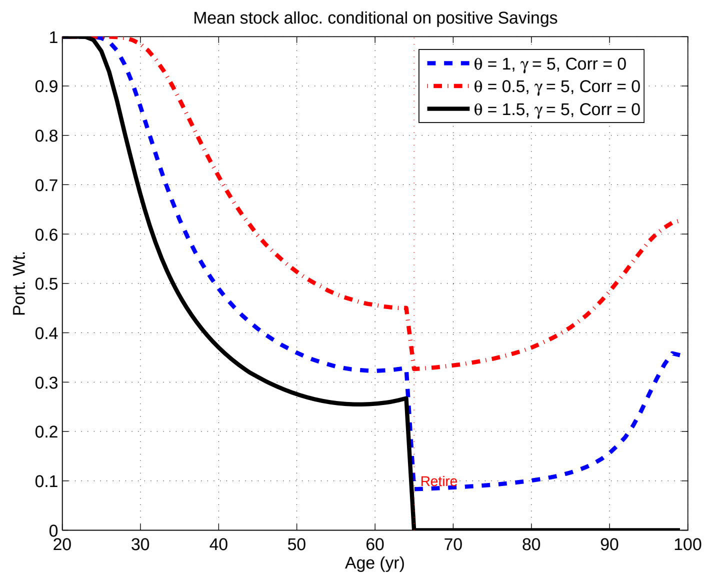


Figure 6.2: The cross-sectional average of the fraction of savings invested in risky assets at all ages over the life of the agents. The plots are for DA preferences with different values for $\theta$.

| $\theta$ | 0.5 | 1.5 | 1.0 |
|---|---|---|---|
| $\Delta$Peak accumulated savings | -6% | 1% | Baseline |
| $\Delta$Average post-retirement savings | 9% | -6% | Baseline |
| $\Delta$Average pre-retirement savings | -12% | 5% | Baseline |
| $\Delta$Equivalent constant consumption | 21% | -13% | Baseline |
| Average stock fraction over age 36 - 50 | 68% | 36% | 47% |
| Average stock fraction over age 51 - 65 | 48% | 26% | 33% |

Table 6.1: Comparison of savings, welfare, and the stock share in the portfolio for varying disappointment aversion ($\theta$). The difference in savings and the equivalent constant consumption stream is relative to the DA preference with $\theta = 0$.

The overall lower stock allocation with the increasing $\theta$ forces the agent to use the risk-free returns to smooth consumption. The lower return on risk-free asset (relative to average returns on stock) implies that the agent must use a higher level of savings to achieve the same degree of consumption smoothing. I tabulate the comparison relative to the benchmark case of $\theta = 1$ in Table 6.1. The peak accumulated savings are higher by 1% for $\theta = 1.5$ and lower by 6% for $\theta = 0.5$. The average savings before retirement follow a similar pattern although this pattern flips after the retirement. The agent facing mortality risk decumulates savings. The lower stock investments due to higher$\theta$ reduce the expected benefits from savings and thus inducing the agent to decumulate the savings even faster. The stock investments are not only lower for $\theta = 1.5$ but drop to zero after retirement. The stronger first-order risk aversion ramps up the aversion to small gambles to such an extent that even the risk reward trade-off of an 18% standard deviation and a 6% risk-premium is not attractive enough and the agent refrains from investing in stocks. In fact, as long as the average excess stock returns are positive, an agent with $\theta$ greater than an approximate threshold $\hat{\theta^*}$ does not participate in the stock market after retirement. The approximate threshold depends solely on the parameters of the return distribution. The proof is in Appendix E.

$$\hat{\theta^*} = -\frac{E_t\left[R_{e,t+1}\right]}{E_t\left[R_{e,t+1}|R_{e,t+1} < 0\right] \times \Phi(R_{e,t+1} < 0)}$$

Ang, Bekaert, and Liu (2005) obtain the same threshold for non-participation in a one-period model without the non-tradable wealth. The threshold however, approximately, characterizes non-participation after retirement for the life-cycle problem.

The above described effect of $\theta$ on savings behavior is also mirrored in the consumption-wealth ratio. Figure 6.3 plots the average of the consumption to wealth ratios from simulations at every age over the life. The increase in $\theta$ increases the savings until retirement and hence a lower consumption-wealth ratio. The increasing consumption-wealth ratio after retirement with the increasing $\theta$ echoes the more rapid decumulation pattern of savings in retirement. The plots for simulated savings and consumption are in figures I.4 and I.3 in the appendix.

Heterogeneity in disappointment aversion leads to heterogeneity in savings for retirement and stock investments before and after retirement. A value of $\theta$ above the threshold generates non-participation in the stock markets in old age. This result is in contrast to CRRA preferences, which produce at least some investment in stocks since the premium on risky assets is positive. The other explanations for the drop in risky holdings in old age are based on the changes in environment with age or a preference for bequest. The bequest motive deters the agent from exhausting all of his savings and thus restrains the agent from increasing his stock share of his savings. Health risks are another reason to be cautious with investments in old age (Yogo (2009) and references therein). Love and Smith (2009) use the Health and Retirement Study to address the causal effect of health on portfolio choice. They find that after controlling for unobserved heterogeneity, health does not significantly affect portfolio choice among single households. There is, however, a small (2-3 percentage points) effect for

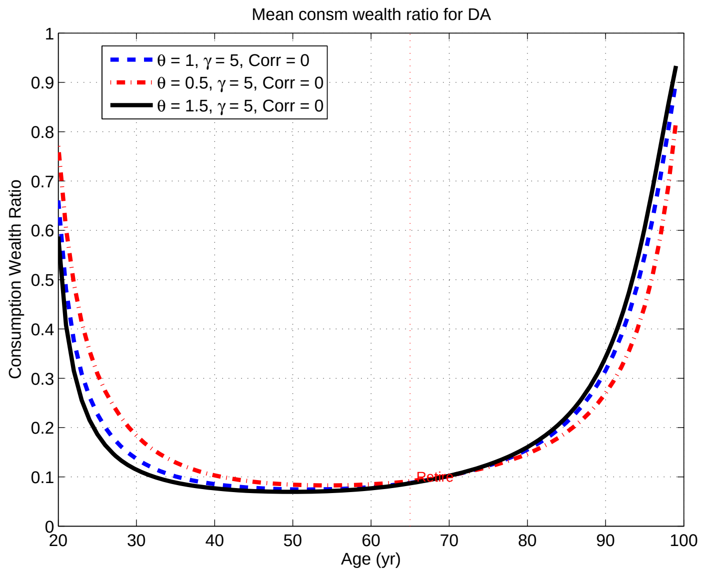


Figure 6.3: The cross-sectional average of the consumption-wealth ratio at all ages over the life of the agents. The plots are for DA preferences with differing values for $\theta$.

married households and for those in the lowest health categories. The lower flexibility of labor supply in old age is also another rationale for conservative investment.

## 6.3 Tapered Drop in Income Uncertainty

The benchmark case represents retirement as an abrupt drop in uncertainty. This drop in uncertainty is associated with a sharp decline in stock investments around retirement. I consider a variation of the benchmark case that can help match the observed gradual drop in the portfolio share of stocks in the data. I consider a progressive decline in income uncertainty as the agent nears retirement.

I taper down the size of labor income shocks over the ages 61 through 65. The agent retires at age 65. The magnitude of the shocks at age 61,62,63,64, and 65 relative to the magnitude at age 60 are 85%, 65%, 45%, 25%, and 5%, respectively. The consumption profile remains qualitatively similar to that in the benchmark case. I plot the cross-sectional averages of risky allocations at every age over the life in Figure 6.4. The plot includes results for both the benchmark case and the one with the tapered down labor income risk. The shrinking uncertainty is associated with the gradually diminishing stock investment as the agent nears retirement. Thus in contrast to the benchmark labor income process a declining uncertainty prior to retirement produces a better match to the observed drop in portfolio allocation around the age of retirement.

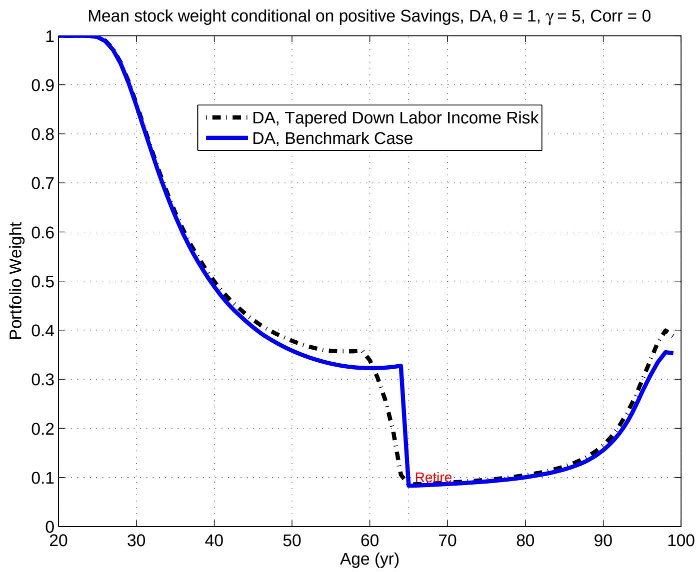


Figure 6.4: The cross-sectional average of the fraction of savings invested in risky assets at all ages over the life of the agents. The plots are for DA preferences with the same preference parameters but differing pattern of labor income risk over the age of 61 through 65.

## 6.4 Defined Contribution

I consider the effects of changes in labor income process on the portfolio allocation. In particular I analyze the portfolio allocation in the absence of pension, social security or other non-tradable source of income in retirement. The labor income through working-life is the same as that in the bechmark case. The agent, however, does not receive any retirement income. These modifications create a simplified representation of labor income under defined contribution.

I plot the cross sectional averages of stock allocation conditional on positive savings in Figure 6.5. In contrast to the case with risk-free retirement income the agents reduce stock allocation more rapidly as they approach retirement. The retirement income creates a non-tradable risk-free holding. In the absence of this implicit risk-free asset the agents rapidly lower the risky holdings as the human capital declines.

The risky share of the savings after retirement is almost the same as that prior to retirement for CRRA preferences. The risky share is slightly higher prior to retirement due to remaining human capital that is more akin to a risk-free asset than the stock index. The risky share, however, significantly drops around retirement for DA preferences. The agent may completely withdraw from the stock market if he is sufficiently disappointment averse. The portfolio weights after retirement are similar to the ones under the complete-market case. Thus the agent with the CRRA preferences and finite relative risk aversion does not completely

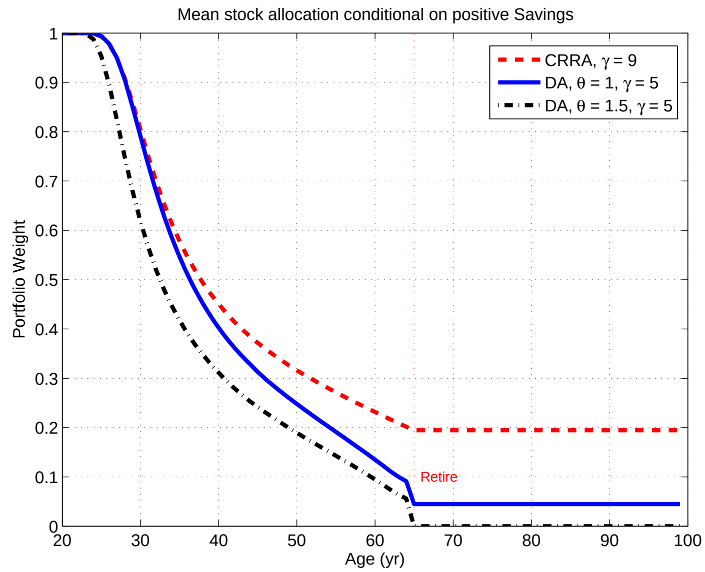


Figure 6.5: The cross-sectional average of the fraction of savings invested in risky assets at all ages over the life of the agents. The plots are for DA preferences with different values for $\theta$, including $\theta = 0$ case which corresponds to CRRA preference.

withdraw from the stock market[33].

The drop in stock investment around retirement for DA preferences is moderate compared to the benchmark case. The retirement income in the benchmark case is a fraction of the permanent income prior to the retirement. The unceratinty resolved around retirement significantly affects the welfare after retirement. However in the present case the agent does not receive any retirement income. Thus, as the risky labor income vanishes, limited uncertainty resolved around retirement results in a smaller drop in the risky asset holdings. $\hat{\theta}^*$, the approximation for the critical disappointment aversion parameter in retirement for the benchmark case, $\theta^*$, is in fact accurate in the present case. The agents with the disappointment aversion parameter $\theta \geq \theta^*$ choose not to participate in the stock market in retirement. The critical value solely depends on the distribution of excess returns (see Appendix F).

$$\theta^* = -\frac{E_t\left[R_{e,t+1}\right]}{E_t\left[R_{e,t+1}|R_{e,t+1} < 0\right] \times \Phi(R_{e,t+1} < 0)}$$

I plot the cross-sectional averages of consumption at every age over the life in Figure 6.6. The consumption for the DA agent peaks at retirement, whereas it continues to rise into the retirement for the CRRA agent. Thus the consumption profile qualitatively still remains the same as that in the benchmark case, but the retirement consumption puzzle is

[33]Presuming positive equity premium.

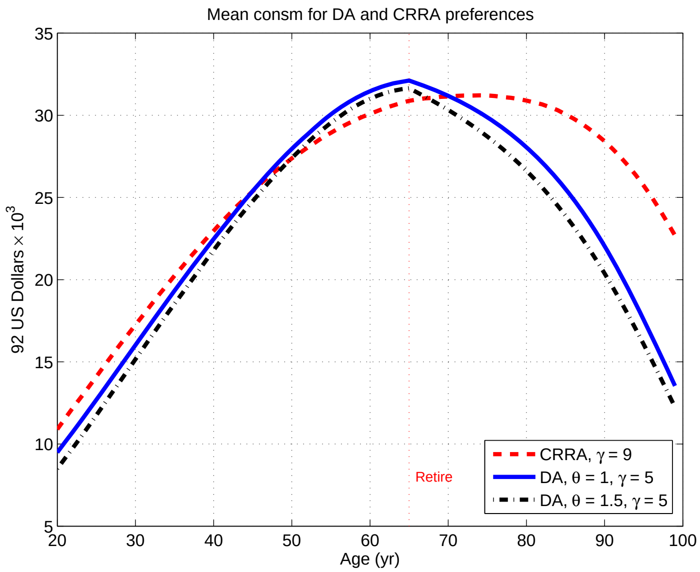


Figure 6.6: The cross-sectional mean of the consumption at every age over the life of the agents. The plots are for CRRA and DA preferences with varying values for $\theta$.

stronger for the CRRA agent. The consumption profiles are, however, smoother compared to the benchmark case. This is due to the absent non-tradable risk-free retirement income. The retirement income in the benchmark case in part ensures furture consumption and hence the agent rapidly draws down on the savings creating an uneven profile. In the absence of a certain source of income the agent consumes the savings less rapidly.

## 6.5 Stock Crashes and Depressions

Lynch and Tan (2004) explore the effects of the pro-cyclical variation in labor income growth and the counter-cyclical variation in income volatility on stock investments for agents with CRRA preferences. They find that this variation helps explain the low stock investments of the young and especially those with little wealth. In the following, I explore the effects of increased labor growth uncertainty in the troughs of the business cycle using stock market crashes and disastrous draws for the permanent income.

Barro and Ursua (2009) estimate the probabilities of stock market crashes and macroeconomic depressions. They find that the stock-market crashes occur more often than the depressions. The crashes have an associated probability of 22% for a minor depression and a 3% probability of a major depression[34]. Storesletten, Telmer, and Yaron (2004), using PSID

[34]The minor depressions are multiyear declines of consumption or GDP of 10% or more in magnitude. The declines of 25% or more in consumption or GDP constitute major depressions. Stock market crashes are

| | |
|---|---|
| Correlation of labor income shocks with stock returns, | 0.15 |
| Probability of Depression, | 3.8% |
| Size of permanent shock in depression, | $-2 \times \sigma_u$ |
| Probability of stock crash, | 5.8% |
| Size of crash, | -41% |

Table 6.2: The parameter values for the case with stock market crashes and macroeconomic depressions. The values in the table are the additional parameters or the parameters that are different from the benchmark case in Table 5.1.

data, find that the standard deviation of the shock to permanent log labor income increases by about 75% as the economy rides over the peak to the trough[35]. I model the effect of depression on labor income as a large negative permanent income shock. I set the magnitude of the shock to be twice the standard deviation of the shock in normal times, $-2 \times \sigma_u$. Following Barro and Ursua (2009), I set the depression probability at 3.8% . However, I restrict focus solely on the U.S. stock market crashes to obtain an estimate of the crash probability and the size of the crash. I list the crashes and returns in Table I.1 in the appendix. The restriction to the U.S. markets yields a lower estimate for the probability of crash at 5.8% and the size of the crash at 41%. I list the parameters in Table 6.2.

The optimal stock investment rules as a function of the scaled tradable wealth before retirement are in Figure 6.7. The portfolio rules indicate the agents do not invest any of their savings in stocks at low levels of tradable wealth. The greater resemblence of the capitalized labor income to the stock index than to the risk-free asset is the reason for the aversion to stocks. The permanent component of labor income has a correlation of 0.15 with stock returns in the absence of disasters. Labor income subject to disastrous shocks that correlate with crashes in the stock market further reduce the attractiveness of stock investments. The proportion of labor income in total wealth is higher at low levels of tradable wealth. Precautionary needs restrict the agent from investing in stocks at low levels of wealth. Risky investments rise with wealth and asymptote to the all-tradable-wealth solution. The all-tradable-wealth solution for CRRA preferences is a positive investment in stocks given the positive premium on stocks. The optimal investment in stocks for DA preferences, however, asymptote toward zero investment. The difference in investment strategy persists into retirement. The optimal decision under the DA preference is not to participate in the stock market at all. CRRA preferences, however, yield positive investment in stocks. The optimal investment is in fact higher than the all-tradable-wealth solution when labor income is not negligible compared to the tradable wealth.

The above differences between the DA and the CRRA preferences map into the simulated

characterized by multi-year real returns of -25% or less.

[35]The standard deviation increases from 0.12 to 0.21.

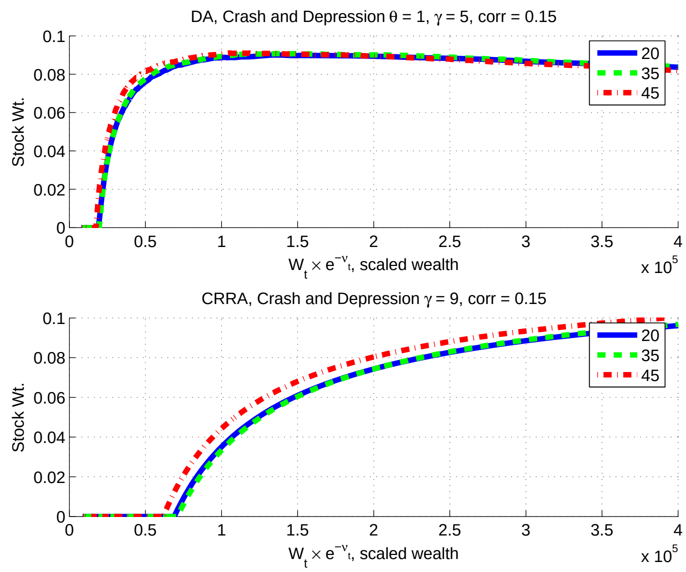


Figure 6.7: The stock investment rules as a function of wealth scaled by the permanent income before retirement for DA and CRRA preferences.

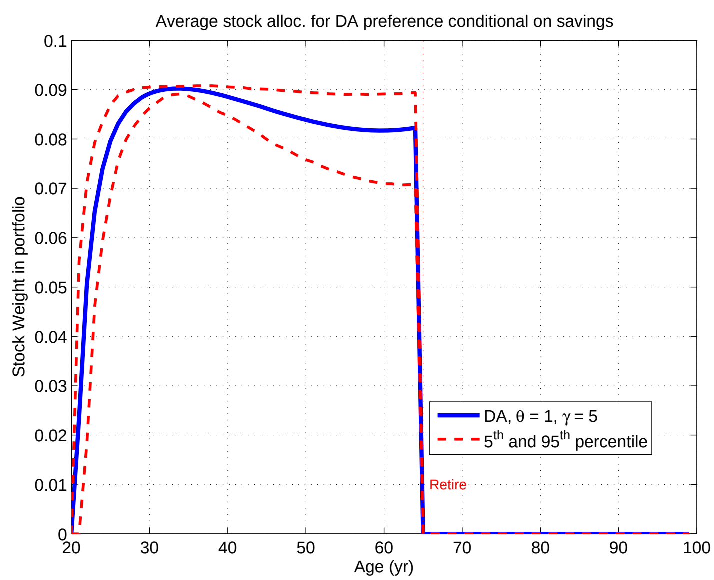


Figure 6.8: The cross-sectional average of the fraction of savings invested in stocks at all ages over the life of the agent with DA preferences. The fractions are computed conditional on positive savings.

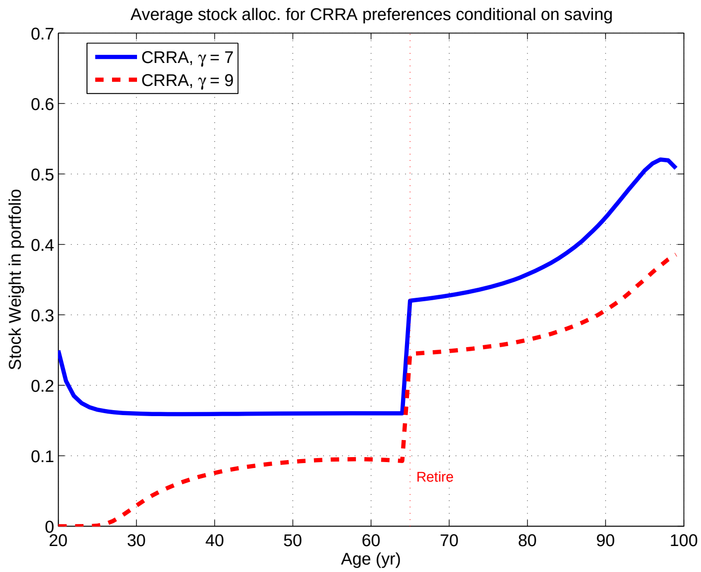


Figure 6.9: The cross-sectional average of the fraction of savings invested in stocks at all ages over the life of the agents with CRRA preferences. The fractions are computed conditional on positive savings.

stock investments over life in figures 6.8 and 6.9. Low financial wealth at young age restrict stock investments due to precautionary reasons as stocks and the capitalized labor appear quite similar. Accumulated savings, however, increase the investment in stocks in mid-adult life. Still, CRRA and DA preferences generate very different investment strategies after retirement. Agents with CRRA preferences favor stock participation on account of the positive risk premium. The DA agent, given the risk of disastrous stock returns, prefers to stay out of the market. Thus DA preferences in the presence of market crashes and disastrous shocks to permanent income help match lower investments in stocks at young and old age at the same time. Differences in retirement age across agents in the economy can help reconcile the discrete drop in average risky allocation suggested by the simulation with the gradual drop observed in the data.

## 6.6 Bequest

I expand upon the benchmark case by adding the bequest motive to the preferences[36]. In the absence of a bequest motive the agents prefer to run down the assets due to uncertain mortality and higher risky allocation in the old age for the benchmark labor income process. The bequest motive involves preferences over the gambles in the event of death and the

[36] See Nardi (2004) for evidence on the motive to bequeath wealth.

utility gained from passing the savings to future generations. I use the following preference formulation that includes bequest.

$$
\begin{aligned}
\frac{J_t(W_t,\nu_t)^{1-\gamma}}{1-\gamma} &= \max_{C_t,x_t} \frac{C_t^{1-\gamma}}{1-\gamma} + p_t\beta\frac{\mu_t(J_{t+1}(W_{t+1},\nu_{t+1}))^{1-\gamma}}{1-\gamma} && \text{(6.1a)}\\
&\quad +(1-p_t)\beta b\frac{\mu_{b,t}(W_{t+1}/b)^{1-\gamma}}{1-\gamma} \\
\mu_t^{1-\gamma} &= E_t\left[J_{t+1}(W_{t+1},\nu_{t+1})^{1-\gamma}\right] && \text{(6.1b)}\\
&\quad -\theta \underset{J_{t+1}<\mu_t}{E_t}\left[\mu_t^{1-\gamma} - J_{t+1}(W_{t+1},\nu_{t+1})^{1-\gamma}\right] \\
\mu_{b,t}^{1-\gamma} &= E_t\left[(W_{t+1}/b)^{1-\gamma}\right] && \text{(6.1c)}\\
&\quad -\theta \underset{\frac{W_{t+1}}{b}<\mu_{b,t}}{E_t}\left[\mu_{b,t}^{1-\gamma} - (W_{t+1}/b)^{1-\gamma}\right].
\end{aligned}
$$

The agent lives until age 100 and the terminal valuation due solely to bequest is encoded as,

$$
\frac{J_{T+1}(W_t,\nu_t)^{1-\gamma}}{1-\gamma} = b\frac{(W_{T+1}/b)^{1-\gamma}}{1-\gamma}.
$$

The intensity of bequest motive is parametrized in $b$. I retain all other parameters from the benchmark case. Gomes and Michaelides (2005) find that a value of 2.5 for the bequest motive, $b$ with Epstein-Zin (EZ) preference specification is a better match for the wealth distribution. However, EZ preferences generate high savings and risky allocation over the old age. The DA preferences, on account of lower risky allocation in old age, generate lower savings in old age. Hence, I use a higher value for the bequest motive so that the accumulated savings may better match the wealth distribution observed in the data.

I plot the average portfolio allocation at every age in Figure 6.11, average savings in Figure 6.10 and average consumption in Figure 6.12. The agent, instead of running down the savings in old age for his own consumption, now trades-off against the motive to leave inheritance to heirs. The bequest motive also raises the savings over the working life. The overall effect of the preference to bequeath wealth to future generations is to increase savings. The effect is prominently observable in old age (Figure 6.10). The increasing chances of mortality with age strengthen the motive to bequeath wealth. This increasing intensity of bequest results in a sharp rise in savings relative to the capitalized retirement income (not included).

The relatively higher proportion of the tradable wealth or the savings with the increasing age induces the agent to choose lower risky allocation in old age (Figure 6.11). The other features of stock investments in the benchmark case including the decline in stock investments around retirement are unchanged in the presence of bequest motive. The consumption profile qualitatively remains unchanged as well. The lower average benefits to savings in old age still

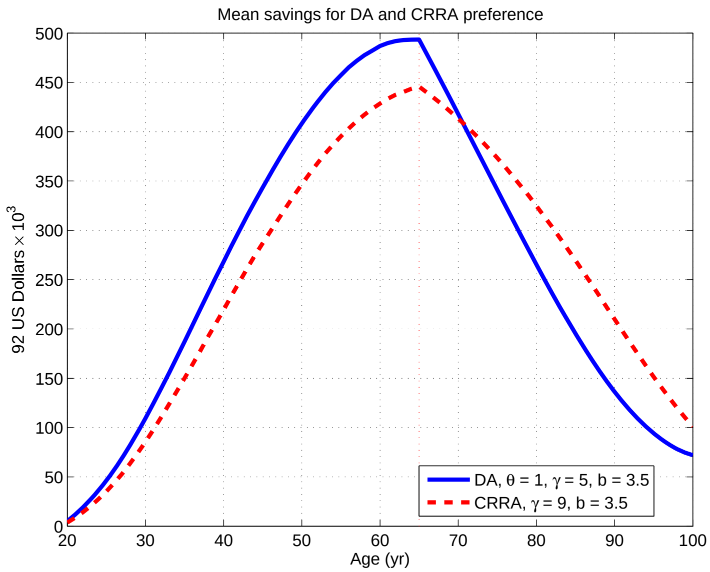


Figure 6.10: The cross-sectional mean savings at every age over the life of the agents with CRRA and DA preferences.

result in a consumption drop and an uneven profile around retirement for DA preferences. The consumption profile for the CRRA preference, on the other hand, is still smooth without a decline in the consumption around retirement.

# 7 Conclusion

This article provides a preference based explanation for the conservative investment strategy around retirement and of the elderly. Disappointment Aversion (DA) preferences motivated by the Allais paradox generate aversion to risky investments when the background uncertainty is below a threshold level. The background risk limits the alternative gambles available to the agent. The absence of background risk after retirement provides a sure alternative to the risky stock investment. The agent's behavior, consistent with the Allais paradox, is to choose a low risk portfolio and hence a lower stock allocation after retirement. The optimal choice when the agent is sufficiently disappointment averse is to not take up any risky investments at all. The preferences also imply greater savings for retirement and a drop-off in consumption post-retirement. DA preferences also generate an increased perceived correlation between the labor income and the stock returns.

DA preferences in the presence of disastrous returns and disastrous shocks to permanent labor income yield stock investment rules that are increasing in the wealth of the agent. Overweighting of the disappointing states and the disastrous permanent shocks to income

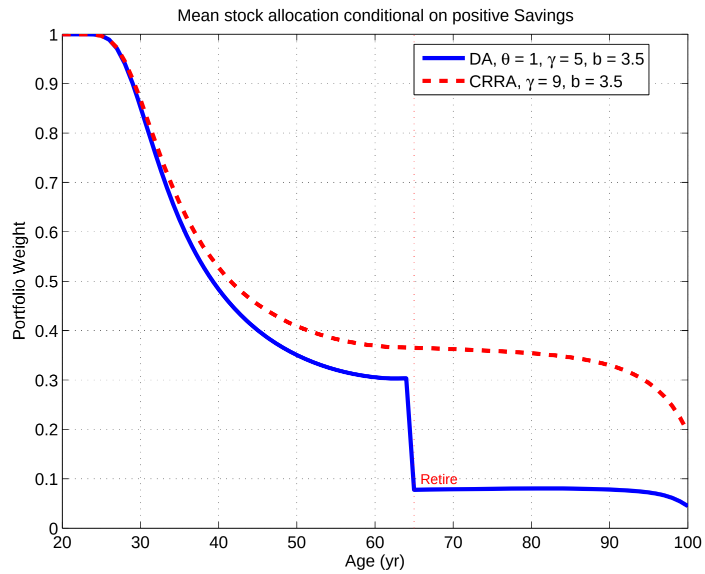


Figure 6.11: The cross-sectional average of the fraction of savings invested in risky assets at all ages over the life of the agents.

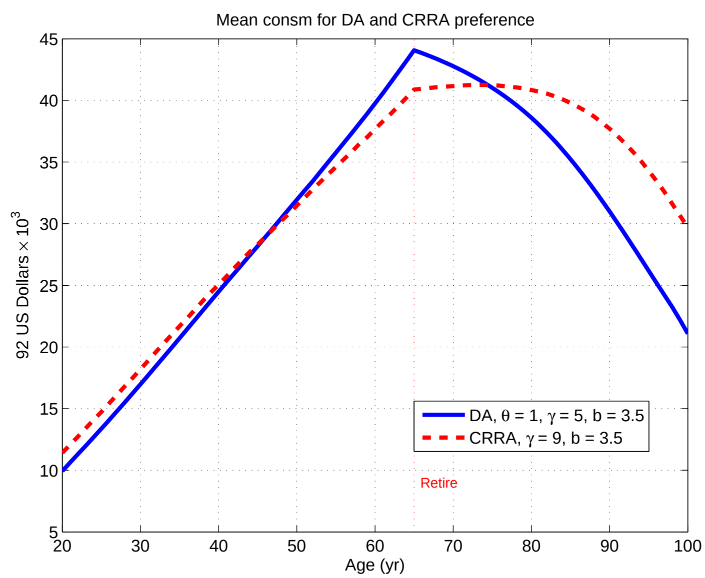


Figure 6.12: The cross-sectional mean of the consumption at every age over the life of the agents.

imply that the agent's human capital is more akin to a stock than a risk-free asset. Young agents with little savings and borrowing constraints choose not to increase investment in a correlated asset. DA preferences combined with environment of disastrous shocks yield a drop in risky investment at young age and a drop in investment after retirement without resorting to bequest motive or health risks.

# A Numerical Method

I restate the problem. The optimization program for any period $t$ is given by,

$$\frac{J_t^\nu (W_t^\nu)^{1-\gamma}}{1-\gamma} = \max_{C_t^\nu, x_t} \frac{(C_t^\nu)^{1-\gamma}}{1-\gamma} + p_t \beta \frac{\mu_t (\exp(u_{t+1}) J_{t+1}^\nu (W_{t+1}^\nu))^{1-\gamma}}{1-\gamma}.$$

The wealth evolves according to $W_{t+1}^\nu = (W_t^\nu - C_t^\nu)\left[R_{e,t+1} x_t + R_f\right] e^{-u_{t+1}} + Y_{t+1}^\nu$, with the constraints that $C_t^\nu \leq W_t^\nu$ and $1 \geq x_t \geq 0$. The certainty equivalent $\mu_t$ is,

$$\begin{aligned} \mu_t^{1-\gamma} &= E_t \left[ \left[ e^{u_{t+1}} J_{t+1}^\nu (W_{t+1}^\nu) \right]^{1-\gamma} \right] \\ &\quad - \theta \underset{e^{u_{t+1}} J_{t+1}^\nu < \mu_t}{E_t} \left[ \mu_t^{1-\gamma} - \left[ e^{u_{t+1}} J_{t+1}^\nu (W_{t+1}^\nu) \right]^{1-\gamma} \right]. \end{aligned}$$

The terminal valuation is $J_T^\nu(W_T^\nu) = W_T^\nu$. The agent consumes all the wealth $W_T^\nu$ at time $T$. Thus $C_T^\nu = W_T^\nu$ and the agent has no savings ($A_T^\nu = W_T^\nu - C_T^\nu$), and hence the portfolio problem for the terminal period (choosing optimal $x_T$) does not exist. I numerically solve for the optimal policy rules for all other periods. I use the endogenous grid method by Carroll (2006). Consider the period $T-1$. The certainty equivalent $\mu_{T-1}$ is,

$$\begin{aligned} \mu_{T-1}^{1-\gamma} &= E_{T-1} \left[ \left[ e^{u_T} W_T^\nu \right]^{1-\gamma} \right] \\ &\quad - \theta \underset{e^{u_T} W_T^\nu < \mu_{T-1}}{E_{T-1}} \left[ \mu_{T-1}^{1-\gamma} - \left[ e^{u_T} W_T^\nu \right]^{1-\gamma} \right]. \end{aligned}$$

The budget equation is $W_T^\nu = (W_{T-1}^\nu - C_{T-1}^\nu)\left[R_{e,T} x_{T-1} + R_f\right] e^{-u_T} + Y_T^\nu$, which I rewrite as $W_T^\nu = A_{T-1}^\nu \left[R_{e,T} x_{T-1} + R_f\right] e^{-u_T} + Y_T^\nu$. I choose a grid on the savings $A_{T-1}^\nu$. I solve for the optimal $x_{T-1}$ that maximizes $\mu_{T-1}$ for a given value of savings $A_{T-1}^\nu$.

I describe the approach for computing $\mu_{T-1}$ for a chosen $x_{T-1}$ and $A_{T-1}^\nu$. The gamble the investor faces is $G = e^{u_T} W_T^\nu$.

$$\begin{aligned} W_T^\nu &= A_{T-1}^\nu \left[R_{e,T} x_{T-1} + R_f\right] e^{-u_T} + Y_T^\nu \\ R_{e,T} &= R_T - R_f \\ Y_T^\nu &= \exp(g_T + \epsilon_T) \quad \epsilon_T \sim N(0, \sigma_\epsilon^2) \text{ and } u_T \sim N(0, \sigma_u^2) \\ R_T &= \overline{R} \exp(\eta_T) \quad \eta_T \sim N(\mu_r, \sigma_r^2), \quad \text{with} \quad \mu_r = -\sigma_r^2/2 \\ \Rightarrow W_T^\nu &\equiv W_T^\nu(u_T,\ \epsilon_T,\ \eta_T) \\ G &= e^{u_T} W_T^\nu \\ \Rightarrow G &\equiv G(u_T,\ \epsilon_T,\ \eta_T) \end{aligned}$$

The gamble $G$ is dependent on the three shocks: permanent labor income shock $u_T$, idiosyncratic labor income shock $\epsilon_T$, and return shock $\eta_T$. I approximate the continuous distribution of these shocks by a finite number of draws. I use the equi-distributed Sobol sequences (Judd (1998)) to generate these draws. I choose equi-distributed sequences over pseudo-random number generator sequences to achieve faster convergence of the computed integrals[37]. This method reduces the problem to computing, $\mu$, where $\mu$ is the fixed zero of $f(\mu) \equiv \mu^{1-\gamma} + \theta \underset{G<\mu}{E}\left[\mu^{1-\gamma} - G^{1-\gamma}\right] - E\left[G^{1-\gamma}\right]$. I use bisection algorithm to obtain the $\mu$.[38] This provides a method to compute $\mu_{T-1}(A^{\nu}_{T-1},\ x_{T-1})$ for a given portfolio weight and saving. I choose a search method to find the maximum of $\mu_{T-1}(x_{T-1})$ as a function of the weight $x_{T-1}$ in place of first order condition to obtain the optimal portfolio weight, $x^*_{T-1}(A^{\nu}_{T-1})$. The reason for the choice is that the first order condition requires computation of $\mu_{T-1}$ and also the computation of an additional integral in the form of expectation, which increases the computational cost. I use the golden section search with parabolic interpolation[39] to search for the optimal portfolio weight $x^*_{T-1}(A^{\nu}_{T-1})$ with the bounds $0 \leq x_{T-1} \leq 1$.

I use the above method to compute the optimal portfolio weight $x^*_{T-1}(A^{\nu}_{T-1})$ at every grid point of the savings. The optimal portfolio weight at zero saving is not defined and the certainty equivalent $\mu_{T-1}$ is a constant for zero saving as it solely depends on the exogenous labor income gamble. I use the first order condition in Appendix B to compute the optimal consumption, $C^{\nu,*}_{T-1}(A^{\nu}_{T-1})$. These values for given savings map to a specific value of wealth. Thus, $C^{\nu,*}_{T-1}(W^{\nu}_{T-1}) = C^{\nu,*}_{T-1}(A^{\nu}_{T-1}) + A^{\nu}_{T-1}$ and $x^*_{T-1}(W^{\nu}_{T-1}) = x^*_{T-1}(A^{\nu}_{T-1})$ provide the consumption and portfolio policy rules over a grid of wealth points.[40] The optimal consumption for zero saving is equal to wealth and the portfolio weight not defined.

The above method for computing the optimal portfolio weight and the optimal consumption is applicable for any time period $t$. However, other time periods require interpolation of consumption policy rules $C^{\nu,*}_{t+1}(W^{\nu}_{t+1})$ and the valuation $J^{\nu}_{t+1}(W^{\nu}_{t+1})$. I collect the optimal consumption values and the valuations for time period $t+1$ at all wealth grid points and use the linear interpolation to compute values at intermediate wealth points.

---

[37] I performed computations using $100 \times 10^3$ draws with both sequences and found no difference in the policy rules.

[38] The function, $f$ is monotonic in $\mu$ since $f'/(1-\gamma) > 0$. Thus the problem $f(\mu) = 0$ is amenable to a bisection algorithm. The two end points of the search are, $[G_{min},\ G_{max}]$. Note that, $f(G_{min}) \times f(G_{max}) < 0$, thus the solution to $f(\mu) = 0$, lies in $[G_{min},\ G_{max}]$. The starting guess I use, however, is not $(G_{min} + G_{max})/2$. The starting guess is $E\left[G^{1-\gamma}\right]^{1/(1-\gamma)}$, which is the certainty equivalent for the case of $\theta = 0$ or the CRRA preference, with same $\gamma$.

[39] This method is fast if the solution is interior, but quite slow for the corner solutions.

[40] I also verify that the value function $V^{\nu}_{T-1}(W^{\nu}_{T-1}) = \frac{J^{\nu}_{T-1}(W^{\nu}_{T-1})^{1-\gamma}}{1-\gamma}$, decreases by either increasing or decreasing the consumption level around $C^{\nu,*}_{T-1}(W^{\nu}_{T-1})$.

# B Optimal Consumption and Portfolio weights

The value function with the scaled variables is,

$$\frac{J_t^{\nu}(W_t^{\nu})^{1-\gamma}}{1-\gamma} = \max_{C_t^{\nu}, x_t} \frac{(C_t^{\nu})^{1-\gamma}}{1-\gamma} + p_t \beta \frac{\mu_t(\exp(u_{t+1}) J_{t+1}^{\nu}(W_{t+1}^{\nu}))^{1-\gamma}}{1-\gamma}.$$

Where, the scaled variables are,

$$W_t^{\nu} = W_t \exp(-\nu_t), \; C_t^{\nu} = C_t \exp(-\nu_t), \; Y_t^{\nu} = Y_t \exp(-\nu_t) = \exp(g_t + \epsilon_t).$$

The budget equation with the scaled variables is,

$$W_{t+1}^{\nu} = (W_t^{\nu} - C_t^{\nu}) \left[R_{e,t+1} x_t + R_f\right] \exp(-u_{t+1}) + Y_{t+1}^{\nu}.$$

## 2.1 Period T-1

The value function for the last but one period $T-1$ is,

$$\frac{J_{T-1}^{\nu}(W_{T-1}^{\nu})^{1-\gamma}}{1-\gamma} = \max_{C_t^{\nu}, x_t} \frac{\left(C_{T-1}^{\nu}\right)^{1-\gamma}}{1-\gamma} + p_{T-1} \beta \frac{\mu_{T-1}(e^{u_T} J_T^{\nu}(W_T^{\nu}))^{1-\gamma}}{1-\gamma}.$$

The terminal valuation is $J_T^{\nu}(W_T^{\nu}) = W_T^{\nu}$. The budget equation is,
$W_T^{\nu} = (W_{T-1}^{\nu} - C_{T-1}^{\nu}) \left[R_{e,T} x_{T-1} + R_f\right] e^{-u_T} + Y_T^{\nu}$,
and the constraints are $C_{T-1}^{\nu} \leq W_{T-1}^{\nu}$ and $1 \geq x_{T-1} \geq 0$.
The certainty equivalent $\mu_{T-1}$ is,

$$\begin{aligned} \mu_{T-1}^{1-\gamma} &= E_{T-1}\left[\left[e^{u_T} J_T^{\nu}(W_T^{\nu})\right]^{1-\gamma}\right] \\ &\quad -\theta \underset{e^{u_T} J_T^{\nu} < \mu_{T-1}}{E_{T-1}} \left[\mu_{T-1}^{1-\gamma} - \left[e^{u_T} J_T^{\nu}(W_T^{\nu})\right]^{1-\gamma}\right]. \end{aligned}$$

### 2.1.1 Optimal Portfolio Weight

I differentiate certainty equivalent $\mu_{T-1}$ with respect to $x_{T-1}$ and obtain the following[41].

[41] The integrand is zero over at the boundary and hence no need to compute the differential of the boundary.

$$
\begin{aligned}
\mu_{T-1}^{-\gamma}\frac{\partial \mu_{T-1}}{\partial x_{T-1}} &= E_{T-1}\left[\left[e^{u_T}J_T^{\nu}(W_T^{\nu})\right]^{-\gamma}\left[e^{u_T}\frac{\partial J_T^{\nu}}{\partial W_T^{\nu}}\frac{\partial W_T^{\nu}}{\partial x_{T-1}}\right]\right] \\
&\quad -\theta \underset{e^{u_T}J_T^{\nu}<\mu_{T-1}}{E_{T-1}}\left[\mu_{T-1}^{-\gamma}\frac{\partial \mu_{T-1}}{\partial x_{T-1}} - \left[e^{u_T}J_T^{\nu}(W_T^{\nu})\right]^{-\gamma}\left[e^{u_T}\frac{\partial J_T^{\nu}}{\partial W_T^{\nu}}\frac{\partial W_T^{\nu}}{\partial x_{T-1}}\right]\right] \\
\mu_{T-1}^{-\gamma}\frac{\partial \mu_{T-1}}{\partial x_{T-1}} &= \frac{E_{T-1}\left[\left[e^{u_T}J_T^{\nu}(W_T^{\nu})\right]^{-\gamma}\left[e^{u_T}\frac{\partial J_T^{\nu}}{\partial W_T^{\nu}}\frac{\partial W_T^{\nu}}{\partial x_{T-1}}\right](1+\theta I(e^{u_T}J_T^{\nu}<\mu_{T-1}))\right]}{\left[1+\theta E_{T-1}\left[I(e^{u_T}J_T^{\nu}<\mu_{T-1})\right]\right]}
\end{aligned}
$$

The partial derivatives for the last period are (1) $\frac{\partial J_T^{\nu}}{\partial W_T^{\nu}} = \frac{\partial\left(W_T^{\nu}\right)}{\partial W_T^{\nu}} = 1$ where $J_T^{\nu}(W_T^{\nu}) = W_T^{\nu}$ and (2) $\frac{\partial W_T^{\nu}}{\partial x_{T-1}} = A_{T-1}^{\nu}R_{e,T}e^{-u_T}$ where $A_{T-1}^{\nu} = W_{T-1}^{\nu} - C_{T-1}^{\nu}$. However at the optimal portfolio weights, $\left.\frac{\partial \mu_{T-1}}{\partial x_{T-1}}\right|_{x_{T-1}=x_{T-1}^{*}} = 0$. Thus the first order condition for the last period reduces to Eqn. B.1.

$$
0 = E_{T-1}\left[\left[e^{u_T}W_T^{\nu}\right]\left[R_{e,T}\right]\left(1+\theta I(e^{u_T}W_T^{\nu}<\mu_{T-1}^{*})\right)\right] \tag{B.1}
$$

### 2.1.2 Optimal Consumption

The first order condition for the optimal consumption in the last but one period is,

$$
\begin{aligned}
0 &= \left(C_{T-1}^{\nu}\right)^{-\gamma} + p_{T-1}\beta\left(\mu_{T-1}^{*}\right)^{-\gamma}\frac{\partial \mu_{T-1}}{\partial C_{T-1}^{\nu}} \\
\left(C_{T-1}^{\nu,*}\right)^{-\gamma} &= -p_{T-1}\beta\times\left(\mu_{T-1}^{*}\right)^{-\gamma}\left.\frac{\partial \mu_{T-1}^{*}}{\partial C_{T-1}^{\nu}}\right|_{C_{T-1}^{\nu}=C_{T-1}^{\nu,*}}
\end{aligned}
$$

The differential of the certainty equivalent with respect to the consumption involves similar steps as in the sub-section above.

$$
\mu_{T-1}^{-\gamma}\frac{\partial \mu_{T-1}}{\partial C_{T-1}^{\nu}} = \frac{E_{T-1}\left[\left[e^{u_T}J_T^{\nu}(W_T^{\nu})\right]^{-\gamma}\left[e^{u_T}\frac{\partial J_T^{\nu}}{\partial W_T^{\nu}}\frac{\partial W_T^{\nu}}{\partial C_{T-1}^{\nu}}\right](1+\theta I(e^{u_T}J_T^{\nu}<\mu_{T-1}))\right]}{\left[1+\theta E_{T-1}\left[I(e^{u_T}J_T^{\nu}<\mu_{T-1})\right]\right]}
$$

The two partial derivatives in the expectation are, $\frac{\partial J_T^{\nu}}{\partial W_T^{\nu}} = \frac{\partial\left(W_T^{\nu}\right)}{\partial W_T^{\nu}} = 1$ and $\frac{\partial W_T^{\nu}}{\partial C_{T-1}^{\nu}} = -e^{-u_T}R_{p,T}$. I substitute for these two partials and obtain the optimal consumption in period $T-1$ to be,

$$
\left(C_{T-1}^{\nu,*}\right)^{-\gamma} = p_{T-1}\beta\times\frac{E_{T-1}\left[\left[e^{u_T}W_T^{\nu}\right]^{-\gamma}R_{p,T}\left(1+\theta I(e^{u_T}W_T^{\nu}<\mu_{T-1}^{*})\right)\right]}{\left[1+\theta E_{T-1}\left[I(e^{u_T}W_T^{\nu}<\mu_{T-1}^{*})\right]\right]}.
$$

## 2.2 Any other time period

The value function for any period other than $T-1$ is,

$$\frac{J_t^\nu (W_t^\nu)^{1-\gamma}}{1-\gamma} = \max_{C_t^\nu, x_t} \frac{(C_t^\nu)^{1-\gamma}}{1-\gamma} + p_t \beta \frac{\mu_t (e^{u_{t+1}} J_{t+1}^\nu (W_{t+1}^\nu))^{1-\gamma}}{1-\gamma}.$$

The budget equation is,
$W_{t+1}^\nu = (W_t^\nu - C_t^\nu) \left[ R_{e,t+1} x_t + R_f \right] e^{-u_{t+1}} + Y_{t+1}^\nu,$
and the constraints are $C_t^\nu \leq W_t^\nu$ and $1 \geq x_t \geq 0$.
The certainty equivalent $\mu_t$ is,

$$\begin{aligned} \mu_t^{1-\gamma} &= E_t \left[ \left[ e^{u_{t+1}} J_{t+1}^\nu (W_{t+1}^\nu) \right]^{1-\gamma} \right] \\ &\quad -\theta \underset{e^{u_{t+1}} J_{t+1}^\nu < \mu_t}{E_t} \left[ \mu_t^{1-\gamma} - \left[ e^{u_{t+1}} J_{t+1}^\nu (W_{t+1}^\nu) \right]^{1-\gamma} \right]. \end{aligned}$$

### 2.2.1 Optimal Portfolio Weight

I differentiate certainty equivalent $\mu_t$ with respect to $x_t$ and obtain the following. The steps are similar to those for period $T-1$.

$$\mu_t^{-\gamma} \frac{\partial \mu_t}{\partial x_t} = \frac{E_t \left[ \left[ e^{u_{t+1}} J_{t+1}^\nu (W_{t+1}^\nu) \right]^{-\gamma} \left[ e^{u_{t+1}} \frac{\partial J_{t+1}^\nu}{\partial W_{t+1}^\nu} \frac{\partial W_{t+1}^\nu}{\partial x_t} \right] \left( 1 + \theta I (e^{u_{t+1}} J_{t+1}^\nu < \mu_t) \right) \right]}{\left[ 1 + \theta E_t \left[ I (e^{u_{t+1}} J_{t+1}^\nu < \mu_t) \right] \right]}$$

One of the partials is, $\frac{\partial W_{t+1}^\nu}{\partial x_t} = (W_t^\nu - C_t^\nu) R_{e,t+1} e^{-u_{t+1}}$, which I rewrite as, $e^{u_{t+1}} \frac{\partial W_{t+1}^\nu}{\partial x_t} = A_t^\nu R_{e,t+1}$, where $A_t^\nu = (W_t^\nu - C_t^\nu)$. Thus the first order condition for the optimal portfolio weight $\left. \frac{\partial \mu_t}{\partial x_t} \right|_{x_t = x_t^*} = 0$ reduces to,

$$0 = E_t \left[ \left[ e^{u_{t+1}} J_{t+1}^\nu (W_{t+1}^\nu) \right]^{-\gamma} \left[ \frac{\partial J_{t+1}^\nu}{\partial W_{t+1}^\nu} R_{e,t+1} \right] \left( 1 + \theta I (e^{u_{t+1}} J_{t+1}^\nu < \mu_t^*) \right) \right]. \qquad \text{(B.2)}$$

Now, I compute the remaining partial in the integral. Given that $J_{t+1}^\nu$ is ,

$$\frac{J_t^\nu (W_t^\nu)^{1-\gamma}}{1-\gamma} = \max_{C_t^\nu, x_t} \frac{(C_t^\nu)^{1-\gamma}}{1-\gamma} + p_t \beta \frac{\mu_t (e^{u_{t+1}} J_{t+1}^\nu (W_{t+1}^\nu))^{1-\gamma}}{1-\gamma}.$$

The differential ignoring the maximization with respect to consumption is,

$$(J_t^\nu)^{-\gamma} \frac{\partial J_t^\nu}{\partial W_t^\nu} = p_t \beta \left( \mu_t^* \right)^{-\gamma} \frac{\partial \mu_t^*}{\partial W_t^\nu}.$$

But, note that,

$$\frac{\partial \mu_t}{\partial W_t^\nu} = \frac{\partial \mu_t}{\partial A_t^\nu}\frac{\partial A_t^\nu}{\partial W_t^\nu} = \frac{\partial \mu_t}{\partial A_t^\nu}\frac{\partial (W_t^\nu - C_t^\nu)}{\partial W_t^\nu} = \frac{\partial \mu_t}{\partial A_t^\nu},$$

and also,

$$\frac{\partial \mu_t}{\partial C_t^\nu} = \frac{\partial \mu_t}{\partial A_t^\nu}\frac{\partial A_t^\nu}{\partial C_t^\nu} = \frac{\partial \mu_t}{\partial A_t^\nu}\frac{\partial (W_t^\nu - C_t^\nu)}{\partial C_t^\nu} = -\frac{\partial \mu_t}{\partial A_t^\nu}.$$

Hence, $\frac{\partial \mu_t^*}{\partial W_t^\nu} = -\frac{\partial \mu_t^*}{\partial C_t^\nu}$. I substitute this relation in the equation with the differential $\partial J_t^\nu / \partial W_t^\nu$ and obtain,

$$(J_t^\nu)^{-\gamma} \frac{\partial J_t^\nu}{\partial W_t^\nu} = -p_t \beta \left(\mu_t^*\right)^{-\gamma} \frac{\partial \mu_t^*}{\partial C_t^\nu}.$$

However $J_t^\nu$ is based on the maximization over consumption and portfolio weight. The first order condition for the consumption optimization is,

$$(C_t^{\nu,*})^{-\gamma} = -p_t \beta \times (\mu_t^*)^{-\gamma} \frac{\partial \mu_t^*}{\partial C_t^\nu}.$$

Combining the above two equations amounts to the envelope condition and I obtain the formula for $\partial J_t^\nu / \partial W_t^\nu$, which is,

$$\frac{\partial J_t^\nu}{\partial W_t^\nu} = \left(\frac{C_t^{\nu,*}}{J_t^\nu}\right)^{-\gamma}. \tag{B.3}$$

I substitute this envelope condition result by incrementing the time forward in Eqn. B.2 and obtain the following first order condition for the optimal portfolio weights.

$$0 = E_t\left[\left[e^{u_{t+1}} C_{t+1}^{\nu,*}\right]^{-\gamma} \left[R_{e,t+1}\right] \left(1 + \theta I(e^{u_{t+1}} J_{t+1}^\nu < \mu_t^*)\right)\right]$$

### 2.2.2 Optimal Consumption

As noted above the first order condition for the optimal consumption is given by,

$$(C_t^{\nu,*})^{-\gamma} \quad = \quad -p_t \beta \times (\mu_t^*)^{-\gamma} \frac{\partial \mu_t^*}{\partial C_t^\nu}.$$

The differential with respect to the consumption can be transformed into the differential with respect to the savings as shown below,

$$\frac{\partial \mu_t}{\partial C_t^\nu} = \frac{\partial \mu_t}{\partial A_t^\nu}\frac{\partial A_t^\nu}{\partial C_t^\nu} = \frac{\partial \mu_t}{\partial A_t^\nu}\frac{\partial (W_t^\nu - C_t^\nu)}{\partial C_t^\nu} = -\frac{\partial \mu_t}{\partial A_t^\nu}.$$

Thus, rewriting the first order condition for optimal consumption I obtain,

$$(C_t^{\nu,*})^{-\gamma} = p_t\beta \times (\mu_t^*)^{-\gamma}\,\frac{\partial \mu_t^*}{\partial A_t^\nu}.$$

I follow steps similar to those for the optimal portfolio weights and obtain the differential of the certainty equivalent with respect to savings to be,

$$(\mu_t^*)^{-\gamma}\,\frac{\partial \mu_t^*}{\partial A_t^\nu} = \frac{E_t\left[\left[e^{u_{t+1}}C_{t+1}^{\nu,*}\right]^{-\gamma} R_{p,t+1}^*\left(1+\theta I(e^{u_{t+1}}J_{t+1}^\nu < \mu_t^*)\right)\right]}{\left[1+\theta E_t\left[I(e^{u_{t+1}}J_{t+1}^\nu < \mu_t^*)\right]\right]}.$$

Thus the first order condition for the optimal consumption is,

$$(C_t^{\nu,*})^{-\gamma} = p_t\beta \times \frac{E_t\left[\left[e^{u_{t+1}}C_{t+1}^{\nu,*}\right]^{-\gamma} R_{p,t+1}^*\left(1+\theta I(e^{u_{t+1}}J_{t+1}^\nu < \mu_t^*)\right)\right]}{\left[1+\theta E_t\left[I(e^{u_{t+1}}J_{t+1}^\nu < \mu_t^*)\right]\right]}.$$

# C Matching the DA and CRRA Parameters

I compare the attitudes towards risk of the agents with the disappointment averse and CRRA preferences using the risk premiums for a simple log-normal gamble on wealth (or consumption). The parameters for DA preference are set at $\gamma_{da} = 5$ and $\theta = 1$. I plot the uncertainty of the future gambles again in Figure C.1 for convenience. The standard deviation of the log of the future gamble facing the investor at young age is between 10% and 11% for the benchmark case. I choose the value for $\gamma_{crra}$ for CRRA preferences that best matches the risk premium and marginal incremental premium at this standard deviation for a simple log-normal gamble. This method yields the match of $\gamma_{crra} = 8$ for the risk parameters of CRRA preferences. The match based simply on the risk premium results in $\gamma_{crra} = 9$. I illustrate the matching method based on risk premium and the incremental premium below.

The log-normal gamble on wealth, $W(\epsilon)$ is given by $\ln(W(\epsilon)) = k + \sigma\epsilon$, where $\epsilon$ is a standard normal random variable. The expected outcome from the gamble on the wealth is, $\ln(E(W)) = k + \frac{\sigma^2}{2}$. I consider the log of the ratio of the certainty equivalent, (the least certain-value gamble that the agent will be willing to trade for the uncertain gamble) to the expected wealth outcome as a measure of the risk premium the agent demands to hold the gamble[42]. The certainty equivalent under the power utility preference, $\mu_{crra}$ is given by $\ln(\mu_{crra}) = k + \frac{(1-\gamma_{crra})\sigma^2}{2}$, where $\gamma_{crra}$ measures the relative risk aversion. The risk premium $\ln(E(W)/\mu_{crra})$ is $RP^{crra} = \gamma_{crra}\sigma^2/2$. The risk premium is proportional to the variance of the gamble. The incremental premium or the incremental reward per unit rise in the risk is proportional to the size or the standard deviation of the gamble $d(RP^{crra})/d\sigma = \gamma_{crra}\sigma$. The certainty equivalent for the disappointment averse preferences is given by,

[42]The idea is the same as that in Backus, Routledge, and Zin (2004).

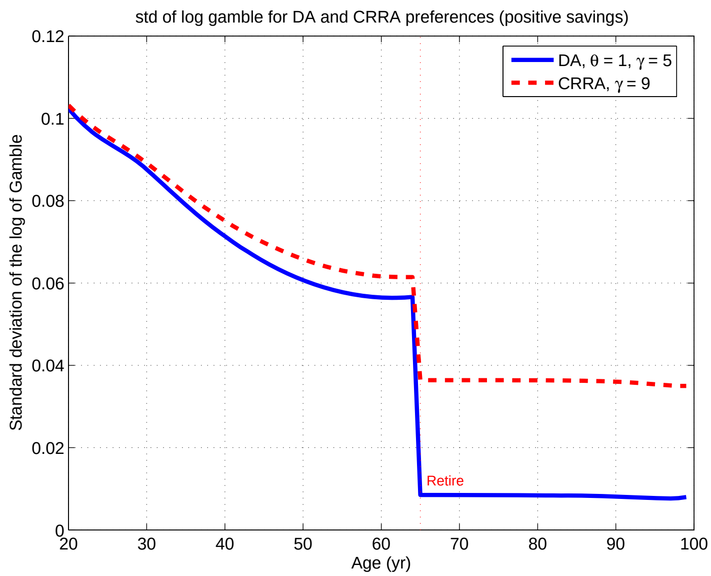


Figure C.1: The future gamble outcomes at time $t$ with optimal portfolio weight $x^*_{t+1}$ and consumption $C^{*,\nu}_{t+1}$ are given by $e^{u_{t+1}} J^{\nu}_{t+1}(W^{*,\nu}_{t+1})$. The plot is the cross-sectional average of the standard deviation of the log of this future gamble facing the investor. The plot is for the benchmark case of no correlation between the labor income and portfolio returns. Further, the average is conditional on positive savings.

$$\mu_{da}^{1-\gamma} = E\left[W^{1-\gamma}\right] - \theta \underset{W<\mu_{da}}{E}\left[\mu_{da}^{1-\gamma} - W^{1-\gamma}\right].$$

Although no explicit formula is available an implicit formula for $\mu_{da}$ exists. I numerically compute the certainty equivalent $\mu_{da}$ and the risk premium $RP^{da} = \ln(E(W)/\mu_{da})$ using this implicit formula (Eqn. C.1).

$$\begin{aligned}
\mu_{da}^{1-\gamma} &= \exp\left((1-\gamma_{da})k + \frac{(1-\gamma_{da})^2\sigma^2}{2}\right) \\
&\quad +\theta\left[\exp\left((1-\gamma_{da})k + \frac{(1-\gamma_{da})^2\sigma^2}{2}\right)\Phi\left(\frac{\ln\mu_{da} - k - (1-\gamma_{da})\sigma^2}{\sigma}\right)\right. \\
&\quad \left. -\mu_{da}^{1-\gamma}\Phi\left(\frac{\ln\mu_{da} - k}{\sigma}\right)\right] \qquad \text{(C.1)}
\end{aligned}$$

I numerically compute the certainty equivalent $\mu_{da}$ and the risk premium $RP^{da}$ using this implicit formula. The risk premium is defined as, $RP^{da} = \ln(E(W)/\mu_{da})$. I set the constant $k$ at 0.01. The constant, however, is inconsequential for the risk premium as the certainty equivalents for both the preferences are scalable by the constant multiplying the gamble.

Let $a = RP^{da}\big|_{\gamma=5,\theta=1}$, $b = \frac{d(RP^{da})}{d\sigma}\Big|_{\gamma=5,\theta=1}$. The same for the CRRA preferences is, $RP^{crra} = \gamma_{crra}\sigma^2/2$ and $\frac{d(RP^{crra})}{d\sigma} = \gamma_{crra}\sigma$. I find find the $\gamma^*_{crra}$ such that,

$$\begin{aligned}
\gamma^*_{crra} &= \arg\min_{\gamma_{crra}}(RP^{crra} - a)^2 + \left(\frac{d(RP^{crra})}{d\sigma} - b\right)^2. \\
\gamma^*_{crra} &= \arg\min_{\gamma_{crra}}\left(\frac{\gamma_{crra}\sigma^2}{2} - a\right)^2 + (\gamma_{crra}\sigma - b)^2
\end{aligned}$$

The matched $\gamma^*_{crra}$ at $\sigma = 0.10$ is 7.87. Fig. C.2 plots the best fit $\gamma^*_{crra}$ at different values of the standard deviation of the log-gamble.

# D Primer on First Order Risk Aversion

The first order risk aversion refers to the risk premium being proportional to first order terms that characterize the gamble. Consider a small equally probable two outcome gamble, with the outcomes, $1-\sigma$ and $1+\sigma$. The expected value of the outcome is 1. I compute the approximate values for the certainty equivalent (CE) of gamble under both the preferences. Let $\gamma$ represent the relative risk aversion and $\theta$ the disappointment aversion parameter. Eqn. D.2 and D.1 are the exact formulas for the certainty equivalent under the two preferences.

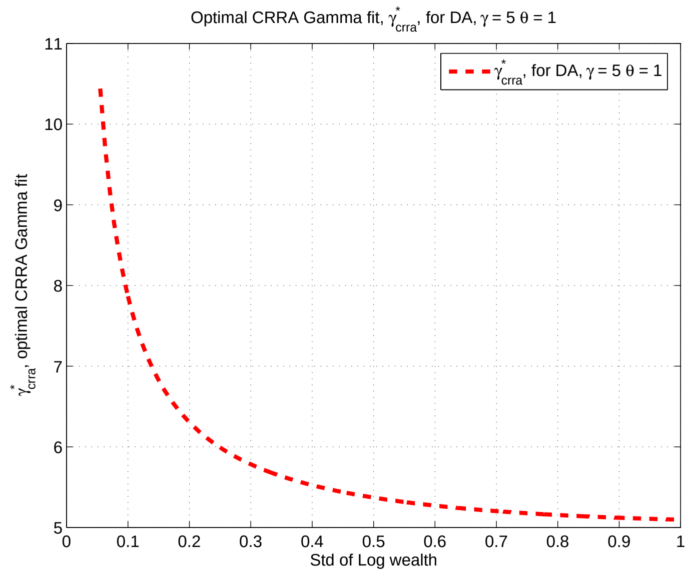


Figure C.2: The optimal $\gamma^*_{crra}$ for CRRA preferences that matches the risk reward attitude of DA preferences with $\gamma = 5$ and $\theta = 1$. The fit is based on the risk premium and the incremental premium for a given standard deviation of the log gamble.

The DA preferences overweight the disappointing outcomes. For this simple two state gamble the $(1 - \sigma)$ outcome is the disappointing outcome.

$$\mu_{pow} = \left[0.5 \times (1-\sigma)^{1-\gamma} + 0.5 \times (1+\sigma)^{1-\gamma}\right]^{1/1-\gamma} \tag{D.1}$$

$$\mu_{da} = \left[\frac{0.5(1+\theta)}{1+0.5\theta} \times (1-\sigma)^{1-\gamma} + \frac{0.5}{1+0.5\theta} \times (1+\sigma)^{1-\gamma}\right]^{1/1-\gamma} \tag{D.2}$$

$$\mu_{pow} \approx 1 - \frac{\gamma\sigma^2}{2} \tag{D.3}$$

$$\mu_{da} \approx 1 - \frac{\theta}{\theta+2}\sigma - 4\gamma\frac{\theta+1}{(\theta+2)^2}\frac{\sigma^2}{2} \tag{D.4}$$

The approximate expressions for the certainty equivalents, Eqn. D.3 and D.4, indicate that the premium for the power utility preferences is proportional to the variance of the gamble whereas the same for the disappointment averse preferences also includes a term proportional to the standard deviation of the gamble.

When the gamble is small the variance is negligible and the the second order terms are too small to have any sizeable contribution to the risk premium. However, the DA preferences also include terms proportional to the standard deviation of the gamble indicating significant aversion to small gambles as long as the disappointment aversion parameter is not close to zero.

# E Critical DA parameter for Non-Participation in Retirement

The retirement income is not stochastic once the agent enters the last period, age 65, he/she spends in the labor market. The retirement income is in fact a constant fraction of the permanent income in the last period the agent participates in the labor market.

$$\begin{aligned}
\ln(Y_t) &= \ln(\lambda) + g_K + \nu_K; \quad \forall K+1 \leq t \leq T \\
\Rightarrow Y_t &= \lambda e^{g_K + \nu_K} \\
K &= 65, \text{ the last period agent works}
\end{aligned}$$

The the retirement income scaled by the permanent income is in fact, $Y^{\nu}_{t+1} = Y_{t+1} e^{-\nu_K} = \lambda e^{g_K} = Y_{\lambda, g_K}$. The budget equation is,

$$W^{\nu}_{t+1} = \left(W^{\nu}_t - C^{\nu}_t\right)\left[R_{e,t+1} x_t + R_f\right] e^{-u_{t+1}} + Y^{\nu}_{t+1}.$$

In retirement, however, as the income is certain the permanent shocks are zero, i.e. $u_{t+1} = 0$. Thus the budget equation simplifies to,

$$\begin{aligned}
W^{\nu_K}_{t+1} &= \left(W^{\nu_K}_t - C^{\nu_K}_t\right)\left[R_{e,t+1} x^{\nu_k}_t + R_f\right] e^0 + Y^{\nu}_{t+1} \\
W^{\nu_K}_{t+1} &= A^{\nu_K}_t \left[R_{e,t+1} x^{\nu_k}_t + R_f\right] + Y_{\lambda, g_K}. \qquad \text{(E.1)}
\end{aligned}$$

Where, $A^{\nu_K}_t = W^{\nu_K}_t - C^{\nu_K}_t$ are the savings. The value function with the scaled variables is,

$$\frac{J^{\nu}_t (W^{\nu}_t)^{1-\gamma}}{1-\gamma} = \max_{C^{\nu}_t, x_t} \frac{(C^{\nu}_t)^{1-\gamma}}{1-\gamma} + p_t \beta \frac{\mu_t (e^{u_{t+1}} J^{\nu}_{t+1}(W^{\nu}_{t+1}))^{1-\gamma}}{1-\gamma}.$$

However, as the permanent shocks are set to zero ($u_{t+1} = 0$), the value function in retirement is,

$$\frac{J^{\nu_K}_t (W^{\nu_K}_t)^{1-\gamma}}{1-\gamma} = \max_{C^{\nu}_t, x_t} \frac{(C^{\nu_K}_t)^{1-\gamma}}{1-\gamma} + p_t \beta \frac{\mu_t (J^{\nu_K}_{t+1}(W^{\nu_K}_{t+1}))^{1-\gamma}}{1-\gamma}.$$

I approximate $J^{\nu_K}_t (W^{\nu_K}_t)$ around $x_t = 0$ using Taylor expansion and obtain,

$$J^{\nu_K}_{t+1}(W^{\nu_K}_{t+1}) \approx a + b A^{\nu_K}_t R_{e,t+1} x_t. \qquad \text{(E.2)}$$

Where the constants are, $a = J^{\nu_K}_{t+1}(W^{\nu_K}_{t+1})\big|_{R_{e,t+1}x_t=0}$ and $b = \left(J^{\nu_K}_{t+1}\right)'\Big|_{R_{e,t+1}x_t=0}$. Note that $b > 0$ since the marginal utility of wealth is positive. Also, note that the certainty equivalent of the future gamble facing the agent at zero portfolio weight is a constant and equal to $a$. This result however does not depend on the approximation. The future wealth at zero portfolio weight is a constant, $W^{\nu_K}_{t+1}\big|_{x_t=0} = A^{\nu_K}_t R_f + Y_{\lambda,g_K} = constant$. The future value function dependent on the wealth at zero portfolio weight is in fact the first term in the approximation, $J^{\nu_K}_{t+1}(W^{\nu_K}_{t+1})\big|_{x_t=0} = J^{\nu_K}_{t+1}(W^{\nu_K}_{t+1})\big|_{R_{e,t+1}x_t=0} = a$. Since the future value function is a sure thing and not a gamble the certainty equivalent is in fact equal to this sure outcome. Thus the certainty equivalent at zero portfolio weight is, $\mu_t(J^{\nu_K}_{t+1}(W^{\nu_K}_{t+1}))\big|_{x_t=0} = a$. This is same result if the approximation Eqn. E.2 for $J^{\nu_K}_{t+1}(W^{\nu_K}_{t+1})$ was used instead. I apply this result and the approximation Eqn. E.2 to the inequality $J^{\nu_K}_{t+1} < \mu_t$ and simplify the inequality.

$$\begin{aligned}
J^{\nu_K}_{t+1} &< \mu_t \\
a + bA^{\nu_K}_t R_{e,t+1}x_t &\lesssim \mu_t \\
\lim_{x_t \to 0^+} a + bA^{\nu_K}_t R_{e,t+1}x_t &\lesssim \lim_{x_t \to 0^+} \mu_t \\
a + bA^{\nu_K}_t R_{e,t+1}x_t &\lesssim a \\
\lim_{x_t \to 0^+} R_{e,t+1}x_t &\lesssim 0 \\
R_{e,t+1} &\lesssim 0
\end{aligned} \tag{E.3}$$

Thus I obtain the result that the inequality, $J^{\nu_K}_{t+1} < \mu_t$ as the portfolio weight tends to zero is approximately the same as $R_{e,t+1} < 0$. Also, note that the policy rule for the next period's consumption, $C^{\nu_K,*}_{t+1}$, dependent on the next period's wealth, $W^{\nu_K}_{t+1}$, reduces to a constant number if the date $t$ portfolio weight $x_t$ is zero. Thus I obtain, $\lim_{x^{\nu_K}_t \to 0^+} C^{\nu,*}_{t+1}(W^{\nu_K}_{t+1}) = c = constant$. I collect all the results above and apply them to the first order condition for optimal portfolio weight in retirement. The differential of the certainty equivalent with respect to the portfolio weight is,

$$\begin{aligned}
\mu_t^{-\gamma}\frac{\partial \mu_t}{\partial x_t} &= \frac{E_t\left[\left[e^{u_{t+1}}C^{\nu,*}_{t+1}\right]^{-\gamma}\left[R_{e,t+1}\right]\left(1+\theta I(e^{u_{t+1}}J^{\nu}_{t+1} < \mu^*_t)\right)\right]}{\left[1+\theta E_t\left[I(e^{u_{t+1}}J^{\nu}_{t+1} < \mu^*_t)\right]\right]} \\
\mu_t^{-\gamma}\frac{\partial \mu_t}{\partial x_t} &= \frac{E_t\left[\left[C^{\nu,*}_{t+1}\right]^{-\gamma}\left[R_{e,t+1}\right]\left(1+\theta I(J^{\nu}_{t+1} < \mu^*_t)\right)\right]}{\left[1+\theta E_t\left[I(J^{\nu}_{t+1} < \mu^*_t)\right]\right]}
\end{aligned} \tag{E.4}$$

The second equality follows from the fact that the permanent shock in zero as the income is certain in retirement. I take the right limit as the portfolio weight tends to zero.

$$
\begin{aligned}
\lim_{x_t \to 0^+} \mu_t^{-\gamma} \frac{\partial \mu_t}{\partial x_t} &\approx \frac{E_t\left[c^{-\gamma}\left[R_{e,t+1}\right]\left(1+\theta I(R_{e,t+1}<0)\right)\right]}{\left[1+\theta E_t\left[I(R_{e,t+1}<0)\right]\right]} \\
\lim_{x_t \to 0^+} \left(\frac{\mu_t}{c}\right)^{-\gamma} \frac{\partial \mu_t}{\partial x_t} &\approx \frac{E_t\left[\left[R_{e,t+1}\right]\left(1+\theta I(R_{e,t+1}<0)\right)\right]}{\left[1+\theta E_t\left[I(R_{e,t+1}<0)\right]\right]} \\
\lim_{x_t \to 0^+} \frac{\partial \mu_t}{\partial x_t} &\lesssim 0 \quad \forall \quad \theta > \hat{\theta}^* \text{ and if } E_t\left[R_{e,t+1}\right] > 0
\end{aligned}
$$

Where, the formula for $\hat{\theta}^*$ depends solely on the distribution parameters of the asset returns.[43]

$$
\hat{\theta}^* = -\frac{E_t\left[R_{e,t+1}\right]}{E_t\left[R_{e,t+1} | R_{e,t+1} < 0\right] \times \Phi(R_{e,t+1} < 0)}
$$

Thus $\hat{\theta}^*$ characterizes the approximate value of $\theta^*$ such that the agent does not invest any amount in stocks for values of $\theta > \hat{\theta}^*$. The marginal improvement in the certainty equivalent is negative if the agent invests even a small amount in stocks from his/her savings. Thus the certainty equivalent, being concave, cannot be improved further by reallocating any more of the savings from the risk-free asset to the risky asset.

I do not consider the case of negative portfolio weights as the negative weight implies shorting the stock index. The returns on stock index are log-normal and thus the investor faces unlimited downside if he/she shorts the stock index. In other words, for any given negative portfolio weight, the probability of zero or negative wealth is turns out to be a finite non-zero value[44]. The marginal utility is however infinite for zero wealth and not defined for negative wealth.

If however the agent has limited liability and the approximation in Eqn. E.2 and the marginal welfare in Eqn. E.4 are applicable, the agent would still not prefer to short the stock index. This is because all the the positive excess return states turn into the disappointing states.

[43] The negative excess return states are still the disappointing states if I perform a Taylor expansion on welfare, $\mu_t$ in Eqn. E.3. This implies that the limiting $\theta$ parameter is positive given the average premium is positive.

[44] If the portfolio weight, $x_t^{\nu_k}$ is and negative, the states with excess returns $R_{e,t+1}$ such that, $R_{e,t+1} > \frac{(Y_{\lambda,g_K}/A_t^{\nu_K})+R_f}{\left|x_t^{\nu_k}\right|}$ result in wealth that is non positive.

$$
\begin{aligned}
J_{t+1}^{\nu_K} &< \mu_t \\
a + bA_t^{\nu_K} R_{e,t+1} x_t &\lesssim \mu_t \\
\lim_{x_t \to 0^-} a + bA_t^{\nu_K} R_{e,t+1} x_t &\lesssim \lim_{x_t \to 0^-} \mu_t \\
a + bA_t^{\nu_K} R_{e,t+1} x_t &\lesssim a \\
\lim_{x_t \to 0^-} R_{e,t+1} x_t &\lesssim 0 \\
R_{e,t+1} &\gtrsim 0
\end{aligned}
$$

If all the positive excess return states are disappointing then the marginal welfare is positive at zero portfolio weight and the agent prefers not to short the stock index.

$$
\lim_{x_t \to 0^-} \frac{\partial \mu_t}{\partial x_t} \gtrsim 0 \quad \forall \quad \theta > 0 \text{ and if } E_t \left[ R_{e,t+1} \right] > 0
$$

# F Critical DA Parameter for Defined Contribution

The agent does not receive any social security, pension or other source of non-tradable income under the simplified representation of Defined Contribution. The budget equation therefore simplifies to,

$$
\begin{aligned}
W_{t+1} &= \left( W_t - C_t \right) \left[ R_{e,t+1} x_t + R_f \right]; \quad \forall K \leq t \leq T. \\
K &= 65, \text{ the last period agent works}
\end{aligned}
$$

The value function in retirement is,

$$
\frac{J_t(W_t)^{1-\gamma}}{1-\gamma} = \max_{C_t, x_t} \frac{\left( C_t \right)^{1-\gamma}}{1-\gamma} + p_t \beta \frac{\mu_t (J_{t+1}(W_{t+1}))^{1-\gamma}}{1-\gamma}; \quad \forall K \leq t \leq T.
$$

The optimal portfolio weights in retirement are time invariant and simply involve maximization of function $H(x) = \frac{\mu(R_p)^{1-\gamma}}{1-\gamma}$ (See Lemma F.1). $R_p$ is the portfolio return, where $R_p = R_e x + R_f$. $R_e = R_{e,t+1}$ since the investment opportunity set does not change. Thus optimal portfolio weight is a solution to,

$$
x* = \arg\max_x \frac{\mu(R_p)^{1-\gamma}}{1-\gamma}.
$$

Ang, Bekaert, and Liu (2005) obtain the critical parameter $\theta^*$, such that $x^* = 0$ for all $\theta \geq \theta^*$. Transforming the notation the critical parameter is given by,

$$\theta^* = -\frac{E_t\left[R_{e,t+1}\right]}{E_t\left[R_{e,t+1}|R_{e,t+1}<0\right]\times\Phi(R_{e,t+1}<0)}.$$

**Lemma F.1.** *The value function in retirement obtains the form $J_t(W_t) = \chi_t W_t$, where $\chi_t$ is a positive constant, $\forall\ K \le t \le T$. The optimal portfolio allocation $x_t^*$ is the same through out the retirement period, and the optimal consumption to wealth ratio $CW_t^* = C_t^*(W_t)/W_t$ varies only with age.*

Note that, without the non-tradable source of income, the value function is scalable in consumption and wealth.

*Proof.* Assume that $J_{t+1}(W_{t+1}) = \chi_{t+1}W_{t+1}$, where $\chi_{t+1}$ is a positive constant. The budget equation is, $W_{t+1} = (W_t - C_t)\left[R_{e,t+1}x_t + R_f\right] = (W_t - C_t)R_{p,t+1}$.

Thus,

$$\begin{aligned}
\frac{\mu_t(J_{t+1}(W_{t+1}))^{1-\gamma}}{1-\gamma} &= \frac{\mu_t(\chi_{t+1}W_{t+1})^{1-\gamma}}{1-\gamma} \\
&= \chi_{t+1}^{1-\gamma}\frac{\mu_t((W_t-C_t)R_{p,t+1})^{1-\gamma}}{1-\gamma} \\
&= (W_t-C_t)^{1-\gamma}\chi_{t+1}^{1-\gamma}\frac{\mu_t(R_{p,t+1})^{1-\gamma}}{1-\gamma}.
\end{aligned}$$

Inserting above result in the value function for period $t$, I obtain,

$$\begin{aligned}
\frac{J_t(W_t)^{1-\gamma}}{1-\gamma} &= \max_{C_t,x_t}\frac{(C_t)^{1-\gamma}}{1-\gamma} + p_t\chi_{t+1}^{1-\gamma}\beta(W_t-C_t)^{1-\gamma}\frac{\mu_t(R_{p,t+1})^{1-\gamma}}{1-\gamma} \\
\frac{J_t(W_t)^{1-\gamma}}{1-\gamma} &= W_t^{1-\gamma}\max_{CW_t,x_t}\frac{(CW_t)^{1-\gamma}}{1-\gamma} + p_t\chi_{t+1}^{1-\gamma}\beta(1-CW_t)^{1-\gamma}\frac{\mu_t(R_{p,t+1})^{1-\gamma}}{1-\gamma} \\
\frac{J_t(W_t)^{1-\gamma}}{1-\gamma} &= W_t^{1-\gamma}\max_{CW_t}\frac{(CW_t)^{1-\gamma}}{1-\gamma} + p_t\chi_{t+1}^{1-\gamma}\beta(1-CW_t)^{1-\gamma}\frac{\mu_\chi^{1-\gamma}}{1-\gamma} \\
&= \frac{W_t^{1-\gamma}}{1-\gamma}\left[(CW_t^*)^{1-\gamma} + p_t\chi_{t+1}^{1-\gamma}\beta(1-CW_t^*)^{1-\gamma}\mu_\chi^{1-\gamma}\right] \\
\frac{J_t(W_t)^{1-\gamma}}{1-\gamma} &= \chi_t^{1-\gamma}\frac{W_t^{1-\gamma}}{1-\gamma} \\
J_t(W_t) &= \chi_t W_t
\end{aligned}$$

Where, $\chi_t$ is a constant and,

$$\max_{x_t} \frac{\mu_t(R_{p,t+1})^{1-\gamma}}{1-\gamma} = \frac{\mu_\chi^{1-\gamma}}{1-\gamma}.$$

$\chi_t$ is a constant since (1) portfolio decision is independent of time and level of wealth and (2) the optimal consumption-wealth ratio depends only on time.

The optimization step involves maximizing function $G_t(CW_t,\ x_t)$.

$$\begin{aligned}
G_t(CW_t,\ x_t) &= \frac{(CW_t)^{1-\gamma}}{1-\gamma} + p_t \chi_{t+1}^{1-\gamma} \beta (1-CW_t)^{1-\gamma} \frac{\mu_t(R_{p,t+1})^{1-\gamma}}{1-\gamma} \\
G_t(CW_t,\ x_t) &= G_{1,t}(CW_t) + G_{2,t}(CW_t) \times H_t(x_t)
\end{aligned}$$

Where, (1) $G_{1,t}(CW_t) = \frac{(CW_t)^{1-\gamma}}{1-\gamma}$, (2) $G_{2,t}(CW_t) = p_t \chi_{t+1}^{1-\gamma} \beta (1-CW_t)^{1-\gamma}$ and (3) $H_t(x_t) = \frac{\mu_t(R_{p,t+1})^{1-\gamma}}{1-\gamma}$. But, note that the optimization with respect to portfolio weights simply involves maximizing[45] $H_t(x_t)$. In other words the portfolio optimization and consumption-wealth ratio optimization are separate. Now, note that the optimization of $H_t(x_t) = \frac{\mu_t(R_{p,t+1})^{1-\gamma}}{1-\gamma}$ involves choosing optimal portfolio weight that depends only on the investment opportunity set at time $t$. Thus, $H_t(x_t^*) = H_t^*$, a function of time only. However note that the investment opportunity set is constant and thus the distribution of portfolio returns $R_{p,t+1}$ for a given portfolio weight $x_t$ is the same at all time periods $t$. Thus, I obtain, $\frac{\mu(R_p)^{1-\gamma}}{1-\gamma} = \frac{\mu_t(R_{p,t+1})^{1-\gamma}}{1-\gamma}$ and $H(x) = H_t(x_t)$. Thus the optimal portfolio weight $x^* = x_t^*$ is time invariant.

Further substituting the optimal portfolio weight in the optimization of $G_t(CW_t,\ x_t)$, I obtain,

$$\begin{aligned}
\max_{CW_t, x_t} G_t(CW_t,\ x_t) &= \max_{CW_t} G_{1,t}(CW_t) + G_{2,t}(CW_t) \times \left[\max_{x_t} H_t(x_t)\right] \\
&= \max_{CW_t} G_{1,t}(CW_t) + G_{2,t}(CW_t) \times H_t^* \\
G_t(CW_t^*,\ x^*) &= \max_{CW_t} G_{1,t}(CW_t) + G_{2,t}(CW_t) \times H^*.
\end{aligned}$$

Now the optimization of consumption-wealth ratio $CW_t$ involves only the parameters that are constants. The optimal consumption-wealth ratio depends on preference parameters, return distribution. and also the conditional probability of survival at time $t$, $p_t$. It is the dependence on the conditional probability of survival $p_t$ that results in a consumption-wealth ratio that varies with time. Thus the optimal $G_t(CW_t^*,\ x^*)$ reduces to a constant.

$$\begin{aligned}
\max_{CW_t, x_t} G_t(CW_t,\ x_t) &= G_{1,t}(CW_t^*) + G_{2,t}(CW_t^*) \times H^* \\
G_t(CW_t^*,\ x_t^*) &= \chi_t^{1-\gamma}/1-\gamma
\end{aligned}$$

[45] The constraints $0 \leq CW_t \leq 1$ and the fact that $\chi_{t+1} > 0$ imply that $G_{2,t}(CW_t) > 0$.

Further, note that since $G_{1,t}(CW_t^*),\ G_{2,t}(CW_t^*),\ H^*$ are all positive, $\chi_t$ is also positive.

Now, note that the terminal valuation is given by, $J_T(W_T) = \chi_T W_T$, where $\chi_T = 1$. Hence, $J_t(W_t) = \chi_t W_t$, where $\chi_t$ is a positive constant, optimal consumption-wealth ratio $CW_t^*$ varies only with time and optimal portfolio weight $x_t^*$ is time invariant for all $\forall K \leq t \leq T$. $\square$

$\square$

# G Equivalent Constant Consumption

$$\begin{aligned}
\frac{J_t(W_t,\nu_t)^{1-\gamma}}{1-\gamma} &= \max_{C_t,x_t} \frac{C_t^{1-\gamma}}{1-\gamma} + p_t\beta \frac{\mu_t(J_{t+1}(W_{t+1},\nu_{t+1}))^{1-\gamma}}{1-\gamma} \\
\mu_t^{1-\gamma} &= E_t\left[J_{t+1}(W_{t+1},\nu_{t+1})^{1-\gamma}\right] \\
&\quad -\theta \underset{J_{t+1}<\mu_t}{E_t}\left[\mu_t^{1-\gamma} - J_{t+1}(W_{t+1},\nu_{t+1})^{1-\gamma}\right]
\end{aligned}$$

I restate the optimization program above. The permanent level varies with the permanent shock $\nu_t \equiv \nu_t(u_t)$ and the wealth is dependent on the permanent and the idiosyncratic shocks and also the return shocks, $W_t \equiv W_t(u_t,\ \epsilon_t,\ \eta_t)$. The last period valuation is $J_T = W_T$. Let, $V_t^*(W_t,\nu_t) = \frac{J_t^*(W_t,\nu_t)^{1-\gamma}}{1-\gamma}$ be the maximum value at time $t$ if the agent follows the optimal portfolio and consumption policy rules, $x_t \equiv x_t(W_t,\ \nu_t)$ and $C_t \equiv C_t(W_t,\ \nu_t)$ for all time periods until the end of the optimization program $T$.

I consider the alternative where the agent, with wealth $W_t$ and permanent income $\nu_t$ is instead exogenously provided with a constant consumption in each time period, $C_t = C^{eqv}\ \forall t \leq T$. Since the certainty equivalent of the sure thing is a sure things itself, the valuation under this constant consumption program is $V^{eqv} = k' \times \frac{(C^{eqv})^{1-\gamma}}{1-\gamma}$.

I define the equivalent constant consumption for a given wealth and permanent income level as the minimum constant consumption stream over the period of the optimization program such that the agent is indifferent between accepting the labor income and stock return gamble and accepting the constant consumption stream. In other words the agent achieves the same level of utility irrespective of whether he takes on the chances with the uncertain labor income and stock returns and follows the optimal policy rules or receives a sure consumption in every period.

If the maximum utility for a given wealth $W_t$ and permanent income level $\nu_t$ is, $\frac{J_t^*(W_t,\nu_t)^{1-\gamma}}{1-\gamma}$, the equivalent constant consumption is given by,

$$\begin{aligned}
k' \times \frac{(C^{eqv,*})^{1-\gamma}}{1-\gamma} &= \frac{J_t^*(W_t,\nu_t)^{1-\gamma}}{1-\gamma} \\
C^{eqv,*} &= (k')^{\gamma-1} \times J_t^*(W_t,\nu_t) \\
C^{eqv,*} &= k \times J_t^*(W_t,\nu_t) \quad \text{Where } k = constant.
\end{aligned}$$

Thus the equivalent constant consumption $C^{eqv,*}$ is proportional to the maximum welfare $J_t^*(W_t, \nu_t)$ that the agent with beginning period wealth $W_t$ and permanent income level $\nu_t$ can achieve.

# H Approximate optimal weight

The optimal portfolio weights are the same as those for the power utility investor except that the agent uses a distorted distribution for optimization. The welfare and the budget constraints are as follows.

$$
\begin{aligned}
\mu_t^{1-\gamma} &= E_t\left[W_{t+1}^{1-\gamma}\right] - \theta \underset{W_{t+1}<\mu_t}{E_t}\left[\mu_t^{1-\gamma} - W_{t+1}^{1-\gamma}\right] \qquad \text{(H.1)}\\
W_{t+1} &= A_t R_{p,t+1} + Y_{t+1}\\
R_{p,t+1} &= R_{e,t+1} x_t + R_f\\
R_{e,t+1} &= R_{t+1} - R_f
\end{aligned}
$$

The implicit formula for welfare Eqn. H.1 re-written as Eqn. H.2 shows that the agent is evaluating the gamble with a distorted probability distribution[46]. The distorted probability formulation is easier for the interpretation of the first order condition.

$$
\begin{aligned}
\mu_t^{1-\gamma} &= \frac{E_t\left[W_{t+1}^{1-\gamma}\left(1+\theta I(W_{t+1}<\mu_t)\right)\right]}{1+\theta\Phi(W_{t+1}<\mu_t)} \qquad \text{(H.2)}\\
\mu_t^{1-\gamma} &= \sum_{W_{t+1}} \hat{p}(W_{t+1}) W_{t+1}^{1-\gamma}\\
\hat{p}(W_{t+1}) &= \frac{1+\theta I(W_{t+1}<\mu_t)}{1+\theta\sum_{W_{t+1}}\hat{p}(W_{t+1}) I(W_{t+1}<\mu_t)} \times p(W_{t+1})
\end{aligned}
$$

I differentiate Eqn. H.1 with respect to portfolio weight $x_t$ and use $\frac{dW_{t+1}}{dx_t} = A_t R_{e,t+1}$.

$$
\begin{aligned}
\mu_t^{-\gamma}\frac{d\mu_t}{dx} &= E_t\left[W_{t+1}^{-\gamma}\frac{dW_{t+1}}{dx_t}\right] - \theta \underset{W_{t+1}<\mu_t}{E_t}\left[\mu_t^{-\gamma}\frac{d\mu_t}{dx} - W_{t+1}^{-\gamma}\frac{dW_{t+1}}{dx_t}\right]\\
\frac{d\mu_t}{dx_t} &= A_t E_t\left[\left(\frac{W_{t+1}}{\mu_t}\right)^{-\gamma} R_{e,t+1}\frac{\left(1+\theta I(W_{t+1}<\mu_t)\right)}{1+\theta\Phi(W_{t+1}<\mu_t)}\right]
\end{aligned}
$$

[46] $\sum_{W_{t+1}} \hat{p}(W_{t+1}) = 1$ and $\hat{p}(W_{t+1}) \geq 0$, hence $\hat{p}(W_{t+1}) \leq 1$ and $\hat{p}(W_{t+1})$ represents a probability distribution.

The first order condition for the optimization is, $\frac{d\mu_t}{dx_t} = 0$[47].

$$
\begin{aligned}
0 &= E_t\left[\left(W^*_{t+1}\right)^{-\gamma} R_{e,t+1}\frac{\left(1+\theta I(W^*_{t+1} < \mu^*_t)\right)}{1+\theta\Phi(W^*_{t+1} < \mu^*_t)}\right] \\
0 &= \hat{E}_t\left[\left(W^*_{t+1}\right)^{-\gamma} R^*_{e,t+1}\right] \\
\hat{E}_t\left[\left(W^*_{t+1}\right)^{-\gamma} R^*_{t+1}\right] &= \hat{E}_t\left[\left(W^*_{t+1}\right)^{-\gamma} R_f\right] \qquad \text{(H.3)}
\end{aligned}
$$

Thus the first order condition is same as that for the power utility except for the distortion of the data generating process for the labor income and stock returns. I use the log-linearization technique of Campbell and Viceira (2002) to obtain an approximate formula for the optimal weights. The budget constraint is $W_{t+1} = A_t R_{p,t+1} + Y_{t+1}$. The portfolio returns and the labor income have log-normal distributions. The sum of log-normal distributions is not log-normal. Thus without an explicit formula for the distribution of terminal wealth it is hard to obtain a closed form formula for the optimal portfolio weights. The log-linearization approximates the budget equation and helps obtain log-normal approximation for the terminal wealth. I use lower case letters to represent the natural logarithms. The log-linearized version of the budget constraint is Eqn. H.4.

$$
\begin{aligned}
w_{t+1} &\approx k + \hat{\rho}(a_t + r_{p,t+1}) + (1-\hat{\rho})y_{t+1} \qquad \text{(H.4)} \\
\frac{1}{\hat{\rho}} &= 1 + \frac{\exp(y)}{\exp(a+r_p)}; \qquad 0 < \hat{\rho} < 1
\end{aligned}
$$

The quantity $\hat{\rho}$ represents the elasticity of the terminal consumption with respect to financial wealth. It is evaluated using the expected value of log-labor income $\hat{E}_t\left[y_{t+1}\right] \equiv y$ and the expected value of log-financial wealth $\hat{E}_t\left[a_t + r_{p,t+1}\right] \equiv a + r_p$. I follow Campbell and Viceira (2002) and use the budget equation approximation of log-normal terminal wealth[48] and rewrite the first-order condition Eqn. H.3.

[47] The welfare is concave and first order condition is sufficient.

[48] Computing the expectations in the first order condition yields the formula in terms of the covariance with the terminal wealth/consumption.

$$
\begin{aligned}
\hat{E}_t\left[\left(W^*_{t+1}\right)^{-\gamma} R^*_{t+1}\right] &= \hat{E}_t\left[\left(W^*_{t+1}\right)^{-\gamma} R_f\right] \\
\hat{E}_t\left[\exp(-\gamma w^*_{t+1} + r_{t+1})\right] &= \hat{E}_t\left[\exp(-\gamma w^*_{t+1} + r_f)\right] \\
\hat{E}_t\left[\exp(-\gamma w^*_{t+1} + r_{t+1} - r_f)\right] &= \hat{E}_t\left[\exp(-\gamma w^*_{t+1})\right]
\end{aligned}
$$

I compute the expectations assuming a log-normal distribution for the terminal wealth.

$$\hat{E}_t\left[r_{t+1}-r_f\right]+\frac{1}{2}\hat{\sigma}^2_{r,t}=\gamma \hat{Cov}_t(w_{t+1},r_{t+1}) \tag{H.5}$$

Further, substituting Eqn. H.4 into the first order condition Eqn. H.5 I obtain the approximate formula for the optimal portfolio weights[49].

$$\begin{aligned} x_t &\approx \frac{1}{\hat{\rho}}\left(\frac{\widehat{\overline{r_{t+1}-r_f}}+\hat{\sigma}^2_{r,t}/2}{\gamma\hat{\sigma}^2_{r,t}}\right)+\left(1-\frac{1}{\hat{\rho}}\right)\frac{\hat{\sigma}_{y,r,t}}{\hat{\sigma}^2_{r,t}} \\ x_t &\approx \frac{1}{\hat{\rho}}\left(\frac{\widehat{\overline{r_{t+1}-r_f}}+\hat{\sigma}^2_{r,t}/2}{\gamma\hat{\sigma}^2_{r,t}}\right); \qquad \text{If } \hat{\sigma}_{y,r,t}\approx 0 \end{aligned}$$

---

$$\begin{aligned} \hat{E}_t\left[\exp(-\gamma w^*_{t+1}+r_{t+1}-r_f)\right] &= \exp(-\gamma\overline{w^*_{t+1}}+\frac{\gamma^2}{2}var_t(w^*_{t+1})) \\ &\quad \times\exp(\hat{E}_t\left(r_{t+1}-r_f\right)+\frac{1}{2}\hat{\sigma}^2_{r,t}-\gamma\hat{Cov}_t(w^*_{t+1},r_{t+1})) \\ \hat{E}_t\left[\exp(-\gamma w^*_{t+1})\right] &= \exp(-\gamma\overline{w^*_{t+1}}+\frac{\gamma^2}{2}var_t(w^*_{t+1})) \end{aligned}$$

Equating the two expectations, I obtain the familiar formula, but with a distorted probability distributions.

$$\hat{E}_t\left[r_{t+1}-r_f\right]+\frac{1}{2}\hat{\sigma}^2_{r,t}=\gamma \hat{Cov}_t(w_{t+1},r_{t+1})$$

[49]I also use following approximation for the portfolio returns:

$$r_{p,t+1}\approx x_t(r_{t+1}-r_f)+r_f+\frac{1}{2}x_t(1-x_t)\hat{\sigma}^2_{r,t}.$$

# I Additional Plots and Tables

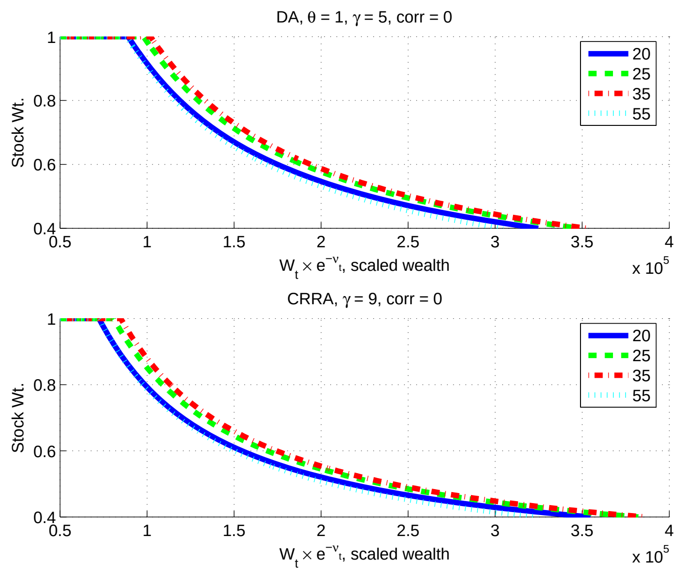


Figure I.1: The optimal portfolio policy rules as a function of wealth scaled by the permanent income for the young and the mid adult life for DA and CRRA preferences. These plots are for the case of no correlation between the labor income and stock returns.

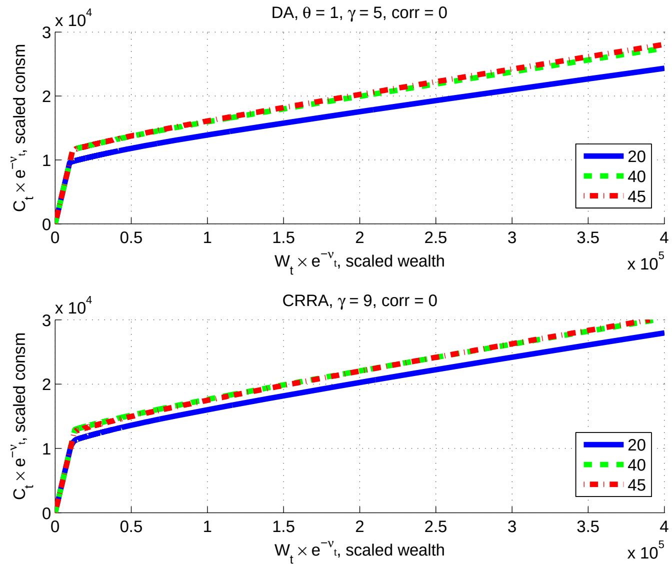


Figure I.2: The optimal consumption policy rules as a function of wealth scaled by the permanent income for the young and the mid adult life for DA and CRRA preferences. The consumption on y-axis is scaled by the permanent income. These plots are for the case of no correlation between the labor income and stock returns.

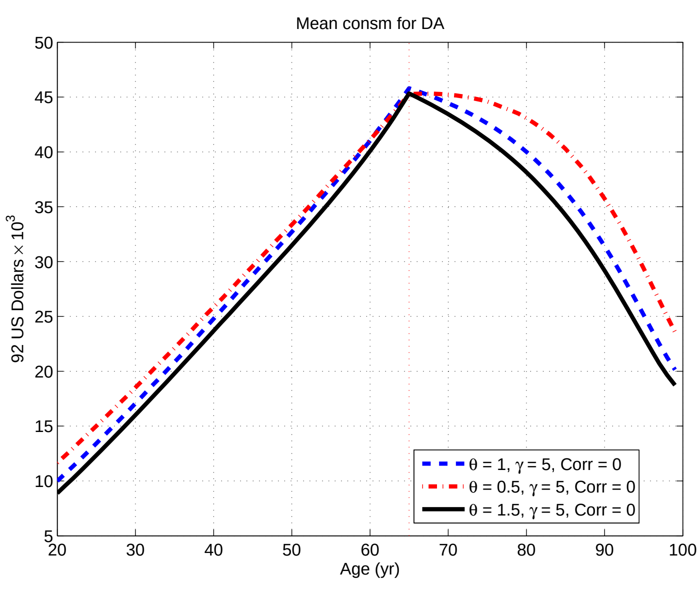


Figure I.3: The cross-sectional mean consumption at every age over the life of the agents with DA preferences and varying $\theta$.

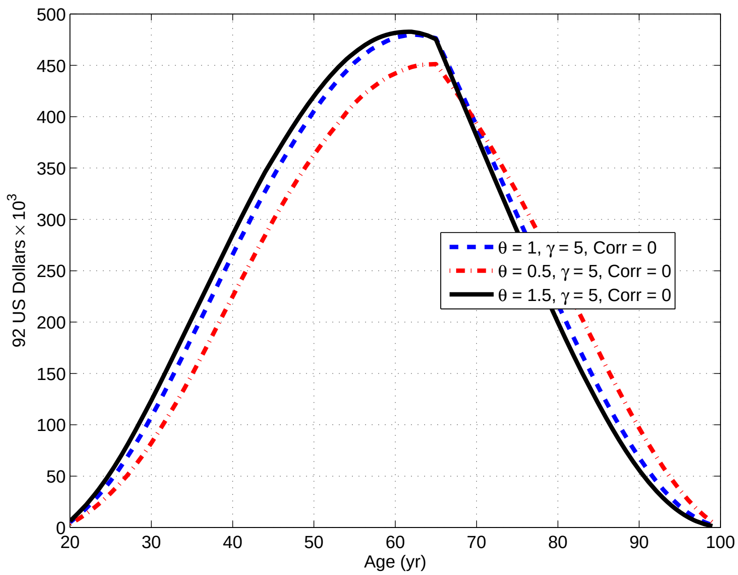


Figure I.4: The cross-sectional mean savings at every age over the life of the agents with DA preferences and varying $\theta$.

US stock market crashes since 1869

| Period | \|Real stock returns\| |
|---|---:|
| 1907-07 | 0.32 |
| 1916-20 | 0.47 |
| 1929-31 | 0.55 |
| 1937-37 | 0.37 |
| 1939-41 | 0.29 |
| 1973-74 | 0.49 |
| 2000-02 | 0.42 |
| 2008-08 | 0.37 |

Table I.1: The stock market crashes and the magnitude of cumulative real returns over the same periods. The table is reproduced from Barro and Ursua (2009), Table 1.

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