Loan Commitments

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Abstract: We propose a new model for the valuation of loan commitments and some of their main features including the MAC (Material Adverse Change) clause. We employ a two-period contingent claim approach. The advantage of this approach is that it is based on rational economic considerations that are utility-free.

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Loan Commitments

Introduction

One major tool to finance corporations is via credit line commitments. These are short term credit lines that firms can withdraw from their banks, up to a certain, predetermined ceiling, at a certain cost, usually above the interest rate on long term credit. The credit lines serve corporations to finance, usually, short term financial needs, when cash outflows are greater than cash inflows. Also credit lines are used as a buffer against unexpected short term gaps as well as to take advantage of unexpected investment opportunities. Thus,loan commitments (LC) can be employed defensively as well as offensively by the corporations.

Firms need cash for future, uncertain uses. Financial markets and many intermediaries provide an access to borrowing or other forms of raising capital. This access to liquidity can be expensive, take time or can require revealing private information. Most companies keep cash, or liquid reserves to serve their immediate needs and provide a buffer for future potential uses. The two common forms to safeguard liquidity is first, by keeping enough liquid reserves, or, second, by signing a credit line (CL), or loan commitment (LC) agreements. Since there is an uncertainty about future needs, one should establish an optimal policy that takes into account the firm's expected cashflow and their uncertain nature, its ability to raise new capital and the costs of various forms and sources of funds. Part of the problem of raising liquidity can be dealt with by using callable bonds (raising capital in advance with an ability to return it if eventually there is no need), adjusting dividend payments and retaining earnings.

Empirically, we observe that loan commitments are heavily used. In Figure 1 we show data from the FDIC depicting the amount of unused loan commitments on the balance sheets of US insured banks for the period 1984-2016. In 2007 just before the sub-prime crisis erupted, the total reached over 8 trillion dollars, which is a substantial amount compared to total bank loans. The amount of unused loan commitments went to below 6 trillion dollars in the period 2010-2012. Sufi (2009) reports that firms from all major industries heavily utilize lines of credit, with wholesale and retail trade being the industries with the highest fraction of firms with lines of credit.

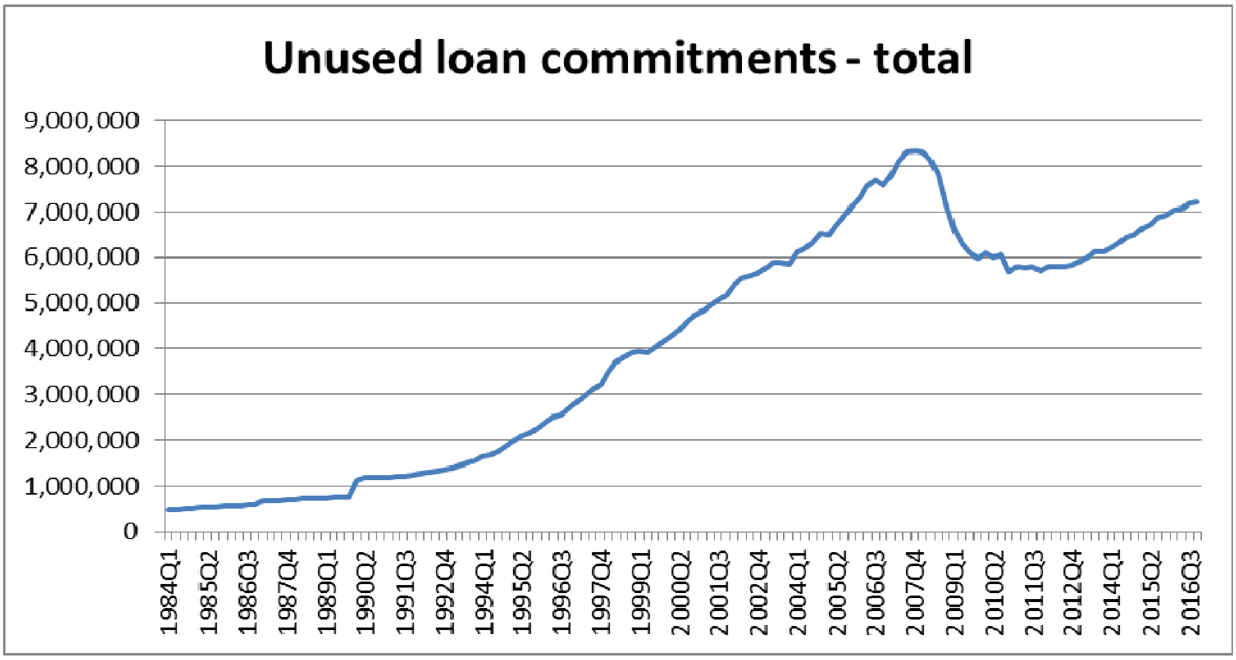

Figure 1. Total unused loan commitments by US banks, source, FDIC, Quarterly Banking Profile.

Theoretical studies present alternative explanations for the main function of credit lines. Campbell (1978), Thakor (1982), James (1982), and Melnik and Plaut (1986) argue that credit lines improve the completeness of financial markets. Boot et al. (1987), Berkovitch and Greenbaum (1991) and Holmstrom and Tirole (1998) argue that lines of credit serve as interest rate or liquidity protection while mitigating moral hazard problems. Morgan (1993) provides an asymmetric information model in which a LC dominates an ordinary debt contract when investment projects have random returns that are costly to observe and random costs that are completely unobservable.

A few studies address the question of pricing LCs. Turnbull (2003) values loans with perfectly foreseen drawdowns using a reduced-form valuation methodology proposed by Jarrow and Turnbull (1995, 1997). Hughston and Turnbull (2002) consider default as a point process with intensity depending on both the unique characteristics of the borrower and the state of the economy. They incorporate state dependent drawdowns. The breakeven loan spread is such that it equalizes the present value of the revenues and costs of the loan. Loukoianova et al. (2006) model non-committed credit lines as an option on the credit spread combined with a reverse knock-out option. Jones (2001) models credit quality evolution as a jump-diffusion process; process parameters are estimated using monthly bond data for 105 firms.

Jones and Wu (2009) examine credit lines using the reduced form approach of Duffie and Singleton (1997), rather than the structural model approach that we employ. They also assume that credit quality follows a mixed jump-diffusion process, while we assume that the firm's assets follow a stationary diffusion process. We assume a given leverage ratio and credit risk or quality that can change due to the firm's value increasing or decreasing over time.

Sufi (2009) takes an empirical approach, examining the factors that determine whether firms use credit lines or cash in managing their liquidity. He concludes that the LC is a viable liquidity substitute only for firms with high cashflow. Demiroglu and James (2011) also review the evidence on the use of bank LC. They conclude that access to lines of credit is not a perfect substitute to cash, since they are contingent on the credit quality of the borrower as well as on the financial conditions of the bank.

While these valuation models take into account the credit quality of the borrower and its evolution over time, they ignore the credit riskiness of the line's issuer (the potential lender) and its ability to supply the needed credit upon request at time of liquidity crisis or financial distress of the line issuer. The basic assumption in most of these models is a risk free issuer. However, such models do not fully capture the clustering in default correlations, sometimes called "credit contagion".

Credit contagion has been at the heart of the financial crisis that started in 2007. Federal Reserve Chairman Bernanke justified in 2008 the rescue of Bear Stearns by explaining that "the company's failure could also have cast doubt on the financial positions of some of Bear Stearns' thousands of counterparties and perhaps of companies with similar businesses." Second-generation models attempt to provide structural explanations for this default clustering. For instance, Duffie et al. (2009) estimate a "frailty" model where defaults are driven by an unobserved time-varying latent variable, which partially explains the observed default clustering. Another extension would be to consider multiple factors effect, or industry factors. When a firm defaults, other firms in the same industry could suffer from contagion effects, reflecting shocks to cash flows that are common to that industry, see Lang and Stulz (1992) and Jorion and Zhang (2007).

The major cost to credit line borrowers is the interest charges on the drawn down amount. Interest accrues between payment dates at a fixed contractual spread above the level of a default-free reference rate or as a fixed rate. Hence, credit line contracts contain a valuable embedded option to the borrower. When his cost of debt is high, the borrower may use the credit line as a cheaper source of capital. It should be noted that banks also charge a fee when making loan commitments to firms.

The two major explanations for using the LC, whether to manage liquidity or to exploit unexpected investment opportunities assume implicitly market imperfection in the sense that the firm cannot instantly raise the liquidity it needs. Nevertheless, the models employed to price the LC are based on the perfect capital market (PCM) assumption. Our approach is fully consistent with M&M propositions and the PCM assumption. We model LC as a tool to potentially finance future expected and unexpected cash needs. One major issue related to LC is whether by exercising the LC and using the money, the riskiness of the firm is expected to change significantly. Our explicit assumption is that the volatility of the expected future distribution of returns on the firm stays constant. Most papers on this subject did not control for the potential change in the riskiness of the firm. The MAC to some extent mitigates such a potential increase in the volatility of the firm.

In this paper we propose a contingent claims type model for valuing loan commitments. This approach allows us to take into account the future uncertainty and to derive the present value of the LC. The option-like approach for valuation of credit lines was first used by Thakor, Hong and Greenbaum (1981). They introduce the contingent claim approach by pricing LC as an option, when the market interest rate for similar loans is stochastic. We take a similar approach, but relate the LC to the debt structure and the riskiness of the firm which purchased the LC. Kozak, Aaron and Gauthier (2006) apply the contingent claims approach to Canadian firms. We make some simplifying assumptions about the way that loan commitments are used by corporations. This allows us to derive an economic valuation model of the optional elements of the LC. We show how banks should price each specific LC as well as how to determine the MAC (materially adverse change) clause, and how this clause affects the value of the LC. The value of the LC in our model is firm specific.

The model

We start with a simple model of a loan commitment from the point of view of bank B providing the LC to a company, C. We make, initially, the following assumptions: Firm C at time t=0 has total assets worth V_0, and it decides to finance them with debt, B_0, and equity $S_0=V_0-B_0$. Debt is assumed to mature at T_1, with face value F_1 (which includes both principal and interest amounts). Debt will be refinanced at time T_1 for an additional period τ (till $T_2=T_1+\tau$). We also assume that firm D, is identical to C in all parameters, except that C also purchases LC from Bank B.

The LC is acquired at time t=0 for the same period of time – till T_1. At T_1 firm C must decide whether to exercise the LC or not. If firm C exercises the line of credit and takes the guaranteed loan, it borrows the amount guaranteed by LC for a the period of time τ, at a promised, fixed interest rate R^*. We further assume that the LC is a proportion α of other term loans of the firm on a pari passu basis, and when LC is exercised the money is used to refinance the firm's old debt. We denote by V_1, S_1 and B_1 the value of the assets of the firm, its equity and debt at time T_1, before the LC is exercised. Both the amount of LC and the guaranteed rate R* will be discussed below.

We evaluate the LC from the bank perspective, and it is done recursively. At time T_1, there are three states: the first state is that firm C is bankrupt at T_1. This bankruptcy occurs when $V_1 < F_1$ and the firm cannot fully pay its obligations to debtholders, F_1. The third state is when the value of assets is so high that the firm decides not to exercise the LC, and the second state is when C exercises the LC.

Let us assume that at time t=0, there is another firm, D, identical to firm C in its assets and leverage ratio. Hence D has the same assets $V_0^D = V_0^C$ and its equity and debt are the same. At this stage we consider only the productive assets of both firms and ignore the cost and value of the LC. Firm D is assumed to issue one-period debt. At T_1, it must refinance its initial debt of F_1 (if it is not in a state of bankruptcy, i.e. $V_1^D > F_1$). It was shown in Merton (1974) and Galai and Masulis (1976) that for firm D at time T_1 we have

(1) $$S_1^D = V_1^D N(d_1) - L_2^D e^{-r\tau} N(d_2)$$

(2) $$B_1^D = V_1^D - S_1^D = V_1^D N(-d_1) + L_2^D e^{-r\tau} N(d_2)$$

where $d_1 = \dfrac{\ln\left(\dfrac{V_1^D}{L_2^D}\right) + \left(r + \dfrac{\sigma^2}{2}\right)\tau}{\sigma\sqrt{\tau}}$, and $d_2 = d_1 - \sigma\sqrt{\tau}$, and $\tau = T_2 - T_1$. By assumption at T_1 the value of the new debt - B_1^D equals the face value of the initial loan F_1, and L_2^D is the face value of this new debt issued at T_1 to pay out the old one. Hence L_2^D is endogenously determined such that given V_1^D, σ, r and L_2^D we get $B_1^D = F_1$. Here σ is the standard deviation of the rate of return on the assets of the firm, and r is the riskless interest rate. Both r and σ are assumed to be given and constant.[1] L_2^D is the face value of the new debt of the firm, to be paid at T_2 if the firm is solvent.

It can be shown that the equilibrium yield to maturity of the new debt at T_1 is given by

(3) $$\overline{R} = r - \frac{1}{\tau}\ln\left(N(d_2) + \frac{V_1^D}{L_2^D e^{-r\tau}} N(-d_1) \right)$$

This yield to maturity is of course contingent on the realization of V_1^D at T_1.

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[1] It should be noted that we can easily relax the assumption that the riskless rate is constant and, following Merton (1973) replace $e^{-r\tau}$ by the present value of a pure discount bond maturing at τ. The model, therefore, can be extended to the more general case when the risk-free rate is also uncertain.

Firm C is identical to firm D except that at t=0 it purchased the LC. We assume it raised additional funds from the initial shareholders to finance this cost, but as far as productive assets, it is identical to D, $V_0^D = V_0^C$. We also assume that firm C raised (zero-coupon) debt with face value of F_1 at T_1. In addition we assume that firm C purchased at t=0 a line of credit, which allows it to roll over its debt at time T_1 at the yield R*. In the appendix we consider the case when the line of credit covers only a part of the total amount F_1.

Since the use of LC is optional, the firm C will make a decision at T_1 based on the realized value of its assets V_1. Figure 2 shows three different areas.

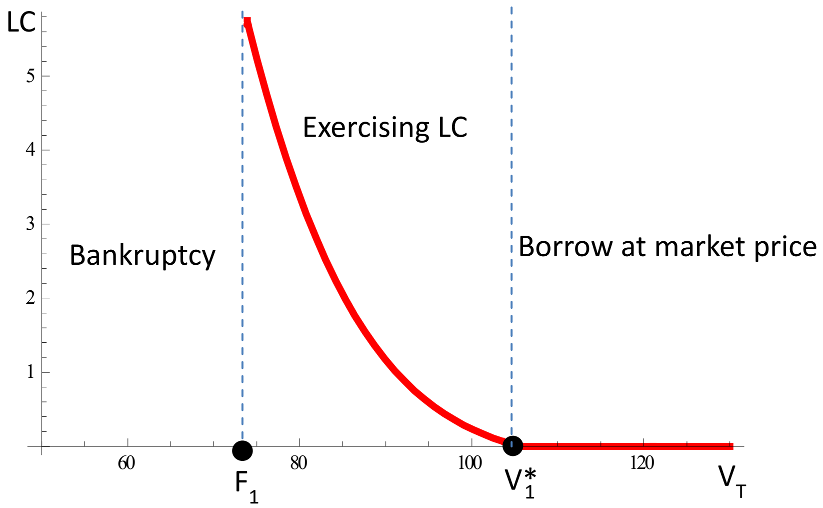

Figure 2. Value of LC at T_1 as a function of V_T

In Figure 2 we show the three states at time T_1: In the first state, when $V_1 \le F_1$, the firm is bankrupt and no new debt is raised. In the second state where $F_1 < V_1 \le V_1^*$ it is worth exercising the LC since $\overline{R} > R^*$. The critical firm value, V_T^*, is the value of assets at T_1 such that $\overline{R} = R^*$. In the third case, when $V_1 > V_1^*$, the firm is doing well and there is no incentive to exercise the LC since the market cost of debt is cheaper than the rate R* promised by the LC.[2] The intuition behind the analysis at T_1 is that, given the leverage selected at t=0, and the need to refinance the debt, if the firm realizes a low value $V_1 < F_1$, it faces bankruptcy. But if it realizes a very high value $V_1 > V_1^*$, it is doing very well and hence can refinance its debt on even better terms than the LC guarantees.

[2] We can add to the complexity by adding the possibility that the bank will be distressed and will not be able to honor the promised LC in the second state.

In order to find the present value of the LC at time t=0 we calculate the value of LC at time T_1 as a function of V_1 and then use the option pricing theory to find the present value of LC at t=0. This can be done either numerically (as a simple integration with the risk-neutral density of V_1) or analytically using the compound option model (based on the two-dimensional normal density function).

At time T_1 the firm needs to refinance its debt by paying the amount F_1. In order to do so it can issue a new bond maturing at T_2. The present value of the bond must be F_1 in order to fully refinance the old debt, thus the equation for F_2 is:

(4) $$F_1 = F_2 e^{-r\tau} - Put(V_1, F_2, \tau, r, \sigma)$$

By solving this equation with respect F_2 to we get $F_2(V_1)$ and as a result also the required yield on the new debt: $y_2^D = \frac{1}{\tau} Ln\left(\frac{F_2}{F_1}\right)$. There is a critical value of the firm's assets - V_1^*, above which it does not make sense to use the LC and it is cheaper to issue a new debt at market conditions. This critical value can be found by solving the following equation with respect to V_1^*:

(5) $$F_1 = F_1 e^{-R^*\tau} - Put(V_1^*, F_1 e^{-R^*\tau}, \tau, r, \sigma)$$

The value V_1^* is such that if the value of assets is equal V_1^* at T_1 then the required yield on a debt refinancing is exactly equal to R* - the promised yield on LC.

The value of the LC at time T_1 is given by

(6) $$LC_1(V_1) = \begin{cases} 0, & if\ V_1 \le F_1 \\ F_1 - \left(F_1 e^{R^*\tau} e^{-r\tau} - Put(V_1, F_1 e^{R^*\tau}, \tau, r, \sigma)\right), & if\ \ F_1 < V_1 < V_1^* \\ 0, & if\ V_1 \ge V_1^* \end{cases}$$

Expression (6) is depicted in Figure 2. In the second area where $F_1 < V_1 \le V_1^*$, the value of LC is equal to the difference between the value of a new loan (without an LC) and the promised loan at the yield R*. Obviously this is a declining function of V_1 – the higher the value of firm's assets the less valuable is the ability to take a loan at a pre-set rate R*.

In order to value LC at time t=0 we need to integrate the expression (6) with respect to all possible realizations of V_1. We demonstrate this by a numerical simulation below.

Simulation Results

In order to illustrate our model, and analyze the results we use the following basic numerical example:

Initial value of productive assets: V_0 = \$100, r = 5% risk free interest rate in both period 1 and period 2,

$T_1 = 1$ (one year),

$T_2 = 2$, and thus $\tau = T_2 - T_1 = 1$,

Volatility of assets' rates of return $\sigma = 20\%$. We assume that the required initial leverage is $\frac{B_0}{V_0} = 70\%$.

Based on the above parameters we get the following results for firm D (without LC): From the debt valuation formula in the first period $70 = F_1 e^{-r\tau} - Put(V_0, F_1, \tau, r, \sigma)$ we get that the face value of debt at T_1 is F_1 = \$73.86 and its YTM is y_1=5.37%. Then, at the end of the first period if $V_1 \le F_1$ there is a default of the firm, it repays partially its debt. Otherwise, if $V_1 > F_1$, the firm is solvent, it will issue a new debt based on V_1 - its realized value at T_1 so that the new debt will fully refinance the old one:

$$F_1 = F_2 e^{-r\tau} - Put(V_1, F_2, \tau, r, \sigma)$$

Denote the solution of this equation as $F_2^D(V_1)$. This is the face value promised to bondholders of C at T_2=2 in order to finance the full repayment of the debt at T_1. We plot $F_2^D(V_1)$ and the corresponding yield to maturity $y_2^D = \frac{1}{\tau} Ln\left(\frac{F_2(V_1)}{F_1}\right)$ in the following Figures 3 and 4.

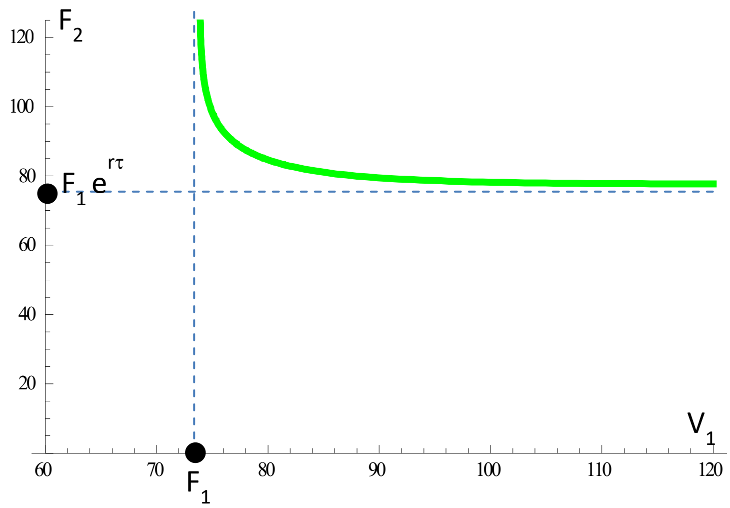

Figure 3. The face value of the new debt required for refinancing the old one.

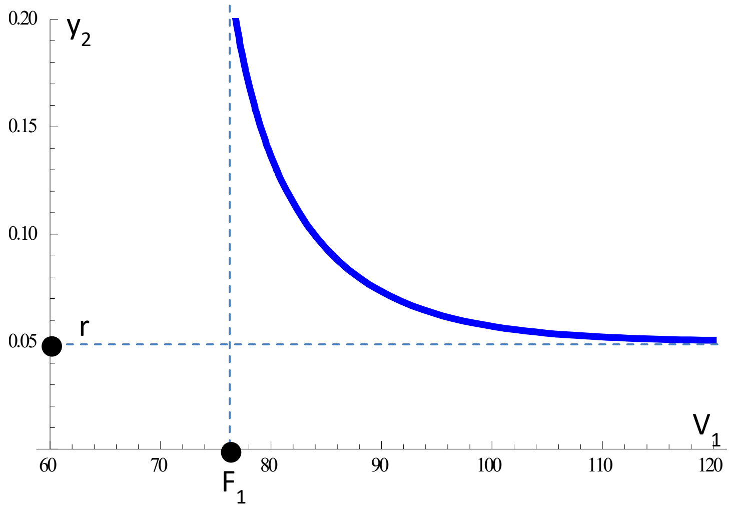

Figure 4. The yield to maturity of the new debt required as a function of V_1.

The higher is the realized value of V_1, the lower is the required yield and the corresponding face value that it has to pay at T_2, such that the economic value of this promise at T_1 is F_1. Given the initial debt at t=0 that must be refinanced at T_1=1, the higher is the realized V_1 the safer is the new debt, and its yield will converge to the riskless interest rate of 5%. At the end of the first period the (risk-neutral) probability of default is 4.8%.

Now we consider a similar firm C that purchased a LC (using additional equity raised by shareholders) at time 0. We assume that the size of LC is equal the whole amount F_1 and has the same guaranteed yield as the initial loan R*=5.37%. We also assume in the base case the bank must provide the loan as long as the firm is not bankrupt and is willing to use it at time T_1. This LC gives an option to raise the required amount of capital ($73.86) at time T_1 at the pre-determined yield R*. Of course the higher is R*, the lower is the value of the LC.

The critical value of V_1^* above which the LC will not be used is defined by $y_2^C(V_1^*) = R^*$ or alternatively by equation (5). For R*=5.37% we get $V_1^* = \$105.52$. The value of this LC at T_1 is shown in Figure 5 as a function of R* (recall that R* is bigger than the risk free rate of 5%).

Figure 2 shows the value of the LC as a declining function of the realized value of V_1. As V_1 increases the required yield $\overline{R}$ on the new debt is declining and approaches R*. Hence, the implied benefit of the LC is also declining as V_1 is increasing, and it becomes zero when $V_1 \geq V_1^*$ (see Figure 6).

The value of this LC at time zero can be calculated numerically by integrating the payoff with the risk-neutral probability of realizing V_1. Assuming the standard Geometrical Brownian motion we get that the value of such LC at time t=0 is $0.63. It means that in order to guarantee at time 0, a loan of $73.86 at time T_1=1 at the rate of R*=5.37% (while the risk free rate is 5%), the firm has to pay (at time zero) \$0.63, or 0.63/73.86 = 0.85% option premium. The shareholders should incur this cost since they benefit from potentially exercising the option at T_1=1. For example, if at T_1 the value of the productive asset of the company is, say \$80, the market required interest on the loan will be $y_2^D(80) = 13.6\%$. In this case the option to raise the same debt at 5.37% is very valuable (this option will be worth \$3.37 in this case). This last result is due to the high probability of default in the second period when V_1=80 and we take the PD into account when calculating the effect of the LC.

If the firm/shareholders would like to guarantee LC with R*=6%, the initial cost of LC will be lower, only \$0.49 and $V_1^* = \$97.12$. For the guarantee at R*=7% it will only be \$0.36. Here R* serves as a strike price of an option. Figure 5 describes the range of possible R* and the corresponding values of LC. In Figure 6 we show the yield to maturity with and without the LC. In Figure 7 we depict the probability of default for both firms C and D at time T_1=1 for the next year as a function of the realized value of productive assets V_1.

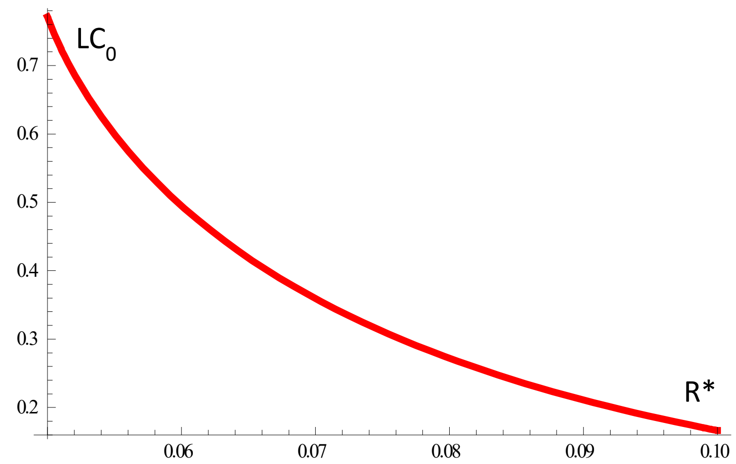

Figure 5. The value of LC_0 as a function of the promised yield R*.

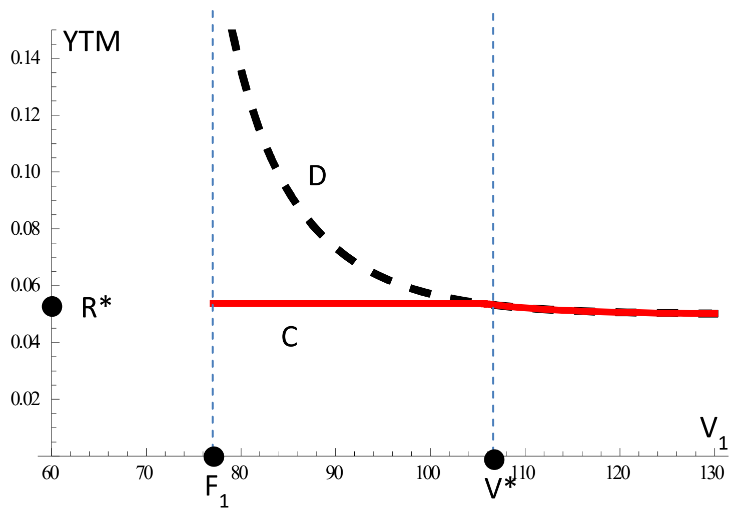

Figure 6. The yield on debt with and without LC as a function of V_1 when R*=5.37%.

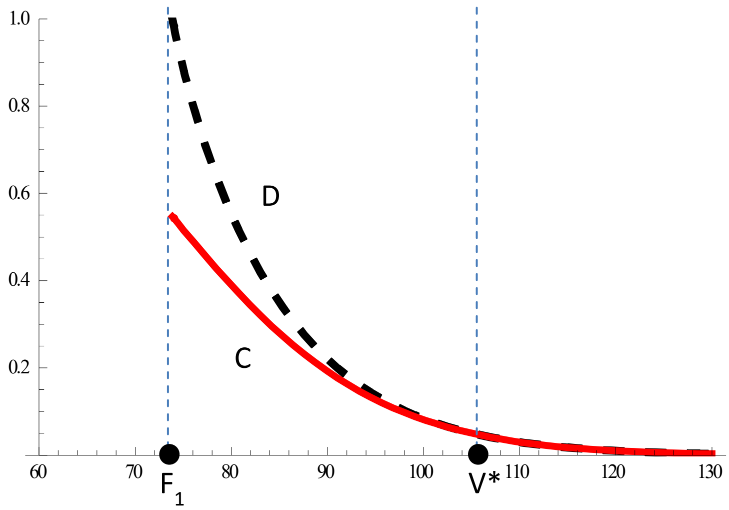

Figure 7. The PD of the firm during the second period with and without LC as a function of V_1 when R*=5.37%.

Table 1 below shows the simulation results for the present value of the LC for alternative initial leverage values, which represent the financial risk of the debt and equity, and the volatility of assets, which reflects the business risk of the firm. Each cell in the Table is based on a different promised rate R*, which is determined based on the risky YTM for each combination of B_0/V_0 and σ. For our base case of B_0/V_0=70% and σ=20%, we calculated R*=5.37% and the value of LC is \$0.63.

Leverage\volatility	15%	20%	25%	30%
60%	\$0.04	\$0.24	\$0.58	\$1.24
70%	\$0.25	\$0.63	\$0.98	\$1.23
80%	\$0.62	\$0.88	\$1.00	\$1.06
90%	\$0.53	\$0.51	\$0.46	\$0.42

Table 1. The present value of LC for alternative leverage ratios and volatilities.

Table 1 shows the current value of the LC for alternative initial leverage values and the volatility of assets of the firm. The value of the LC is very sensitive to these two parameters of the financial risk and business risk respectively. In each case we assume that R* equals the promised rate in LC is set at the level of the YTM of the debt in period 1.

It is interesting to note that while the cost is going up as expected with the leverage ratio and σ, for very high leverage ratio it is declining. This result is due to the fact that the default zones are much bigger for leverage ratios of 80% and 90%. Hence there is a smaller region for which the LC is relevant. Also in these cases R* is much higher, and affects the potential "subsidy" implicit in the LC. The R* implied in each case are given in the Table 2 below.

Leverage\volatility	15%	20%	25%	30%
60%	5.00%	5.04%	5.25%	5.74%
70%	5.05%	5.37%	6.14%	7.40%
80%	5.55%	6.76%	8.68%	11.23%
90%	8.16%	11.39%	15.42%	20.12%

Table 2. The first year yield of the loan.

Table 2 shows the YTM of the first year loan given the leverage ratio and the volatility of the firm's assets. We use these rates in Table 1 as the promised rates on the LC in the second period in each corresponding cell.

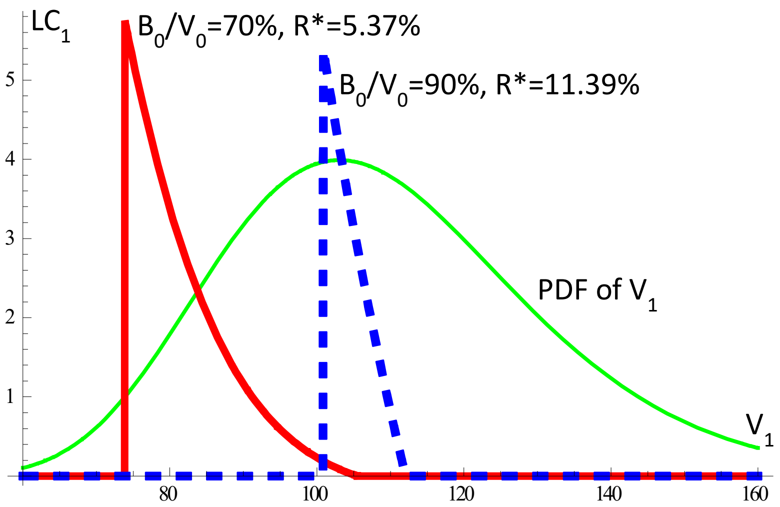

Figure 8. Value of LC as a function of V_1 for the base case (solid line) and for the case when the leverage ratio is 90% and the promised yield R*=11.39%.

In Figure 8 we depict the T_1 value of the LC as given by expression (6), for the base case (leverage ratio of 70%, volatility of 20% and R*=5.37%) as well as for the case with the leverage ratio of 90% (and hence R*=11.39%) and the same volatility of 20%. Since the volatility of the assets is the same, the distribution of the value of assets at T_1 is the same, but the range of the non-zero LC for the high leverage shifts to the right and it also narrows. However, it falls under more probable area of the distribution of V_1. All these three effects, shifting to the right, falling under higher probability and narrowing the range for non-zero LC translates to a lower cost of LC despite the higher leverage. In other words, a model is needed to consider all these effect simultaneously in order to determine the initial value of LC, as described in Table 1.

It should be noted that a better alternative for determining R*, rather than the YTM during the first period, is to calculate the forward yield for the bond in the second period. For example, for our base case the YTM in period 1 is 5.37%, and the YTM of the bond in the second period is 5.95%, therefore the forward yield for period 2 is 6.54%. For this forward rate the value of the LC drops from \$0.63 to \$0.41. As one can see the value of the LC is very sensitive to the level of promised yield.

Looking at some published data on a sample of loan commitments, as in Greenbaum and Thakor (1995), which is also reproduced in Ergungor (2001), we see that commitment fees ran between zero and 157 basis points. Our simulation results in Table 1 are consistent with the empirical data, though the sample lacks information on leverage and riskiness of the assets. Table 2 of Ergungor (reproduced from Shockley and Thakor (1997)) shows the average upfront fee for a much larger sample, classified by the stated use of the LC. For liquidity purposes the fee was 24 basis points (for duration of 28 months) while for leveraged buyouts it was 90 basis points and duration of 65 months. It is reasonable to believe that also the average leverage ratio is higher for the LBO.

Introducing the MAC (Material Adverse Change) condition to the model

In many LC agreements, the bank reserves the right to ignore the LC if the firm's situation changed in a material way *in the opinion of the bank*. The MAC (Material Adverse Change) clause is very important in real life (see for example Ergungor (2001)). In our framework we can incorporate MAC by conditioning the LC on the realized value of V_1 or on the corresponding credit risk of the firm at $T_1=1$ as measured by $y_2^C(V_1)$. In the first case we can extend the first zone in Figure 6 to be above the point F_1 but also below some value V_{MAC}, below which the credit risk is considered to be too high. Obviously the irrevocable LC is much more expensive as in this case the loan must be provided even if the firm is bankrupt in the first period.

The MAC condition plays an important role in risk mitigation as it allows the bank to fulfill its obligation to provide loan only if the firm is solvent and its credit risk did not deteriorate. It can also mitigate the risk of the firm changing its risk profile by investing in new ventures with high volatility.

We model the MAC condition by a MAC factor (MACF). When MACF = 0 we assume that the loan is irrevocable and must be provided regardless of the value of assets at T_1. Even if the firm is in default the bank will give a loan of F_1 (thus fully repaying the old debt) and will wait till T_2 hoping that something good will happen with the company's assets and it will repay its debt (at least partially). Such a loan commitment is very expensive. When MACF>0 we assume that the loan commitment will be respected by the bank only if $V_1 > F_1 \cdot MACF$. When MACF >1 we assume that the bank requires a certain cushion in the form of minimal capital in order to provide a new debt. Our basic case corresponds to MACF =1 but in practice MACF should be bigger than 1.

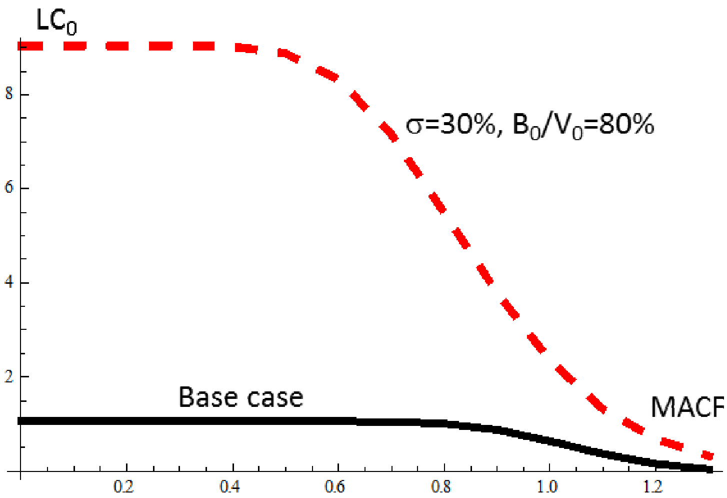

Figure 9. The value of LC as a function of the MAC factor (MACF), for the base case $R^*=5.37\%$, $\sigma=20\%$, leverage ratio $B_0/V_0=70\%$ and the dashed line shows the value of CL when $R^*=5.37\%$, but the firm is more risky with $\sigma=30\%$, and leverage ratio $B_0/V_0 = 80\%$.

In Figure 9 we show the value of the LC if the bank introduces MAC. We compare our base case to the case of higher leverage bank with $B_0/V_0=80\%$, and higher business risk $\sigma=30\%$. As can be expected the cost of the LC is falling rapidly with the increase of the MAC factor (on the horizontal axis). The more restrictive are the conditions, the lower is the value of the LC.

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Appendix A

Partial Line of Credit

We considered the case when the line of credit covers the whole amount that has to be repaid at T_1. One can use a partial line of credit that is cheaper but promises only a certain portion of the debt. Denote by K the amount of new debt that can be raised at time T_1 at the yield R*, we assume that $K<F_1$. At T_1 there are again three cases. If $V_1<F_1$ the firm is in bankruptcy and can not use the LC. If the value of assets is high enough, it will prefer raising a new debt without the LC. If the value of assets is between the two critical values, it will use the LC and raise only the remaining portion of the debt independently.

If the LC guarantees only a portion of the required debt at T_1, then we will use a pari passu assumption, meaning that both LC and the new debt have equal seniority in a case of a default. In this case at time T_1 the firm has assets with value V_1 and must repay F_1 its first period debt. Out of this amount K can be raised by using the loan commitment and the remaining amount F_1-K should be raised at market terms as a new debt. In order to find the terms of the new debt we must solve the following equation with respect to F_2:

$$F_1 - K = F_2 e^{-r\tau} - \frac{F_2}{F_2 + Ke^{R^*\tau}} Put(V_1, F_2 + Ke^{R^*\tau}, \tau, r, \sigma)$$

This equation shows that the missing amount is equal to the fraction of the total debt that can be raised in a free market.

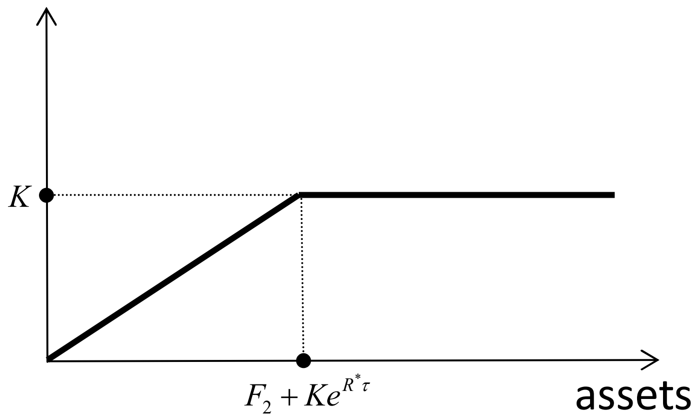

Figure A1. Payment of a partial loan at T_2 as a function of the value of the assets.

In order to decide whether to use the LC at time T_1 or not the firm can look at the following expression:

$$F_1 e^{R^*\tau} e^{-r\tau} - Put(V_1, F_1 e^{R^*\tau}, \tau, r, \sigma)$$

As long as its value is above F_1 the firm should not use the LC and it is cheaper to raise a new debt independently.