Relative Performance Banker Compensation
and Systemic Risk

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Abstract. This paper shows that in the presence of investment opportunities that are correlated, risk sharing between bank shareholders and bank managers leads to compensation contracts that include relative performance evaluation and to investment decisions that are biased to the more correlated opportunities, leading to systemic risk. We analyse various policy recommendations regarding managerial pay at major banks and show that relative performance evaluation undoes most of the intended risk-reducing effects of the policies, demonstrating their ineffectiveness in curbing systemic risk.
1. Introduction

Central banks around the world are entering unchartered territory by regulating pay of bank Chief Executive Officers (CEOs). These actions are a response to the view that bank executives’ compensation packages are one of the main culprits of the risk taking in the banking industry that preceded the recent financial crisis (e.g., International Monetary Fund, 2014). Loosely speaking, excessive risk can arise because bank CEOs are shielded from significant negative shocks to their own banks because of poorly designed compensation packages (e.g., Admati and Hellwig, 2013; International Monetary Fund, 2014; and Geithner, 2010). This paper provides a model with a novel mechanism through which pay of bank executives can lead to systemic risk in the banking industry as it leads banks to take on correlated actions. It then uses the model to analyze the effectiveness of many of the new regulatory actions by central banks in reducing systemic risk.

The question we ask is whether optimally designed compensation packages, that are not misaligned in any way by managerial entrenchment, can lead to systemic risk even in the absence of bailout guarantees. The goal is to identify contractual features in compensation that can potentially lead to systemic risk and that thus warrant pay regulation by a central bank who values social welfare losses from systemic risk.

We model two identical banks each bank with a risk-neutral principal (the shareholders) and a risk-averse agent (the CEO). Each bank has access to two investment opportunities, one with only idiosyncratic risk and another that carries risk that is correlated across banks. The agent is required to spend costly unobservable effort to increase the return of the projects available to the bank and makes an unobservable portfolio allocation of how much of each investment opportunity to pursue. To focus on risk alone we assume equal expected returns to both projects and thus an equal contribution of effort to expected returns.

As in the classical principal-agent setting with hidden action, in our model the agent is induced to deploy unobservable effort by linking her pay to the bank’s performance. However, because the agent is risk-averse, this contract can be improved by reducing the volatility in the compensation of the manager by incorporating relative performance evaluation (RPE). Having compensation depend on relative performance rather than on absolute performance works to reduce volatility of pay and is particularly effective when there is a high degree of correlation among the performance of the bank with its rival.

The novelty in the model arises from the strategic interactions between the two banks and the endogeneity of the industry return. The presence of relative performance in the compensation scheme leads the manager to choose to put more weight on investments that are common to the rival bank, as opposed to bank-specific investments subject to idiosyncratic risks. In our model there is no excess risk taking at the individual bank level but, nonetheless, there is excessive risk at the industry wide level. In addition, the weights placed by each bank in the common project are strategic complements. The more one bank chooses to invest in the common project
the greater the correlation of the banks’ overall returns if the other bank also chooses to invest more in the common project. With greater correlation comes less overall risk in pay for the same amount of relative performance in each contract. In turn, this gives rise to a strategic complementarity in the amounts of RPE in the compensation of the managers of the two rival banks: if one bank designs a compensation package with more RPE, the optimal response of the shareholders of the rival bank is to increase the RPE in the compensation of their own manager. With more relative performance and a greater weight on the common project, the manager’s pay volatility decreases but at the cost of an increasing amount of systemic risk associated with the increased likelihood of joint bank failure that comes with the greater investment in the common project.

The model offers several predictions. First, RPE in executive compensation should be common in banking, allowing shareholders to grant more powered incentives that lead CEOs to work harder, increasing bank productivity and returns. While the earlier literature of RPE produced mixed results across industries, more recent evidence from both the implicit and explicit use of RPE suggests that the generality of firms use RPE in CEO pay (see, for example, Albuquerque, 2009, on implicit RPE and Angelis and Grinstein (2016), on explicit RPE). Finance, in particular, has been found to be an industry where RPE is pervasive: Albuquerque (2014) estimates that the finance industry has one of the highest average level of RPE in CEO pay, second only to utilities firms; Angelis and Grinstein (2016) find that 37% of firms in their Money industry subsample use RPE against a corresponding figure of 34% for the overall sample; Ilic et al. (2015) examines the usage of RPE in a sample of non-US large international banks, finding that 60% disclose the usage of RPE and that the likelihood of RPE adoption increases with bank size. The usage of RPE in banking has also been shown to have increased following the deregulation of banking in the early eighties, accompanying a parallel increase in the pay-for-performance sensitivity of bank CEOs (Crawford, 1999). Moreover, as predicted by our model, empirical studies on implicit RPE usage uncover evidence of RPE only when peers are chosen narrowly to capture firms exposed to similar exogenous shocks (such as on the basis of industry and size as in Crawford (1999) and Albuquerque (2009). In the same vein, explicit RPE studies show that firms disclosing the usage of RPE based on custom peer groups select peers carefully to filter out common shocks to performance (Bizjak et al., 2016).

Second, the usage of RPE in the pay of bank executives should be accompanied by herding in the choice of risk exposures across banks, creating systemic risk. In line with this prediction, Bhattacharyya and Purnanandam (2011) report that between 2000 and 2006 — that is, the period preceding the financial crisis — the idiosyncratic

1. A related issue is whether RPE determines management turnover. Barro and Barro (1990) and Barakova and Palvia (2010) find that RPE plays an important role in the dismissal decisions of bank executives. Barakova and Palvia (2010), however, document that, in an industry downturn, absolute performance plays a more important role than relative performance in determining executive turnover, a result which they interpret as evidence that “bad times reveal the quality of management.”
risk of US commercial banks dropped by half, whereas the systematic risk doubled. This prediction is shared with the models of Acharya and Yorulmazer (2008) and Farhi and Tirole (2012) because there, too, an implicit bailout guaranty leads banks to take on correlated risk. Third, executive pay volatility decreases as industry volatility increases on account of the RPE effect. This prediction is new as it related directly to executive pay as a source of systemic risk: it can help identify our mechanism from other sources of systemic risk like bailout guarantees. Fourth, the endogenous variables of our model — intensity of incentive pay, intensity of RPE, degree of herding in bank risk exposures and amount of systemic risk — should vary over time as a function of the availability of correlated projects. In particular, the lowering of barriers to bank competition (such as regulatory impediments to competition across different geographies business lines or, yet, impediments to international trade) that enhance the creation of a unified global banking market, should produce more extreme outcomes for the model’s endogenous variables.

The second part of the paper takes a normative perspective, examining how different constraints on the compensation of bank executives either already adopted or currently being considered by regulators affect the equilibrium of the model — i.e., the endogenous optimal compensation package of managers and the endogenous optimal structure of banks’ investment portfolio — and the level of systemic risk resulting thereof. In this regard, we argue that without a regulatory constraint on the amount of RPE received by bank executives, some of the restrictive measures on executive compensation that are usually considered by regulators are ineffective in reducing systemic risk. For example, imposing a cap on equity incentives leads banks in the model to change the amount of relative performance pay in such a way as to keep incentives unchanged regarding the amount invested in the common project and hence the amount of systemic risk. On top of the inability to affect systemic risk, an unintended consequence of a cap on equity incentive pay is the reduction of the amount of managerial effort and thus on measured productivity in the industry. We view this ineffectiveness result as a formalization of the argument put forth in Posner (2009, p. 297) that “Efforts to place legal limits on compensation are bound to fail, or to be defeated by loopholes, or to cause distortions in the executive labour market and in corporate behaviour.” More than a “loophole,” we argue that different dimensions of executive pay will adjust to an artificial regulation of one dimension in isolation; and that, as a result, no positive effect will take place in terms of systemic risk; rather, a negative effect (a “distortion”) may take place in “corporate behaviour.” Murphy (2009) and Ferrarini (2015) hypothesize unintended consequences of regulating executive pay on the quality of the workforce and the productivity of the industry. Kleymenova and Tuna (2015) provide evidence that an unintended consequence of the increased regulation in the U.K. is that compensation contracts have become more complex for U.K. banks relative to other firms in the U.K. In the same spirit, the French et al. (2010) suggests that governments should not regulate the level of executive pay in financial firms because markets are better at setting prices.
Literature Review. A large literature examines the motivations for herding in managerial decisions. Within this literature only a few authors study the choice of projects or business activities by banks and the systemic risk resulting from correlated choices, but none that we know go on to study the implications of constraining parameters of the compensation contract.

The papers that are closer to us associate endogenous executive compensation with endogenous investment choices. Zwiebel (1995) assumes that managers have private information about their ability and make an unobservable choice between a standard industry project and a non-standard project that delivers a higher mean return. Relative performance evaluation in managerial pay filters out from realized project returns systematic industry factors, thus improving the inference with respect to managerial quality particularly so if the manager chooses the standard project. Zwiebel shows that managers of average quality herd in the standard project while managers of either high or low ability choose the non-standard project. Ozdenoren and Yuan (2014) analyze a generic industry populated by a continuum of principal-agent pairs, where each pair faces a classical moral hazard problem. The novelty of the model is the assumption that the return obtained by each pair depends on the effort made by the agent and on an unobservable aggregate shock in a multiplicative fashion, and on a firm-specific shock. The aggregate return therefore equals the aggregate shock times the average effort level in the industry. As in Zwiebel, the closer the agent’s effort level is to the industry’s average, the more informative is the industry return and the more valued is relative performance evaluation. The main difference with our setting is that in Ozdenoren and Yuan the choice of risk is tied to the choice of return; agents’ effort choices become correlated and systemic risk is higher when expected industry productivity is high. In contrast to Zwiebel and Ozdenoren and Yuan, in our setting correlated strategies are optimal even when expected returns are equated across projects.

Another set of papers focuses on government guarantees and their role in creating incentives for banks to choose correlated strategies (Kane, 2010). In Acharya and Yorulmazer (2008) the benefit of engaging in correlated strategies arises when banks are underperforming and the central bank bails them out. The cost of engaging in correlated strategies is the additional rent that can be garnered by a surviving bank after buying the failed bank. In Farhi and Tirole (2012), the time consistent decision of the banking regulator is to bailout banks in the event of a shock if the extent of the banking crisis is big enough. This regulatory moral hazard makes banks’ choices of balance sheet risk strategic complements and banks take on correlated risks. Acharya et al. (2015) model a risk shifting problem when there is too much debt and an inadequate loan monitoring problem when there is too little debt. They show that bailout guarantees can arise in an equilibrium where banks take on excessive debt, engage in risk shifting, and fail together. Our model does not require bailout guarantees to generate systemic risk, but bailout guarantees would magnify the mechanism we describe. Our paper points to optimal private incentives to generate systemic risk in the absence of a regulator.
Other, less related mechanisms have been suggested as a way to generate correlated choices of agents in the banking industry. Acharya and Yorulmazer (2008) model banks that in order to minimize their cost of borrowing seek to minimize the information content about their exposure to systematic risk conveyed by the performance of rivals’ loan portfolio. They show that the optimal bank strategy is to undertake correlated investments. In Acharya (2009), the failure of one bank entails a recessionary spillover on surviving banks, creating an incentive among banks to fail and survive together. Allen et al. (2012) propose a model where banks diversify their idiosyncratic risks by swapping assets. There is an equilibrium clustered structure where banks hold correlated assets. Imperfectly informed creditors do not roll over short term debt in the presence of adverse signals and banks default together. Martinez-Miera and Suarez (2014) study dynamic incentives of banks and show that correlated strategies, which yield higher returns in good states, are more likely to occur after extended good aggregate periods that allow banks to accumulate capital to be used to meet potential future capital regulatory constraints.

There is a growing literature that studies restrictions on bank executive pay aimed at limiting risk taking (see for example, Hauswald and Senbet, 2009; Thanassoulis, 2012, 2014; Chaigneau, 2012; Kolm et al., 2014; Bolton et al., 2015; Hilscher et al., 2016; Asai, 2016). These papers are cast in the context of single-bank models and thus cannot disentangle bank-specific risk from systemic risk. For “too-big-to-fail” institutions bank-specific risk may be equated to systemic risk. Our focus on correlated actions as the driver of systemic risk points to a complementary concern for regulators, one that we show is intertwined with contractual features in executive compensation. Specifically, we argue that to evaluate whether risk taking at the level of individual banks translates into systemic risk one has to determine whether the risks taken by banks, large and small, are diversifiable at the industry level. If not, the problem is much more serious since the integrity of the banking system is threatened. Optimal compensation packages designed with private incentives in mind will fail to mitigate the exposure of bank portfolios to correlated risks creating a potential concern for a banking regulator.

Finally, our paper is related to a literature that studies spillovers in governance through compensation packages and the labor market for executives. As in our paper, Acharya and Volpin (2010) and Dicks (2012) show that compensation choices of firms are strategic complements and thus the weakening governance in one firm that raises pay to its CEO induces other firms to also raise pay to their CEOs and to weaken governance. Cheng (2011) shows that RPE can cause correlated choices in governance across firms when managers have career concerns. Levit and Malenko (2016) show that directors’ willingness to serve on multiple boards creates correlated choices in governance.
2. Model

Consider an industry with two banks, denoted \( i = 1, 2 \). Suppose that bank \( i \)'s CEO has a utility function \( \exp(-w_i + d_i) \), where \( w_i \) is CEO compensation and \( d_i \) the CEO's disutility from effort \( e_i \). By assuming an exponential utility function, we assume back CEOs are risk averse.\(^2\) By contrast, we assume bank shareholders are risk neutral.\(^3\)

Compensation is a linear function of own and rival bank performance:

\[
w_i = k_i + a_i r_i - b_i r_j
\]

where \( j \neq i \) and we assume \( a_i, b_i > 0 \) are compensation coefficients to be determined by shareholders as part of the CEO contract. In particular, \( b_i \) corresponds to Relative Performance Evaluation (RPE), the central issue of our analysis.

We assume the CEO's disutility of effort is quadratic:

\[
d_i = \frac{1}{2} \gamma_i e_i^2
\]

The bank's return, \( r_i \), is a combination of: effort, \( e_i \); return on an activity of a type that is available to the whole industry, \( c_i \); and return on an activity that is available to the bank alone, \( s_i \). Until Section 5 we exclude the possibility of levering.\(^4\) This implies that each bank's assets are equal to its equity; and the CEO's portfolio choice is limited to determining the fraction \( x_i \) of assets invested in common assets, where \( x_i \in [0, 1] \). We thus have

\[
r_i = e_i + x_i c_i + (1 - x_i) s_i
\]

Since our focus is on risk and correlation induced by joint portfolio choices, we assume that all underlying assets have the same expected value and variance. Specifically, we assume that \( c_i \) and \( s_i \) are normally distributed with mean \( \mu \) and variance \( \sigma^2 \); and with no further loss of generally we assume \( \sigma^2 = 1 \).

Our crucial assumption regarding the underlying assets is that, while \( s_1 \) and \( s_2 \) are independent, \( c_1 \) and \( c_2 \) are positively correlated. Specifically, we denote by \( \psi \) the covariance of \( c_1 \) and \( c_2 \) and assume that \( \psi \in [0, 1] \). We also assume that \( s_i \) is independent of \( c_i \) (as well as \( c_j \) and \( s_j \)).

The timing of the game proceeds as follows. In a first stage, risk-neutral shareholders simultaneously determine their CEO’s compensation parameters: \( k_i, a_i \) and \( b_i \). We assume that \( (k_i, a_i, b_i) \) is observed by bank \( i \)'s CEO but not by other banks.

\(^2\) We also assume that the coefficient of risk aversion is equal to 1. Our results can be generalized to bank CEOs with a coefficient of risk aversion equal to \( \eta \in \mathbb{R}_+ \).

\(^3\) We consider the shareholders’ problem below. Our risk-neutrality assumption is not innocuous: the collapse of the banking system would have to be a risk that cannot be diversified away, whereas under the risk-neutrality assumption we implicitly assume that shareholders would be able to do so.

\(^4\) Most of our results are present in a world without leverage; and the model without leverage is considerably easier to solve and analyze.
This assumption reflects the fact that compensation contracts are typically observed with considerable noise. Next, CEOs simultaneously choose effort $e_i$ and portfolio structure $x_i$. Finally, Nature generates the values of $c$ and $s_i$; and payoff as paid.

We derive the Nash equilibrium of this multi-stage game, providing conditions such that the equilibrium exists and is unique; and compare it to the benchmark where RPE is not present (that is, $b_i = 0$).

3. Portfolio choice without leverage

Substituting (2) for $r_i, r_j$ in (1), we get

$$w_i = k_i + a_i (e_i + x_i c_i + (1 - x_i) s_i) - b_i (e_j + x_j c_j + (1 - x_j) s_j)$$

(3)

For simplicity, we assume that $\mu = 0$. It follows that the first and second moments of CEO compensation are given by:

$$\mathbb{E}(w_i) = k_i + a_i e_i - b_i e_j + (a_i - b_i) \mu$$

(4)

$$\mathbb{V}(w_i) = a_i^2 x_i^2 + b_i^2 x_j^2 - 2 a_i b_i x_i x_j \psi + a_i^2 (1 - x_i)^2 + b_i^2 (1 - x_j)^2$$

(5)

Since $w_i$ is linear in $r_i$ and $r_j$; and since the latter are normally distributed; it follows that the CEO’s utility maximization problem is equivalent to

$$\max_{e_i, x_i} \mathbb{E}(w_i) - \frac{1}{2} \mathbb{V}(w_i) - \frac{1}{2} \gamma_i e_i^2$$

(6)

The first-order condition with respect to $e_i$ is given by

$$a_i - \gamma e_i$$

and so

$$e_i^* = a_i / \gamma_i$$

(7)

where a * denotes optimal (or best-response) value. This is a standard principal-agent result: effort is increasing in performance evaluation and decreasing in the disutility of effort parameter. We next move to the CEO’s optimal portfolio choice. The first-order condition with respect to $x_i$ is given by

$$-a_i (a_i x_i - \psi b_i x_j) + a_i^2 (1 - x_i) = 0$$

(8)

(Notice the second-order condition is satisfied if and only if $a_i > 0$.) It follows that

$$x_i^* = \frac{1}{2} + \frac{\psi b_i x_j}{2 a_i}$$

(9)

If there is no RPE — that is, if $b_i = 0$ — then $x_i^* = \frac{1}{2}$. This corresponds to the standard result of risk lowering by portfolio diversification. Since the assets $c_i$ and $s_i$ are identically and independently distributed, it is optimal to split the portfolio equally across them. By contrast, setting $b_i > 0$ induces a demand for hedging: by increasing the value $x_i$, bank $i$’s CEO decreases the variance of its compensation. An immediate implication of (9) is that
Proposition 1. $x_i^*$ is increasing in $x_j$.

The intuition is that, under relative performance evaluation (that is, with $b_i > 0$) choosing the common asset $c_i$ is a form of “insurance” by bank $i$’s CEO. Specifically, under relative performance evaluation, a high value of $c$ is bad news for firm $i$’s CEO to the extent that firm $j$’s CEO has chosen that asset. In order to hedge against this adverse outcome, bank $i$’s CEO optimally chooses to place a greater weight on asset $c$ as well. In other words, Proposition 1 states that $x_i$ and $x_j$ are strategic complements: bank $i$’s CEO benefits from investing in $c$ because bank $j$’s CEO does so. In fact, this allows us to characterize the equilibrium of the portfolio-choice game as well as its comparative statics with respect to performance evaluation parameters:

Proposition 2. If $a_i \geq b_i > 0$, then the portfolio-choice game has a unique equilibrium. Moreover, the equilibrium levels $\tilde{x}_k$ are strictly increasing in $b_i$.

In other words, CEOs choose the common asset to the extent that rival CEOs choose the common asset and compensation is based on relative performance.

We now turn to the analysis of overall industry returns, which are given by

$$R \equiv \sum_{i=1,2} r_i = \sum_{i=1,2} e_i + x_i c_i + (1 - x_i) s_i$$  \hspace{1cm} (10)

We define systemic risk as the variance of overall industry returns, $\mathbb{V}(R)$. The next result, which is a corollary of Proposition 2, characterizes $\mathbb{V}(R)$.

Proposition 3. An increase in $b_i$ leads to an increase in systemic risk.

In words, Proposition 3 encapsulates one of our main results: relative performance evaluation may lead to an increase in systemic risk. The irony of Corollary 3 is that the increase in overall risk results from the CEOs desire to reduce their individual risk.

4. Corporate governance

We now take one step back and consider the optimal (and equilibrium) choices by shareholders. Bank $i$’s shareholders, who we assume are risk neutral, choose $k, a_i, b_i$ so as to maximize the expected $r_i - w_i$. Specifically, the maximization problem is given by

$$\max_{k_i, a_i, b_i} \mathbb{E}(r_i - w_i)$$

s.t. \hspace{0.5cm} $\mathbb{E}(w_i) - \frac{1}{2} V(w_i) - d_i(e_i) \geq u_i$

\hspace{1cm} $e_i = e^*_i(a_i)$

\hspace{1cm} $x_i = x^*_i(a_i, b_i; x_j)$  \hspace{1cm} (11)
Our first result in this section provides conditions such that relative performance emerges in equilibrium. First, we note that, from (9), portfolio choices are only a function of the ratio

\[ p_i \equiv b_i/a_i \]

That is, \( p_i \) measures the intensity of relative performance evaluation at bank \( i \). Given this definition, the best-response mapping (9) may be re-written as

\[ x_i^* = \frac{1}{2} \left( 1 + \psi p_i x_j \right) \]  
(12)

Notice that (12) confirms Proposition 3: an increase in relative performance by firm \( i \) (measured by \( p_i \)) leads to an increase in \( x_i \) and \( x_j \): Equation (12) shows that the partial effect is to increase \( x_i \); and supermodularity implies that both \( x_i \) and \( x_j \) increase in the resulting subgame equilibrium. As one would expect, if \( p_i = 0 \), then the CEO’s optimal portfolio choice is \( x = \frac{1}{2} \): a mean-variance-utility CEO’s optimal portfolio is to place equal weights on i.i.d. projects.

**Proposition 4.** *In equilibrium \( a_i, b_i > 0 \) (and so \( p_i > 0 \))*

Risk-neutral shareholders are indifferent with respect to their bank’s portfolio composition. However, the need to compensate risk-averse CEOs leads shareholders to “internalize” the CEO’s risk aversion. Specifically, an increase in \( b_i \) leads to a decrease in the variance of CEO pay, which in turn allows shareholders to lower base pay. In other words, the thrust of Proposition 4 is that shareholders are willing to go along with the CEO’s desire to reduce risk; and relative performance evaluation enables CEOs to follow a risk-reducing portfolio strategy.

**Comparative statics.** Proposition 4 states that, in equilibrium, relative performance evaluation is enacted. However, it does not say much regarding the level of relative performance evaluation, \( p_i \equiv a_i/b_i \), or regarding the equilibrium portfolios chosen by bank managers. The following result addresses these issues:

**Proposition 5.** *There exists a unique symmetric equilibrium. It has the property that \( x \) and \( p \) are strictly increasing in \( \psi \), ranging from \((p = 0, x = \frac{1}{2}) \) when \( \psi = 0 \) and \((p = 1, x = 1) \) when \( \psi = 1 \). Moreover, if \( \psi < 1 \) then \( p < \psi \).*

As expected, if \( \psi = 0 \), that is, if there is no correlation between the CEO’s outcome (even when they invest in the same asset), then there is no point in offering RPE \((p = 0)\): in fact, RPE would only add noise to the system without creating any additional incentive.

**The strategic nature of relative performance evaluation.** Earlier we showed that the portfolio \( x_i \) choices are strategic complements. A similar question may be asked regarding the choices of RPE, \( p_i \).
Proposition 6. There exist $\psi', \psi'' \in (0, 1)$ such that, if $\psi < \psi'$ (resp. $\psi > \psi''$), then $p_1$ and $p_2$ are strategic complements (resp. substitutes).

The simpler intuition for Proposition 6 corresponds to the case when $\psi$ is small. When that is the case, an increase in $p_2$ leads to an increase in $p_1$: RPE choices are strategic complements. By (12), an increase in $p_2$ leads to an increase in $x_2$. Given that $x_2$ is greater, the potential for variance decrease by increasing $x_1$ is greater. As a result, the incentive for Bank 1’s shareholders to increase RPE also increase.

Formally, the proof of Proposition 6 develops along the following lines. As shown in the Proof of Proposition 4, the first-order condition for shareholder $i$ payoff maximization with respect to $b_i$ implies

$$p_i = \frac{\psi x_i x_j}{x_j^2 + (1-x_j)^2}$$

(13)

In other words, it’s as if shareholder $i$ “anticipates” the values of $x_i, x_j$ and, accordingly, adjusts the choice of $p_i$. Now suppose that $\psi$ is small, specifically close to zero. Then $x_j$ is close to $\frac{1}{2}$. It follows that a small change in $x_j$ has little effect on the denominator of (13). Therefore, all of the action is in the numerator, which is increasing in $x_i$ and $x_j$. An increase in $p_j$ leads to an increase in $x_j$ (cf (12)), and supermodularity implies that $x_i$ increases as well. Together, this implies an increase in $p_i$. At the opposite extreme, if $\psi$ is close to 1, then the denominator is increasing in $x_j$ (at a high rate), which more than compensates for the increase in the numerator and implies that the increase in $x_j$ leads to a decrease in $p_i$. The idea is that the increase in $x_j$ increases the variance in pay from choosing the common project to such a high level that shareholders are better off by placing less weight on relative payoff.

5. Leverage

Up to now we assumed that, in addition to effort, the bank manager’s choice is limited to the allocation of $\$1$ across two different assets. This precludes the possibility of leverage. By contrast, in this section we assume that the bank’s assets, $x_{ci} + x_{si}$, may be greater than the bank’s equity, which we continue to assume is fixed at $\$1$. For simplicity, we maintain the assumptions that $\mu_i = \mu_c = \mu$ and that $\sigma_i = \sigma_c = \sigma = 1$. These assumptions allow us to focus on the strategic motives leading bank managers to choose a given portfolio (that is, motives different from each asset’s intrinsic value). Finally, we continue to assume that $\psi$ measures the correlation between individual and common asset returns.

Introducing leverage shows that some of the intuitions presented earlier are remarkably robust; it also brings new ideas to the fore. Accordingly, in this section we focus primarily on differences with respect to the previous analysis. Assuming that the bank is able to borrow at the risk-free rate $r_b$, the bank’s return is now given by

$$r_i = e_i + x_{ci} \tilde{c}_i + x_{si} \tilde{s}_i + (1-x_{ci} - x_{si}) r_b$$
We can then write
\[ r_i = e_i + x_{ci} (\tilde{c}_i - r_b) + x_{si} (\tilde{s}_i - r_b) + r_b \]
or, defining \( c_i = \tilde{c}_i - r_b, s_i = \tilde{s}_i - r_b, \)
\[ r_i = e_i + x_{ci} c_i + x_{si} s_i + r_b \tag{14} \]
Asset allocations are constrained by \( x_{ci}, x_{si} > 0 \). Leverage occurs when \( x_{ci} + x_{si} > 1 \).

Below we give conditions for positive leverage.

\section*{Leverage ratios and balance sheet.}
As mentioned earlier, our setup assumes that the bank has $1 of equity to invest. In the benchmark model (without leverage) the bank’s assets are given by \( x + (1 - x) = \$1 \). With leverage, however, assets equal equity plus debt, and so total assets can be larger than equity. Specifically, assets equal \( x_c + x_s \), whereas leverage equals \( (x_c + x_s) - \$1 > 0 \) (a negative number is the bank holds cash or a safe asset).

In this more general framework, the dollar amounts invested in the common and idiosyncratic projects (\( c \) and \( s \)) can no longer also be seen as percentages of the value of equity, as in the benchmark model. Instead, we now express portfolio choices as percentages of total assets, \( x_c + x_s \):
\[ z \equiv x_c + x_s \]
\[ x \equiv x_c/z \]
\[ 1 - x = x_s/z \]
The return
\[ r_i = e_i + x_{ci} \tilde{c}_i + x_{si} \tilde{s}_i + (1 - x_{ci} - x_{si}) r_b \]
should therefore be interpreted as the return on equity, since
\[ e_i + x_{ci} \tilde{c}_i + x_{si} \tilde{s}_i \]
is now the return on assets, and,
\[ \frac{(x_c + x_s) - \$1}{\$1} = z - 1 \]
is now the debt/equity ratio, and,
\[ r_b \]
is the return on debt.

\section*{Compensation.}
Similarly to (3), bank \( i \) manager’s compensation is given by
\[ w_i = k_i + a_i r_i - b_i r_j \]
\[ = k_i + a_i (e_i + x_{ci} c_i + x_{si} s_i + r_b) - b_i (e_j + x_{cj} c_j + x_{sj} s_j + r_b) \]
\[ = k_i + a_i e_i - b_i e_j + a_i x_{ci} c_i - b_i x_{cj} c_j + a_i x_{si} s_i - b_i x_{sj} s_j + a_i r_b - b_i r_b \]
Similarly to (4), mean and variance of bank manager’s pay are given by

\[
\begin{align*}
\mathbb{E}(w_i) &= k_i + a_i e_i - b_i e_j + (a_i(x_{ci} + x_{si}) - b_i(x_{cj} + x_{sj})) \mu + (a_i - b_i) r_b \\
\mathbb{V}(w_i) &= a_i^2 x_{ci}^2 + b_i^2 x_{cj}^2 - 2 a_i b_i x_{ci} x_{cj} \psi + a_i^2 x_{si}^2 + b_i^2 x_{sj}^2
\end{align*}
\]

**Leverage and portfolio composition.** Our first results provides conditions such that the equilibrium composition of bank asset portfolios is the same with and without leverage.

**Proposition 7.** For given RPE ratios \(p_i \equiv a_i/b_i\), portfolio composition choices \(x_i\) are invariant with respect to the degree of leverage.

In the proof of Proposition 7, we show that the best-response mappings for bank managers regarding portfolio structure choices \(x_i\) remain the same when we introduce leverage. This implies that the intuitions developed earlier, namely those regarding strategic complementarity, remain valid.

Before we forced the level of leverage to be zero, that is, we forced total assets to add up to 1. The next result characterizes the endogenous value of leverage chosen by bank managers if they have the freedom to do so.

**Proposition 8.** In a symmetric equilibrium \((a_1 = a_2 = a, b_1 = b_2 = b)\), bank leverage \(z - 1\) is positive if and only if \(\mu > a \frac{(1 - \psi p)}{(2 - \psi p)}\), where \(p \equiv b/a\).

Our results regarding the level of leverage follow the basic intuition regarding portfolio choice by a risk-averse agent; everything else constant, the greater the value of \(\mu\), the greater the attraction of increasing asset levels; likewise, everything else constant, the lower the combined risk of asset investments, the greater the level of assets chosen by bank managers.
6. Public Policy

At different banking jurisdictions, the recent regulatory trend has been to fix criteria for the design of pay structures that meet the international principles and standards issued by the Financial Stability Board in 2009 (FSB Principles for Sound Compensation Practice, 2009). These standards were formulated at a sufficient level of abstraction so as to allow for the smoothing of conflicts among members countries and insert flexibility in implementation. For example, with respect to the structure of pay, the FSB simply advocates the alignment of compensation with prudent risk taking, with the latter encompassing all types of risks.

In Europe the FSB standards were implemented through detailed rules enacted by primary legislation. The most important is the 4th Capital Requirements Directive (CRD IV, 2013), which states that variable compensation cannot exceed 100% of fixed pay, with at least 40% of it deferred for a minimum of 3 years.\(^5\) The European Banking Authority (EBA) subsequently issued detailed technical standards to clarify and interpret the rules enshrined in CRD IV. EBA takes a broad interpretation of variable compensation, including in it all compensation that is not contractually predetermined. It states that variable pay should be based on risk-adjusted performance and that the criteria to gauge performance may include measures of absolute performance as well as measures of relative performance vis-à-vis industry peers.\(^6\)

An extreme position is being taken by Israeli legislators, who have approved a cap on total pay (of around 650,000 USD). In contrast to Europe and Israel, the US has followed a regulatory approach based on the ex-post supervision of banks to check for consistency of FSB principles on sound compensation policies. Hence no specific quantitative limits on pay (such as caps on variable pay or floors on deferred pay) have been set.\(^7\)

In this section we use the model developed in the previous sections to remark on the strengths and weaknesses of some of these public policy measures and proposals. Our analysis suggests that they grossly omit the role that RPE plays in creating systemic risk, as shown in the previous sections.

CEO compensation includes several components: specifically, total pay is equal to fixed pay, \(k_i\), plus variable pay (or pay for performance), \(a_i r_i - b_i r_j\). Variable pay, in turn, is equal to incentive pay, \(a_i r_i\), plus RPE pay, \(-b_i r_j\). In what follows, we consider regulations that address each of these components of CEO compensation.

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5. Member states can set more stringent limits on variable pay. Member states may also allow shareholders to approve a higher maximum (up to 200%) by a supermajority vote (see article 94, (g) (ii)).

6. EBA also states that “relative measures could encourage excessive risk taking and need always to be supplemented by other metrics and controls” (Executive Summary, 44), but is unclear as to whether excessive risk refers to bank idiosyncratic risk or industry-wide risk.

7. An exception are financial institutions that were bailed out through TARP. The highest paid executives of these firms had their salaries capped at 500,000 USD while under the support of the US Treasury.
Caps on incentive pay. Consider first a cap in the form $a_t \leq \pi$, that is, an upper bound on the own-performance variable pay coefficient. The following result provides an irrelevance result that speaks to the ineffectiveness of incentive pay regulation.

**Proposition 9.** Consider a cap on incentive pay: $a_t \leq \pi$, where $\pi > 0$. In the model without leverage, the cap does not change the equilibrium level of systemic risk. In the model with leverage, there is $\gamma^* > 0$ such that for $\gamma < \gamma^*$ the cap increases the equilibrium level of systemic risk.

Recall that in both models, the share of assets invested in the common project only depends on the ratio $p_i \equiv b_i/a_i$; and $p_i$ is thus a sufficient statistic for systemic risk. In the model without leverage, the variance of pay can be written as $\mathbb{V}(w_i) = a_i^2 f(p_i, x_{ci}^*(p_i), x_{cj})$, so the choice of $b_i$, which minimizes the variance of pay, is proportional to the choice of $a_i$. Thus, any active constraint on $a_i$ leads to a proportional change in $b_i$ that keeps $p_i$ constant and systemic risk unchanged. In the model with leverage, an active constraint capping the value of $a_i$ leads to a change in $b_i$ that is less than proportional, and $p_i$ increases. Intuitively, all else equal, a lower $a_i$ leads to an increase in the dollar value invested in the common project and in the specific project, and a consequent increase in leverage. This increase in leverage increases the variance of pay, which must be compensated by an increase in $b_i/a_i$, which further increases the investment in the common project. Thus, a cap on $a_i$ leads to an increase in systemic risk.

Strictly speaking the actual proposal in CRD IV is not to cap $a_i$, but rather to cap variable pay at 100% of fixed pay, that is $a_i r_i - b_i r_j \leq k_i$. This leads to a compensation level given by

$$w_i = k_i + \min \{a_i r_i - b_i r_j, k_i\}$$

The second component of pay is equivalent to the payout from shorting a put with the put’s underlying being $a_i r_i - b_i r_j$ and its strike price being $k_i$. Under this constraint, compensation is weakly increasing and concave on $a_i r_i - b_i r_j$. As the utility function is increasing and concave over $w_i$, the utility function remains increasing and concave over $a_i r_i - b_i r_j$. The shareholder therefore still cares about the negative effect that the volatility of $a_i r_i - b_i r_j$ has on the manager’s utility, and will try to use RPE to reduce that volatility. While the specific implications from a constraint that introduces a kink in compensation are hard to derive analytically in our setting, the mechanism in the previous sections should still apply, generating investments in the common project that are strategic complements and that increase in the amount of RPE.

While the effect of incentive-pay regulation does not seem to improve systemic risk in the model, it may actually have a strictly negative overall efficiency effect. A
binding constraint that causes \( a_i \) to be lower than the equilibrium outcome reduces the amount of effort by bank executives and lowers the value added of the financial industry.\(^9\)

We view our ineffectiveness result as an illustration of the argument put forth in Posner (2009, p. 297) that

Efforts to place legal limits on compensation are bound to fail, or to be defeated by loopholes, or to cause distortions in the executive labour market and in corporate behaviour.

More than a “loophole,” we argue that different dimensions of an executive pay will adjust to an artificial regulation of one dimension in isolation; and that, as a result, no positive effect will take place in terms of systemic risk; rather, a negative effect (a “distortion”) may take place in “corporate behavior.”

The above discussion comes with a caveat. Our relatively simple model of banking competition is purposely simple and ignores potentially important features of the banking industry. Some of these may provide an independent justification for caps on variable pay. That possibility notwithstanding, our results suggest a fundamental weakness of the proposed measures: since RPE can be used to reduce the bank executive’s compensation risk, it can also be used to undo at least partly the intended risk-reduction goal of a cap on incentive pay.

Finally, we note that in the model without leverage a cap on variable pay reduces mean total compensation. To see this, recall that the individual participation constraint is given by

\[
\mathbb{E}(w_i) - \frac{1}{2} \mathbb{V}(w_i) - \frac{1}{2} \gamma_i e_i^2 = u_i
\]

In equilibrium

\[
\mathbb{V}(w_i) = a_i^2 x_i^2 + b_i^2 x_j^2 - 2 a_i b_i x_i x_j \psi + a_i^2 (1 - x_i)^2 + b_i^2 (1 - x_j)^2
\]

\[
= a_i^2 \left( x_i^2 + p_i^2 x_j^2 - 2 p_i x_i x_j \psi + (1 - x_i)^2 + p_i^2 (1 - x_j)^2 \right)
\]

Because the term in curved brackets remains unchanged with the cap on \( a_i \) (recall that \( p \) and \( x \) are unchanged), \( \mathbb{V}(w_i) \) decreases with the cap on incentive pay (that is, \( \mathbb{V}(w_i) \) is increasing in \( a_i \)). Likewise \( e \) also decreases. Hence, mean total compensation decreases. Intuitively, the executive in the model is risk averse and cares about volatility. If she faces lower volatility, she does not require as much total pay. This result contrasts with some arguments that mean total pay will not decrease (e.g., Murphy, 2013). In the model with leverage, a cap on \( a \) may result in an increase in leverage that increases volatility of total pay, in which case the executive requires greater compensation.

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\(^9\) This point has been made by several authors in different contexts, specifically regarding the cap on incentive pay in the banking industry (e.g., Bhagat et al., 2008; and Murphy, 2009; Murphy (2013)).
It is reasonable to think of imposing caps on the component of pay for peer performance, $b$, since that’s what’s causing the bias towards the common project and the increase in systemic risk. In fact, we can show that in both models (with and without leverage) an active cap on $b$ leads to a lower $p$. In the model without leverage, this translates into lower investment in the common project and lower systemic risk, since the benefit of hedging is now lower for the executive. For the model without leverage it is not possible to sign the change in investment in the common project, since a lower $b$ also leads to a lower $a$ that pushes leverage up.

**Caps on total pay.** To analyze the implications of a cap on total pay, we re-solve the shareholders problem, (11), imposing an additional constraint on average pay:$^{10}$

**Proposition 10.** Consider a cap on total pay: $\mathbb{E}(w_t) \leq v$, where $v > 0$. In the model without leverage, the equilibrium level of systemic risk is unchanged. In the model with leverage, a cap on total pay that is close to the equilibrium unconstrained mean value of total pay decreases the level of systemic risk.

The implication from the model without leverage is that, along the lines of Posner (2009), market mechanisms can undo the intended actions of the regulators leaving the level of systemic risk unchanged. The fact that in the model with leverage systemic risk decreases with the cap on total pay suggests the cap may act by curbing excessive leverage.

Even if there is no change in systemic risk, by having a different $a$ the equilibrium generates different levels of effort and of productivity in the financial industry. As was true for caps on $a$ alone, a constraint on total pay changes the mix of fixed, pay for performance and relative performance. In the Squam Lake Report (French et al., 2010), the authors recommend governments not to regulate the level of pay, partly due to the lack of evidence linking level of pay and risk-taking, and partly due to unintended consequences of regulating the level of pay, such as affecting the value added of the financial industry.

**Deferred pay.** The Financial Stability Board and the CRD IV call for performance to be evaluated over a multi-year period so as to

Ensure that the assessment process is based on longer-term performance and that the actual payment of performance-based components of remuneration is spread over a period which takes account of the underlying business cycle of the credit institution and its business risks (Article 94 of CRD IV).

In accordance with FSB recommendations, the CRD calls for deferments of 40%–60% of variable pay depending on the size of the pay for at least three years. Deferment

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$^{10}$ The proposed constraint is that $w_t \leq v$, which implies, though is not equivalent to, the constraint we consider.
periods are also being pursued by the UK’s Prudential Regulation Authority and the Financial Conduct Authority, arguing specifically that these are preferred to caps on incentive pay (see also French et al., 2010).

Our model can only be used to assess one of the potential benefits from deferred pay, perhaps not the most relevant one. By making a multi-year assessment, deferred pay excludes elements of business risk that are unrelated to managerial effort. In the limit when performance is measured over an infinite number of periods, there is no uncertainty in the effort-performance relation. In terms of our model, this would correspond to a decrease in the random component of performance to zero (in the limit).

Even this “small” benefit of deferred pay already implies that RPE pay would cease to play a role as a way to reduce CEO risk. As such, we might say that deferred pay and RPE pay are substitutes. This is true too of other types of RPE as in Holmstrom (1979). In this sense, proposals that call for the consideration of more and varied metrics, financial and non-financial, to evaluate executive performance (see CRD IV Article 94(a)), can also act as deferred pay does, so long as they increase the precision with which contracted performance is measured.

However, it is not clear whether deferred pay, as a substitute for RPE, will lead to an industry equilibrium with lower systemic risk. For example, if the expected return on bank-specific projects is lower than that on common projects — even if infinitesimally so —, then the reduction of noise in the effort-performance relationship brought about by deferred pay will reduce the importance of risk diversification, thereby causing banks to load on common projects (as the high expected return alternative). Moreover, the dissipation of noise in the effort-performance relationship resulting from extending the number of periods of performance assessment may occur at a faster rate for common projects than for bank-specific projects, tilting the asset allocation of banks toward common projects too. That would occur, for example, if the noise in the effort-performance relationship associated with common projects features a lower degree of serial correlation than that associated with bank-specific projects.

To conclude this section, we should note that our policy analysis assumes that outside opportunities, denoted in our model by $u_i$, do not change with the proposed policy actions that we consider. However, some commentators (e.g., Murphy, 2013) argue that by lowering the level and structure of pay, pay restrictions reduce the attractiveness of senior management positions in the banking industry vis-à-vis other sectors of activity, decreasing the talent pool and reducing the long-term ability of the financial industry to generate value added for the rest of the economy.

7. Conclusion

Our main point is that, under RPE pay, risk-averse bank CEOs are likely to coordinate on common projects as a means to reduce the variance in pay. Anticipating such behavior, shareholders have an incentive to offer RPE as a means to reduce the
expected value of CEO compensation required to satisfy the CEO’s participation constraint.

In other words, we uncover three sources of strategic complementarity: (a) under RPE pay, the more a CEO invests in a correlated project, the more the rival CEO wants to do the same; (b) the more a bank shareholder offers RPE pay, the more the rival bank’s shareholder wants to do the same; and (c) the more CEOs invest in correlated projects, the more shareholders want to increase the extent of RPE pay.

We derived a number of public policy implications of these results. One additional area that might be worth examining is international spillover effects. Suppose that two banks in two different countries (e.g., Spain and Belgium) compete in the same market; and suppose that one of the countries (e.g., Belgium) enacts regulation that effectively reduces the level of investment in common assets. Even if the other country (Spain, in our example) does not impose and regulatory restriction on its banks, strategic complementarity leads the latter to decrease their investment in common assets, in tandem with Belgium banks.
Appendix

Proof of Proposition 1: The proof follows by direct implication of (9). □

Proof of Proposition 2: Proposition 1 implies that $x_i$ and $x_j$ are strategic complements. Moreover, from (9) and the assumptions that $b_i > 0$ and $a_i \geq b_i$,
\[
\frac{dx_i^*}{dx_j} = \frac{b_i x_j}{2a_i} < \frac{b_i x_j}{a_i} \leq x_j \leq 1
\]
It follows that the reaction curves have a slope of strictly less than 1, which implies there exists a unique equilibrium. □

Proof of Proposition 3: From (10), the variance of industry returns is given by
\[
\mathbb{V}(R) = x_1^2 + x_2^2 + 2\psi x_1 x_2 + (1-x_1)^2 + (1-x_2)^2
\]
Since $x_j$ is chosen based on the belief regarding the value of $b_i$ (not the actual value), it follows that $d\hat{x}_j/db_i = 0$. For the same reason, the change in $x_i$ resulting from a change in $b_i$ is given by the best-response mapping, $x_i^*$, rather than the equilibrium value, $\hat{x}_i$. It follows that
\[
\frac{d\mathbb{V}(R)}{db_i} = 2\left(x_i + \psi x_j - (1-x_i)\right) \frac{dx_i^*}{db_i}
\]
Substituting (9) for $x_i, x_j$, and simplifying, we get
\[
\frac{d\mathbb{V}(R)}{db_i} = 2\left(x_j + \frac{b_i x_j}{a_i}\right) \frac{dx_i^*}{db_i}
\]
If $\mu_c \geq \mu_i$, then the term in brackets is positive. Finally, (9) implies that $dx_i^*/db_i > 0$. □

Proof of Proposition 4: At the optimum, the first constraint in (11) holds as an equality (and determines the value of $k_i$). Moreover $\mathbb{E}(r_i) = e_i$. The maximization problem is therefore equivalent to
\[
\max_{a_i, b_i} e_i - \frac{1}{2} \gamma e_i^2 - \frac{1}{2} \mathbb{V}(w_i(x_i, x_j))
\]
s.t. $e_i = \hat{e}_i(a_i, b_i)$

\[
x_i = x_i^*(x_j; a_i, b_i)
\]

\[
x_j = x_j^*(x_i; \hat{a}_i, \hat{b}_i)
\]
or simply
\[
\max_{a_i, b_i} \hat{e}_i - \frac{1}{2} \gamma \hat{e}_i^2 - \frac{1}{2} \mathbb{V}(w_i(x_i^*, x_j^*)) \tag{17}
\]
where, for simplicity, we omit the arguments of $\hat{e}_i, x_i^*$ and $x_j^*$. 19
Consider the first-order condition with respect to $b_i$. From (7), $\hat{e}_i$ is not a function of $b_i$ or $x_i$. We thus focus on the partial derivative of $\nabla(w_i)$ with respect to $b_i$ as well as the effects through changes in $x_i$.

From (4) we see that $\vartheta \mathbb{E}(w_i) / \partial x_i = 0$. It follows that the first-order condition for (6) that corresponds to $x_i$ is equivalent to $d\nabla(w_i) / dx_i = 0$. Given our assumption that bank $i$’s compensation contract is not observed by bank $j$’s CEO, it follows that $dx_i^* / db_i = 0$. In sum, the effects through CEO portfolio choices are zero. It follows that the first-order condition with respect to $b_i$ is simply given by

$$d\nabla(w_i) / db_i = \vartheta \nabla(w_i) / \partial b_i = 0$$

From (5), this first-order condition is given by

$$\left( a_i x_i \psi - b_i x_j \right) x_j - b_i (1 - x_j)^2 = 0$$

which leads to

$$b_i = \frac{\psi a_i x_i x_j}{x_j^2 + (1 - x_j)^2}$$

(18)

By the same argument as before, when computing the first-order condition with respect to $a_i$ we can ignore the indirect effects through $x_i$ and $x_j$. We thus have

$$(1 - \gamma_i e_i) \frac{de_i}{da_i} - \frac{1}{2} \frac{\partial \nabla(w_i)}{\partial a_i} = 0$$

(19)

From (7), $e_i = a_i / \gamma_i$ and $de_i / da_i = 1 / \gamma_i$. From (5)

$$\frac{\partial \nabla(w_i)}{\partial a_i} = 2 x_i \left( a_i x_i - \psi b_i x_j \right) + 2 a_i (1 - x_i)^2$$

Substituting the above equalities into (19) and simplifying, the first-order condition with respect to $a_i$ is given by

$$\frac{1 - a_i}{\gamma_i} - x_i \left( a_i x_i - b_i x_j \psi \right) - a_i (1 - x_i)^2 = 0$$

Solving for $a_i$, we get

$$a_i = \frac{1 + \gamma_i \psi b_i x_i x_j}{1 + \gamma_i x_i^2 + \gamma_i (1 - x_i)^2}$$

(20)

Finally, (18) and (20) imply that $a_i, b_i > 0$ for $x_i, x_j > 0$.

**Proof of Proposition 5:** Symmetry implies that $x_i = x_j = x$ and $p_i = p_j = p$, which in turn implies that (9) turns into

$$x = \frac{1}{2} (1 + \psi p x)$$

20
Solving for $p$ we get
$$p = \frac{2x - 1}{\psi x} \quad (21)$$

The first-order condition with respect to the relative-performance parameter $b_i$ is given by
$$\frac{\partial V(w_i)}{\partial b_i} = 2b_i x_j^2 - 2\psi a_i x_i x_j + 2b_i (1 - x_j)^2 = 0$$

At a symmetric equilibrium, this becomes
$$p = \frac{\psi x^2}{x^2 + (1 - x)^2} \quad (22)$$

Define
$$y = \frac{1 - x}{x}$$

(Note that $x$ is strictly decreasing in $y$ and that $x \in (\frac{1}{2}, 1)$ implies that $y \in (0, 1)$.)

Given this change in variable, (21) and (22) may be re-written as
$$\frac{1}{p} = \frac{\psi}{1 - y}$$
$$\frac{1}{p} = \frac{1 + y^2}{\psi}$$

(Note that either equation implies that $p$ is strictly decreasing in $y$.) Together, these equations imply
$$(1 - y)(1 + y^2) = \psi^2 \quad (23)$$

Computation establishes that (23) has two imaginary roots and a real root. Setting $\psi = 0$, the real root is $y = 1$, whereas setting $\psi = 1$ we get $y = 0$. Moreover, the derivative of the left-hand side with respect to $y$ is given by $1 + y(2 - 3y)$, which is strictly positive for $y \in (0, 1)$, implying (by the implicit function theorem) that $y$ is decreasing in $\psi$. Since $p$ and $x$ are increasing in $y$, it follows that $p$ and $x$ are strictly increasing in $\psi$. Finally, from (23),
$$\frac{1 - y}{\psi} = \frac{\psi}{1 + y^2}$$

It follows that
$$\frac{1}{p} = \frac{\psi}{1 - y} = \frac{1 + y^2}{\psi} > \frac{1}{\psi}$$

where we use the fact that $x \in (0, 1)$ and thus $y > 0$. It follows that $p < \psi$. □

**Proof of Proposition 6:** The first-order condition with respect to $b_i$ implies:
$$(x_i \psi - p_i x_j) x_j - p_i (1 - x_j)^2 = 0$$
Solving (12) for \( x_i \), we get
\[
\hat{x}_i = \frac{2 + \psi p_i}{4 - \psi^2 p_i p_j}
\]
and
\[
1 - \hat{x}_i = \frac{2 - \psi p_i (1 + \psi p_j)}{4 - \psi^2 p_i p_j}
\]
Substituting into the first-order condition and simplifying,
\[
\Phi_i \equiv (2 + \psi p_i) (2 + \psi p_j) - p_i (2 + \psi p_j)^2 - p_i \left(2 - \psi p_i (1 + \psi p_j)\right)^2 = 0 \quad (24)
\]
Differentiating with respect to \( p_i \), we get
\[
\frac{\partial \Phi_i}{\partial p_i} = \psi (2 + \psi p_j) - (2 + \psi p)^2 - (2 - \psi p_i (1 + \psi p_j))^2 \\
+ 2 p_i \left(2 - \psi p_i (1 + \psi p_j)\right) \psi (1 + \psi p_j)
\]
At a symmetric equilibrium, \( p_i = p_j = p \). Moreover, Proposition 5 implies that \( p = 0 \) if \( \psi = 0 \) and \( p = 1 \) if \( \psi = 1 \). Therefore
\[
\left. \frac{\partial \Phi_i}{\partial p_i} \right|_{\psi = 0} = -8, \quad \left. \frac{\partial \Phi_i}{\partial p_i} \right|_{\psi = 1} = -6
\]
The implicit-function theorem implies that, in the neighborhoods of \( \psi = 0 \) and \( \psi = 1 \), the sign of the slope of \( B_i(p_j) \), shareholder \( i \)'s best-response mapping, is the same as the sign of \( \partial \Phi_i / \partial p_j \). Differentiating (24), we get
\[
\frac{\partial \Phi_i}{\partial p_j} = \psi (2 + \psi p_i) - 2 \psi p_i (2 + \psi p_j) + 2 \psi^2 p_i^2 (2 - \psi p_i (1 + \psi p_j))
\]
which implies
\[
\left. \frac{\partial \Phi_i}{\partial p_j} \right|_{\psi = 0} = 2 \psi, \quad \left. \frac{\partial \Phi_i}{\partial p_j} \right|_{\psi = 1} = -3
\]
The result then follows by continuity. (Notice in particular that, at \( \psi = 0, \partial \Phi_i / \partial p_j = 0 \), but in the right neighborhood where \( \psi > 0 \) we have \( \partial \Phi_i / \partial p_j > 0 \).)

**Proof of Proposition 7:** Bank managers solve:
\[
\max_{e_i, x_i, x_{si}} \mathbb{E}(w_i) - \frac{1}{2} \mathbb{V}(w_i) - \frac{1}{2} \gamma_i e_i^2
\]
Similarly to (7), the first-order condition with respect to \( e_i \) leads to
\[
\hat{e}_i = a_i / \gamma_i
\]
Similarly to (9), the first-order condition with respect to $x_{ci}$ implies

$$x_{ci}^* = \frac{\mu + \psi b_i x_{cj}}{a_i}$$  \hspace{1cm} (25)$$

The first-order condition with respect to $x_{si}$, in turn, implies

$$x_{si} = \frac{\mu}{a_i}$$  \hspace{1cm} (26)$$

From (25) and (26), we derive the value of $x$, the relative weight of common assets in total assets:

$$x_i^* = \frac{1}{2} + \frac{\psi b_i x_j}{2 a_i}$$

which is the same as (9). ■

**Proof of Proposition 8:** In a symmetric equilibrium, (25)–(26) imply

$$x_c = \frac{\mu + \psi b x_c}{a}$$

$$x_s = \frac{\mu}{a}$$

Adding up and simplifying, we get

$$z = x_c + x_s = \frac{\mu}{a} \frac{2 - \psi p}{1 - \psi p}$$

Solving $z > 1$ yields the expression in the proposition. ■

**Proof of Proposition 9:** Consider first the model without leverage. Assume that the constraint on incentive pay is active, $a_i = \bar{a}$, otherwise there would be no change in the game's equilibrium outcome. Note that the equilibrium is still characterized by the solution $(p, x)$ that solves (21)–(22), because (22) results from the first-order condition for $b_i$ that still holds with equality. Once the value of $p$ is determined in equilibrium, $b_i$ (and $b_j$) can be appropriately adjusted for any given $a_i$ (and $a_j$). Thus a binding constraint on $a$ affects the value of $b$ but not the value of $p$. Therefore, it does not change the equilibrium allocation to the project available to the whole industry, $x$, keeping the level of systemic risk unchanged.

Consider now the model with leverage. We show first that there is $\gamma' > 0$ such that, for $\gamma < \gamma'$, the equilibrium is characterized by $\bar{a} < \frac{1}{4}$ and $\bar{p} < \frac{1}{3} \psi^{-1}$. We start by showing that if $\bar{a} < \frac{1}{4}$, then $\bar{p} < \frac{1}{3} \psi^{-1}$. To do so, we re-write equation (20) as

$$(\psi^2 - 1) p + \frac{\psi}{a} (1 - p \psi) - p (1 - p \psi)^2 = 0$$

First we show that, for each $a$, there exists a unique $p$ that solves this equation. Denote the left-hand side by $f(p; a)$. We have that $f(0; a) = \frac{\psi}{a} > 0$. Also, it can
be shown that the derivative of $f$ with respect to $p$ is everywhere negative (it has a maximum at $p = \frac{2}{3} \psi^{-1}$, at which it is negative). Hence, if there is a zero of $f(p)$, it must be unique. Next we show that if $a < \frac{1}{3}$, then $p < \frac{1}{3} \psi^{-1}$. Evaluating $f(\frac{1}{3} \psi^{-1}) = -\frac{1-4a}{3a} \psi - \frac{13}{27} \psi^{-1}$. If $a < \frac{1}{3}$, then $f(\frac{1}{3} \psi^{-1}) < 0$. With $f(0) > 0$, the zero of the function must be such that $p < \frac{1}{3} \psi^{-1}$.

Now we show that we can pick $\gamma$ small so that $a < \frac{1}{3}$ if $p < \frac{1}{3} \psi^{-1}$. Consider now equation (19) and assume that $p < \frac{1}{3} \psi^{-1}$. Note that $\frac{2-p \psi}{1-p \psi}$ is increasing in $p$. Thus $\frac{2-p \psi}{1-p \psi} < \frac{5}{2}$. The second term in the equilibrium condition

$$0 = (1-a) a^2 \frac{1}{\gamma} - \mu^2 \frac{2-p \psi}{1-p \psi}$$

is bounded between $2 \mu^2$ when $p$ is zero and $5 \mu^2/2$ when $p = \frac{1}{3} \psi^{-1}$. A low enough $\gamma$ increases the first term so that $a$ has to decrease if the equality is to hold. Lower $\gamma$ means that less incentives need to be granted to induce the CEO to work equally as hard. (Note that $(1-a) a^2$ is increasing in $a$ for $a < \frac{2}{3}$.) We conclude that there is $\gamma' > 0$ such that for $\gamma < \gamma'$, the equilibrium is characterized by $a < \frac{1}{3}$ and $p < \frac{1}{3} \psi^{-1}$.

Consider now the effect of a cap on $a$ when $\gamma < \gamma'$. Assume that the constraint on incentive pay is active, which replaces the equilibrium condition (19). Use equation (20) and the implicit function theorem to get

$$\frac{\partial p}{\partial a} = \frac{\frac{\psi}{a} (1-p \psi)}{-\psi^2 \frac{1-a}{a} - 2 + 4 p \psi - 3 p^2 \psi^2}$$

If the denominator is negative, then $\frac{\partial p}{\partial a} < 0$. When $p < \frac{1}{3} \psi^{-1}$, the denominator is such that

$$-\frac{1-a}{a} \psi^2 - 2 + 4 p \psi - 3 p^2 \psi^2 < -\frac{1-a}{a} \psi^2 - 2 + 4 \frac{1}{3} - 3 p^2 \psi^2 < 0$$

We’ve shown that for low enough $\gamma$, $\frac{\partial p}{\partial a} < 0$. Hence a cap on $a$ causes $p$ to increase. As a consequence, systemic risk increases. In fact,

$$x_c^* = \frac{\mu}{a (1-\psi p)}$$

and so

$$\frac{\partial x_c^*}{\partial a} = \frac{-\mu}{a^2 (1-\psi p)} + \frac{\psi \mu}{a (1-\psi p)} \frac{\partial p}{\partial a}$$

$$= \frac{-\mu}{a (1-\psi p)} \left( \frac{1}{a} - \psi \frac{\partial p}{\partial a} \right)$$

$$< 0$$

which concludes the proof. ■
Proof of Proposition 10: Suppose that $E(w_i) > v$, where $w_i^*$ corresponds to the unconstrained solution. Then the cap matters, that is, (21) holds as an equality. Consider first the model without leverage. Then (11) may be written as

$$\max_{a_i, b_i} e_i - v$$

subject to the participation constraint,

$$v - \frac{1}{2} V(w_i) - d_i(e_i) \geq u_i$$

as well as the constraint that $e_i$ and $x_i$ belong to the best-response mappings.

Notice that $b_i$ is not present in the objective function: from (7), $e_i$ is a function of $a_i$ but not $b_i$. It follows that the optimal $b_i$ maximizes the slack in the participation constraint. As shown in the proof of Proposition 4, this implies $\frac{\partial V(w_i)}{\partial b_i} = 0$, which in turn determines the value of $p_i = a_i / b_i$. It follows that the same value of $p_i$ obtains as in the problem without the condition (21).

Consider now the model with leverage. The problem faced by shareholders is:

$$\max_{k, a_i, b_i} E(r_i - w_i)$$

subject to

$$E(w_i) - \frac{1}{2} V(w_i) - d_i(e_i) \geq u_i$$

$$E(w_i) = v$$

and that $e_i$ and $x_c$ and $x_s$ belong to the best-response functions. Let $k_i$ be such that $E(w_i) = v$ and rewrite the problem as:

$$\max_{a_i, b_i} E(r_i) - v$$

subject to

$$v - \frac{1}{2} V(w_i) - \frac{1}{2} \gamma_i^{-1} a_i^2 \geq u_i$$

Letting $\lambda \geq 0$ be the Lagrange multiplier associated with the constraint, we have

$$\max_{a_i, b_i} E(r_i) + \lambda \left( v - \frac{1}{2} V(w_i) - \frac{1}{2} \gamma_i^{-1} a_i^2 - u_i \right)$$

Notice that this problem has the same solution to the unconstrained problem

$$\max_{a_i, b_i} E(r_i) - \frac{1}{2} V(w_i) - \frac{1}{2} \gamma_i^{-1} a_i^2 - u_i$$

if $\lambda = 1$.

We start by evaluating $\lambda$ when the constraint on pay is just binding, i.e., $E(w_i^*) = v$. Use the constraint (27) (with equality) to get implicitly the value of $a$ given $b$, $a = f(b; v)$, and use the first order condition w.r.t. $a$ to get the value of $\lambda$:

$$\lambda = \frac{dE(r_i) / da}{\gamma_i^{-1} a_i}$$

$$= \frac{1}{a_i} - \gamma \frac{2 - p \psi \mu^2}{1 - p \psi a_i^2}$$

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where we have used the equilibrium solution for $x_c$ and assumed symmetry. When the cap on pay is just binding, equation (19) holds we get that $\lambda = 1$. Now compute

$$\frac{d\lambda}{da} = \frac{\partial \lambda}{\partial a} + \frac{\partial \lambda}{\partial p} \frac{dp}{da}$$

$$= -\frac{1}{a_i^2} + 3\gamma 2 \frac{1 - p\psi}{1 - p\psi \frac{\mu^2}{a_i^4} - \gamma} \psi \frac{\mu^2}{a_i^3} \frac{dp}{da}$$

To evaluate the last term, use the implicit function theorem to get

$$\frac{df}{db} = -\frac{\frac{1}{2} \frac{\partial \mathcal{V}(w_i)}{\partial b}}{\frac{1}{2} \frac{\partial \mathcal{V}(w_i)}{\partial a} + \gamma^{-1} a_i}$$

Below we show that $\frac{\partial \mathcal{V}(w_i)}{\partial a} = 0$ and from the first order condition w.r.t. $b$, it can be shown that $\frac{\partial \mathcal{V}(w_i)}{\partial b} > 0$. Thus

$$\frac{dp}{da} = \frac{1}{a} \frac{db}{da} - \frac{p}{a} < 0$$

When the cap on pay is just binding, The sum of the two first terms in $d\lambda/da$ is positive and so $d\lambda/da > 0$.

Consider now a cap that is infinitesimally below $\mathbb{E}(w_i^*)$. It must be that the Lagrange multiplier on the constraint (27) increases, since we are below the unconstrained optimum. Because $d\lambda/da > 0$, to achieve an increase in $\lambda$ while at the same time trading off $b$ and $a$ so as to keep constraint (27) with equality, $a$ must increase and $b$ decrease. Hence $p$ decreases. Overall, systemic risk is reduced.

To conclude the proof, we show that $\frac{\partial \mathcal{V}(w_i)}{\partial a_i} = 0$. We know that

$$\mathcal{V}(w_i) = a_i^2 x_{ci}^2 + b_i^2 x_{cj}^2 - 2a_i b_i x_{ci} x_{cj} \psi + a_i^2 x_{si}^2 + b_i^2 x_{sj}^2$$

$$a_i x_{ci}^* = \mu + \psi b_i x_{cj}$$

$$x_{si}^* = \frac{\mu}{a_i}$$

$$e_i^* = a_i/\gamma_i$$

Therefore,

$$\frac{\partial \mathcal{V}(w_i)}{\partial a_i} = 2 a_i x_{ci}^2 - 2 \psi b_i x_{ci} x_{cj} + 2 a_i x_{si}^2$$

$$-2 a_i^2 x_{ci} \frac{\mu + \psi b_i x_{cj}}{a_i^2} + 2 \psi a_i b_i x_{cj} \frac{\mu + \psi b_i x_{cj}}{a_i^2} - 2 a_i^2 x_{si} \frac{\mu}{a_i^2}$$

which, after replacing the equilibrium values for $x_{ci}$ and $x_{si}$, gives the desired result.

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11. The first order condition w.r.t. $b$ yields:

$$\mu \frac{dx_c}{db} - \lambda \frac{1}{2} \frac{d\mathcal{V}(w_i)}{db} = 0$$

If $\lambda \geq 0$ and $\frac{dx_c}{db} = \frac{\psi x_{cj}}{a} > 0$, then $\frac{d\mathcal{V}(w_i)}{db} \geq 0$. 

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References


