

# The Cyclicity of Search Intensity in a Competitive Search Model\*

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## Abstract

Reasonably calibrated versions of the Diamond-Mortensen-Pissarides search and matching model of unemployment underpredict, by a wide margin, the volatility of vacancies, unemployment, and the vacancies-unemployment ratio – variables at the heart of this model. These shortcomings motivate the introduction of worker search effort that permits the unemployed to take direct action to affect the outcome of their labor market search. With this modification in place, the benchmark model captures the bulk of the standard deviation of unemployment, vacancies, and the vacancies-unemployment ratio.

**Keywords:** Variable Search Effort, Unemployment and Vacancies, Endogenous Matching Technology, Time Use, Wage Posting, Competitive Search, Unemployment Volatility by Education

**JEL Codes:** E24, E32, J63, J64

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# 1 Introduction

The Diamond-Mortensen-Pissarides (DMP) model of search and matching is a widely accepted model of equilibrium unemployment. Yet, reasonably calibrated versions of this model fail – by a wide margin – to deliver sufficient cyclical variability in key labor market variables that are central to this theory, namely vacancies, unemployment, and the vacancy-unemployment ratio (often referred to in the literature as labor market tightness); see [Andolfatto \(1996\)](#), [Merz \(1995\)](#) and [Shimer \(2005\)](#).<sup>1</sup>

This paper departs from the textbook DMP model by adding worker search effort, modeled as in [Pissarides \(2000, Ch. 5\)](#). As a result, workers can take direct action to affect the outcome of their labor market search, a channel absent from most previous studies of the DMP model, an exception being [Merz \(1995\)](#). Calibration of the search cost function is disciplined by data from the American Time Use Survey (ATUS), including some innovative work by [Krueger and Mueller \(2010\)](#) that provides evidence concerning the elasticity of search with respect to unemployment benefits. The latter work builds on a sizable micro-labor literature on the responses of the unemployed to the policy parameters of unemployment insurance programs; important early contributions include [Katz and Meyer \(1990\)](#) and [Meyer \(1990\)](#). The results below show that a model with endogenous worker search effort can account for the bulk in the cyclical variation in vacancies, unemployment, and the vacancies-unemployment ratio.

A second, innocuous change to the DMP framework is to drop what [Rogerson, Shimer and Wright \(2005\)](#) refer to as the black box of the Nash bargaining solution determination of wages in favor of competitive search which entails wage posting by firms and directed search

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<sup>1</sup>[Hagedorn and Manovskii \(2008\)](#) show that when calibrated so that the flow utility while unemployed is very close to the level of labor productivity, the textbook DMP model *can* deliver sufficient volatility in labor market variables. [Mortensen and Nagypál \(2007\)](#) persuasively argue that the [Hagedorn and Manovskii](#) calibration is unreasonable since it implies that workers and firms bargain over a surplus of around 5% of labor productivity.

on the part of the unemployed; see [Moen \(1997\)](#) and [Rogerson \*et al.\* \(2005\)](#).<sup>2</sup> Wage posting is motivated by two facts. First, as documented by [Hall and Krueger \(forthcoming\)](#), wages of newly-hired workers with less than college education are predominantly determined through wage posting, not bargaining. Second, working with data from the Current Population Survey (CPS) reveals that over 85% of the cyclical variation in unemployment is due to individuals with less than college education; see [Figures 1 and 2](#). In summary, the bulk of the cyclical variation in unemployment can be attributed to individuals with less than college education, and their wages are primarily determined by wage posting.

In the search and matching literature with Nash bargaining, the so-called [Hosios \(1990\)](#) condition is often applied so that the allocation is efficient. In brief, the [Hosios](#) condition requires that the value of the worker's bargaining parameter is equal to the elasticity of the matching function with respect to unemployment; when the matching function is Cobb-Douglas, this elasticity is the parameter on unemployment. [Section 5](#) derives an appropriate Hosios condition for the model with endogenous worker search effort and stochastic productivity.<sup>3</sup> Such a derivation is entirely unnecessary under competitive search since the allocation is always efficient; it is in this sense that using competitive search can be said to be innocuous. Furthermore, applying the Hosios condition when wages are determined by Nash bargaining is straightforward only when the matching function is Cobb-Douglas. While the Cobb-Douglas form is often chosen, other forms have been used in the literature; see, for example, [den Haan, Ramey and Watson \(2000\)](#) and [Hagedorn and Manovskii \(2008\)](#). For these alternative functional forms, imposing the Hosios condition would mean changing the worker's bargaining parameter over the cycle in order for the allocation to be efficient. This complication is avoided with competitive search.

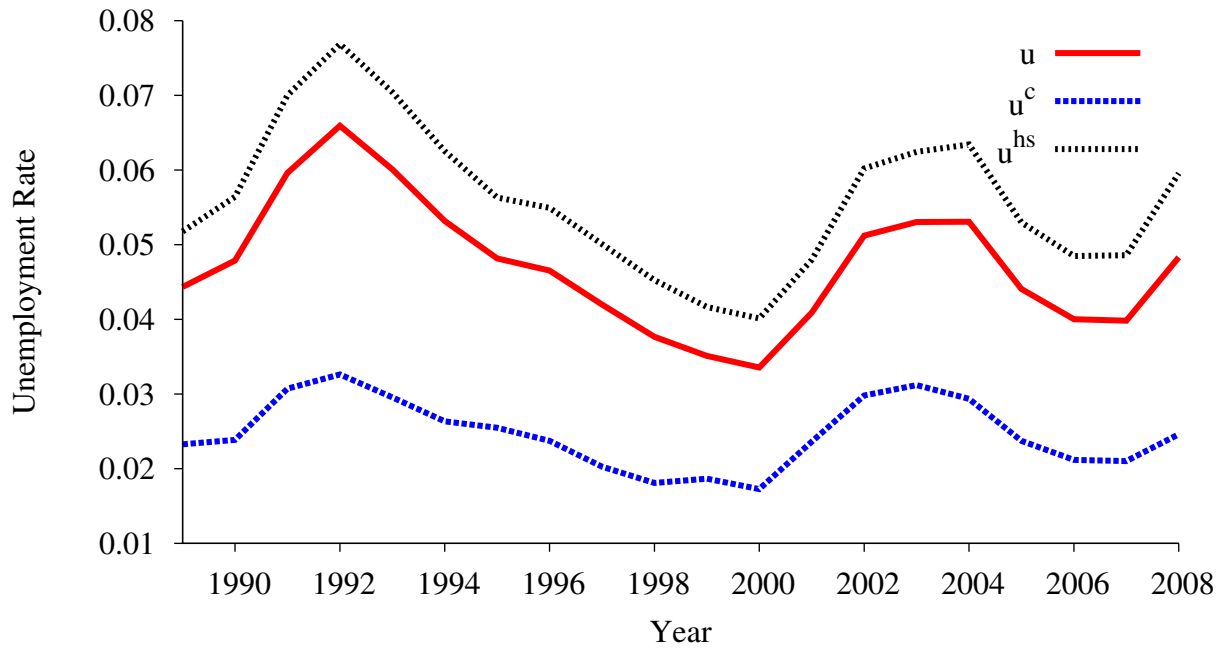
In [Shimer \(2005\)](#), the DMP model with Nash bargaining determination of wages and

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<sup>2</sup>As pointed out by [Veracierto \(2009\)](#), it is unclear what value should be used for the Nash bargaining parameter, nor is it clear how this parameter might change over the business cycle. [Section 5](#) shows how the Nash bargaining parameter should change, in order to achieve allocative efficiency, over the cycle for a popular variant of the matching function.

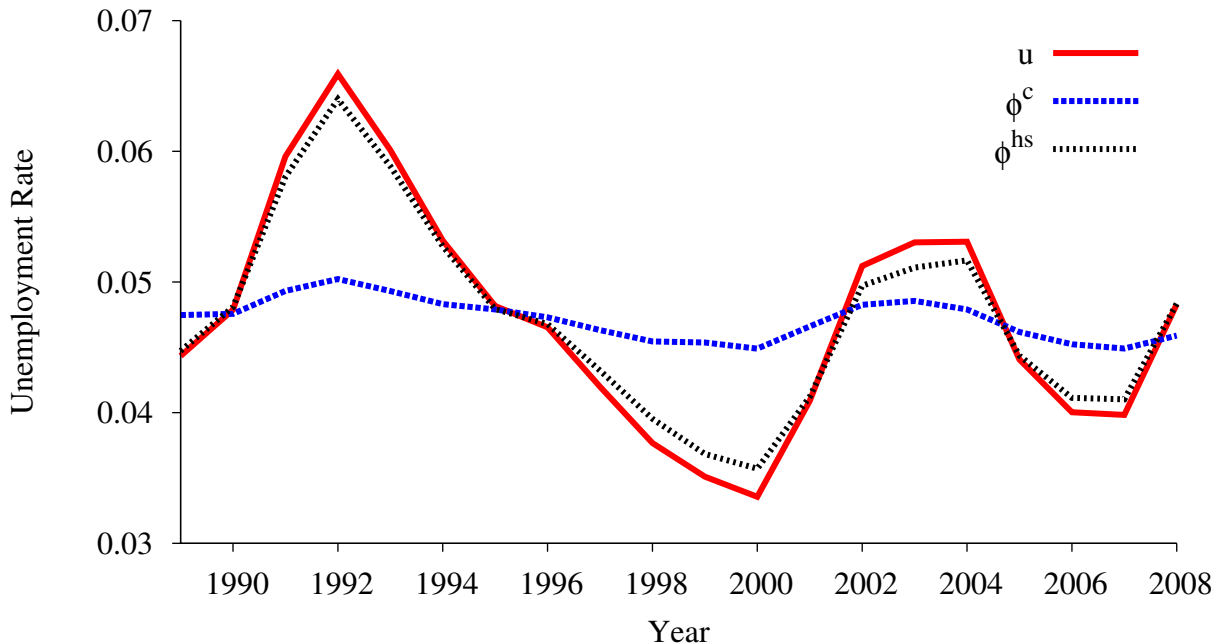
<sup>3</sup>[Shimer \(2005\)](#) extends the Hosios condition to a stochastic setting with fixed search intensity.

**Figure 1: Unemployment by Education**



**Notes:** Source: Authors' calculations from the Current Population Survey of the Bureau of Labor Statistics, which is available from the NBER website. The sample includes adult civilians aged 20-65 years who are in the labor force.  $u^c$  refers to unemployment of individuals who have at least a college degree;  $u^{hs}$  refers to unemployment of those who have less than a college degree; and  $u$  is unemployment measured using all individuals in the sample. The key observation is that aggregate unemployment follows that of less educated workers more closely than that of highly educated workers.

**Figure 2:** Decomposition of Variation of Aggregate Unemployment



**Notes:** As in Figure 1,  $u$  refers to overall unemployment.  $\phi_t^c$  measures that portion of the cyclical variation in the overall unemployment rate that can be attributed to college educated individuals. Specifically,  $\phi_t^c = \omega_t u_t^c + (1 - \omega_t) \bar{u}^{hs}$  where  $\omega_t$  is the fraction of the labor force that is college educated and  $\bar{u}^{hs}$  is the average unemployment rate of high school educated workers. Similarly,  $\phi_t^{hs} = \omega_t \bar{u}^c + (1 - \omega_t) u_t^{hs}$  gives the proportion of overall unemployment variation attributable to high school educated workers. This figure shows that aggregate unemployment fluctuations are mainly driven by unemployment of less educated workers. The coefficient of variation of these two time series over the sample period are  $CV(\phi^c) = 0.035$  and  $CV(\phi^{hs}) = 0.154$  whereas the coefficient of variation of overall unemployment is  $CV(u) = 0.182$ . In other words, unemployment of the less educated group accounts approximately 85% of aggregate unemployment variation over the sample period.

fixed worker search effort accounts for roughly 1/10 of the standard deviation of vacancies, unemployment and the vacancies-unemployment ratio. In contrast, the benchmark calibration explains just over 70% of the standard deviation of unemployment and the vacancies-unemployment ratio, and almost 80% of that of vacancies. The mechanism behind this results is as follows. An increase in productivity raises firms' incentives to post more vacancies. Such a productivity improvement also leads firms to increase their posted wages in order to increase their hiring rates. Higher wages lead workers to search more intensively. As a result, search intensity is procyclical, a result common in DMP models with endogenous search effort. This prediction is at variance with the empirical evidence in [Shimer \(2004\)](#). [Tumen \(2012\)](#) has re-examined [Shimer's](#) results. He argues that the unemployed use search methods sequentially, not simultaneously, and so [Shimer's](#) measure of search intensity – the number of search methods employed during an unemployment spell – is flawed because the number of search methods is positively associated with the length of an unemployment spell. [Tumen](#) uses the number of search methods used per week of unemployment as his measure of search intensity; he finds that search intensity is procyclical. The model's prediction of procyclical search intensity is also consistent with [Krueger and Mueller \(2010\)](#) who found that worker search intensity increases with job seekers' expected wages. Moreover, [Marinescu \(2012\)](#) shows that the number of weeks of unemployment benefits has a negative impact on the number of job applications at the state level; this result is consistent with [Krueger and Mueller \(2010\)](#) and [Tumen \(2012\)](#).

Recalibrating so that search intensity is fixed, the model accounts for about 14% of the standard deviation of unemployment, 38% of the variability in the vacancies-unemployment ratio, and 59% of the volatility in vacancies. That is to say, endogenous search effort is an important ingredient of the model, with its effects working most strongly through unemployment, and so the vacancies-unemployment ratio.

The model is also recalibrated so as to match the observed volatility of the vacancy-unemployment ratio. As shown in section 4.2, the resulting parameter values are not unrea-

sonable. This calibration can, in addition, account for virtually all of the cyclical variability in both vacancies and unemployment.

The results in this paper would be vacuous if we were unconstrained in our choice of the search cost function. Section 3.4 shows analytically that the properties of the cost function are constrained by the elasticity of the matching function with respect to the vacancy-unemployment ratio. Empirical plausibility then places strong restrictions on the cost of search function.

Yashiv (2000) appears to be the only paper that estimates the matching technology when search intensity is endogenous; he used Israeli data.<sup>4</sup> In general, ignoring search intensity may be an important oversight. The results in Section 6 show that neglecting search intensity introduces a large upward bias in the elasticity of the number of matches with respect to vacancies; this result is consistent with the empirical work of Yashiv. For the benchmark calibration, omitting search effort would lead one to erroneously conclude that a 10% increase in vacancies would increase the number of matches by more than 5% whereas the actual impact is less than 2%. Such a discrepancy should make one cautious in interpreting results from equilibrium search and matching models with fixed search intensity, particularly when quantitatively evaluating the effects of alternative public policies such as the effects of unemployment benefits, employment subsidies, and job search assistance and counseling.

Another, even more important implication of the findings in Section 6 concerns the Nash bargaining parameter, which is central to standard search and matching theory. In the literature, the Nash bargaining parameter is usually inferred from data on unemployment and vacancies (Shimer, 2005; Mortensen and Nagypál, 2007). Specifically, guided by the Hosios (1990) condition, a worker's bargaining power is set to the the elasticity of matching function with respect to unemployment. The results in Section 6 suggest that the common

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<sup>4</sup>Yashiv's 2000 principle contributions are to estimate the various frictions in the matching process, including the matching function, firm search, and worker search. He does not perform a quantitative evaluation of the model like that contained herein.

method of estimating bargaining power exhibits a strong downward bias. For example, the numerical results show that when the elasticity of matching with respect to unemployment is 0.456, the worker's bargaining power parameter required to achieve the constrained efficient allocation is not 0.456, but rather 0.802. Conversely, picking the bargaining parameter based on the measured elasticity of the matching function with respect to unemployment or vacancies cannot always guarantee constrained efficiency; see the earlier discussion regarding matching functions that deviate from the usual assumption of Cobb-Douglas.

An important issue that has arisen in the recent literature on the DMP model is the size of the surplus of a match (the difference between the value of the match, and the outside options of the firm and worker). While [Hagedorn and Manovskii \(2008\)](#) show that the DMP model can successfully explain the cyclical volatility of vacancies and unemployment, they do so by shrinking the surplus of a match to around 5% (the flow value of being unemployed is roughly 95% of labor productivity). [Mortensen and Nagypál \(2007\)](#) argue that this surplus is implausibly small since it implies that workers choose to work for an increase in their income of 2.3% of productivity over what they would receive while unemployed. In the benchmark model, the surplus of a match is around 12.5% of productivity, and workers increase their flow utility by more than 12% over that received while unemployed.

The outline of the rest of the paper is as follows. Section 2 presents a dynamic, stochastic model of equilibrium unemployment incorporating variable search intensity into a competitive search model. Section 3 explores the steady-state properties of the model. The steady state analysis is important for calibrating a number of key parameters in the model. The model is calibrated and simulated in Section 4 in order to establish the model's business cycle properties. Two experiments are conducted. The first suppresses search intensity in order to measure the impact of endogenous search on the moments of key labor market variables. In the second experiment, the model is recalibrated so as to match the volatility of the vacancy-unemployment ratio observed in the U.S. data. Section 5 extends the Hosios condition to a setting with stochastic productivity and endogenous search intensity. Impli-



cations of variable search intensity on the aggregate matching technology are discussed in Section 6. Section 7 concludes. The appendices provide detailed analytical derivations as well as a description of the numerical procedures used to simulate the model.

## 2 Model

### 2.1 Environment

The economy is populated by a measure one of infinitely-lived, risk-neutral workers and a continuum of infinitely-lived firms. Individuals are either employed or unemployed. An unemployed worker looks for a job by exerting variable search effort. The cost of searching for a job depends on how intensively the worker searches. Let  $s_i \geq 0$  be the search intensity of worker  $i$ . The cost of  $s_i$  units of search is  $c(s_i)$  where  $c$  is a twice continuously differentiable, strictly increasing and strictly convex function. Flow utility of unemployed worker  $i$  is given by  $z - c(s_i)$  where  $z \geq 0$ . Normalize the cost of search so that  $c(0) = 0$ , implying that  $z$  is flow utility of an unemployed worker who exerts zero search intensity. Flow utility of an employed worker is the wage,  $w$ . Workers and firms discount their future by the same factor  $0 < \beta < 1$ .

Each firm employs at most one worker. Per-period output of each firm-worker match is denoted by  $p$  and evolves according to a stationary and monotone Markov transition function  $G(p'|p)$  given by

$$p' = 1 - \varrho + \varrho p + \sigma_\varepsilon \varepsilon \tag{1}$$

where  $\varepsilon$  is an *iid* standard normal shock,  $0 < \varrho < 1$  and  $\sigma_\varepsilon > 0$ .<sup>5</sup> There is free entry for firms. A firm finds its employee by posting a vacancy, at the per period cost  $k$ , when looking for workers. All matches are dissolved at an exogenous rate  $\lambda$ . Matches are formed at random; the matching technology is discussed shortly.

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<sup>5</sup>As in [Shimer \(2005\)](#), mean productivity is normalized to 1.

## 2.2 Wage determination

Wages are determined via competitive search instead of Nash bargaining. The setup follows Rogerson *et al.* (2005). Given current productivity,  $p$ , a firm decides whether or not to post a vacancy. If it does, the firm decides what wage to offer in order to maximize its expected profits. An unemployed worker directs her search towards the most attractive job given the current aggregate labor market condition.

Since separations are exogenous the shape of the wage-tenure profile has no impact on job search so long as the expected present discounted value of the wage stream,  $Y$ , remains the same. Assuming that the wage of a particular match does not change over time,<sup>6</sup> the per-period wage is  $w = (1 - \beta(1 - \lambda))Y$ . Then, a vacant job is fully characterized by  $(p, w)$ . Let  $\mathcal{W}(p)$  denote the set of wages posted in the economy when aggregate productivity is  $p$ .

## 2.3 Matching technology

Matching between firms and workers operates as follows. Let  $s_{i,j}$  denote search effort by worker  $i$  for job type  $j = (p, w)$  where it is understood that  $s_{i,j}$  can be non-zero for at most one  $j$  if the person is unemployed and zero for all  $j$  if the person is employed (there is no on-the-job search). Since a worker searches for at most one type of job,  $s_i = \max_j \{s_{i,j}\}$ . Total search intensity for a job of type  $j$  is  $S_j = \int_0^1 s_{i,j} di$ . Denote total vacancies of type  $j$  by  $v_j$ . As in Pissarides (2000, Ch. 5), the total number of matches formed for a particular job type is given by the Cobb-Douglas function,

$$M_j = \mu v_j^\eta S_j^{1-\eta} \quad (2)$$

where  $0 < \eta < 1$  and  $\mu > 0$ . The (effective) queue length for a type  $j$  vacant job is given by  $q_j = S_j/v_j$ , and the probability that a particular job is filled is given by  $\alpha(q_j) = \mu q_j^{1-\eta}$ .

Denoting the measure of unemployed workers exerting non-zero search intensity for a type

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<sup>6</sup>Shifts in the average wage are driven by the share of new jobs and their wages relative to the wages of the old jobs. Section A.6 allows the wage of old matches to respond to productivity shocks.

$j$  job by  $u_j$ , the vacancy filling rate is increasing in average search intensity  $\bar{s}_j = S_j/u_j$ , and decreasing with labor market tightness  $\theta_j = v_j/u_j$ . The probability that an unemployed worker  $i$  finds a job of type  $j$  is  $\tilde{f}(q_j, s_{i,j}) = (M_j/S_j) s_{i,j} = f(q_j) s_{i,j}$  where  $f(q_j) = \mu/q_j^\eta$ . For notational brevity, the individual index  $i$  is omitted for the rest of the paper.

## 2.4 Value functions

Let the value of searching for a job paying wage  $w$  when aggregate productivity is  $p$  be given by

$$\begin{aligned} \tilde{U}(w, p) \equiv \max_{s_{w,p}} \left\{ z - c(s_{w,p}) + \beta f(q_{w,p}) s_{w,p} \int W(w, p') dG(p'|p) \right. \\ \left. + \beta (1 - f(q_{w,p}) s_{w,p}) \int U(p') dG(p'|p) \right\}. \end{aligned} \quad (3)$$

Then,

$$U(p) \equiv \max_{w \in \mathcal{W}(p)} \{ \tilde{U}(w, p) \} \quad (4)$$

where we anticipate the result that there are a finite number of elements in  $\mathcal{W}(p)$ . In other words, an unemployed worker can search for only one type of job, and will choose to search for the job that yields the highest expected utility. A worker's search effort is, then, the optimal level associated with that job type.

The value of being employed is given by

$$W(w, p) = w + \beta \lambda \int U(p') dG(p'|p) + \beta (1 - \lambda) \int W(w, p') dG(p'|p). \quad (5)$$

A firm's value of a filled job paying the wage  $w$  is given by the following asset-pricing equation:

$$J(w, p) = p - w + \beta (1 - \lambda) \int J(w, p') dG(p'|p). \quad (6)$$

Finally, the value of a vacancy when productivity is  $p$  is given by

$$V(p) = \max_w \left\{ -k + \beta \alpha(q_{w,p}) \int J(w, p') dG(p'|p) \right\}. \quad (7)$$

The remainder of the paper establishes the main properties of the equilibrium.<sup>7</sup> For expositional purposes, the analysis proceeds in two steps. Section 3, analyzes the steady state of the model. In particular, setting  $p = 1$ , the model is solved analytically and key parametric relationships that are important to the calibration of the model are obtained. In Section 4, productivity is once again stochastic, and the main properties of the model are established numerically.

### 3 Steady state analysis

#### 3.1 Workers

When there are no shocks to productivity, that is when  $p = 1$ , a job is fully characterized by its per-period wage  $w$ . Given any posted wage  $w \in \mathcal{W}$ , the value of being unemployed is given by

$$U = \max_{s_w} \{ z - c(s_w) + \beta f(q_w) s_w (W(w) - U) + \beta U \} \quad (8)$$

and the value of being employed with wage  $w$  is

$$W(w) = \frac{w + \beta \lambda U}{1 - \beta(1 - \lambda)}. \quad (9)$$

A worker will take the queue length,  $q_w$ , as given. Differentiating the right hand side of equation (8) with respect to search effort,  $s_w$ , gives

$$c'(s_w) = \beta f(q_w)(W(w) - U).$$

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<sup>7</sup>The formal definition of the labor market equilibrium is provided in Appendix A.

Combining this result with equations (8) and (9), it can be shown that, for any posted wage  $w$ , the optimal search intensity must satisfy the following:

$$w - z = \frac{1 - \beta(1 - \lambda)}{\beta f(q_w)} c'(s_w) + c'(s_w) s_w - c(s_w). \quad (10)$$

### 3.2 Firms

Firms making their vacancy posting decision will take equation (10) as given. Following Rogerson *et al.* (2005), substitute equation (10) into equation (7) for  $w$  and thereby reduce a firm's problem to the following:

$$\max_{q_w} \left\{ \alpha(q_w) \left( p - z - \frac{1 - \beta(1 - \lambda)}{\beta f(q_w)} c'(s_w) - c'(s_w) s_w + c(s_w) \right) \right\}. \quad (11)$$

Solving for the optimal  $q_w$  yields

$$p - z = \frac{1 - \beta(1 - \lambda)}{\beta \alpha'(q_w)} c'(s_w) + c'(s_w) s_w - c(s_w). \quad (12)$$

### 3.3 The steady state equilibrium

Subtract equation (10) from equation (12) to obtain

$$\frac{p - w}{1 - \beta(1 - \lambda)} = \frac{c'(s_w)}{\beta} \left( \frac{1}{\alpha'(q_w)} - \frac{1}{f(q_w)} \right). \quad (13)$$

The left hand side of equation (13) is merely the value of a matched firm; see equation (6).

On the other hand, using equation (7) and the free entry condition gives

$$k = \beta \alpha(q_w) \frac{p - w}{1 - \beta(1 - \lambda)}. \quad (14)$$

Now, combining equations (13) and (14) leaves

$$k = c'(s_w) \left( \frac{\alpha(q_w)}{\alpha'(q_w)} - q_w \right). \quad (15)$$

The labor market equilibrium is characterized by equations (12), (14) and (15). It is not immediately clear that these equations yield a unique solution for a given productivity level,  $p$ . For example, if workers search with less intensity, firms create fewer jobs. Similarly, if workers search with greater intensity, firms create more jobs. Therefore, there is a possibility of non-unique wages. However, the following proposition rules out that possibility.

**Proposition 1 (Same jobs).** *All firms posting a vacancy choose the same wage.*

*Proof.* Strict convexity of the worker cost of search function  $c$  along with equation (15) implies that worker search intensity,  $s_w$ , and queue length,  $q_w$ , have a negative relationship. Combining this fact with equation (12), it can be shown that  $q_w$  is unique across vacancies. Then, equation (14) implies that  $w$  is the same across all jobs.  $\square$

Proposition 1 implies that there is only one type of job in steady state. The uniqueness of  $q_w$  along with equation (15) implies that all unemployed workers exert the same search intensity. Given these results, we drop the subscripts of  $s$ ,  $q$  and  $\theta$ .

### 3.4 The role of variable search intensity

Now consider the response of the model economy to a permanent shift in productivity.

**Proposition 2 (Permanent shock).** *An increase in productivity raises both search intensity and the vacancy-unemployment ratio and therefore raises the job-finding rate.*

*Proof.* Given the inverse relationship between queue length,  $q$ , and worker search intensity,  $s$ , the right hand side of equation (12) is strictly increasing in  $s$ . Therefore,  $s$  increases with productivity,  $p$ . A higher  $s$  and a lower  $q$  means a higher vacancy-unemployment ratio.

More vacancies per unemployed worker along with higher search intensity imply a higher job-finding rate.  $\square$

Re-write equation (15) in the following form:

$$\theta = \frac{\eta}{(1-\eta)k} sc'(s). \quad (16)$$

Given the strict convexity of  $c$ , equation (16) implies that market tightness,  $\theta$ , increases strictly with search intensity,  $s$ . More importantly, in light of Proposition 2, equation (16) suggests that volatility of the vacancy-unemployment ratio is closely related to the search cost. This relation is quantified in the following section.

Before going to the numerical analysis, we extend the analytical results in Hagedorn and Manovskii (2008) and Mortensen and Nagypál (2007) to the model with variable search intensity. Specifically, we calculate the elasticity of the vacancy-unemployment ratio to productivity, defined as  $\frac{d \ln \theta}{d \ln p}$ , and compare it with that in a standard model or, equivalently, the model with fixed search intensity.

Let  $\tilde{\eta}$  denote the implied (or empirical) elasticity of the job-finding rate with respect to the vacancy-unemployment ratio, that is,

$$\tilde{\eta} = \frac{d \ln(f(q)s)}{d \ln \theta} = \frac{d \ln(q\alpha(q)s)}{d \ln \theta} = \frac{d \ln(\theta\alpha(q))}{d \ln \theta} = 1 + \frac{d \ln \alpha(q)}{d \ln \theta}. \quad (17)$$

Since  $\ln \theta = \ln s - \ln q$ , equation (17) can be written as

$$\tilde{\eta} - 1 = \frac{\epsilon_{q,s}}{1 - \epsilon_{q,s}} \frac{d \ln \alpha(q)}{d \ln q}, \quad (18)$$

where  $\epsilon_{q,s} = \frac{d \ln q}{d \ln s}$ . Recalling that  $\theta = s/q$ , differentiation of equation (16) gives  $\epsilon_{q,s} = -\frac{sc''(s)}{c'(s)}$  in equilibrium.

Differentiate  $\ln \theta = \ln s - \ln q$  with respect to  $\ln p$  to obtain the elasticity of the vacancy-

unemployment ratio  $\theta$  with respect to productivity  $p$ :

$$\frac{d \ln \theta}{d \ln p} = (1 - \epsilon_{q,s}) \frac{d \ln s}{d \ln p}, \quad (19)$$

Now, without loss of generality, normalize the search intensity to 1 so that  $s = 1$ .<sup>8</sup> Then, by taking logs in equation (12) and differentiating the result with respect to  $\ln p$ , it can be shown that

$$\frac{d \ln s}{d \ln p} = \frac{p}{p - z} \times \frac{\frac{1-\beta(1-\lambda)}{\beta f(q)(1-\tilde{\eta})} \frac{c'(1)}{c''(1)+c'(1)} + \frac{c'(1)-c(1)}{c''(1)}}{\frac{1-\beta(1-\lambda)}{\beta f(q)} + 1}. \quad (20)$$

Now combining equations (19) and (20) along with  $\epsilon_{q,s} = -\frac{c'(1)}{c''(1)}$ , it can be seen that

$$\frac{d \ln \theta}{d \ln p} = \frac{p}{p - z} \times \frac{\frac{1-\beta(1-\lambda)}{\beta f(q)(1-\tilde{\eta})} + \left(1 - \frac{c(1)}{c'(1)}\right) \left(1 + \frac{c'(1)}{c''(1)}\right)}{\frac{1-\beta(1-\lambda)}{\beta f(q)} + 1}. \quad (21)$$

Given convexity of the search cost function it follows that  $0 < \frac{c(1)}{c'(1)} < 1$  and  $\frac{c'(1)}{c''(1)} > 0$ , and therefore,  $C \equiv \left(1 - \frac{c(1)}{c'(1)}\right) \left(1 + \frac{c'(1)}{c''(1)}\right) > 0$ . In steady state, the unemployment rate is  $\frac{\lambda}{\lambda+f(q)}$ . Given that the average unemployment rate for the U.S. is around 6% (Shimer, 2005), it follows that  $\frac{\lambda}{\lambda+f(q)} \simeq 0.06$  which implies  $f(q) \gg \lambda$ . When the model period is relatively short,  $\beta$  is close to 1 and so  $\frac{1-\beta(1-\lambda)}{\beta f(q)} \simeq \frac{\lambda}{f(q)}$  is much smaller than 1. Further, the observed elasticity  $\tilde{\eta} \simeq 0.5$  (Petrongolo and Pissarides, 2001; Mortensen and Nagypál, 2007) and so  $\frac{1-\beta(1-\lambda)}{\beta f(q)} \frac{1}{1-\tilde{\eta}} \simeq \frac{\lambda}{f(q)} \frac{1}{1-\tilde{\eta}}$  is also much smaller than 1. The upshot is that the magnitude of the elasticity  $\frac{d \ln \theta}{d \ln p}$  is dictated by  $\frac{p}{p-z}$  and  $\left(1 - \frac{c(1)}{c'(1)}\right) \left(1 + \frac{c'(1)}{c''(1)}\right)$ .

Clearly, the magnitude of this elasticity can be made arbitrarily large by assuming a cost function such that  $\frac{c(1)}{c'(1)} \ll 1$  and  $\frac{c'(1)}{c''(1)} \gg 1$ . However, doing so will lead to highly counterfactual implications for the matching technology. Specifically, using equation (18),

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<sup>8</sup>Suppose that the search intensity is normalized to  $x > 0$ . Let the associated search cost function be  $\tilde{c}$ . Denote the vacancy cost and the coefficient of the matching function by  $\tilde{k}$  and  $\tilde{\mu}$ , respectively. The equilibrium allocations continue to be characterized by equations (12) and (15). Then, it can be seen that the same allocation is obtained by choosing the cost function to satisfy  $\tilde{c}'(x)x - \tilde{c}(x) = c'(1) - c(1) > 0$  while setting  $\tilde{k} = \frac{x\tilde{c}'(x)}{c'(1)}k$  and  $\tilde{\mu} = \frac{x^\eta\tilde{c}'(x)}{c'(1)}\mu$ .



the fact that  $\frac{d \ln \alpha(q)}{d \ln q} \leq 1$ , and  $\epsilon_{q,s} = -\frac{sc''(s)}{c'(s)}$ ,

$$C = \left(1 - \frac{c(1)}{c'(1)}\right) \left(1 + \frac{c'(1)}{c''(1)}\right) < 1 + \frac{c'(1)}{c''(1)} = \frac{1}{1 - \tilde{\eta}} \frac{d \ln \alpha(q)}{d \ln q} \leq \frac{1}{1 - \tilde{\eta}} \simeq 2. \quad (22)$$

So, the empirical elasticity of the matching function,  $\tilde{\eta}$ , dictates that  $C$  can not be much larger than 2. In fact, if search costs are given by a power function – a commonly-used specification<sup>9</sup> – then the value of  $C$  is much lower than 2. Specifically, let the function  $c$  given by the following power function:

$$c(s) = \chi s^\gamma, \quad (23)$$

where  $\chi > 0$  and  $\gamma > 1$ . Under such a parametric specification,  $C = 1$ . In other words, empirical plausibility places strong restrictions on the magnitude of  $C$  and so on the cost function  $c$ .

With the cost function in equation (23), equation (21) becomes

$$\frac{d \ln \theta}{d \ln p} = \frac{p}{p - z} \times \frac{\frac{1 - \beta(1 - \lambda)}{\beta f(q)(1 - \tilde{\eta})} + 1}{\frac{1 - \beta(1 - \lambda)}{\beta f(q)} + 1}. \quad (24)$$

For comparison purposes, we also calculate the above elasticity for the model with fixed search intensity. Here the model with fixed search intensity refers to the model where search intensity is fixed to 1 while the elasticity of the matching function and the unemployment rate are matched with their empirical counterparts. Under fixed search intensity, the elasticity is given by<sup>10</sup>

$$\frac{d \ln \theta^F}{d \ln p} = \frac{p}{p - \tilde{z}} \times \frac{\frac{1 - \beta(1 - \lambda)}{\beta f(q)(1 - \tilde{\eta})} + 1}{\frac{1 - \beta(1 - \lambda)}{\beta f(q)} + 1}, \quad (25)$$

where  $\tilde{z} = z - c(1)$ . Comparing equations (24) and (25) leads to the following key observa-

<sup>9</sup>See, for example, Christensen, Lentz, Mortensen, Neumann and Werwatz (2005).

<sup>10</sup>See Appendix B.4 for derivation. It can be seen that the elasticity given by equation (25) is the same as that obtained by Hagedorn and Manovskii (2008) and Mortensen and Nagypál (2007) after imposing the Hosios condition while setting the flow utility of unemployment to  $z - c(1)$ .

tions:

- a) The elasticity of vacancy-unemployment ratio with respect to productivity is mainly determined by  $\frac{p}{p-z}$ ,<sup>11</sup> which is somewhat consistent with Shimer (2005) and Hagedorn and Manovskii (2008).
- b) However, an important difference is that the net flow utility of an unemployed worker in the model with variable search intensity is  $z - c(1)$  while that in the models with fixed search intensity is simply  $z$ .

## 4 Business cycle properties

Here, productivity is stochastic and the model is solved numerically. Details of the numerical solution of the stochastic model are provided in Appendix A. Before calibrating the model, Proposition 1 is extended to the stochastic case.

**Proposition 3 (Uniqueness).** *Given current productivity, all firms choose to post the same wage, and unemployed workers looking for a job in a given time period exert the same search intensity.*

*Proof.* See Appendix A. □

### 4.1 Calibration

#### Standard parameters

The length of the time period is a quarter of a month, which will be referred to as a week. The discount factor  $\beta$  is set to  $1/1.04^{1/48}$ , a value consistent with an annual real interest rate of 4%. The separation rate is set to that in Shimer (2005); normalizing it to a weekly frequency,  $\lambda = \frac{0.1}{12} = 0.0083$ .

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<sup>11</sup>It should be noted that one could consider an alternative specification of the search cost so that the magnitude of  $C$  is close to 2. However, the numerical analysis in Section 4 shows that the function in equation (23) works well for the model in the sense that the implied volatility of unemployment and vacancies are remarkably close to their empirical counterparts; these volatilities are not targeted during the calibration.

The productivity process  $G(p'|p)$  is approximated by a five-state Markov chain using the method of [Rouwenhorst \(1995\)](#).<sup>12</sup> The following targets for the productivity process are taken from [Hagedorn and Manovskii \(2008\)](#): the quarterly autocorrelation of 0.765, and the unconditional standard deviation of 0.013 for the HP-filtered productivity process with a smoothing parameter of 1600. At a weekly frequency, these targets require setting:  $\varrho = 0.9903$  and  $\sigma_\varepsilon = 0.0033$ .

## Normalization

Following [Shimer \(2005\)](#), the target for the mean vacancy-unemployment ratio is 1.<sup>13</sup> Then, it follows that the effective queue length is  $q = 1$  in steady state and therefore, recalling that productivity,  $p$ , has been normalized to 1, equations (12) and (15) can be rewritten as

$$z = 1 - \frac{(1 - \beta(1 - \lambda))\chi\gamma}{\beta(1 - \eta)\mu} - \chi(\gamma - 1) \quad (26)$$

and

$$k = \chi\gamma \frac{\eta}{1 - \eta}. \quad (27)$$

Given the rest of the parameter values,  $z$  and  $k$  are chosen according to equations (26) and (27). The value of  $\mu$ , the scaling parameter in the matching function, is chosen by targeting an average unemployment rate of 5.7% ([Shimer, 2005](#)).

## The elasticity of matches to vacancies

The key parameter of the matching technology is the elasticity of matches with respect to vacancies,  $\epsilon_{M,v} = \frac{\partial \ln M}{\partial \ln v}$ . When search intensity is fixed, this elasticity is given by  $\eta$ . There is a large literature that estimates  $\eta$  under the assumption of fixed search intensity; see,

<sup>12</sup>[Galindev and Lkhagvasuren \(2010\)](#) show that for highly persistent autoregressive processes, the method of [Rouwenhorst \(1995\)](#) outperforms other commonly-used discretization methods.

<sup>13</sup>As in [Shimer \(2005\)](#), this normalization is inconsequential to the results. Consider another value, say  $\bar{\theta}$ , for the mean vacancy-unemployment ratio. Then, it can be seen that multiplying  $k$  and  $\mu$  by  $\bar{\theta}$  and  $\bar{\theta}^\eta$ , respectively, leaves the equilibrium allocations given by equations (12) and (15) unaffected.

for example, [Petrongolo and Pissarides \(2001\)](#) for a recent survey of empirical studies on the matching technology. However, to the best of our knowledge, only [Yashiv \(2000\)](#) has estimated the matching technology with endogenous search intensity. Section 6, shows that when search intensity is allowed to vary, the measured elasticity of matches to vacancies,  $\epsilon_{M,v}$ , differs from  $\eta$ .

Appendix A shows that equation (15) holds even in the case of stochastic productivity. Thus, combining equation (15) with equations (23) and (27) gives

$$s^\gamma = \theta. \tag{28}$$

On the other hand, given the uniqueness result in Proposition 1, total search intensity is simply  $S = us$  where  $u$  denotes unemployment. Therefore, equations (2) and (28) imply that, under variable search intensity, the equilibrium number of matches is given by

$$M = \mu v^{1-(1-\eta)(1-\frac{1}{\gamma})} u^{(1-\eta)(1-\frac{1}{\gamma})}. \tag{29}$$

At this point, there are two important conclusions:

- a) The property that the matching function is constant returns to scale with respect to unemployment and vacancies is preserved under variable search intensity. This result is consistent with the fact that empirical studies do not reject constant returns to scale in the matching functions; see the survey of [Petrongolo and Pissarides \(2001\)](#).<sup>14</sup>
- b) Under endogenous job search effort, the implied elasticity of matches with respect to vacancies is given by

$$\epsilon_{M,v} = 1 - (1 - \eta) \left( 1 - \frac{1}{\gamma} \right). \tag{30}$$

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<sup>14</sup>The fact that the number of matches exhibits constant returns to scale is not specific to the Cobb-Douglas matching function. To see this, consider a matching function  $\tilde{M}(v, us)$  which exhibits a constant returns to scale. Using equation (15), it can be seen that  $s = F(\frac{v}{u})$  for some function  $F$ . Then, the total number of matches is given by  $\tilde{M}(v, uF(\frac{v}{u}))$  which, in turn, exhibits constant returns to unemployment and vacancies.

Given the value of  $\gamma$ ,  $\eta$  is chosen such that  $\epsilon_{M,v} = 0.544$ , an elasticity estimate obtained by [Mortensen and Nagypál \(2007\)](#).

### **The search cost: time spent on job search**

The scaling parameter,  $\chi$ , is calibrated such that the average disutility per minute of work is equal to the average disutility per minute of job search. Given that average search intensity is normalized to one, the average flow cost of job search is approximately  $\chi$ . The average disutility of job search is, then,  $\chi/T_u$  where  $T_u$  is time spent on job search. Recall that  $z$  represents the per-period flow of utility of an unemployed worker who exerts zero search effort. This flow utility consists of unemployment benefits received during unemployment,  $b$ , and the imputed value of leisure,  $\ell$ :

$$z = b + \ell. \tag{31}$$

$b$  is set to 0.3 as in [Mortensen and Nagypál \(2007\)](#). The disutility of work is, then,  $\ell$  and the disutility per unit time is  $\ell/T_w$  where  $T_w$  is time spent working. Consequently, the parameter  $\chi$  is set such that

$$\frac{\chi}{T_u} = \frac{\ell}{T_w}. \tag{32}$$

According to the 2008 ATUS, a typical unemployed worker spends 40.47 minutes per weekday on job search activities. The same survey reveals that employed workers spend, on average, 39.99 hours per week working. These numbers imply that the ratio of search time to work time is  $T_u/T_w = 0.0844$ .

The only remaining parameter is  $\gamma$  which governs the curvature of the workers' search cost function. This parameter is important to the analysis since the responsiveness of time spent on job search to aggregate productivity depends heavily on the value of  $\gamma$ . For example, if  $\gamma$  is very high, there will be very low variation in search intensity and the model will behave similarly to those in the previous studies. Ideally, the value of  $\gamma$  would be pinned down using

data on the cyclical volatility of time spent on job search. However, the ATUS only covers a relatively short period of time – 2003 to 2009 – making it difficult to measure the cyclical volatility of time spent on job search since the data cover less than one business cycle.<sup>15</sup> Equations (12) and (15) provide useful insight into how to quantify the responsiveness of search intensity to the labor market conditions. These two equations give

$$p - (b + \ell) = \frac{1 - \beta(1 - \lambda)}{\beta\mu} \left(\frac{k}{\eta}\right)^\eta \left(\frac{\chi\gamma}{1 - \eta}\right)^{1-\eta} s^{(\gamma-1)(1-\eta)} + \chi(\gamma - 1)s^\gamma, \quad (33)$$

where, as above,  $\ell$  is the imputed value of leisure. The curvature parameter  $\gamma$  is pinned down by calibrating to the elasticity of job search with respect to unemployment benefits, denoted  $\epsilon_{s,b}$ . Using cross-state differences, [Krueger and Mueller \(2010\)](#) find that time spent on job search is inversely related to the generosity of unemployment benefits. They estimate that the elasticity of time spent on job search with respect to unemployment benefits is between  $-2.235$  and  $-1.579$ . The calibration target for  $\epsilon_{s,b}$  is  $-1.907$ , the value halfway between the two estimates.<sup>16</sup> This elasticity is computed in the model as follows. The stochastic model is solved for the benchmark value of unemployment benefits,  $b$ . The model is, then, re-solved for a value of  $b$  that is 1% higher than its benchmark value. The elasticity is then calculated as the percentage change in (average) search time divided by the percentage change in unemployment benefits.

The value of  $\gamma$ , 2.3195, means that worker search costs are roughly quadratic. This finding is consistent with the estimates of [Yashiv \(2000\)](#) using Israeli data, and with [Christensen et al. \(2005\)](#) who used micro data on wages and employment.

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<sup>15</sup>Furthermore, there are only a few hundred unemployed workers in the ATUS each year, and the amount of time devoted to searching for a job each day differs substantially across workers; see [Krueger and Mueller \(2010, 2011\)](#). Therefore, time series estimates using this data will be prone to bias (or subject to large confidence intervals.) [Krueger and Mueller \(2010\)](#) get around these limitations by using cross-sectional variation in unemployment benefits, eligibility and dependent allowances across states while controlling for a rich set of worker characteristics.

<sup>16</sup>Among workers with lower annual income, [Krueger and Mueller \(2010\)](#) find that the elasticity of search intensity is  $-2.7$ . This value implies that search intensity is more elastic among lower educated workers, who have much higher cyclical unemployment than their more educated counterparts. This result is reassuring in that [Figure 2](#) shows that the bulk of U.S. unemployment variability is due to workers with low educational attainment.

**Table 1:** Calibration Targets of the Benchmark Model

	Model	Data
unemployment, $u$	0.057	0.057
the elasticity of matches w.r.t. vacancies, $\epsilon_{M,v}$	0.544	0.544
average job search time relative to work hours, $\mathbb{E}(\chi s^\gamma)/(z - b)$	0.084	0.084
the elasticity of time spent on job search w.r.t. benefits, $\epsilon_{s,b}$	-1.907	-1.907

**Table 2:** Parameters of the Benchmark Model

Parameter	Value	Description
$\beta$	0.9992	the time discount factor ( $= 1/1.04^{1/48}$ )
$\lambda$	0.0083	the separation rate ( $= 0.1/12$ )
$\varrho$	0.9903	persistence of the productivity shock
$\sigma_p$	0.0241	standard deviation of the productivity shock
$\mu$	0.1438	the coefficient of the matching technology
$k$	0.0301	the vacancy creation cost
$b$	0.3	unemployment insurance benefit
$z$	0.9212	flow utility of unemployment when search intensity is zero
$\chi$	0.0524	the average search cost
$\eta$	0.1984	the parameter of the matching technology
$\gamma$	2.3195	the power of the search cost function

## 4.2 Results

For the remainder of the paper, the current calibration will be referred to as the benchmark model. Table 1 shows that the model matches the calibration targets.<sup>17</sup> Table 2 displays the parameters of the benchmark model. The average search cost,  $\mathbb{E}(\chi s^\gamma)$ ,<sup>18</sup> measured relative to labor productivity, is 0.05. Average flow utility while unemployed is approximately 0.869, expressed relative to labor productivity, implying that the employment surplus is approximately 13.1 percent of labor productivity.

<sup>17</sup>To measure the targeted moments and predictions of the model, the model economy is simulated for 5,000,000 weeks. In computing moments, all variables are converted to a quarterly frequency as in Hagedorn and Manovskii (2008). To measure the business cycle fluctuations, the simulated quarterly data is detrended by logging and applying the HP-filter with a smoothing parameter of 1600.

<sup>18</sup> $\mathbb{E}(X)$  denotes the time average of an arbitrary variable  $X$ .

**Table 3:** Hagedorn and Manovskii’s (2008) summary statistics of quarterly US data

	$u$	$v$	$v/u$	$p$	
Standard deviation	0.125	0.139	0.259	0.013	
Autocorrelation	0.870	0.904	0.896	0.765	
Cross-correlation	$u$	1	-0.919	-0.977	-0.302
	$v$		1	0.982	0.460
	$v/u$			1	0.393
	$p$				1

**Table 4:** Results from the Benchmark Model

	$u$	$v$	$v/u$	$s$	$p$	
Standard deviation	0.089	0.108	0.186	0.080	0.013	
Autocorrelation	0.829	0.614	0.763	0.763	0.765	
Cross-correlation	$u$	1	-0.777	-0.931	-0.931	-0.915
	$v$		1	0.953	0.953	0.918
	$v/u$			1	1	0.972
	$s$				1	0.972
	$p$					1

**Notes:** As in Hagedorn and Manovskii (2008), the moments are measured after converting the variables to a quarterly frequency and then removing their trends using the HP-filter with a smoothing parameter of 1600.

Table 3 presents Hagedorn and Manovskii’s (2008) summary statistics of quarterly U.S. data, 1951:1–2004:4. Table 4 shows predictions of the benchmark model. The results show that the benchmark model accounts for 70 percent of the observed volatility of both the vacancy-unemployment ratio and unemployment, and nearly 80% of the percentage standard deviation of vacancies. Search intensity is pro-cyclical over the business cycle with a standard deviation of 8 percent.<sup>19</sup> To put the variability of search effort into perspective, the benchmark model’s prediction for this moment is close to its prediction for the standard deviation of unemployment. The model performs reasonably well along other dimensions.

<sup>19</sup>In the spirit of the business cycle literature which defines the cycle as deviations of output from trend, here the cycle is defined as deviations of labor productivity from trend.



Hagedorn and Manovskii (2008) estimate that the elasticity of wages with respect to productivity is 0.449 and target this elasticity in their calibration. In the benchmark model, the elasticity of wages of new matches with respect to productivity is  $\epsilon_{w,p} = 0.472$ . This elasticity is somewhat comparable to the one targeted by Hagedorn and Manovskii (2008), although it should be noted that the elasticity of the wages of new matches is calculated under the assumption that the wage of a particular match does not change over time. In section A.6, this assumption is relaxed; depending on how frequently wages of old matches are re-negotiated, the elasticity of the average wage with respect to productivity ranges between 0.041 (if the wages of old matches are not re-negotiated at all) and 0.967 (if the wages of old matches are re-negotiated following each aggregate shock). Therefore, it makes a little sense to target the elasticity of the wage with respect to productivity in a model with wage posting. At the same time, it also means that one can target any level of the elasticity in the above range by introducing an additional parameter for the frequency of wage renegotiation.

## The net impact of variable search intensity

How much of the success of the benchmark model can be attributed to variable search intensity? To answer this question, the model is solved while fixing the search intensity. The solution method of the stochastic model with fixed search intensity is provided in Appendix B. In the absence of shocks to productivity, the model with fixed search intensity is identical to the competitive search model of Rogerson *et al.* (2005).

Two cases are considered. First, the model is solved while fixing the search intensity at one. This restriction is referred to as F-1. Table 5 shows the implications of fixed search intensity on the volatility of labor market variables. Fixing search intensity reduces the percentage standard deviation of the vacancy-unemployment ratio by almost a half. While the percentage standard deviation of vacancies falls by around 20%, that of unemployment is reduced by a factor of 80%, leaving its volatility at just under 15% of that seen in the data

**Table 5:** Labor Market Volatility Under Fixed Search Intensity

	$u$	$v$	$v/u$
Data	0.125	0.139	0.259
Benchmark Model	0.089	0.108	0.186
Fixed Search Intensity: F-1	0.018	0.082	0.099
Fixed Search Intensity: F-2	0.051	0.059	0.104

**Notes:** As in [Hagedorn and Manovskii \(2008\)](#), the moments are measured after converting the variables to a quarterly frequency and then removing their trends by logging then applying the HP-filter with a smoothing parameter of 1600.

(compared to over 70% for the benchmark model).

The results show that approximately 34% ( $\simeq \frac{0.186-0.099}{0.259}$ ) of the observed volatility of the vacancy-unemployment ratio is explained by variable search effort. Repeating the same calculation for unemployment, search intensity explains approximately 57% ( $\simeq \frac{0.089-0.018}{0.125}$ ) of the volatility of cyclical unemployment. By this same metric, search intensity accounts for about 19% ( $\simeq \frac{0.108-0.082}{0.139}$ ) of the volatility of vacancies. In other words, search intensity has a much larger impact on the percentage standard deviation of unemployment than vacancies.

Clearly, under specification F-1, the implied elasticity of matches with respect to vacancies is  $\epsilon_{M,v} = \eta = 0.1984$ , which is much lower than its empirical counterpart 0.544. To target the latter under fixed search intensity, the model is simulated while setting  $\eta$  to 0.544 and keeping the search intensity at one. This restriction is referred to F-2 in [Table 5](#). The results show that fixing search intensity while targeting the elasticity of the matching function has a smaller effect on unemployment but a slightly larger impact on vacancies when compared to a model which only fixes search intensity. However, the volatility of the vacancy-unemployment ratio remains almost the same with what is obtained under F-1.

## Targeting the volatility of the vacancy-unemployment ratio

How volatile must search intensity be in order to generate the observed volatility of the vacancy-unemployment ratio? Here, the model is recalibrated for a variety of values of  $\gamma$ ,

**Table 6:** Calibration: Targeting the volatility of the vacancy-unemployment ratio

	Model	Data
unemployment, $u$	0.057	0.057
the elasticity of matches w.r.t. vacancies, $\epsilon_{M,v}$	0.544	0.544
time spent on job search relative to work hours, $\mathbb{E}(\chi s^\gamma)/(z - b)$	0.084	0.084
the standard deviation of the vacancy-unemployment ratio	0.259	0.259

**Table 7:** Parameters: Targeting the volatility of the vacancy-unemployment ratio

$\mu$	$k$	$z$	$\chi$	$\eta$	$\gamma$
0.1489	0.0110	0.9383	0.0539	0.0925	2.010

**Notes:** The table displays the parameter values that are different than those in the benchmark model. The parameters  $\beta$ ,  $\lambda$ ,  $\varrho$  and  $\sigma_p$  are common with the benchmark model. See Table 2.

the curvature parameter in the workers' cost of search. The key observation is that there *is* a value of  $\gamma$  such that the model's prediction for the volatility of the vacancy-unemployment ratio matches that observed in the data; see Tables 6 to 8 for details. The value of  $\gamma$  that achieves this feat is somewhat smaller than in the benchmark model (2.010 compared to 2.3195). The other parameters that end up substantially changed are:  $k$ , the cost of posting a vacancy (0.0110, down from 0.0301 in the benchmark calibration), and  $\eta$ , the weight on vacancies in the matching function (0.0925, compared to 0.1984). Remarkably, while only the volatility of the vacancy-unemployment ratio was targeted during this exercise, the model comes close to matching the volatility of both unemployment (0.123 versus 0.125 in the data) and vacancies (0.153 compared to 0.139).

A key issue in [Hagedorn and Manovskii \(2008\)](#) is how much surplus workers are bargaining over. In their calibration, the flow value of unemployment is 95.5% of productivity. [Mortensen and Nagypál \(2007\)](#) persuasively argue that this value is implausibly large since it implies that workers are bargaining over a surplus of 4.5% of productivity, and that workers end up choosing to work to increase their income by 2.3% of productivity. Table 7 shows that flow utility while unemployed,  $\mathbb{E}(z - \chi s^\gamma)$ , is 88.4% of productivity. Those who do

**Table 8:** Predictions: Targeting the volatility of the vacancy-unemployment ratio

	$u$	$v$	$v/u$	$s$	$p$	
Standard deviation	0.123	0.153	0.259	0.129	0.013	
Autocorrelation	0.831	0.609	0.760	0.760	0.765	
Cross-correlation	$u$	1	-0.760	-0.923	-0.923	-0.895
	$v$		1	0.951	0.951	0.881
	$v/u$			1	1	0.945
	$s$				1	0.945
	$p$					1

**Notes:** See notes to Table 5.

work do so to increase their flow utility by 11.5% of productivity, much larger than that in [Hagedorn and Manovskii](#).

Under this calibration, the cyclical variation of search intensity is nearly 13%, compared to 8% for the benchmark model. The elasticity of search intensity with respect to benefits is  $-2.9$  which is slightly higher (in absolute terms) than was targeted in the benchmark model. However, this value is close to  $-2.7$ , the elasticity estimate reported by [Krueger and Mueller \(2010\)](#) for workers with lower annual income. Moreover, this value is within the 95% confidence interval of the smaller (in absolute terms) elasticity estimate,  $-1.6$ , reported by [Krueger and Mueller](#) for all workers. These observations suggest that the model calibrated to the cyclical volatility of the vacancy-unemployment ratio does not overstate the responsiveness of search intensity to labor market conditions.

## 5 Nash bargaining and the Hosios condition

Suppose, instead, that wages are determined via Nash bargaining. That is,

$$w = \operatorname{argmax}_w [W(w, p) - U(p)]^{1-\psi} [J(w, p) - V(p)]^\psi$$

where  $\psi$  is the bargaining power of the firm.

Deriving the Hosios condition for the model with endogenous worker search effort requires the first-order condition of equation (4) with respect to  $s$ :

$$c'(s) = \beta f(q) \mathbb{E}_p(W(w, p') - U(p')). \quad (34)$$

The free entry condition implies  $V(p) = 0$ ; equation (7) can, then, be expressed as

$$k = \beta \alpha(q) \mathbb{E}_p J(w, p'). \quad (35)$$

Now let  $\Xi_p$  denote the match surplus, that is,  $\Xi_p = W(w, p) + J(w, p) - U(p)$  for all  $p$ . Then, as in [Mortensen and Nagypál \(2007\)](#), the match surplus can be written

$$\frac{W(w, p) - U(p)}{1 - \psi} = \Xi_p = \frac{J(w, p)}{\psi} \quad (36)$$

for all  $p$ . Using equations (34) to (36), it can be seen that

$$k = \frac{\psi}{1 - \psi} c'(s) q. \quad (37)$$

As mentioned earlier, equation (15) holds even in the case of stochastic productivity. Then, combining that equation with equation (37), it can be seen that the solutions obtained under competitive search and Nash bargaining coincide when  $\eta = \psi$ . The latter extends the Hosios condition to a setting with stochastic productivity and endogenous search intensity.

It is important to note that the key to this equivalency is the Cobb-Douglas matching function. Thus, it is misleading to say that the equivalency result always holds. For example, under the matching function  $m(v, S) = \frac{vS}{(v^a + S^a)^{1/a}}$ , where  $a > 0$ , ([den Haan \*et al.\*, 2000](#); [Hagedorn and Manovskii, 2008](#)), equation (15) becomes

$$k = c'(s) q^{1+a}. \quad (38)$$

Therefore, the equivalency does not hold unless the bargaining power  $\psi$  in equation (37) varies with the queue length  $q$  according to the following equation:

$$\psi = \frac{q^a}{1 + q^a}. \quad (39)$$

More importantly, using equations (12) and (15), it can be seen that for the allocation to be efficient, the bargaining power must change with search intensity,  $s$ , the vacancy-unemployment ratio,  $\theta$ , and aggregate productivity,  $p$ .

## 6 Implications on the matching technology

As stated earlier, only Yashiv (2000) has estimated the matching technology when search intensity is endogenous, and he used Israeli data, not U.S. data. In this section, we analyze the implications of search intensity on labor-market matching.

### 6.1 Interdependence of matching and search intensity

Earlier, it was shown that under endogenous job search effort, the elasticity of the number of matches with respect to vacancies is given by

$$\epsilon_{M,v} = 1 - (1 - \eta) \left( 1 - \frac{1}{\gamma} \right).$$

When  $\gamma$  is infinity (equivalently, search intensity is fixed), the elasticity of matches with respect to vacancies is  $\eta$ . Consequently, the matching technology with fixed search effort used in the previous studies can be thought of as a special case of the one considered in this paper.

Another important observation is that the matching technology parameter  $\eta$  differs substantially from  $\epsilon_{M,v}$ , the elasticity measured directly from data on cyclical unemployment, vacancies and matches (see equation (29)). Specifically, since  $\gamma > 1$  and  $0 < \eta < 1$ ,  $\epsilon_{M,v} > \eta$ .

For example, for the benchmark calibration,  $\eta = 0.1984$  and  $\epsilon_{M,v} = 0.544$ . Thus, if one abstracts from variable search intensity, one would make an erroneous conclusion that a one percent increase in vacancies will raise the number of matches by more than 0.5 percent whereas the actual impact could be less than 0.2 percent.

These results show that the matching technology and the search cost are intimately related. To estimate the two functions simultaneously requires an equilibrium framework that allows for endogenous search effort. This paper offers one such a framework.

## 6.2 Shifts in the Beveridge curve

Throughout this paper, labor market fluctuations have been modeled as productivity shocks. However, [Mortensen and Nagypál \(2007\)](#) point out that the correlation between labor productivity and the vacancy-unemployment ratio is less than unity and emphasize the importance of other omitted driving forces. Consistent with their finding, empirical studies that employ linear and log-linear regressions to estimate matches as a function of unemployment and vacancies yield  $R^2$ s in the range of 0.4 to 0.9. In other words, a sizable fraction of the variation of matches is not explained by shifts in unemployment and vacancies.<sup>20</sup>

The results in this paper suggest that part of these unexplained shifts in matches (or, equivalently, changes in the parameters of the matching functions) can be attributed to shifts in the costs associated with vacancy creation and job search. For instance, equation (33) shows that an increase in the cost parameters  $k$ ,  $\chi$  and  $\gamma$ , reduces equilibrium search intensity. Therefore, in general, the total number of matches is given by

$$M(k, \chi, \gamma, v, u) = A(k, \chi, \gamma)v^\eta u^{1-\eta}, \quad (40)$$

where  $A$  is a decreasing function of its arguments. As a result, the number of matches for a given level of unemployment and vacancies can shift with these cost parameters.

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<sup>20</sup>In this context, variation of matches means overall shifts in the number of matches, which includes both cyclical fluctuations and the trend.

Equation (40) has the following important implications. First, [Lubik \(2011\)](#) argues that a decline in the match efficiency parameter  $A$  is consistent with the outward shift of the U.S. Beveridge curve in the aftermath of the Great Recession. His finding, along with Equation (40), raises the possibility that the above cost parameters might account for a part of persistently high unemployment despite an increased number of vacancies during the recent recovery.

Second, cross-country data show that there are substantial differences in unemployment across countries. Empirical studies have tended to focus on whether taxes or benefits can explain these cross-country unemployment differences; see, for example, [Prescott \(2004\)](#) and [Ljungqvist and Sargent \(2006\)](#). Time spent on job search also differs substantially across countries. For example, according to [Krueger and Mueller \(2010\)](#), an average unemployed worker spends 41 minutes a day searching for a job in the U.S., compared with just 12 minutes in the average European country. The results in this paper suggest that differences in time spent on job search, or equivalently, differences in search or vacancy creation costs, may account for a substantial part of the cross-country differences in unemployment.

## 7 Conclusion

[Shimer \(2005\)](#) showed that the DMP model of search and unemployment underpredicts the standard deviation of key labor market variables – namely vacancies, unemployment and the vacancies-unemployment ratio – by an order of magnitude; see also [Andolfatto \(1996\)](#) and [Merz \(1995\)](#). This observation motivated looking at two modifications to the textbook DMP model. The first and most important modification was to add worker search intensity, allowing workers to directly affect the outcome of their job search. The second change was more innocuous: dropping Nash bargaining determination of wages in favor of competitive search (a combination of wage posting and directed search). An important advantage of using competitive search is that the allocations are efficient. In contrast, under



Nash bargaining, allocations are efficient only under the [Hosios \(1990\)](#) condition: the value of the worker's bargaining parameter equals the elasticity of the matching function with respect to unemployment.

The benchmark model predicts standard deviations for unemployment and the vacancies-unemployment ratio that are 70% of that seen in the U.S. data; for vacancies, the model captures nearly 80% of the observed volatility. Almost all of the improvement with respect to the variability of unemployment – and about half of that for the vacancies-unemployment ratio – can be traced to the introduction of endogenous worker search effort. The volatility of vacancies is not much affected by worker search intensity.

When the model is recalibrated to match the observed volatility of the vacancies-unemployment ratio, the model also captures virtually all of the variability in unemployment and vacancies. The principal differences relative to the benchmark calibration are: (1) a lower cost of posting vacancies, (2) a smaller weight on vacancies in the matching function, and (3) a smaller curvature parameter on the cost of worker search intensity. This curvature parameter is directly related to the elasticity of search effort with respect to unemployment benefits. For this calibration, this elasticity is larger (in absolute value) than the estimates in [Krueger and Mueller \(2010\)](#), but is nonetheless within a 95% confidence of their smallest (in absolute value) point estimate.

To date, endogenous worker search effort has been largely overlooked when estimating the matching technology; a notable exception is [Yashiv \(2000\)](#). Section 6 shows that this omission can lead to an overestimate, by a factor of 2.5, of the effects on job matching of an increase in vacancies. This problem is not merely of academic interest since it has implications for public policies aimed at reducing unemployment. The results also suggest that when wages are determined by Nash bargaining, choosing the bargaining power of workers based on estimates of matching functions alone is premature and cannot always guarantee constrained efficiency.

The model can also be used to analyze cross-country differences. [Krueger and Mueller](#)

(2010) report that time spent on job search is much higher in the U.S. than in Europe. On the other hand, it is well known that unemployment is substantially lower in the U.S. than in Europe. Since the model establishes a negative correlation between time spent on job search and unemployment in an equilibrium framework, it may be interesting to use the model to examine whether cross-country differences in worker search costs can account for the cross-country differences in unemployment.

# A Model with variable search intensity

## A.1 The definition of the labor market equilibrium

Since unemployed workers are intrinsically identical, it follows that  $U(p)$  is common to all unemployed workers. Further,  $\tilde{U}(w, p)$  must be the same for all jobs for which workers actually search. It then follows that the queue length,  $q_{w,p}$ , must be unique for all jobs with positive worker search: The compensation for searching for a lower wage job is a higher probability of being matched, that is, a lower queue length. Using equations (3) and (4), it can be seen that search intensity,  $s_{w,p}$ , must also be unique for each job type  $(w, p)$ . Introducing the following functions,  $s(w, p) = s_{w,p}$ ,  $q(w, p) = q_{w,p}$ ,  $v(w, p) = v_{w,p}$ ,  $u(w, p) = u_{w,p}$  and  $S(w, p) = S_{w,p}$  for any  $(p, w)$  such that  $w \in \mathcal{W}(p)$ , the labor market equilibrium can now be defined.

**Definition A.1.** *The equilibrium is a set of value functions,  $\{U, W, J, V\}$ , a decision rule  $s$ , wage sets  $\mathcal{W}$ , the measures,  $\{u, v\}$ , the total search intensity,  $S$ , and the queue length,  $q$ , such that*

1. *unemployed: given  $q$  and  $W$ , the decision rule  $s(w, p)$  and the value functions  $U(p)$  and  $\tilde{U}(w, p)$  solve equations (3) and (4) for any  $w \in \mathcal{W}(p)$ ;*
2. *employed: given  $U$ , the value function  $W(w, p)$  solves equation (5);*
3. *matched firm: the value function  $J(w, p)$  solves equation (6);*
4. *vacancy: given  $q$  and  $J$ , the wage  $w$  and value function  $V(p)$  solve equation (7) with  $w \in \mathcal{W}(p)$ ;*
5. *free entry: for any real number  $x$ ,*

$$\begin{cases} v(x, p) > 0 \text{ and } V(p) = 0 & \text{if } x \in \mathcal{W}(p), \\ v(x, p) = 0 \text{ and } V(p) \leq 0 & \text{if } x \notin \mathcal{W}(p) \text{ or } \mathcal{W}(p) = \emptyset; \text{ and} \end{cases} \quad (\text{A.1})$$

6. *consistency: the total search intensity  $S$  and the queue length  $q$  are consistent with*

individuals' and firms' behavior:  $S(w, p) = u(w, p)s(w, p) = v(w, p)q(w, p)$  for  $w \in \mathcal{W}(p)$ .

## A.2 Workers

Let  $\mathbb{E}_p X(p')$  denote the expected value of an arbitrary variable  $X$  conditional on the current state  $p$ . Then, the value of being employed with wage  $w$  can be written as

$$W(w, p) = \frac{w}{1 - \beta(1 - \lambda)} + \mathbb{E}_p K(p') \quad (\text{A.2})$$

where  $K$  is given by the following recursive equation conditional on  $U$ :

$$K(p) = \beta\lambda U(p) + \beta(1 - \lambda)\mathbb{E}_p K(p'). \quad (\text{A.3})$$

Then, it can be seen that  $\mathbb{E}_p W(w, p') - \frac{w}{1 - \beta(1 - \lambda)}$  does not vary with  $w$ . Thus, let

$$D(p) = \mathbb{E}_p W(w, p') - \mathbb{E}_p U(p') - \frac{w}{1 - \beta(1 - \lambda)}. \quad (\text{A.4})$$

A worker will take the queue length  $q = S/v$  as given. The first-order condition with respect to search intensity,  $s$ , in equation (4) is

$$c'(s) = \beta f(q) \left( D(p) + \frac{w}{1 - \beta(1 - \lambda)} \right) \quad (\text{A.5})$$

which can be rewritten as

$$\frac{w}{1 - \beta(1 - \lambda)} + D(p) = \frac{c'(s)}{\beta f(q)}. \quad (\text{A.6})$$

### A.3 Firms

Let  $Z(p)$  denote the value of the expected output streams of a firm when the current state is  $p$ :

$$Z(p) = p + \beta(1 - \lambda)\mathbb{E}_p Z(p'). \quad (\text{A.7})$$

Furthermore, let  $y(p) = (1 - \beta(1 - \lambda))\mathbb{E}_p Z(p')$ . Then, equation (6) can be rewritten as

$$V(p) = \max_w \left\{ -k + \beta\alpha(q) \frac{y(p) - w}{1 - \beta(1 - \lambda)} \right\}. \quad (\text{A.8})$$

Analogous to Rogerson *et al.* (2005), substituting equation (A.6) into equation (A.8) and taking the first-order condition with respect to  $q$  yields

$$\frac{y(p)}{1 - \beta(1 - \lambda)} + D(p) = \frac{c'(s)}{\beta\alpha'(q)}. \quad (\text{A.9})$$

Combining equations (A.6) and (A.9) gives

$$\frac{y(p) - w}{1 - \beta(1 - \lambda)} = \frac{c'(s)}{\beta} \left( \frac{1}{\alpha'(q)} - \frac{1}{f(q)} \right). \quad (\text{A.10})$$

The left hand side is simply  $\mathbb{E}_p J(p', w)$ . Therefore, the free entry condition  $k = \beta\alpha(q)\mathbb{E}_p J(p', w)$  implies that

$$k = c'(s)\alpha(q) \left( \frac{1}{\alpha'(q)} - \frac{1}{f(q)} \right). \quad (\text{A.11})$$

Since  $\alpha(q)/\alpha'(q) = q/(1 - \eta)$  and  $\alpha(q)/f(q) = q$  under the Cobb-Douglas matching function (see equation (2)), it follows that

$$k = \frac{\eta}{1 - \eta} c'(s)q. \quad (\text{A.12})$$

Substituting the latter into equation (A.9) for  $q$ , it can be shown that

$$\frac{y(p)}{1 - \beta(1 - \lambda)} + D(p) = \frac{k^\eta}{\beta\mu\eta^\eta(1 - \eta)^{1-\eta}}(c'(s))^{1-\eta}. \quad (\text{A.13})$$

#### A.4 Proof of Proposition 3

Given productivity  $p$ , the left hand side of equation (A.13) is unique among unemployed workers. Thus, the strict convexity of  $c(s)$  will imply that every unemployed worker will search with same intensity. Then, given  $p$ , equation (A.12) implies that  $q$  is unique.

#### A.5 Algorithm

Given the results above, the following algorithm is used to solve the stochastic dynamic model with variable search intensity:

1. Calculate  $y(p)$  using equation (A.7).
2. Form a guess on  $D(p)$ .
3. Find  $s$  using equation (A.13).
4. Using equation (A.12), find  $q$ .
5. Using equation (A.10), find  $w$ .
6. Using  $q$ ,  $w$  and  $s$ , calculate  $U(p)$  and  $W(w, p)$ . Also calculate  $\mathbb{E}_p U(p')$  and  $\mathbb{E}_p W(w, p')$ .
7. Using the expected values and  $w$ , calculate  $D(p)$  according to equation (A.4).
8. Iterate until the gap between the initial guess and the updated value of  $D(p)$  is sufficiently small.

Given the equilibrium values of  $\tilde{f}(p)$ , unemployment at time  $t$  is given recursively by  $u_{t+1} = (1 - \lambda)u_t + \tilde{f}(p_t)(1 - u_t)$ .

## A.6 Elasticity of wages with respect to productivity

Given productivity  $p$ , Proposition 3 implies that the expected present discounted value of the wage stream of new matches is unique. Let  $Y(p)$  denote this unique value. Then, under the assumption that the wage of a particular match does not change over time, the wage of new matches is given by

$$w(p) = (1 - \beta(1 - \lambda))Y(p). \quad (\text{A.14})$$

For the benchmark calibration, the elasticity of  $w(p)$  with respect to productivity is  $\epsilon_{w,p} = 0.472$ ; this value is comparable to the one targeted by Hagedorn and Manovskii (2008). Since most of the matches are old, the elasticity of the *average* wage in the model economy must be much smaller. In fact, under the assumption of constant within-match wages, the elasticity of the *average* wage with respect to productivity is  $\hat{\epsilon}_{w,p} = 0.041$ .

Suppose instead that the wages of old matches are allowed to evolve over time. Specifically, let the wages of all matches, new or old, be the same and respond to each aggregate shock. Let  $\tilde{w}$  be the wage determined in such a way. Then, it can be seen that

$$Y(p) = \tilde{w}(p) + \beta(1 - \lambda)\mathbb{E}_p Y(p'). \quad (\text{A.15})$$

Combining the latter with equation (A.14) gives

$$\tilde{w}(p) = \frac{1}{1 - \beta(1 - \lambda)} (w(p) - \beta(1 - \lambda)\mathbb{E}_p w(p')). \quad (\text{A.16})$$

In this case, the elasticity of  $\tilde{w}$  with respect to productivity is  $\tilde{\epsilon}_{w,p} = 0.967$ . Therefore, depending on how frequently wages of old matches are renegotiated after each aggregate shock, the elasticity of the average wage with respect to productivity ranges from 0.041 to 0.967.<sup>21</sup>

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<sup>21</sup>This range is for the benchmark model. For the calibration that matches the volatility of the vacancies-unemployment ratio, the measured elasticities do not differ substantially; specifically,  $\epsilon_{w,p} = 0.478$ ,  $\hat{\epsilon}_{w,p} = 0.041$  and  $\tilde{\epsilon}_{w,p} = 0.979$ .

## B Model with fixed search intensity

### B.1 Workers

When search intensity is fixed at one, the flow utility of unemployment becomes

$$\tilde{z} = z - c(1).$$

Then, the value of being unemployed is given by

$$U(p) = \tilde{z} + \beta f(q) (\mathbb{E}_p W(w, p') - \mathbb{E}_p U(p')) + \beta \mathbb{E}_p U(p'). \quad (\text{B.1})$$

The value of being employed is as before:

$$W(w, p) = w + \beta(1 - \lambda) \mathbb{E}_p W(w, p') + \beta \lambda \mathbb{E}_p U(p'). \quad (\text{B.2})$$

Given  $U$ , let  $K$  be given by the recursive equation (A.3). Let

$$H(p) = \mathbb{E}_p (\mathbb{E}_{p'} K(p'')) - \mathbb{E}_p U(p'). \quad (\text{B.3})$$

Then, equation (B.1) can be written as

$$U(p) = \tilde{z} + \beta f(q) \left( \frac{w}{1 - \beta(1 - \lambda)} + H(p) \right) + \beta \mathbb{E}_p U(p'). \quad (\text{B.4})$$

Therefore, for any posted wage  $w \in \mathcal{W}(p)$ ,

$$\frac{w}{1 - \beta(1 - \lambda)} + H(p) = \frac{U(p) - \tilde{z} - \beta \mathbb{E}_p U(p')}{\beta f(q)}. \quad (\text{B.5})$$



## B.2 Firms

As in Rogerson *et al.* (2005), substituting equation (B.5) into equation (A.8) for  $w$  and taking the first order condition with respect to  $q$  yields

$$\frac{y(p)}{1 - \beta(1 - \lambda)} + H(p) = \frac{U(p) - \tilde{z} - \beta \mathbb{E}_p U(p')}{\beta \alpha'(q)}. \quad (\text{B.6})$$

Combine equations (B.5) and (B.6) to obtain

$$\frac{y(p) - w}{1 - \beta(1 - \lambda)} = \frac{\eta}{\mu \beta (1 - \eta)} (U(p) - \tilde{z} - \beta \mathbb{E}_p U(p')) q^\eta. \quad (\text{B.7})$$

Combining this result with the free entry condition,

$$\frac{1 - \eta}{\eta} k = (U(p) - \tilde{z} - \beta \mathbb{E}_p U(p')) q. \quad (\text{B.8})$$

## B.3 Algorithm

Using the results above, the following algorithm is used to solve the model under fixed search intensity:

1. Calculate  $y(p)$ .
2. Form a guess on  $U(p)$ .
3. Calculate  $\mathbb{E}_p U(p')$ .
4. Find  $q$  using equation (B.8).
5. Using equation (B.7), find  $w$ .
6. Using  $q$  and  $w$ , calculate  $U(p)$ . Iterate until the gap between the initial guess and the updated value of  $U(p)$  is sufficiently small.

## B.4 Elasticity of the vacancy-unemployment ratio with respect to productivity

In the absence of the aggregate shock, equation (A.3) converts into

$$K = \frac{\beta\lambda}{1 - \beta(1 - \lambda)}U. \quad (\text{B.9})$$

Therefore, equation (B.3) becomes

$$H = -\frac{1 - \beta}{1 - \beta(1 - \lambda)}U. \quad (\text{B.10})$$

Then, using these equations, the equilibrium conditions given by equations (B.6) and (B.8) can be rewritten as

$$\frac{p - (1 - \beta)U}{1 - \beta(1 - \lambda)} = \frac{(1 - \beta)U - (z - c(1))}{\beta\alpha'(q)} \quad (\text{B.11})$$

and

$$\frac{1 - \eta}{\eta} \frac{k}{q} = (1 - \beta)U - (z - c(1)), \quad (\text{B.12})$$

respectively. Note that equation (B.11) uses the fact that  $y(p) = p$  under permanent shock.

Combining these two equations and that  $q = 1/\theta$ , one can arrive at

$$p - (z - c(1)) = \frac{1 - \eta}{\eta} k \left( \theta + \frac{1 - \beta(1 - \lambda)}{\beta\mu(1 - \eta)} \theta^{1-\eta} \right). \quad (\text{B.13})$$

As before, by taking logs and differentiating the result with respect to  $\ln p$  while taking into account the steady-state normalization  $\theta = 1$  and the fact that  $\tilde{\eta} = \eta$ ,

$$\epsilon_{\theta,p}^F = \frac{\partial \ln \theta}{\partial \ln p} = \frac{1 + \frac{1}{1-\tilde{\eta}} \frac{1-\beta(1-\lambda)}{\beta\mu}}{1 + \frac{1-\beta(1-\lambda)}{\beta\mu}} \times \frac{p}{p - (z - c(1))}. \quad (\text{B.14})$$

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