SET THEORY

Summary

1. The sets

A ***set*** is a collection of well defined objects that we call ***elements***.

Example

- Given Ω, the set of all results obtained by adding the two die :

$\Omega = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\};$

The values $2, 3, 4, \dots, 12$ are elements of the set

The sets of numbers employed most frequently (and which thus require particular attention) are the following:

- The set of natural numbers $\mathbb{N} = \{0, 1, 2, 3, 4, 5 \dots\};$
- The set of whole numbers $\mathbb{Z} = \{\dots, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, \dots\};$
- The set of rational numbers $\mathbb{Q}$ (all numbers that can be written as a fraction) ;
- The set of real numbers $\mathbb{R}$ (all rational and irrational numbers)**.**

By convention, the symbols $\mathbb{N}, \mathbb{Z}, \mathbb{Q}$ **and** $\mathbb{R}$ will denote these sets.

1.1. The empty set

We denote the empty set, that has no elements, as $\emptyset$.

1.2. Element of a set

The symbol $\in$ indicates that an element belongs to a set. Inversely, the symbol $\notin$ identifies an element that does not belong to a set.

Example

- $a \in \{a, e, i, o, u, y\}$
- $j \ \notin \ \{a, e, i, o, u, y\}$
- $2 \ \in \mathbb{N}$
- $0{,}5 \ \notin \mathbb{N}$

1.3. Subsets

The set $\boldsymbol{A}$ is said to be a ***subset*** of $\boldsymbol{B}$ if and only if all elements of $\boldsymbol{A}$ are also elements of $\boldsymbol{B}$. We then say that the set $\boldsymbol{A}$ is ***included*** in the set $\boldsymbol{B}$. The notation $\boldsymbol{A} \subseteq \boldsymbol{B}$ is employed to symbolize the inclusion of $\boldsymbol{A}$ in $\boldsymbol{B}$.

The symbol $\not\subset$ indicates that a set is not included in another set. $\boldsymbol{C} \not\subset \boldsymbol{D}$ expresses that at least one element of $\boldsymbol{C}$ is not an element of $\boldsymbol{D}$.

Example

Given Ω, the set of possible numbers for the sum of two die

$$\Omega = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$$

Here are a few subsets of Ω:

$\boldsymbol{A}$ $=$ the set of even results = $\{2, 4, 6, 8, 10, 12\}$;

$\boldsymbol{B}$ $=$ the set of results inferior or equal to 6 = $\{2, 3, 4, 5, 6\}$;

$\boldsymbol{C}$ $=$ the set of results greater than 12 = $\{\emptyset\}$;

$\boldsymbol{D}$ $=$ the set of results that can be divided by 11 = $\{11\}$.

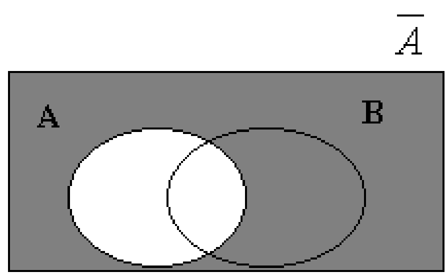
Example

- $\mathbb{N} \subseteq \mathbb{Z}$
- $\mathbb{N} \subseteq \mathbb{Q}$
- $\{0, 1, 2, 3, 4, 5\} \subseteq \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
- $\mathbb{Z} \not\subset \mathbb{N}$
- $\{0, 1, 2, 3, 4, 5\} \not\subset \{1, 3, 5, 7, 9, 11\}$

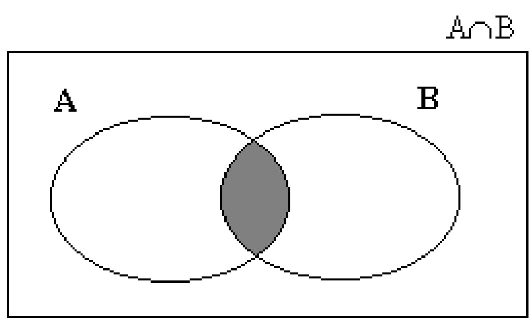
1.4. Operations on sets

We must sometimes carry out operations on sets. For example, perhaps we may want to find elements common to multiple sets or those that belong to only one of these sets. The following section will present the most important set operations, as well as their symbolic notation.

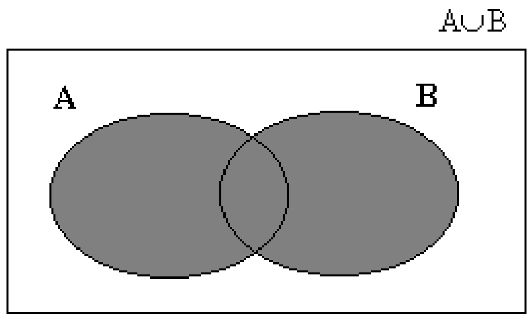
The complement of $\boldsymbol{A}$ $(\overline{\boldsymbol{A}})$: the set of all elements that are not in $\boldsymbol{A}$.



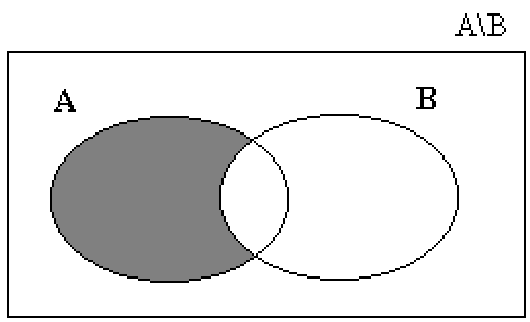
The intersection of two sets $(\boldsymbol{A} \cap \boldsymbol{B})$**:** the set of all elements belonging both to $\boldsymbol{A}$ and to $\boldsymbol{B}$.



The union of two sets $(\boldsymbol{A} \cup \boldsymbol{B})$**:** the set of all elements belonging to either ***A***, or ***B*** or to both $\boldsymbol{A}, \boldsymbol{B}$.



The difference of two sets **(**$\boldsymbol{A} \setminus \boldsymbol{B}$**)** : the set of all elements of $\boldsymbol{A}$ that do not belong to $\boldsymbol{B}$.



Summary exercise

Consider the set Ω, the set of all results that can be obtained by adding the results of two die.

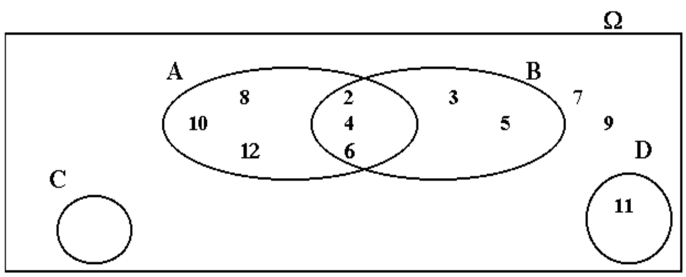
Given the following subsets of Ω:

$\boldsymbol{A}$ = the result is even

$\boldsymbol{B}$ = the result ≤ 6

$\boldsymbol{C}$ = the result >12

$\boldsymbol{D}$ = the results can be divided by 11



Write the elements of the subsets obtained by the following operations:

The complementary sets

$\bar{A}$ = the odd results

$\bar{B}$ = the results > 6

$\bar{C}$ = the results ≤ 12

$\bar{D}$ = the results that cannot be divided by 12

<u>*Solutions*</u>

a. $\{3, 5, 7, 9, 11\}$
b. $\{7, 8, 9, 10, 11, 12\}$
c. $\{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$
d. $\{2, 3, 4, 5, 6, 7, 8, 9, 10, 12\}$

Intersections

$A \cap B$: *the even results and* ≤ 6
$A \cap D$: *the even results and* > 12
$B \cap B$: *the results* ≤ 6 *and* ≤ 6

$B \cap C$: $the\ results \leq 6\ and > 12$
$\Omega \cap D$: $all\ the\ results\ and\ the\ results\ that\ can\ be\ divided\ by\ 11$

<u>*Solutions*</u>

a. $\{2, 4, 6\}$
b. $\emptyset$
c. $\{2, 3, 4, 5, 6\}$
d. $\emptyset$
e. $\{11\}$

Unions

$A \cup B$: $the\ even\ results\ or \leq 6$
$A \cup D$: $the\ even\ results\ or\ divisible\ by\ 11$
$B \cup C$: $t\ \ he\ results \leq 6\ or > 12$

note : the elements belonging both to $\boldsymbol{A}$ and to $\boldsymbol{B}$ are only written once in the new subset.

<u>*solutions*</u>

a. $\{2, 3, 4, 5, 6, 8, 10, 12\}$
b. $\{2, 4, 6, 8, 10, 11, 12\}$
c. $\{2, 3, 4, 5, 6\}$

Differences

$A \setminus B$: $even\ results\ except\ those \leq 6$
$\Omega \setminus A$: $all\ the\ results\ except\ those\ that\ are\ even$
$A \setminus A$: $even\ results\ except\ those\ that\ are\ even$
$A \setminus \emptyset$: $even\ results\ except\ the\ empty\ set$

<u>*Solutions*</u>

a. $\{8, 10, 12\}$
b. $\{3, 5, 7, 9, 11\}$
c. $\emptyset$
d. $\{2, 4, 6, 8, 10, 12\}$