SOLUTION OF A AN EQUATION IN ONE VARIABLE

Summary

An equation is a propositional form that expresses the equality, that can be either true of false, between two mathematical expressions.

Example

a. $2 = 5 - 3$
b. $2x + 4y = 3x - 22z + 4$
c. $3x^2 + 2x - 4 = 0$

The three examples above represent equations that propose an equality between two expressions. This equality could be true, or not. In example (a), it is easy to see that the equality is verified. In examples (b) and (c), the value that the variables take on will determine if the equality is true or not. These values that make an equation true (and especially the methods that allow us to find them) have a capital importance in mathematics. They are the subject of the following section …

Solution of an equation

The solution of an equation is the value- or values- that a variable can take on such that they make the equation true.

Example

Given the equation $4x - 8 = 4$

The value $x = 3$ is a solution for this equation. When x is substituted by the value 3, the equation is satisfied.

$4(3) - 8 = 4$

$12 - 8 = 4$

$4 = 4$ (true!)

Although replacing x by the value 4 would not make this equation true :

$4(4) - 8 = 4$

$16 - 8 = 4$

$8 = 4$ (false!)

It is possible for an equation to have more than one solution. This occurs when the degree of the polynomials used is not 1.

Example

Given the equation $x^2 + 6x = 16$.

The solutions for this equation are $x = -8$ and $x = 2$ since :

If $x = -8 \rightarrow (-8)^2 + 6(-8) = 16$

- $64 + (-48) = 16$
- $64 - 48 = 16$
- $16 = 16$ (true)

If $x = 2 \rightarrow (2)^2 + 6(2) = 16$

- $4 + 12 = 16$
- $16 = 16$ (true)

If an equation contains more than one variable, then the set of values for each variable that makes an equation true, constitutes a solution.

Example

Let us consider the two variable equation $2x + 3y = 8$

$x = 1, y = 2$ is a solution to the equation $2x + 3y = 8$.

When variables x and y are substituted by 1 and 2 respectively, the equation is satisfied:

$2(1) + 3(2) = 8$

$2 + 6 = 8$

$8 = 8$ (true)

$x = 4, y = 0$ is also a solution to the equation $2x + 3y = 8$.

When the variables x and y are substituted by 0 and 4 respectively, the equation is satisfied :

$2(4) + 3(0) = 8$

$8 + 0 = 8$

$8 = 8$ (true)

It would be false to state that $x = 1, y = 1$ is a solution.

$2(1) + 3(1) = 8$

$2 + 3 = 8$

$5 = 8$ (false)

1- Solution of linear equations in one variable

Finding the values that a variable can be to satisfy a given equation is a delicate process. The operations used to resolve an equation and the choice of the order in which we execute these operations requires experience and practice. The following section will give you a method allowing to resolve linear equations (i.e. of degree 1) in one variable.

A equation that expresses the equality between two expressions is comparable to a balance in equilibrium. To maintain equality, we must carry out the same operations on both sides of the equal sign at all times. We can :

- Add or substract the same value on **both sides of the equation ;**
- Multiply or divide **both sides of the equation** by the same value.

1.1. Solution method

In the case of single variable linear equations, the order in which we chose to carry out the operations will be relatively the same for each problem.

1. Carry out all distributions ;
2. Regroup the variables on one side of the equal sign and the numbers on the other ;
3. Divide both sides by the coefficient of the variable.

In order to lighten up the notation, it is useful to carry out all possible simplifications as you go along.

Example

Resolve the equation $4(\boldsymbol{x} - 5) + 2\boldsymbol{x} = 3\boldsymbol{x} + 7$.

- ***Carry out all the distributions ;***

$4\boldsymbol{x} - 20 + 2\boldsymbol{x} = 3\boldsymbol{x} + 7$ (distribution)

$6\boldsymbol{x} - 20 = 3\boldsymbol{x} + 7$ (simplification)

- ***Regroup the variables on one side of the equal sign and the numbers on the other ;***

This operation must be done while respecting the equilibrium between the expressions on the left and on the right. In order to regroup the variables on the left side, we must eliminate the term $3\boldsymbol{x}$ from the right-hand side.

$6x - 20 - 3x = 3x + 7 - 3x$ (subtraction of the same value on both sides)

$3x - 20 = 7$ (simplification)

In order to regroup the numbers on the right-hand side, we must eliminate the term -20 that is on the left.

$3x - 20 + 20 = 7 + 20$ (addition of the same value on both sides)

$3x = 27$ (simplification)

- ***Divide the two sides by the coefficient of the variable.***

$\frac{3x}{3} = \frac{27}{3}$ (division of both sides of the equation by the same value)

$x = 9$ (simplification)

The solution is thus $x = 9$. To validate this result, we must substitute the value obtained in the original equation :

$4(x - 5) + 2x = 3x + 7$

$4(9 - 5) + 2(9) = 3(9) + 7$

$4(4) + 18 = 27 + 7$

$16 + 18 = 34$

$34 = 34$ (true)

Example

Resolve the equation $6(2x + 3) + x - 7 = 3(5x + 7) + 2x$.

- ***Carry out all distributions ;***

$12\boldsymbol{x} + 18 + \boldsymbol{x - 7} = 15\boldsymbol{x} + 21 + 2\boldsymbol{x}$ (distribution)

$13\boldsymbol{x} + 11 = 17\boldsymbol{x} + 21$ (simplification)

- ***Regroup the variables on one side of the equal sign and the numbers on the other ;***

This operation must be carried out while respecting the equilibrium between both left and right expressions. In order to regroup the variables on the left-hand side, we must eliminate the term $17\boldsymbol{x}$ from the term on the right-hand side.

$$13x + 11 - 17x = 17x + 21 - 17x \text{ (substraction on both sides)}$$

$$-4x + 11 = 21 \text{ (simplification)}$$

In order to regroup the numbers on the right-hand side, we must eliminate the term + 11 that can be found on the left-hand side.

$$-4x + 11 - 11 = 21 - 11 \text{ (addition of the same value on both sides)}$$

$$-4x = 10 \text{ (simplification)}$$

- ***Divide both sides of the equality by the coefficient of the variable.***

$$\frac{-4\boldsymbol{x}}{-4} = \frac{10}{-4} \quad \text{(division on both sides by the same value)}$$

$$x = -\frac{5}{2} = -2{,}5 \text{ (simplification)}$$

The solution is therefore $x = -2{,}5$. To verify, let us substitute the value obtained in the original equation:

$$6(2x + 3) + x - 7 = 3(5x + 7) + 2x$$

$$6(2(-2{,}5) + 3) + (-2{,}5) - 7 = 3(5(-2{,}5) + 7) + 2(-2{,}5)$$

$$6(-5 + 3) - 2{,}5 - 7 = 3(-12{,}5 + 7) - 5$$

$$6(-2) - 2{,}5 - 7 = 3(-5{,}5) - 5$$

$$-12 - 2{,}5 - 7 = -16{,}5 - 5$$

$$-21{,}5 = -21{,}5 \text{ (true)}$$

2- Exercises - Solutions of linear equations

Question 1

Circle below all solutions to the equation

$x^2 + 3x = 18$?

a. 6
b. 3
c. -3
d. -6

Solution : b and d

Question 2

Circle below all solutions of the equation $2x - 3y = 4$.

a. $x = 2, y = 0$
b. $x = 0, y = 2$
c. $x = -1, y = -2$
d. $x = 5, y = 2$

Solution : a, c, d

Question 3

Resolve the following equations :

a. $3(x + 4) - 7 = 14x$
b. $5y - 4 = -2(2y + 2)$
c. $8(2z - 3) - 5z = 3z + 8$
d. $2(x + 3) - 9 = 15x$
e. $3z - 7 = 4(z + 2)$

Solution :

a. $x = \frac{5}{11}$
b. $y = 0$
c. $z = 4$
d. $x = -\frac{3}{13}$
e. $z = -15$

Question 4

During a health walk, Jeanne is robbed of thirty dollars. At the casino, she triples the sum that she still has left and now has double the initial amount that she was transporting. How much did she have before being robbed ?

Solution :

$3(x-3) = 2x \rightarrow x = 90$

Question 5

A financial consultant suggests that one should always invest three times more money in "new technology" stocks than in mining investments. How much should a person wishing to invest a total of \$ 2 500 invest in each of these sectors?

Solution :

625 \$ in mining investments, 1875 \$ in new technology stocks

Question 6

A salesman receives a basic weekly salary of \$ 920 and a financial compensation $\boldsymbol{x}$ per km for his traveling by car. During the course of a week, the said salesman traveled 1200 km and had a salary of \$ 1475. Which financial compensation does he get per kilometer?

Solution :

¢ 46,25 per km

Question 7

In 1999, the company VaVite made profits that surpassed by 1,2 millions those of 1998. For these two years, the total profit of VaVite was 15,4 million. Find the profit made in 1998.

Solution :

7,1 million in 1998

Question 8

Considering that the tax rate is 35 % for al revenue exceeding \$ 8 500 (i.e. the first \$ 8 500 are deductible), what is the annual revenue of a person paying \$ 5 250 in taxes ?

Solution :

Given x : the annual revenue

$35\,\%\,(x - 8500) = 5250$

$\rightarrow x = \$\,23\,500$