Inter-Sectorial Risk Pooling

and Income Distributions*

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Abstract

This paper develops a model where two agents in different sectors face uncorrelated income risks and insure each other. We discuss how the rent arising from risk pooling modifies the income distribution in the sector characterized by imperfect competition.

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1 Introduction

Inter-sectorial transfers by members of extended families are an important component of household incomes in most developing countries (World Bank, 1994). These transfers arise in part because of incomplete insurance markets; agents who cannot hedge against crop failure, health problems or unemployment risks use non-market mechanisms as a substitute (Ligon et al., 1997). Typically extended families have a comparative advantage in providing such services because of superior information that mitigate moral hazard and adverse selection problems (Pollak, 1985). Moreover, most of the labor force in those economies is employed in agriculture (Larson and Mundlak, 1997, Table 2). Land, an essential input in that sector, is controlled by a small number of landlords (Tomich et al., 1995) who often use a collusive strategy when hiring workers (Bardhan, 1989).

This paper focuses on the link between inter-sectorial transfers between agents with symmetric information, and income distributions in labor markets characterized by imperfect competition among employers. We first illustrate how such transfers modify the distribution of incomes paid by an employer with some degree of market control. Secondly, we indicate why transfers can imply a transmission mechanism for income distributions across sectors. Finally, we show why, in the absence of inter-sectorial collusion, the economy may be stuck in a poverty trap.

2 Model

Consider an economy with two sectors denoted $i = 1, 2$, each with many identical agents (workers) and a small number of employers. Agents do not have access to a storage technology, while they derive utility from consumption exclusively, and are expected-utility maximizers with monotone increasing and concave sub-utility function $u(\cdot)$.

Let $w_i \in \{\underline{w}_i, \overline{w}_i\}$, where $\underline{w}_i < \overline{w}_i$, denote the stochastic income of a representative agent employed in sector i, with $p_i \equiv \Pr(w_i = \overline{w}_i)$. Inter-sectorial incomes are independent while intra-sectorial incomes are perfectly correlated. We abstract from migration across sectors. The realization of the agents' income is common knowledge to agents, but not to employers. This structure precludes any form of intra-sectorial risk pooling, including self-insurance and market insurance. Consequently, two risk-averse agents may agree to pool risk across sectors and transfer part of their income to each other (Kocherlakota, 1996). Let $t_i > 0$ denote the transfer from a representative agent in sector i (henceforth agent i) to an agent in the other sector. This transfer is determined after some bilateral bargaining process between the two agents and is taken as given in this analysis.

2.1 The agents' problem

We assume that the transfer takes place when only one of the two agents receives the low income in which case, the high-income earner transfers part of his revenues to the low-income one. If both agents receive the high or the low income, no transfers take place. One example of such a transfer scheme is when incomes are equal across the two sectors in each state of nature and both agents have the same bargaining power. In that case, the optimal transfer is that which equalizes the marginal rates of substitution and equals the average income that an agent can make without insurance.[1]

Agreement to the transfer scheme by agent 1 implies that his expected utility is greater under risk pooling than under autarky, i.e.:

$$\begin{aligned} r_1 \equiv \quad & p_1 p_2 u(\overline{w}_1) + p_1(1-p_2)u(\overline{w}_1 - t_1) + (1-p_1)p_2 u(\underline{w}_1 + t_2) + (1-p_1)(1-p_2)u(\underline{w}_1) \\ & - [p_1 u(\overline{w}_1) + (1-p_1)u(\underline{w}_1)] \geq 0. \end{aligned} \tag{1}$$

where r_1 is agent 1's surplus. After some simplification, inequality (1) can be rewritten as:

$$p_2 \geq \frac{p_1[u(\overline{w}_1) - u(\overline{w}_1 - t_1)]}{p_1[u(\overline{w}_1) - u(\overline{w}_1 - t_1)] + (1-p_1)[u(\underline{w}_1 + t_2) - u(\underline{w}_1)]} \equiv p_2^*(p_1), \tag{2}$$

where p_2^* is the lowest probability that agent 2 receives $\overline{w}_2$ such that agent 1 accepts the risk-pooling agreement. Using the equivalent of inequality (2) for agent 2, we obtain:

$$p_1 \geq \frac{p_2[u(\overline{w}_2) - u(\overline{w}_2 - t_2)]}{p_2[u(\overline{w}_2) - u(\overline{w}_2 - t_2)] + (1-p_2)[u(\underline{w}_2 + t_1) - u(\underline{w}_2)]} \equiv p_1^*(p_2). \tag{3}$$

Our first result shows that inequalities (2) and (3) define a convex contract set in the probability space.

[1] In this case $\overline{w}_1 = \overline{w}_2 = \overline{w}$ and $\underline{w}_1 = \underline{w}_2 = \underline{w}$ and both agents have the same bargaining power. Equalizing expected utility across agents yields $\overline{w} - t_i = \underline{w} + t_i$, hence $t_i = 1/2(\overline{w} - \underline{w})$. Note that other factors such as altruism, over which preferences are not defined in our model, could determine the level of transfers.

Lemma 1 (convexity) *A necessary condition for transfers to take place between agents* 1 *and* 2 *is that the marginal benefit from the transfer received in the bad state must be at least as large as the marginal utility loss from paying the transfer in the good state of nature:*

$$u(\underline{w}_i + t_j) - u(\underline{w}_i) \geq u(\overline{w}_i) - u(\overline{w}_i - t_i), \tag{4}$$

for $i, j = 1, 2$ *and* $i \neq j$.

Proof. We derive the condition under which the subspace in the (p_1, p_2) space defined by the intersection of inequalities (2) and (3) is not empty. In this case, transfers may be of mutual interests to both agents. The probability frontiers given by inequalities (2) and (3) can be rewritten as:

$$p_j = \frac{A_i p_i}{A_i p_i + B_i(1 - p_i)}, \tag{5}$$

where $i, j = 1, 2$ for $j \neq i$, and where,

$$\begin{aligned} A_i \equiv & \quad u(\overline{w}_i) - u(\overline{w}_i - t_i), \\ B_i \equiv & \quad u(\underline{w}_i + t_j) - u(\underline{w}_i). \end{aligned}$$

As the utility function is monotone increasing and t_i is positive by assumption, it must be that A_i and B_i are positive. Moreover, equation (5) is continuous and bounded over the domain $p_i \in [0, 1]$, with $p_j(0) = 0$ and $p_j(1) = 1$. Hence we can differentiate equation (5) with respect to p_i and after simplification, we obtain:

$$\frac{\partial p_j}{\partial p_i} = \frac{A_i B_i}{[A_i p_i + B_i(1-p_i)]^2}. \tag{6}$$

As all the terms on the right hand side of equation (6) are positive, $\frac{\partial p_j}{\partial p_i}$ is unconditionally positive. Differentiate equation (6) with respect to p_i to obtain:

$$\frac{\partial^2 p_j}{\partial p_i^2} = \frac{2A_i B_i(B_i - A_i)}{[A_i p_i + B_i(1-p_i)]^3}. \tag{7}$$

Equation (7) is positive if an only if $B_i > A_i$ which simplifies to:

$$u(\underline{w}_i + t_j) - u(\underline{w}_i) \geq u(\overline{w}_i) - u(\overline{w}_i - t_i).$$

Therefore, for positive transfers t_i, monotonicity and concavity of the VNM utility function $u(\cdot)$, condition (4) is sufficient for a convex contract set in the probability space, a necessary condition for risk-pooling agreements to take pace.

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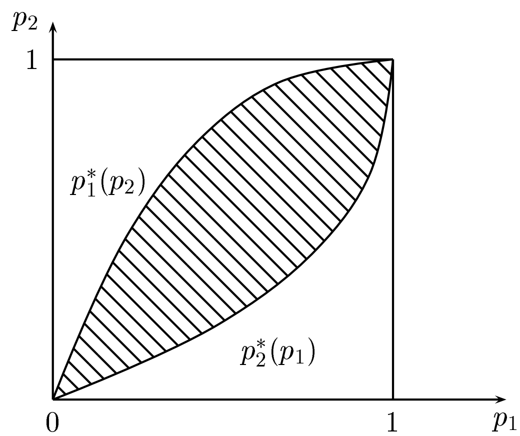
The contract set is illustrated in Figure 1; the transfer scheme occurs only if (p_1, p_2) lie in the shaded region.[2] Observe further that monotonicity of $u(\cdot)$ in equation (1) implies

[2]When inter-sectorial transfers are symmetric, i.e. $t_1 = t_2$, risk aversion guarantees that condition (4) is verified.

Figure 1: Contract set



$p_i \equiv \Pr(w_i = \overline{w}_i), \quad i = 1, 2$

that r_1 is decreasing in p_1:

$$\frac{\partial r_1}{\partial p_1} = (p_2 - 1)\left[u(\overline{w}_1) - u(\overline{w}_1 - t_1)\right] - p_2\left[u(\underline{w}_1 + t_2) - u(\underline{w}_1)\right] < 0.$$

Graphically, in Figure 1, the rent is an increasing function of the distance between any given p_2 and $p_2^*(p_1)$. This result will be useful to characterize the effects of rent extraction on the optimal level of p_1 selected by the principal.

2.2 The principal's problem

Demand for labor in sector 1 is characterized by imperfect competition with collusion among employers. As in Bencivenga and Smith (1997), a representative principal uses a mixed strategy and announces an employment contract which consists of the probability p_1 that agent 1 will obtain $\overline{w}_1$, taking p_2 and all wages as given.

We assume that, because of imperfect competition, the employer is able to extract $G[r_1(p_1;p_2)]$ of the rent created through risk pooling, where $G[\cdot]$ is a monotone increasing function satisfying $G[0] = 0$. We consider two cases to model the employer's problem: (i) the employer is a producer, and (ii) the employer is a rentier.

2.2.1 Principal is a producer

Normalize the size of population to one, and assume that agents are distributed uniformly on the unit interval. We assume that the principal is an employer who selects the number of employees p_1 and $(1-p_1)$ engaged in productive activities. Profits are given by:

$$\pi_1 = F(p_1, 1-p_1) - \overline{w}_1 p_1 - \underline{w}_1(1-p_1) + G[r_1(p_1;p_2)], \tag{8}$$

where $F(\cdot,\cdot)$ is a neoclassical production function with constant returns to scale. If selection of actual employees hired under activities p_1 and $1-p_1$ is done randomly, then this scheme is consistent with our earlier assumptions on uncertainty.

The problem of the employer is to maximize profits subject to the participation constraints to the risk-pooling scheme (2) and (3). Specifically, the Lagrangian for this problem is:

$$\mathcal{L}_1 = \max_{\{p_1\}} \pi_1 + \lambda[p_2 - p_2^*(p_1)] + \mu[p_1 - p_1^*(p_2)], \tag{9}$$

where $\lambda, \mu \geq 0$ are Lagrange multipliers. First-order conditions for an optimum are:

$$\left(\frac{\partial F}{\partial p_1} - \frac{\partial F}{\partial (1-p_1)}\right) - (\overline{w}_1 - \underline{w}_1) + G' r_1' - \lambda p_2^{*\prime} + \mu = 0 \tag{10}$$

where primes ($'$) denote first derivatives.

Consider first the case where no rent from risk-pooling is generated by the choice of employment levels. In this case, since $G[0] = 0$, the optimal p_1^0 satisfies:

$$\left(\frac{\partial F}{\partial p_1} - \frac{\partial F}{\partial (1-p_1)}\right) - (\overline{w}_1 - \underline{w}_1) = 0, \tag{11}$$

when evaluated at $p_1 = p_1^0$. Our next result shows that when the participation constraint for agent 2, eq. (3), is non binding, incorporating the extraction of the risk-pooling rent into the employer's problem necessarily implies a lower p_1 relative to p_1^0.

Proposition 1 *For a level of employment in the other sector p_2 sufficiently low, risk-pooling rent extraction by the employer necessarily implies a lower level of p_1.*

Proof. Since two participation constraints are involved, four cases need to be considered: the constraints for agents 1 and 2 are either binding or non-binding. These cases are illustrated in Figure 2.

(A) $p_2 > p_2^*(p_1)$, $\lambda = 0$ and $p_1 > p_1^*(p_2)$, $\mu = 0$. Substitute in (10). Since by assumption, $G' > 0$ and by monotonicity of preferences $r_1' < 0$, then necessarily:

$$\left(\frac{\partial F}{\partial p_1} - \frac{\partial F}{\partial (1-p_1)}\right) - (\overline{w}_1 - \underline{w}_1) = -G' r_1' > 0.$$

Comparing with (11), by decreasing marginal products, this implies that $p_1 < p_1^0$.

(B) $p_2 = p_2^*(p_1)$, $\lambda > 0$ and $p_1 > p_1^*(p_2)$, $\mu = 0$. Substitute in (10). Since $\lambda > 0$ and from (6) we have shown that $p_2^{*\prime} > 0$, then necessarily:

$$\left(\frac{\partial F}{\partial p_1} - \frac{\partial F}{\partial(1-p_1)}\right) - (\overline{w}_1 - \underline{w}_1) = -G' r_1' + \lambda p_2^{*\prime} > 0.$$

By decreasing marginal products, this implies that $p_1 < p_1^0$.

(C) $p_2 > p_2^*(p_1)$, $\lambda = 0$ and $p_1 = p_1^*(p_2)$, $\mu > 0$. By decreasing marginal products, using (11), $p_1 = p_1^*(p_2) < p_1^0$ if

$$\left(\frac{\partial F}{\partial p_1} - \frac{\partial F}{\partial(1-p_1)}\right)\Bigg|_{p_1 = p_1^*(p_2)} - (\overline{w}_1 - \underline{w}_1) > 0,$$

which is verified for $p_1^*(p_2)$ sufficiently low. Since $p_1^*(\cdot)$ is monotone increasing, and p_1^0 is independent of p_2, there exists a range $p_2 \in (0, p_{2\,\min}]$ where the strict inequality is verified.

(D) $p_2 = p_2^*(p_1)$, $\lambda > 0$ and $p_1 = p_1^*(p_2)$, $\mu > 0$. By convexity of the risk-pooling contract set, this case is impossible for interior p_2.

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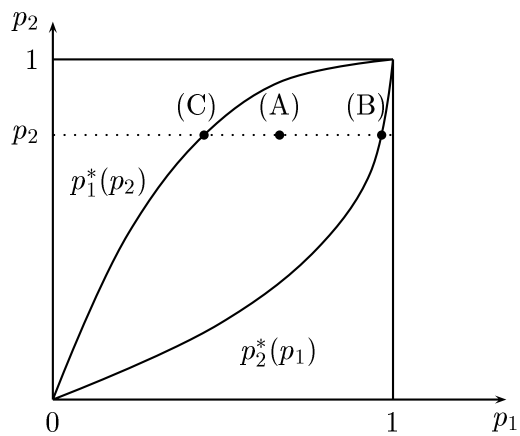
Hence, we have shown that risk-pooling extraction necessarily reduces the level of employment in activities p_1 if that level in the other sector is sufficiently low.

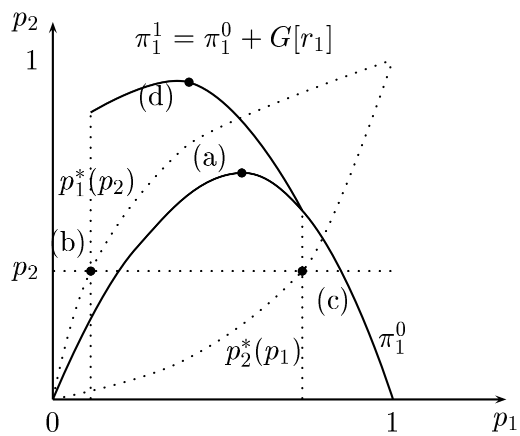
Figure 2: Participation constraints



The principal's problem is illustrated in Figure 3. The profit function in the absence of rent extraction is π_1^0, and yields an optimum at point (a). For a given p_2, when p_1 is within the contract region, i.e. between (b) and (c), the risk-pooling surplus r_1 becomes strictly positive. As mentioned earlier, this rent is an increasing function of the distance between p_2 and $p_2^*(p_1)$. Since the rent extraction function $G(r_1)$ is monotone increasing, this rent is added to the profit function, which shifts upwards to π_1^1. A new optimum obtains at point (d). For a given technology F, an exogenous p_2 that is sufficiently low necessarily implies a lower p_1 at the optimum.

We next turn to the case where the principal's revenues stem only from extracting the surplus created by risk-pooling.

Figure 3: Profit function: Producer



2.2.2 Principal is a rentier

Assume that the employer does not produce, but extracts part of the risk-pooling rent. Consider that agents rent land from the employer and that p_1 is the probability of a good crop. Since $(1 - p_1)$ is the probability of a bad crop, the principal can affect the risk of crop failure by investing in fertilizers, machinery, etc.

The principal's problem is to select the optimal level of p_1 to maximize profits given by his share of the risk-pooling rent $G[r_1]$, minus the cost of adjusting p_1:

$$\pi_1 = G[r_1(p_1; p_2)] - C(p_1), \tag{12}$$

where $C(\cdot)$ is a monotone increasing, convex cost function. Again, rewrite the Lagrangian using (12) for profits, and solve for first-order condition as:

$$G'r_1' - C' - \lambda p_2^{*\prime} + \mu = 0. \tag{13}$$

Our next result shows that the principal in this case always selects the lowest p_1 that satisfies the agent 2's participation constraint.

Proposition 2 *When the principal is a rentier, then at the optimum,*

$$p_1 = p_1^*(p_2)$$

Proof.Again, four cases need to be considered:

(A) $p_2 > p_2^*(p_1)$, $\lambda = 0$ and $p_1 > p_1^*(p_2)$, $\mu = 0$. Substitute in (13). Since by assumption, $G' > 0$ and by monotonicity of preferences $r_1' < 0$, then necessarily:

$$G'r_1' - C' < 0,$$

since costs are increasing. Hence because marginal profits are negative, this cannot be an optimum. The principal reduces p_1.

(B) $p_2 = p_2^*(p_1)$, $\lambda > 0$ and $p_1 > p_1^*(p_2)$, $\mu = 0$. Substitute in (13). Since $\lambda > 0$ and from (6) we have that $p_2^{*\prime} > 0$, then necessarily:

$$G' r_1' - C' - \lambda p_2^{*\prime} < 0,$$

again, this cannot be an optimum. The principal reduces p_1.

(C) $p_2 > p_2^*(p_1)$, $\lambda = 0$ and $p_1 = p_1^*(p_2)$, $\mu > 0$. Substitute in (13). Since $\mu > 0$, then:

$$G' r_1' - C' + \mu = 0.$$

Since the principal cannot reduce further p_1 without destroying the risk-pooling rent, this is the lowest possible p_1 which satisfies the participation constraint for agent 2.

(D) $p_2 = p_2^*(p_1)$, $\lambda > 0$ and $p_1 = p_1^*(p_2)$, $\mu > 0$. By convexity of the risk-pooling contract set, this case is impossible.

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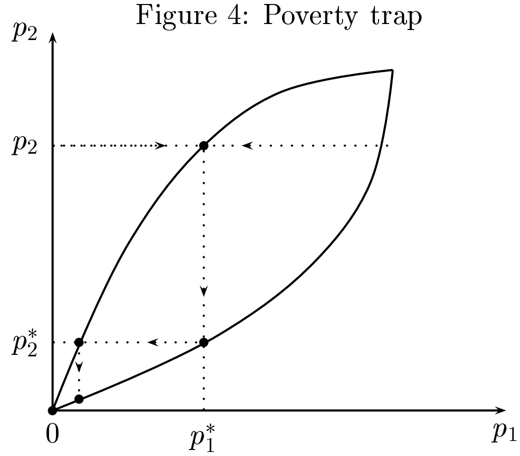
Consequently, the principal chooses the lowest p_1 which satisfies the participation constraint for agent 2 given by inequality (3), and sets $p_1 = p_1^*$ (point (C) in Figure 2). The intuition is straightforward: since costs are increasing in p_1, and the rent accruing to agent 1 decreases in the probability of a good event, then necessarily, the only optimum for the principal is a corner solution.

Figure 4: Poverty trap



2.3 Intertemporal linkages and poverty traps

Following an exogenous increase in p_2, the relative risk of agent 2 of receiving $\underline{w}_2$ decreases compared to the risk faced by agent 1 of obtaining the low income. Hence, a risk-pooling contract becomes less attractive to agent 2. The rentier-principal who wishes to maintain the risk-sharing arrangement between the two agents must also raise the probability of the high outcome for agent 1. The upper envelope of the contract set in this case captures the interactions between the two sectors' distributions. Therefore, risk-sharing between agents in different sectors allows for an *alternative* inter-sectorial transmission mechanism for income distributions.

Moreover, under intra-sectorial collusion but in the absence of inter-sectorial collusion, extracting the rent created by risk pooling may lead to a poverty trap. In Figure 4, if the principal in sector 2 is also a monopsonistic Stackleberg player, its strategies are symmetric to those of sector 1: this employer sets $p_2 = p_2^*$, taking p_1^* as given. Subsequently, the principal in sector 1 responds by choosing a lower p_1. The unique stable equilibrium is $p_1 = p_2 = 0$, with agents exclusively receiving the low incomes $(\underline{w}_1, \underline{w}_2)$.

3 Conclusion

This paper has developed a two-sector model to show how inter-agents transfers can affect the distribution of incomes. When the realization of states is observed only by risk-averse agents with uncorrelated income risk, mutual risk-pooling agreements can arise. The rent thus created can be extracted by a principal who chooses the income distribution taking into account the surplus from risk pooling of his employees which he subsequently extracts. One important implication is a transfers-based inter-sectorial transmission mechanism. This linkage between income distributions across sectors can have perverse effects if both sectors are monopsonistic and do not collude: all employment tends to be concentrated in low-incomes allocations.

Finally, an application of our analysis could be rural and urban sectors where it is in the best interest of a unique rural employer to keep up with the pace of development in the urban sector by increasing rural employment in high income activities.

References

Bardhan, Pranab (1989) *The Economic Theory of Agrarian Institutions* (New York: Oxford University Press)

Bencivenga, Valerie R., and Bruce D. Smith (1997) 'Unemployment, migration and growth.' *Journal of Political Economy* 105, 582–608

Kocherlakota, Narayana R. (1996) 'Implications of efficient risk sharing without commitment.' *Review of Economic Studies* 63, 959–609

Larson, Donald, and Yair Mundlak (1997) 'On the intersectoral migration of agricultural labor.' *Economic Development and Cultural Change* 45(2), 295–319

Ligon, Ethan, Jonathan P. Thomas, and Tim Worrall (1997) 'Informal insurance arrangements in village economies.' mimeo, Agricultural and Resource Economics, University of California at Berkeley

Pollak, Robert (1985) 'A transaction approach to families and households.' *Journal of Economic Literature* 23, 581–608

Tomich, Thomas P., Peter Kilby, and Bruce F. Johnston (1995) *Transforming Agrarian Economies: Opportunities Seized, Opportunities Missed* (Ithaca: Cornell University Press)

World Bank (1994) *Averting the Old Age Crisis: Policies to Protect the Old and Promote Growth* (New-York: Oxford University Press)