Total Wealth and Asset Returns in Canada*

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Abstract

Recent estimation of the aggregate total wealth for Canada by Macklem (1997) explicitly incorporates human net worth. We use these series in a consumption CAPM framework to estimate the Euler equations over the return on total wealth, stocks and bonds. In addition, we specify optimal consumption as a quasi-reduced form that is proportional to wealth. We estimate the model for both separable and nonseparable preferences. Because the wealth beta is larger than the consumption beta, our results show a partial improvement with respect to the main pricing puzzles, and are more favorable to nonseparable compared to separable utility.

JEL classification: G12.

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1 Introduction

The consumption-based Capital Asset Pricing Model (C-CAPM) predicts that the excess returns on risky assets are a function of the risks to the intertemporal marginal rates of substitution (IMRS) of consumption. More precisely, under the joint hypothesis of time and state separability of preferences and iso-elastic utility, the equity premia is proportional to the quantity of consumption risk (i.e. the consumption covariance of returns), times the price of that risk. The latter is given by the representative agent's Arrow-Pratt coefficient of relative risk aversion.

As is well known, this model produces anomalies when confronted to empirical data (Kocherlakota 1996, for example). Hence, the high observed premia on stock can only be reconciled with low quantities of consumption risk by increasing risk aversion to implausible levels (the equity premium puzzle). Moreover, under the null hypothesis of separability, risk aversion is the reciprocal of the elasticity of intertemporal substitution. Highly risk-averse agents are thus extremely reluctant to substitute consumption across periods. A high level of interest rates should be necessary to induce them to save. However, the rate of return on riskfree government bonds is extremely low, suggesting that agents negatively discount future consumption (the riskfree rate puzzle).

More general preferences have been advocated by Epstein and Zin (1989, 1991). These specifications consider the attitudes toward risk and intertemporal substitution as separate. Moreover, contrary to the standard expected utility setting, the agent's utility is nonseparable with respect to time and the state of the world, whereas the agent is not indifferent to the timing of the resolution of uncertainty. The resulting Euler equations incorporate an additional contributor to IMRS risk: The return on the total wealth portfolio. Specifically, the equity premia is then expressed as a weighted average of the consumption risk term, and the static CAPM market portfolio risk term. Under the null of separable preferences, this second risk is not valued by the market, and the expression simplifies to the standard C-CAPM model.

Estimation is complicated by the absence of a reliable proxy for total wealth. Although estimates of nonhuman wealth can be constructed using aggregate financial and tangible wealth, human wealth is not observable. Epstein and Zin (1991) address this issue by resorting to the usual CAPM practice of approximating the return on the market portfolio by the return on a stock market index. They find that nonseparable preferences yield more realistic estimates of the preference parameters, and that the null of separability is in general rejected.

These results however need to be interpreted with caution. First, taking into account conditional heteroscedasticity yields test statistics that do not reject the null that the price of market risk is zero (Jorion and Giovannini 1993, Normandin and St-Amour 1998). More fundamentally, Kocherlakota (1996) points out that using a stock market index to estimate a model with nonseparable preferences is subject to Roll's (1977) critique. Specifically, total wealth incorporates many assets whose returns may be poorly correlated with stocks. In addition, equity accounts for a small portion of total wealth (Heaton and Lucas 1997, Macklem 1997). It follows that using a stock market index as a proxy for total wealth return might overestimate the correlation with individual stocks. By artificially increasing the quantity of market risk, the estimation could therefore reduce the weight placed on consumption risk and result in lower estimates of relative risk aversion. Clearly then, this issue remains an open empirical question requiring better estimates for total wealth.

Recently, Macklem (1997) has produced quarterly estimates for Canadian total wealth (Beach, Boadway and Bruce 1988, provide annual estimates). His constructed series uses financial and tangible wealth to estimate nonhuman wealth. Contemporary human wealth is evaluated as the expected net present value of current and future labor income. This approach implicitly assumes that labor income is the dividend revenue accruing to the holder of human capital, and that the discounting rate of return on this flow can be reasonably be approximated by the real rate of interest

(Shiller 1995, Campbell 1996). The resulting series has the merit of explaining a large portion of consumption (Macklem 1994).

In this paper, we use the total wealth series produced by Macklem in the log-linearized Euler equations obtained under separable and nonseparable preferences. Specifically, we estimate a joint system composed of consumption growth, the returns on total wealth, stock returns and on a riskfree government bond. The quarterly data is for Canada, and covers the sample 1963:2 to 1994:2. Optimal consumption is modeled as quasi-reduced form (QRF) that is proportional to wealth. This formulation is consistent with the assumption of a constant investment opportunity set and the observation that the average propensity to consume shows no discernible trend.

Our first result shows that for separable preferences, incorporating the return on total wealth produces a high coefficient of relative risk aversion that is over 28, and a low subjective discount factor that is close to 76%. Introducing nonlinearities in the optimal consumption schedule does not alter the fundamental results. Indeed, we do not reject the null of a linear consumption rule. Nonetheless, these results constitute a partial improvement compared to the standard practice of postulating an unrestricted reduced form (URF) of constant consumption growth, and estimating the model over stock and bond returns only. Indeed, for our data set, when we follow this approach, the corresponding risk aversion is over 72, while the discount factor is 1.24.

We then generalize the model by incorporating nonseparable preferences. Because of the QRF for optimal consumption, the subjective discount factor is not identifiable, and we therefore fix it to reasonable values to estimate the model's other parameters. Our estimates for relative risk aversion remain unchanged, at over 28, while the estimate for the elasticity of intertemporal substitution is realistic and varies between 1.57 and 0.45, except for very low rates of time discounting.

These results can be interpreted as follows. At the optimum, because consumption and portfolio shares are constant, the excess returns on stock and on the total wealth portfolio are a function of risk aversion only, with nonseparability playing no role in rationalizing the premia. Hence, the only relevant empirical question becomes whether or not the total wealth portfolio risks and variances, which explain the premia, are different from the usual consumption risk. In fact, the data of Macklem are consistent with a larger wealth risk, compared to the consumption beta. Unsurprisingly, this implies that the price of that risk (i.e. the risk aversion) required to replicate the risk premia does not have to be as large.

Nevertheless, nonseparability does play a role in accounting for the riskfree rate. When the discount factor is set at realistic values, we find reasonable estimates for the elasticity of intertemporal substitution. Our empirical results are therefore consistent with the theoretical results of Weil (1989), who showed that nonseparability cannot improve the C-CAPM's performance with respect to the equity premium puzzle, but plays an important role with respect to the riskfree rate puzzle.

The rest of this paper is as follows. In the next section, we present the model and the log-linearized Euler equations, when optimal consumption is linear in wealth. In Section 3, we present the econometric model and discuss the estimation details. The results are presented in Section 4, while a conclusion in Section 5 reviews the main findings.

2 Model

Consider a representative agent whose preferences over uncertain consumption streams are given by:

$$V_t = \left\{ (1-\beta) C_t^{1-1/\psi} + \beta [\mathrm{E}_t(V_{t+1})]^{\frac{1-1/\psi}{1-\gamma}} \right\}^{\frac{1-\gamma}{1-1/\psi}} . \tag{2.1}$$

The agent's problem is to maximize (2.1) by choosing the sequences of consumption C_t and portfolio $\boldsymbol{\omega}_t = [\omega_{i,t}]_{i=1}^n$, subject to:

$$A_{t+1} = (A_t - C_t)(1 + r_{t+1}), \quad r_{t+1} \equiv \sum_{i=1}^{n} \omega_{i,t} r_{i,t+1}. \tag{2.2}$$

In our notation, V_t denotes current utility, $\beta \in (0,1)$ is a subjective discount factor, $\psi > 0$ is the elasticity of intertemporal substitution, $\gamma > 0$ is the coefficient of relative risk aversion, and $\mathrm{E}_t(\cdot) \equiv \mathrm{E}(\cdot|I_t)$ denotes the expectation operator, conditional on the information set I_t. In addition, A_t denotes current total wealth, r_{t+1} is the total wealth portfolio return, $r_{i,t+1}$ denote the rate of return on individual assets $i = 1, \ldots, n$. We assume that the dividend growth on these assets follows a Gaussian, i.i.d. process.

Preferences (2.1) are characterized by nonseparability with respect to time and the state of the world when $\gamma \neq 1/\psi$ (Epstein and Zin 1989, Weil 1990). Specifically, the marginal utility of consumption is not independent of the consumption level in any adjacent periods of time, and/or states of the world. It is straightforward to show that imposing the restriction that risk aversion is the reciprocal of the elasticity of intertemporal substitution ($\gamma = 1/\psi$) yields the usual Von Neuman-Morgenstern preference representation with iso-elastic (i.e. CRRA) sub-utility. The equation (2.2) is the standard intertemporal budget constraint, and implicitly incorporates the revenues accruing to the holder of all capital, whether human or not.

The first-order conditions characterizing an interior optimum for the agent's problem are given by:

$$1 = \mathrm{E}_t \left\{ \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-1/\psi} \right]^{(1-\gamma)/(1-1/\psi)} \left[\frac{1}{1 + r_{t+1}} \right]^{1-(1-\gamma)/(1-1/\psi)} (1 + r_{i,t+1}) \right\}, \tag{2.3}$$

for $i = 1, \ldots, n$. Under the joint assumption that the gross rate of consumption growth and the gross rates of return are conditionally log normal, the log-linearized Euler equation yields the following expression for the equity premia:

$$\mathrm{E}_t r_{e,t+1} - r_{f,t+1} = \frac{1}{\psi}\left(\frac{1-\gamma}{1-1/\psi}\right)\sigma_{ec} + \left[1 - \left(\frac{1-\gamma}{1-1/\psi}\right)\right]\sigma_{er} - 0.5\sigma_{ee}, \tag{2.4}$$

where $\sigma_{ij} \equiv \mathrm{Cov}(r_{i,t+1}, r_{j,t+1}|I_t)$, i, j, r is a covariance term; $r_{e,t+1}$ is a generic risky asset, and $r_{f,t+1}$ is the riskfree asset. From (2.4), it can be seen that for nonseparable preferences, $\gamma \neq 1/\psi$, the premium on a risky asset is a function of two risks: The consumption risk σ_{ec}, and the total wealth portfolio risk σ_{er}. This second source of risk can, in theory, be sufficiently positive to reduce the weight placed on the consumption beta in explaining the high observed premia on stock (Epstein and Zin 1989, Epstein and Zin 1991).

Next, it can be shown that, under the assumption of a constant investment opportunity set, with log-normal returns, individual portfolio shares $\omega_{i,t}$ are constant, such that the market portfolio rate is also log normal, and optimal consumption is proportional to wealth:

$$C_t = cA_t, \quad \forall t, \tag{2.5}$$

where c is a complex function of the model's primitives (Weil 1990, Campbell and Viceira 1999, Normandin and St-Amour 1999). Substituting into the budget constraint (2.2), and using Hicks' approximation, we obtain that consumption growth is affine on the return on the total wealth portfolio:

$$\Delta c_{t+1} = -c + r_{t+1}, \tag{2.6}$$

where $\Delta c_{t+1} \equiv \log(C_{t+1}) - \log(C_t)$ is the consumption growth rate. It follows directly that consumption risk is equal to total wealth portfolio risk, such that the expression for the excess return simplifies to:

$$\mathrm{E}_t r_{e,t+1} - r_{f,t+1} = \gamma\sigma_{er} - 0.5\sigma_{ee}, \tag{2.7}$$

which is exactly the expression obtained under the separable preferences restriction $\gamma = 1/\psi$. Hence, when the investment opportunity set is constant, nonseparability of preferences plays no role in explaining the equity premia at the optimum. This illustrates the well-known result that the equity premium is independent of the elasticity of intertemporal substitution ψ, and is a function of risk aversion only (Weil 1989).

In fact, nonseparability does play a part in explaining the *levels* of returns, and in particular, the riskfree rate. To see this, substitute (2.6) in (2.3) to obtain that (Gilbert 1999) :

$$\mathrm{E}_t\{\Delta c_{t+1}\} = -\frac{c}{1-1/\psi} - \frac{\log(\beta)}{1-1/\psi} - 0.5(1-\gamma)\sigma_{rr}, \tag{2.8}$$

$$\mathrm{E}_t\{r_{t+1}\} = -\frac{c}{\psi-1} - \frac{\log(\beta)}{1-1/\psi} - 0.5(1-\gamma)\sigma_{rr}, \tag{2.9}$$

$$\mathrm{E}_t\{r_{e,t+1}\} = -\frac{c}{\psi-1} - \frac{\log(\beta)}{1-1/\psi} - 0.5\gamma\sigma_{rr} + \gamma\sigma_{er} - 0.5\sigma_{ee}, \tag{2.10}$$

$$\mathrm{E}_t\{r_{f,t+1}\} = -\frac{c}{\psi-1} - \frac{\log(\beta)}{1-1/\psi} - 0.5\gamma\sigma_{rr}. \tag{2.11}$$

Clearly, imposing that $\gamma = 1/\psi$ yields a different expression for all returns. For the riskfree rate of return (2.11), even if the price of risk, γ, is high, low riskfree rates can still be obtained without requiring that $\beta > 1$, through the elasticity of intertemporal substitution ψ and the average propensity to consume c. Again, this is consistent with the theoretical results of Weil (1989) that nonseparability influences mainly the riskfree rate. Finally, observe that subtracting (2.11) from (2.9) yields:

$$\mathrm{E}_t r_{t+1} - r_{f,t+1} = (\gamma - 0.5)\sigma_{rr}, \tag{2.12}$$

which, together with (2.7) identifies the coefficient of relative risk aversion γ. The relevant question for the equity premium puzzle then becomes how the total wealth portfolio risk σ_{er} and variance σ_{rr} compare with the consumption risk σ_{ec} in explaining the excess returns on stocks, and on the total wealth portfolio.

We characterize the average propensity to consume, c, as a QRF. This approach can be justified as follows. In the original pure exchange C-CAPM formulation, nonstorability and identical preferences and endowments implies that equilibrium consumption growth is equal to the growth of dividends on the total wealth portfolio. In corresponding empirical applications, consumption growth is modeled either as purely exogenous (Hansen and Singleton 1983), or equal to the dividend growth on a broad-based stock market index (Cecchetti, Lam and Mark 1990). More precisely, c is estimated from $\mathrm{E}_t[C_{t+1}/C_t] \propto (1-c)$, or from $\mathrm{E}_t[D_{t+1}/D_t] \propto (1-c)$, where D_t are dividends, jointly with the valuation equations for returns. From the budget constraint and (2.6), this approach can be considered as a special case where the returns on the total wealth portfolio are constant: $r_t = r$, $\forall t$. By contrast, we do not impose this restriction, but instead use the fact that the average propensity to consume is uniquely identified from the mean difference between the return on total wealth (2.9) minus consumption growth (2.8):

$$c = \mathrm{E}_t(r_{t+1} - \Delta c_{t+1})$$

independent of preference or scedastic parameters.

3 Estimation

3.1 Econometric Model

We now turn to the estimation of the model. Our approach is similar to Epstein and Zin (1991), Jorion and Giovannini (1993) and Normandin and St-Amour (1998). Specifically, consistent with our assumption of log-normal returns, the econometric model is given by:

$$\begin{aligned}
\Delta c_{t+1} &= \mathrm{E}_t\{\Delta c_{t+1}\} + \epsilon_{c,t+1}, \\
r_{t+1} &= \mathrm{E}_t\{r_{t+1}\} + \epsilon_{r,t+1}, \\
r_{e,t+1} &= \mathrm{E}_t\{r_{e,t+1}\} + \epsilon_{e,t+1}, \\
r_{f,t+1} &= \mathrm{E}_t\{r_{f,t+1}\} + \epsilon_{f,t+1},
\end{aligned} \tag{3.1}$$

where $\boldsymbol{\epsilon}_{t+1} = [\epsilon_{c,t+1}, \epsilon_{r,t+1}, \epsilon_{e,t+1}, \epsilon_{f,t+1}]'$ is a vector of rational-expectations innovations distributed as:

$$\boldsymbol{\epsilon} \sim \text{N.I.D.}\left[\mathbf{0}, \boldsymbol{\Sigma}\right], \tag{3.2}$$

where $\boldsymbol{\Sigma} = [\sigma_{ij}]$ for $i, j = c, r, e, f$. The conditional means $\mathrm{E}_t\{\Delta c_{t+1}\}$, $\mathrm{E}_t\{r_{t+1}\}$, $\mathrm{E}_t\{r_{e,t+1}\}$, and $\mathrm{E}_t\{r_{f,t+1}\}$ are given by (2.8)–(2.11).

In theory, the econometric model (3.1) could be estimated for any arbitrary number of risk assets $r_{e,t+1}$. For simplicity, we limit our attention to a single market index which we discuss below. Under the assumption of Gaussian innovations, we resort to a Maximum Likelihood (ML) estimator. Omitting a constant term, the log-likelihood function for our four-equation system is:

$$\mathcal{L}(\boldsymbol{\theta}) = -0.5 \log |\boldsymbol{\Sigma}| - 0.5 \mathrm{tr}(\boldsymbol{\Sigma}^{-1} \boldsymbol{\epsilon}' \boldsymbol{\epsilon}), \tag{3.3}$$

where $\boldsymbol{\theta} = [c, \gamma, \psi, \beta, \sigma_{ij}]$ is the vector of QRF parameter, preference and scedastic parameters.

Due to the nonlinearities involved, we use two estimation algorithms, BFGS and Simulated Annealing. The latter has the advantage of not resorting to numerical gradients, at the cost however of a much slower convergence. Moreover, to ensure robustness to local extrema, we used several sets of starting values. Our reported results displayed considerable robustness to initial values and convergence problems. Finally, to ensure the covariance matrix $\boldsymbol{\Sigma}$ is positive, semi-definite, we estimated a Cholesky decomposition matrix $\boldsymbol{Q}$, where $\boldsymbol{QQ}' = \boldsymbol{\Sigma}$ instead of estimating $\boldsymbol{\Sigma}$ directly.

3.2 Data

The model (3.1) is estimated using Canadian quarterly data, for the period 1963:2 to 1994:2. The sample period is chosen so as to match the reported wealth series of Macklem (1997). Data sources are from CANSIM.

Our consumption series is measured as real, per-capita expenditures on nondurables and services. Total wealth is also in real, per-capita terms and is taken from Macklem (1997). It is composed from three series: physical, financial and human wealth. Physical and financial wealth include net financial assets (either directly, or indirectly held, e.g. through pension funds), real estate and durables. They are calculated through the flow of funds, and the national accounts. Human wealth is defined by Macklem as the present value of labor income, net of public expenditures. It is based on a bivariate VAR estimate for income and the real interest rate.

The rates of return are obtained for total wealth (r_t), corporate equity ($r_{e,t}$), and the riskfree rate ($r_{f,t}$). The return on the total wealth portfolio is obtained using the budget constraint (2.2):

$$r_{t+1} = \frac{A_{t+1}}{A_t - C_t} - 1.$$

The return on corporate stock is given by the real return on the TSE300 index, adjusted for dividends. Finally, we used the real rates of return on 3-months T-Bills as a proxy for the riskfree rate.

Table 1: Descriptive Statistics

Series	Mean	Std. Err.		Covariances/Correl.		
Δc	5.19e-03	7.67e-03	5.88e-05	3.25e-05	1.54e-04	-1.14e-05
r	3.48e-02	3.22e-02	0.13	1.04e-03	4.29e-04	2.19e-05
r_e	1.39e-02	7.80e-02	0.26	0.17	6.09e-03	4.44e-05
r_f	5.87e-03	9.02e-03	-0.16	0.08	0.06	8.14e-05

Note: Covariances (correlations) in upper (lower) triangle of the variance-covariance matrix.

Descriptive statistics for the series used in the estimation are presented in Table 1. The summary statistics reveal that the market portfolio risk σ_{er} is almost 3 times as large as the consumption risk σ_{ec}. As mentioned earlier, this could imply in theory that the model is able to reproduce the large excess returns on stock without resorting to unrealistic levels of risk aversion. Note further that the return on total wealth is numerically larger than the return on stock, although not significantly different. Finally, the return on total wealth is predictably less volatile than the return on stock.

Table 2 reports preliminary test statistics for the hypotheses of stationarity, normality and conditional homoscedasticity. The three assumptions are not rejected in the data, for the cases of consumption growth, and the return on the total wealth portfolio. The null of stationarity is rejected for Treasury bills under the Augmented Dickey-Fuller test, but not under the Phillips-Perron test. With respect to the Gaussian assumption, the hypothesis of normality is not rejected, except for the Jarque-Bera test in the case of stock. Finally, note that conditional homoscedasticity

Table 2: Preliminary Test Statistics

Test	Δc	r	r_e	r_f
		A. Stationarity		
Augmented Dickey-Fuller (lags)	-4.03(2)	-4.45(4)	-9.52(0)	-1.91*(3)
Phillips-Perron	-10.99	-13.57	-9.43	-5.36
		B. Normality		
Jarque-Bera	1.88	1.78	13.07*	1.84
Kolmogorov-Smirnov	0.99	0.17	0.12	0.82
		C. Conditional Heteroscedasticity		
ARCH(1)	0.14	0.12	0.15	0.02*
ARCH(2)	0.35	0.27	0.02*	0.04*

Note: Star (*) indicates rejection at the 5% level. Number of lags in ADF test optimizes AIC criterion. P-value reported for the K-S normality test and the ARCH tests. ARCH(lags) test presence of autoregressive conditional heteroscedasticity in squared residuals.

is rejected in stock and T-bills. However, for simplicity, we do not incorporate corrections in our estimation.[1]

Finally, Figure 1 plots the consumption-wealth ratio C/A, as well as the returns on the total wealth portfolio r. As the plot makes clear, the assumption that the investment opportunity set is constant, implying that the optimal average propensity to consume is also constant is not unrealistic. Indeed, we find no discernible trend, while the ratio is at best mildly volatile. Below, we nonetheless allow for time-varying consumption wealth ratio, by introducing a nonlinear consumption schedule. As will become clear shortly, the null of constant average propensity to consume is not rejected.

[1] Taking into account potential heteroscedasticity would require allowing for time-varying investment opportunity sets, which would violate the constant average propensity to consume QRF. Moreover, incorporating GARCH structures into asset pricing models with nonseparable preferences has little impact on the estimated parameter values (Jorion and Giovannini 1993, Normandin and St-Amour 1998). Given that the null is not rejected at the 1% level, we do not pursue this approach.

4 Results

This section presents the estimation results. We start our analysis with the separable preference model, followed by the more general nonseparable case. Because the parameters of the covariance matrix are only instrumental to our analysis, we simplify exposition by presenting only the preference parameters, and the QRF average propensity to consume.[2]

4.1 Separable Preferences

We first present the estimated parameters under the imposed restriction that risk aversion is the reciprocal of the elasticity of intertemporal substitution, $\gamma = 1/\psi$. Recall that in this case, preferences simplify to the expected utility representation. Imposing the separable preferences restriction in the log-linearized Euler equations (2.8) – (2.11) yields:

$$\mathrm{E}_t\{\Delta c_{t+1}\} = -\frac{c}{1-\gamma} - \frac{\log(\beta)}{1-\gamma} - 0.5(1-\gamma)\sigma_{rr}, \tag{4.1}$$

$$\mathrm{E}_t\{r_{t+1}\} = -\frac{\gamma c}{1-\gamma} - \frac{\log(\beta)}{1-\gamma} - 0.5(1-\gamma)\sigma_{rr}, \tag{4.2}$$

$$\mathrm{E}_t\{r_{e,t+1}\} = -\frac{\gamma c}{1-\gamma} - \frac{\log(\beta)}{1-\gamma} - 0.5\gamma\sigma_{rr} + \gamma\sigma_{er} - 0.5\sigma_{ee}, \tag{4.3}$$

$$\mathrm{E}_t\{r_{f,t+1}\} = -\frac{\gamma c}{1-\gamma} - \frac{\log(\beta)}{1-\gamma} - 0.5\gamma\sigma_{rr}. \tag{4.4}$$

These conditional means are used in the econometric model (3.1) to estimate the preferences and distributional parameters.

We consider two possible cases for optimal consumption: proportional to wealth, and nonlinear in wealth. Nonlinear consumption schedules have been shown to result in the presence of undiversifiable income risks (Zeldes 1989, Carroll and Kimball 1996, Letendre and Smith 1999). For the

[2]The complete estimation results are available upon request.

second case, we modify the consumption schedule as follows:

$$C_t = c_0 A_t^{c_1}.$$

The corresponding conditional means for consumption growth and returns are given as:

$$\mathrm{E}_t\{\Delta c_{t+1}\} = \frac{c_1 b_t}{1-\gamma c_1} - \frac{c_1 \log(\beta)}{1-\gamma c_1} - 0.5 c_1 (1-\gamma c_1)\sigma_{rr}, \tag{4.5}$$

$$\mathrm{E}_t\{r_{t+1}\} = \frac{\gamma c_1 b_t}{1-\gamma c_1} - \frac{\log(\beta)}{1-\gamma c_1} - 0.5(1-\gamma c_1)\sigma_{rr}, \tag{4.6}$$

$$\mathrm{E}_t\{r_{e,t+1}\} = \frac{\gamma c_1 b_t}{1-\gamma c_1} - \frac{\log(\beta)}{1-\gamma c_1} - 0.5\gamma c_1 \sigma_{rr} + \gamma c_1 \sigma_{er} - 0.5\sigma_{ee}, \tag{4.7}$$

$$\mathrm{E}_t\{r_{f,t+1}\} = \frac{\gamma c_1 b_t}{1-\gamma c_1} - \frac{\log(\beta)}{1-\gamma c_1} - 0.5\gamma c_1 \sigma_{rr}. \tag{4.8}$$

where $b_t \equiv \log(1 - c_0 A_t^{c_1 - 1})$. The linear consumption schedule is obtained as $c = c_0$, $c_1 = 1$.

Finally, to provide a perspective on the impact of introducing total wealth, we also followed the standard practice of setting $\mathrm{E}_t[\Delta c_{t+1}] = 1 - c$, a constant and estimating a trivariate system of consumption growth, stock and bond returns:

$$\mathrm{E}_t\{\Delta c_{t+1}\} = (1-c), \tag{4.9}$$

$$\mathrm{E}_t\{r_{e,t+1}\} = -\log(\beta) + \gamma(1-c) - 0.5(\gamma^2\sigma_{cc} - 2\gamma\sigma_{ce} + \sigma_{ee}), \tag{4.10}$$

$$\mathrm{E}_t\{r_{f,t+1}\} = -\log(\beta) + \gamma(1-c) - 0.5\gamma^2\sigma_{cc}. \tag{4.11}$$

Table 3 reports the estimated parameters for the three models with separable preferences.

In panel A, the estimated parameter values all have the correct sign, and are all significantly different from zero. However, the coefficient of relative risk aversion, γ, is too high with respect to the range usually considered realistic (e.g. between 0 and 10) while the subjective discount factor

Table 3: Estimated Parameters, Separable Preferences

γ	β	c_0	c_1	$(1-c)$	Log-Likelihood
A. Linear Consumption Schedule					
28.66	0.76	0.0296			1247.86
(4.58)	(0.057)	(0.0029)			
B. Nonlinear Consumption Schedule					
24.27	0.74	0.00339	1.17		1248.38
(5.91)	(0.058)	(0.0081)	(0.193)		
C. Constant Consumption Growth (wealth omitted)					
72.78	1.24			0.0052	992.36
(60.39)	(0.10)			(0.0007)	

Note: Standard errors in parentheses. Estimated model is (3.1), with conditional means (4.1)–(4.4) for the linear consumption schedule [panel A], (4.5)–(4.8) for the nonlinear consumption schedule [panel B] and (4.9)–(4.11), for the constant consumption growth model [panel C].

is too low. Still, the model is successful in reproducing the unconditional means, as can be seen from Table 4.

Table 4: Actual and Predicted Means: Separable Preferences with Linear Consumption

Series	Actual mean	Predicted mean	Bias (in %)
Δc_t	5.19e-03	5.18e-03	0.29
r_t	3.48e-02	3.48e-02	0
$r_{e,t}$	1.39e-02	1.33e-02	4.37
$r_{f,t}$	5.88e-03	5.88e-03	2.4e-02

Note: Actual and predicted unconditional means. Estimated model is (3.1), with conditional means (4.1)–(4.4) for the linear consumption schedule.

Allowing for nonlinear average propensity to consume in panel B has little impact on the estimated parameter values. Moreover, a likelihood ratio test that the consumption schedule is linear does not reject the null at the 5% level, with a test statistic of 1.06. Hence, the rest of this analysis treats the consumption wealth ratio as a constant.

Finally, when total wealth is omitted from the analysis in panel C, we estimate $\gamma = 73$ and $\beta = 1.24$. These results are a clear indication of the equity premium puzzle, and the corresponding riskfree rate puzzle (Weil 1989). In comparison, as mentioned earlier, the larger wealth covariance relative to the consumption covariance of stock can justify a high premium without having to inflate the risk aversion parameter to such levels in panels A and B. In that sense, incorporating total wealth provides a partial improvement in the results.

4.2 Nonseparable Preferences

We now turn to the estimation of the model with nonseparable preferences (2.8) – (2.11). From the previous discussion, it is straightforward to demonstrate that the model is underidentified when we use the linear QRF for optimal consumption. Recall that c is identifiable from the mean difference between the total wealth return and the consumption growth rate, while γ is identifiable from the excess returns. However, there remains two preference parameters, β and ψ, to identify from a constant term.

We therefore need to calibrate either β or ψ and estimate the remaining elements of $\boldsymbol{\theta}$. Since for the separable preferences model we found that the subjective discount factor was too low, we fix it to reasonable values and estimate γ, ψ and c. For this purpose, we chose β equal to 0.96, 0.97 and 0.9925, corresponding to annual discount rates of 16%, 10% and 3%, respectively. Table 5 presents the estimation results.

The estimated parameter values are all robust to the choice of starting values for the numerical algorithms. The first observation is that, as anticipated, the coefficient of relative risk aversion and the average propensity to consume remain unchanged with respect to the separable case.

Table 5: Estimated Parameters, Nonseparable Preferences

γ	ψ	c	Log-Likelihood
		A. $\beta = 0.9600$	
28.66	0.45	0.0292	1247.86
(4.56)	(0.10)	(0.0028)	
		B. $\beta = 0.9740$	
28.66	1.57	0.0292	1247.86
(4.56)	(0.65)	(0.0028)	
		C. $\beta = 0.9925$	
28.66	-0.69	0.0292	1247.86
(4.56)	(0.07)	(0.0028)	

Note: Standard errors in parentheses. Estimated model is (3.1), with conditional means (2.8)–(2.11).

Recall from the previous discussion that the equity premium is independent of nonseparabilities in preferences, while c is uniquely identified from the average difference between total wealth return and consumption growth. Secondly, these parameter values are unaffected by the calibrated value for β, as predicted by the model.

Turning to the elasticity of substitution, ψ, we find in panel A a realistic value of 0.45. This estimate is within the range of 0.10 to 0.60 found by others using American data, without a measure for aggregate wealth (Epstein and Zin 1991, Jorion and Giovannini 1993, Normandin and St-Amour 1998). In panel B, when $\beta = 0.974$, the value is larger than 1, although not significantly different from the range usually found by others. Finally, in panel C, ψ is of the wrong sign and significantly different from zero when $\beta = 0.9925$. Interestingly, although a formal test cannot be performed since one of the parameter is fixed, the elasticity of intertemporal substitution is numerically 15 times larger than the inverse of risk aversion. This suggests that the null of separable preferences might not be supported by our data set.

Overall we conclude that the model with nonseparable preferences presents mixed results. One the one hand, the coefficient of relative risk aversion remains excessively high. However, whereas the separable case yields a β that is unreasonable, nonseparability results in realistic estimates for the elasticity of intertemporal substitution when the subjective discount factor is calibrated to reasonable values. Finally, the risk aversion is numerically quite different from the inverse of the elasticity of intertemporal substitution, indicating potential rejection of the separable hypothesis.

5 Conclusion

The objective of this paper has been to introduce recent measures of total wealth in the asset pricing equations obtained under both separable and nonseparable preferences. We estimated the C-CAPM through a quasi-reduced form for optimal average propensity to consume, first by imposing, and then relaxing the separable preferences restriction. Our returns data set included a stock market index, a riskfree rate, as well as the return on the total wealth portfolio calculated by Macklem (1997). We found that separable preferences produce a risk aversion coefficient that is too high and a subjective discount factor that is too low. Allowing for a nonlinear consumption schedule did not alter the results. These results can nonetheless be considered as a partial improvement to the standard practice of using a single consumption beta to measure risk.

Under nonseparable preferences, the subjective discount factor is not identifiable and was calibrated to reasonable values. We found that the coefficient of relative risk aversion remains unchanged, while the elasticity of intertemporal substitution is sensitive to the chosen value of the subjective discount rate, but can be estimated at realistic levels.

Overall, these results come as mixed news for the consumption CAPM model. Allowing for a broader measure of total wealth effectively increases the estimated quantity of systematic risk. Yet this risk remains insufficient to reduce risk aversion to realistic levels. When nonseparable preferences are introduced, the estimated risk aversion is not affected, while it is possible to reproduce the mean returns for acceptable values of the other preference parameters. Finally, the numerical values for the inverse of risk aversion and the elasticity of intertemporal substitution are different, suggesting that the null of separable preferences could be rejected in formal testing.

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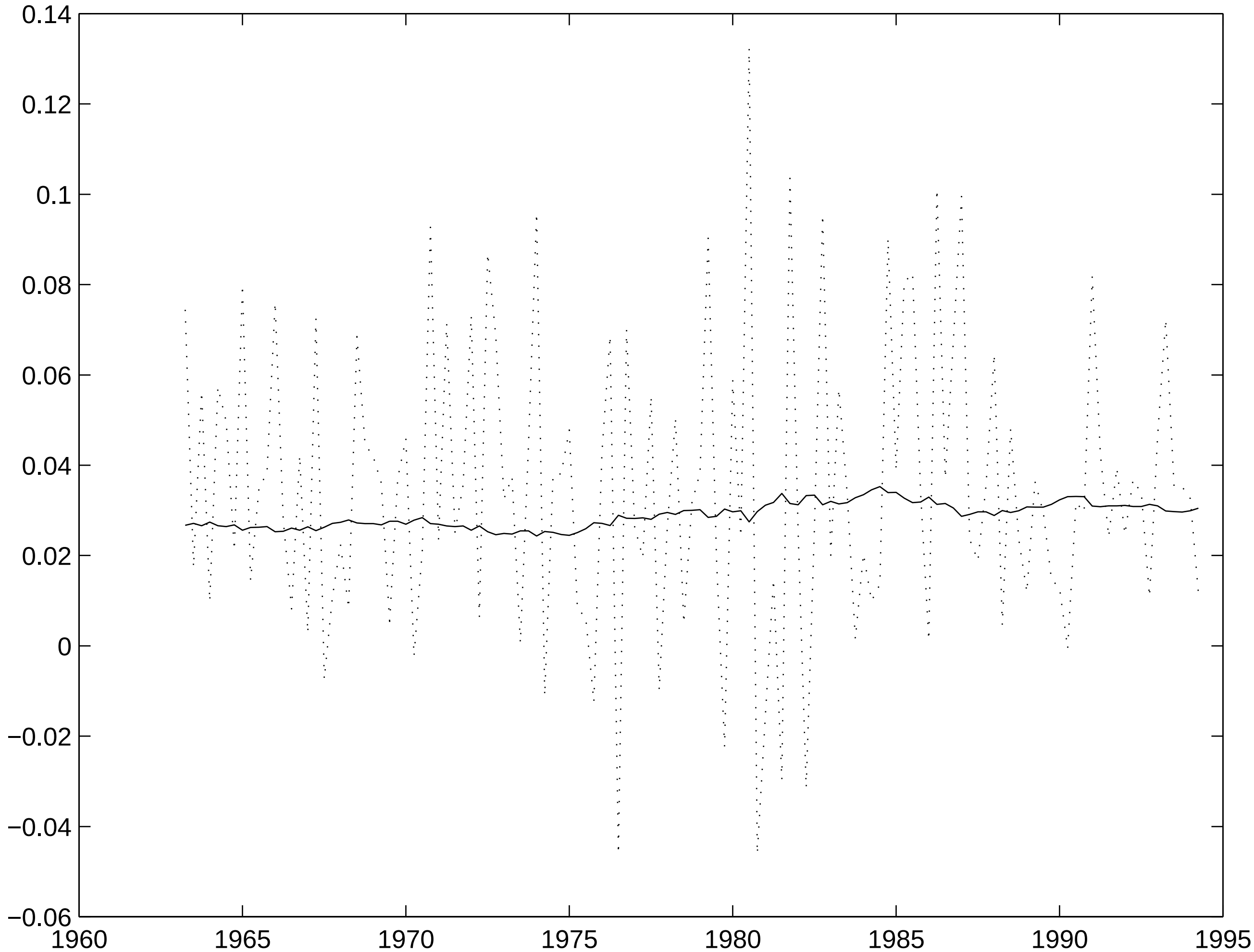

Figure 1: Average Propensity to Consume out of Wealth, and Total Wealth Return

Note: Average propensity to consume C_t/A_t (solid line), and return on total wealth portfolio (dotted line) defined from the budget constraint (2.2) as:

$$r_{t+1} = A_{t+1}/(A_t - C_t) - 1.$$

The sample covers the 1963:2-1994:2 period.