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Newsletter

Groupe d'études et de recherche en analyse de décisions

GERAD

"**Board games** were the springboard for a fast-growing **science**"

Georges Zaccour, HEC Montréal Game Theory and Management Chair holder

Game Theory became a branch of mathematics in the 1940s, and more specifically after the publication of the groundbreaking work *Theory of Games and Economic Behavior* by John von Neumann and Oskar Morgenstern. Its role in economics, however, was only fully recognized fifty years later in 1994 when John F. Nash, John C. Haranyi and Reinhard Selten won the Bank of Sweden Prize in Economics in memory of Alfred Nobel.

"Through their interest in board games like chess, mathematicians proved the theorems that form the basis of Game Theory," notes Georges Zaccour, a GERAD member, holder of HEC's Chair in Game Theory and Management, and Professor at HEC Montréal. "John F. Nash, who discovered the equilibrium named after him and whom we received at GERAD in 2005 for the 25th anniversary, might have had underlying economic ideas, but the curiosity of mathematicians was piqued by board games with their action, reaction and strategy." It is interesting to remember that John Nash's life was the subject of a book and an Oscar-winning film in 2002, *A Beautiful Mind*.

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SUMMARY

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The collaboration between the mathematician John von Neumann and the economist Oskar Morgenstern, both researchers at the famous Princeton University Institute of Advanced Study, remains an inspiration for GERAD, which brings together researchers in different disciplines to solve problems in decision-making.

Game Theory applications are expanding steadily and rapidly as illustrated by the examples in this Newsletter. Applications are found in finance, marketing, telecommunications, power generation management, and even in the movement of large schools of fish. According to Georges Zaccour, this expansion will surely continue particularly in finance, supply chain research, economics, and energy and natural resources management, but it can also be expected to grow in biology where it is "booming", in modelling the spread of cancer, or as a means to explain all sorts of natural phenomena.

Many GERAD researchers apply Game Theory in their work. Moreover, it is interesting to note that Alain Haurie, who founded GERAD, and Georges Zaccour, who headed the centre from 2001 through 2005, are both past presidents of the International Society for Dynamic Games, one of the two international organizations that promote Game Theory.

It is a challenge simply to decline all the different types of games and possible combinations. First there are cooperative games, in which common interest prevails, and non cooperative games, where it is everybody for themselves. Games can be static or dynamic, depending on whether or not the rules and conditions change over time. The players can decide at well defined times (in discrete time) or continuously (differential games, originally developed to solve military problems). Conditions can change in a predictable manner (deterministic games) or randomly (stochastic games).

In cooperative games, the first goal is to choose a strategy that optimizes the common good, points out Georges Zaccour. Then it is decided how to divide up the dividends stemming from cooperation so that all players involved are satisfied. Solutions can be found that are fragile because they to not represent equilibria (i.e., situations from which nobody has an interest in deviating unilaterally). When a solution is not an equilibrium, it is very tempting for players to deviate from the agreement. Researchers are now interested in designing cooperative solutions that are also equilibria. That is the best of both worlds, namely the stability of an equilibrium combined with the dividends stemming from cooperation. OPEC, which is a cartel supposedly operating as a cooperative game, provides an interesting example for studying notions such as coopera-

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John F. Nash (2nd from left) was guest of honour at GERAD's 25th anniversary celebrations in May 2005. With him from left to right, Ms Nash, Michele Breton, and Georges Zaccour, GERAD Director from 2001 to 2005.

Chair in Game Theory and Management

In October 2006, HEC Montréal created the Chair in Game Theory and Management and appointed the first holder, Georges Zaccour. In addition to being a mark of recognition, the Chair represents a significant boost to scientific research on Game Theory, specifically by providing financial support to doctoral and post-doctoral students over a period of several years.

Research initiatives will look at theoretical and algorithmic developments in Game Theory and their applications to management issues. These applications, mainly in marketing and energy and environment, include: conflicts and co-operation in marketing distribution networks; determining marketing strategies in oligopolistic markets; environment cost sharing; coordinating environmental strategies in an international context; coordinating supply chains and the marketing-production interface; and E-commerce and, in particular, its implications for relationships in traditional distribution networks.

Restructuring a company in **financial distress**

Amira Annabi, Michèle Breton, Pascal François*

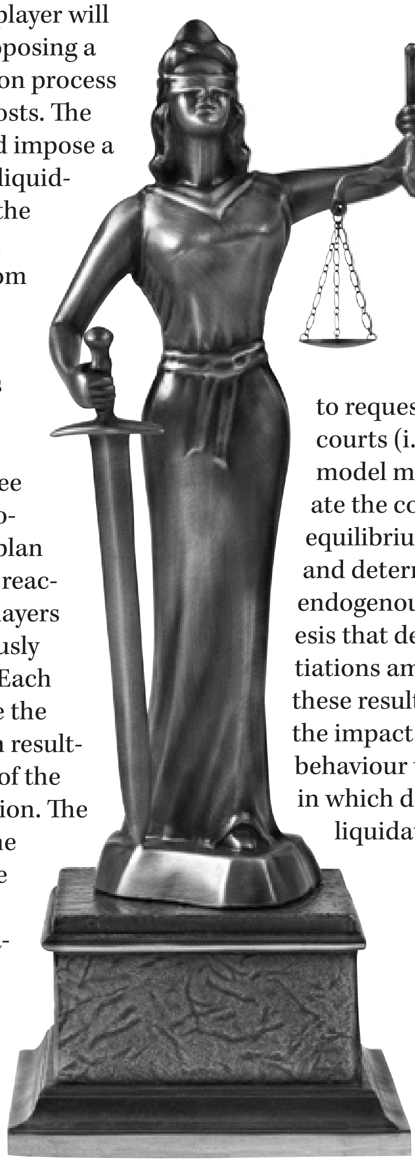
A company defaults when it cannot honour its debt. The default materializes when the value of its assets reaches a certain barrier. Such an event can lead to bankruptcy, but it can also prompt reorganization. The problem before us is to evaluate the company, which must take into account the possibility of reorganizing its debt in case of a default event.

In the economy considered, the value of a company's assets obeys a random process. The company is financed by two classes of subordinate debts. In addition, it pays a coupon to its creditors. The default event is triggered when the value of the assets reaches a given barrier at a random stop time.

The model used for the negotiation process takes the form of a game among the different priority class creditors and the shareholders. Three players are considered: the manager (representing the shareholders), the senior creditor (representing the privileged creditors), and the junior creditor (representing the ordinary creditors). The default event triggers the negotiation process that includes successive rounds. At each round, one of the three players proposes a reorganization plan.

Following this proposal, the two other players can choose to agree to the reorganization plan or to pursue negotiations, while another player will have the privilege of proposing a new plan. The negotiation process takes time and incurs costs. The Court can intervene and impose a reorganization plan, or liquidate the company when the players cannot reach an agreement after a random number of rounds of negotiations.

The negotiation process is thus represented by a series of hierarchical games involving the three players. The "leader" proposes a reorganization plan taking into account the reactions of the two other players who decide simultaneously and non-cooperatively. Each player tries to maximize the share that comes to him resulting from a new sharing of the company, or its liquidation. The value of continuing is the expected result from the equilibrium at the following round of negotiations, whereas the state variable is the value of the company's assets, which in turn is randomly determined.



If a reorganized company emerges from the process, it continues its activities by distributing new coupons to its creditors until the next default event, and that triggers a new negotiation process.

It is important to note that the value of the company depends on the breakdown among shareholders and creditors (or the debt structure). Moreover, the shareholders decide when it is best for a company to request protection from the courts (i.e., default barrier). This model makes it possible to evaluate the company, determine the equilibrium structure of the debt, and determine the default barrier endogenously, based on the hypothesis that default gives rise to negotiations among the parties. With these results, it is possible to assess the impact of the players' strategic behaviour with respect to a model in which default necessarily leads to liquidation. G

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Evaluating **options** embedded in **bonds**

Hatem Ben Ameur, Michèle Breton, Pierre l'Écuyer, Lotfi Karoui*

Many bonds carry embedded options held by different players, such as the bond issuer and holder. The most common embedded options are redemption, put, and conversion. The redemption option enables the issuer to buy back the debt for a preset amount. The conversion option allows the holder to exchange the debt for a preset number of the issuer's securities.

Since the bond is a fixed rate loan contract, its value and that of the redemption and put options depend on the interest rate which varies randomly. The value of the embedded options is difficult to determine directly, since when one player exercises an option he effectively destroys the options held by the other player and also terminates the loan contract.

The model considered is a zero-sum dynamic stochastic game between the issuer and the investor. The issuer seeks to minimize the bond value, whereas the investor seeks to maximize it. Based on the time until maturity and the current interest rate, their respective strategies determine whether it is better or not to terminate the contract by exercising the redemption or put options. The value of this game is the bond value plus the embedded options.

The value of the game is obtained by a dynamic game in discrete time, corresponding to the notification dates when options are exercised. The challenge consists in developing efficient and accurate numerical methods for approximating this value on the state space, which is continuous.

It should be noted that many empirical studies in finance are based on bond price data, from which a large number must be withdrawn, namely those that include embedded options. With our results, it is possible to isolate the bond values and thereby enhance the data bases available.

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tive equilibria and respect of agreements (quotas) by members.

The equilibrium idea was already present in work done by the French mathematician Antoine Cournot, who in 1838 calculated the optimal quantities to be put on the market in the context a duopoly. In the 1920s, Borel's work and above all that of John von Neumann led to major advances in the formalization of key Game Theory concepts (strategy, MiniMax, etc.). The most fundamental result of this theory was probably the non cooperative equilibrium. In 1953, John Nash proved the famous theorem specifying the conditions in which such an equilibrium occurs. Since then, the theoretical and technical developments have multiplied, and applications rapidly followed in a wide variety of areas encompassing economics, management, the military, political science, biology and more.

A career marked by Game Theory

Georges Zaccour joined GERAD in 1982 just after the research centre was founded. As a Master's student, he came to study Game Theory under GERAD founder and director Alain Haurie. Later his doctoral thesis studied Game Theory applications in the energy sector, and particularly interconnected electricity grids and the European natural gas market, a subject that is still making headlines. Georges Zaccour earned his Ph.D. in Administration from HEC Montréal where he has been Professor in the Department of Marketing since 1986. From 2002 to 2006, he was president of the International Society of Dynamic Games, founded in 1990. He is coauthor with Steffen Jørgensen of the book *Differential Games in Marketing* (Springer 2004) and is associate editor of several scientific journals including the *International Game Theory Review*. He is a fellow of the Social Sciences Academy of the Royal Society of Canada and has published more than 85 scientific articles. G

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These networks enable network decentralization by letting all nodes play the role of customer and server. Specifically, data sharing systems make objects all the more available because they are popular, and thus have replicas on the nodes. That makes it possible to reduce the load on the nodes that share popular data, which facilitates the increase in the number of nodes, and thus also the number of data on the network.

The objective, on the one hand, is to develop a realistic optimization model resolved using specialized techniques for solving very large problems. Realistic here means that it is assumed that all information is not necessarily known about the characteristics and types of agents. On the other hand, the objective consists in developing distributed algorithms that meet service quality constraints – bandwidth, time limits – and also scalability constraints. With these two types of techniques, we hope to contribute to the development of efficient peer-to-peer network management algorithms, and also to be able to assess their performance from both a theoretical and a practical standpoint. G

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on an understanding of the statistical behaviour of any individual particle.

The other current is Game Theory, which stems from Mathematics and theoretical economics. In particular in this case, it is the Nash equilibrium notion of stochastic dynamic games. The mathematical apparatus resulting from the synthesis of these two research currents appears to provide solutions to many previously open problems. The central idea, which we call the Nash Certainty Equivalence Principle, is that an average equilibrium trajectory for the mass $r(t)$ must be one such that, if all individuals position themselves optimally with respect to $r(t)$, they will collectively reproduce the very same trajectory $r(t)$. G

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