
EXERCISE 29

OPTIMAL MARK-UP PRICING

This exercise shows how the profit-maximising rule can be compatible with the common practice of fixing prices by adding a mark-up to variable costs.

Based on past experiences of price variations, a firm estimated that the demand for its product could be considered as linear inside some range. The number of units it sells (Q) depending of the price charged (P) is given by the following equation :

$$Q = -\left(\frac{100}{3}\right)P + \frac{1000}{3}$$

The company manager is confident that this relation between prices and quantities is the one he will face in the future as long as $5 < P < 10$.

The company can either make the product itself or import it. If it makes the product, variable costs are estimated by the following function :

$$TC = \left(\frac{1}{60}\right)Q^2 + \frac{16}{3}Q$$

Question 1.

If the manager decides to manufacture the product, which level of output should he chooses and which price should he charge ?

Question 2.

The managers knows that he can also import enough quantity to supply his market, the price at which he will buy is constant and equal to $P_M = 6.50$. What will be his profit-maximising selling price ?

Question 3.

Compute the percentage mark-up charged by the firm if it imports instead of producing.

Question 4.

Using the own-price-elasticity of demand, verify that the above mark-up is indeed optimal (i.e. is profit-maximizing).

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ANSWERS

Question 1.

Given the demand and the total cost functions, the following equations describe the average revenue, the marginal revenue and the marginal cost of the firm.

$$AR = (-3/100)Q + 10$$

$$MR = (-3/50)Q + 10$$

$$MC = (1/30)Q + \frac{16}{3}$$

$$\begin{aligned} \text{Profit is maximised for : } MR = MC & \quad \text{or } 10 - 3Q/50 = 16/3 + Q/30 \\ & \quad \text{or } 14/3 = 14/150 Q \\ & \quad \text{or } \mathbf{Q^* = 50} \end{aligned}$$

and the manager charges a price of $\mathbf{P^* = 10 - 3(50)/100 = 8.5}$

$$\begin{aligned} \text{Total contribution to fixed cost and profit is } P Q - \text{Variable cost}(Q) \\ \text{or } 8.5 * 50 - 50^2/60 + 16(50)/3 = 425 - (41.66 + 266.66) = 425 - 308.33 = \mathbf{116.67} \end{aligned}$$

Question 2.

The monopolist can buy the product at $P_M = 6.5$. He will consider that buying-price as his marginal cost and average cost. On the figure this marginal cost is represented by a horizontal straight line.

$$\begin{aligned} \text{The maximisation rule is : } MR = MC, \text{ but } MC = 6.5 \\ \text{or } -3/50 Q + 10 = 6.5 \quad \text{or } \mathbf{Q^* = 58.33} \end{aligned}$$

If the monopolist imports and sells 58.33 units of the product, the selling price is determined by the market-demand equation : $\mathbf{P^* = -(3/100)(58.33) + 10 = 8.25}$

$$\begin{aligned} \text{Total contribution to fixed cost and profit in this case is equal to :} \\ (8.25 - 6.5)(58.33) = \mathbf{102.07} \end{aligned}$$

The final decision to buy rather than to make will be dictated by the level of profit attainable. As we have no indication on the magnitude of this monopolist fixed cost, we cannot compute the actual level of profit if the monopolist chooses to make rather than to buy. But we do not need to know the fixed cost, because if the monopolist decides to buy the product in the short-run, he has anyway to cover the fixed cost associated whatever his decision to produce or to import. Hence, variable cost is sufficient for the comparison and the monopolist should make the product instead of importing it.

Question 3.

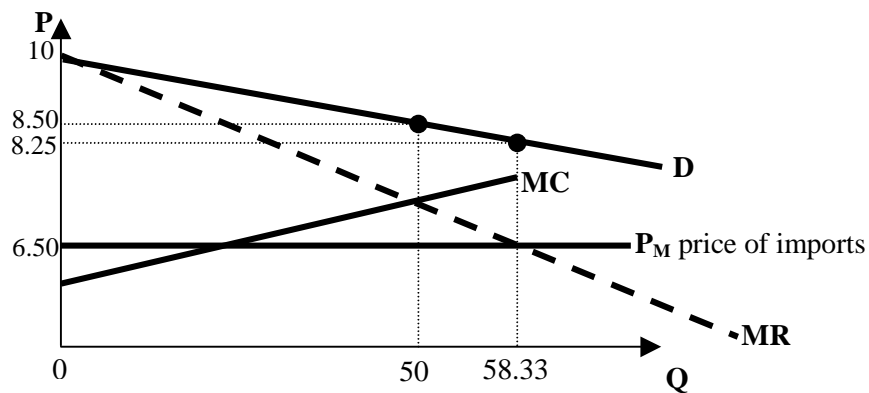
If the monopolist decides to buy instead of producing, he uses a mark-up pricing technique. Mark-up pricing (noted *mu*) is the practice of determining a price by adding a percentage mark-up to average variable cost : $P = AVC + \mu (AVC)$.

$$\text{Here } \mu = \frac{P - AVC}{AVC} = \frac{8.25 - 6.50}{6.50} = 26.9\%$$

Question 4.

We already know that the profit maximising mark-up is equal to $-1/(E_{QP}+1)$ and we must estimate the own-price elasticity at point B on the figure :

$$\frac{dQ}{dP} \frac{P}{Q} = -\frac{100}{3} \cdot \frac{8.25}{58.33} = -4.71 \quad \text{and indeed } -1/(-4.71+1) = -1/-3.71 = 26.9\%$$



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