EXERCISE 21

PROFIT-MAXIMISING PRICE AND ADVERTISING

A producer estimated the dependence of sales volume on advertising expenditures and priced as follow : $\mathbf{Q = 35{,}000 - 5{,}000\ P + 0.8\ A - 0.000025\ A^2}$ where Q is the monthly quantity sold, P the price and A the monthly level of advertising.

The average cost is constant at $2.65 (and hence equal to marginal cost) between 5,000 and 20,000 units produced per month and there is no fixed cost other than the advertising expenditures.

Right now, the price is fixed by a 60% mark-up over average cost or P = 1.60 * 2.65 = $4.24 and the advertising expenditures amount to $9,000 per month.

Question 1. **<u>At the actual level of advertising of $ 9,000</u>** is the $ 4.24 price profit-maximising ?

If not, determine the profit-maximising price and the maximum amount of profit.

Question 2. **<u>At the actual price of $ 4.24</u>** what would be the profit-maximising level of advertising expenditures and what would be the maximum amount of profit ?

Question 3. Verify if the Dorfman-Steiner condition is satisfied for a level of advertising of $ 8.524 and a price of $ 5.33.

Question 4. Using alternatively the procedure used for the first two questions, imagine an iterative method to find the profit-maximising price and promotional expenditures. Computation might be easier on a spread sheet like Excel.

*

ANSWERS

The dependence of sales volume on advertising expenditures and price is :

$$Q = 35{,}000 - 5{,}000\,P + 0.8\,A - 0.000025\,A^2$$

where Q is the monthly quantity sold, P the price and A the monthly level of advertising.

The average cost is constant at $2.65 (and equal to marginal cost) between 5,000 and 20,000 units produced per month and <u>except for the advertising expenditures</u> there is no other fixed cost. Right now, the price is fixed by a 60% mark-up over average cost or P = 1.60 * 2.65 = $4.24 and the advertising expenditures amount to $9,000 per month.

Question 1.

<u>For a given advertising level of $9,000</u>, the demand function condenses to

$Q = 35{,}000 - 5{,}000\,P + 0.8 * 9{,}000 - 0.000025 * 9{,}000^2 = 40{,}175 - 5{,}000\,P$

or P = 8.035 – 0.0002 Q and MR = 8.035 – 0.0004 Q

Profit-maximising quantity equalises marginal cost to marginal revenue

2.65 = 8.035 – 0.0004 Q* or Q* = (8.035-2.65)/0.0004 = 13,462.5 units/month.

<u>P*</u> = 8.035 – 0.0002 * 13,462.5 = **<u>5.3425 > 4.24</u>**

The maximum amount of profit is 13,462.5 * (5.3425 – 2.65) – 9,000 = **<u>27,247.781</u>**

The actual quantity is Q' = 40,175 – 5,000 * 4.24 = 18,975 and the actual profit is

18,975 * (4.24 – 2.65) – 9,000 = 21,170.25 only.

So for that level of advertising the price is too low.

Question 2.

<u>For a price fixed at $ 4.24</u>, the optimality condition is

$\mathbf{\partial \pi / \partial A = 1}$ or $\mathbf{(\partial \pi / \partial Q)(\partial Q / \partial A) = 1}$

$\pi = TR - TC = Q(4.24 - 2.65) = 1.59\,Q$ is the contribution to fixed cost and profit hence

$\partial \pi / \partial Q = 1.59$

and $\partial Q/\partial A = \partial(35{,}000 - 5{,}000\,P + 0.8\,A - 0.000025\,A^2)/\partial A = 0.8 - 0.00005\,A$

For P = 4.24 : $\partial \pi / \partial A$ or $(\partial \pi / \partial Q)(\partial Q/\partial A) = 1.59 * (0.8 - 0.00005\,A) = 1$

or 1,272 – 1 = 0.0000795 A or **A*** = 0.272/0.0000795 = **<u>$ 3,421.38</u>**

It is easy to see that at A = 9,000 :

$\partial \pi / \partial A$ or $(\partial \pi / \partial Q)(\partial Q/\partial A)$ **=** 1.59(0.8 - 0.00005 * 9,000) = 0.5565 < 1 hence for a price of $4.24 the advertising budget should be reduced since the last dollar spent on advertising brings in only $0.5565.

With A = 3,421.38 and P = 4.24, the quantity sold is Q = 16,244.46 units/month and profit **π*** = 16,244.46 (4.24-2.65) – 3.421.38 = **<u>22,407.31</u>** better than 21,170.25 actually.

Question 3.

The profit maximising price P and promotional budget A are such that the ratio of the advertising elasticity to minus the own-price elasticity is equal to the ratio of the advertising budget to total revenue or - $\mathbf{E_{AP}/E_{QP} = A/PQ}$.

Now Q = 35,000 – 5,000 * 5.33 + 0.8 * 8,524 – 0.000025 * 8.524^2 = 13,352.74

The own-price elasticity : $E_{QP} = (\partial Q/\partial P)(P/Q)$ = -5,000 * 5.33/13,352.74 = -1.9958

The elasticity with respect to advertising : $E_{QA} = (\partial Q/\partial A)(A/Q)$

$\partial Q/\partial A$ = (0.8 – 0.00005 A) = 0.8 – 0.4262 = 0.3738

E_{QA} = 0.3738 * 8,524/13,352.74 = 0.2386

$\mathbf{-E_{QA}/Q_{QP}}$ = 0.2386 / -1.9958 = **<u>+ 0.1196</u>**

While **A/PQ** = 8.524/(13,352.74 * 5.33) = **<u>0.1197</u>**

Yes, the firm indeed charges a profit-maximising price of $5.33 and spends a profit-maximising promotional budget of $8,524 simultaneously.

Profit is then : **π*** = 13,352.74 (5.33-2.65) – 8,524 = **<u>27,259.36</u>** ***much better.***

Question 4.

See file : **Profit_Maximising_Price_And_Advertising.xls**

The following four pages is a print of this program. In a few iterations, it converges very rapidly to **P* = 5.52528** and **A* = 8,524.143** for a maximum profit of **27,261.44** ; so P = 5.33 and A = 8,524 are indeed profit-maximising.

If the demand function is not linear or if the cost function is a cubic function, setting equal to zero the partial derivative of profit with respect to price and advertising : $\partial(TR-TC)/\partial P = 0$ and $\partial(TR-TC)/\partial A = 0$ will give a set of two simultaneous NON-LINEAR equations that needs to be solved via an iterative method as shown here.

* *