Aftermarket Power and Basic Market Competition

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Abstract

I revisit the issue of aftermarkets by developing an infinite period model with overlapping consumers. If the aftermarket is characterized by constant returns to scale, then social and consumer surplus are invariant with respect to aftermarket power. Under increasing returns to scale, however, greater aftermarket power leads to: higher social surplus; higher consumer surplus; greater concentration in the basic market; higher barriers to entry.

Keywords: aftermarkets, dynamic price competition, market power.
JEL codes: L1, L4.

1 Introduction

Many consumers complain they pay too much for printer toners. But the same consumers are also happy to purchase printers at fairly low prices. To some extent, lower printer prices compensate for higher toner prices. Or do they?

The printer-toner example in one of many instances of industries characterized by a basic market that is complemented by one or several aftermarkets. Typically, the basic market corresponds to a durable good, whereas the aftermarkets correspond to non-durable products or services. Other than printers, examples include cameras and film, photocopiers and repair service, videogame consoles and games.[1]

In these industries, an interesting policy question is how to treat seller power in the aftermarket. An old argument states that a seller can only have so much market power, and that an increase in aftermarket power is compensated by an equal decrease in power in the basic market: the price of blades may be very high, but razor holders are very cheap. Some authors argue that the conditions for such an equivalence result are very stringent. For example, Borenstein, Mackie-Mason and Netz (1995) claim that "economic theory does not support the argument that strong primary market competition will discipline aftermarket behavior, even without market imperfections" (p. 459). Other authors, while recognizing the welfare reducing effects of market power, suggest that these are rather small in magnitude. For example, Shapiro (1995) concludes that "significant or long-lived consumer injury based on monopolized aftermarkets is likely to be rare, especially if equipment markets are competitive" (p. 485).[2]

In this paper, I revisit the relation between aftermarket power and basic market competition. The novel element of my analysis is to consider a dynamic (infinite period) model with increasing returns to scale in the after-

1. A related, but different, setting consists of secondary markets where the initial product may be resold. See for example Hendel and Lizzeri (1999).
2. Another set of questions relates to the efficiency effects of aftermarket power. For example, Chen and Ross (1993) argue that a seller may use the aftermarket as a "metering device to discriminate between high-intensity, high-value users and low-intensity, low-value users" (p. 139). More recently, Carlton and Waldman (2001) show that "behaviors that hurt competition in aftermarkets can ... be efficient responses to potential inefficiencies that can arise in aftermarkets." See Chen, Ross and Stanbury (1998) for a review on the economics and legal literature on aftermarkets.

market (which may result from economies of scale, indirect network effects, or other causes). I assume consumers' lives overlap with one another. In each period, one consumer is born and joins one of the existing installed bases; next, aftermarket payoffs are received by sellers and consumers; and finally, one consumer dies. I derive the unique symmetric Markov equilibrium of this game and the resulting stationary distribution over states (which correspond to each firm's installed base).

I show that increasing returns in the aftermarket induce increasing dominance in the basic market; that is, under increasing returns a large firm is more likely to capture a new consumer than a small firm. Moreover, an increase in aftermarket power increases the extent of increasing dominance. This in turn has several implications. First, aftermarket power implies a stationary distribution with greater weight on asymmetric states. Second, social welfare is greater with aftermarket power (basically because social welfare is higher at asymmetric states). Third, the value of a small firm (a firm with no installed base) is lower when there is aftermarket power. Fourth, because the difference in value between large firms and small firms widens, firms compete more aggressively to attract new customers when there is aftermarket power. And finally, because of more aggressive price competition, consumer welfare is greater when there is aftermarket power.

Intuitively, my results are related to two important features of dynamic price competition. The first one is the *efficiency* or *joint profit* effect.[3] The idea is that a large firm has more to lose from decreasing its market share than a small firm has to gain from increasing its market share. This induces the large firm to be relatively more aggressive and make the next sale with greater probability than the small firm: increasing dominance. In my model, I show that aftermarket power increases the stakes that firms compete for; and this in turn increases the extent of increasing dominance.

The second feature is what we might call the *Bertrand supertrap* effect.[4] Consider a symmetric bidding game, where the winner receives w and the loser gets l. Equilibrium bids are given by $w - l$; it follows that each player's equilibrium payoff is given by l: if you win, you get w, but you also have to pay $w - l$. In the present context, I show that aftermarket power, while increasing future profits, makes firms so much more competitive that,

3. See Gilbert and Newbery (1982), Budd, Harris and Vickers (1993), Cabral and Riordan (1994), Athey and Schmutzler (2000).

4. See Cabral and Riordan (1994), Cabral and Villas-Boas (2005).

starting from a symmetric state, firms are worse off, whereas consumers are better off. In other words, in terms of future value a large firm is better off with aftermarket power, but a small firm is worse off; and the latter is what matters in terms of present value.

In terms of competition policy, my paper makes two points not previously considered. First, given a set of firms and product offerings, consumers need not be harmed by aftermarket power. In fact, to the extent that there are increasing returns in the aftermarket and the basic market is competitive, consumers can be strictly better off in the presence of aftermarket power. Second, increases in aftermarket power have important implications for market share dynamics. On average, the basic market will be more concentrated; and the barriers to entry of new firms increase. Taken together, these two points suggest that that aftermarket power raises concerns from a consumer welfare point of view, but not for the reasons typically considered in the literature.

The paper is organized as follows. In Section 2, I introduce my dynamic model of basic market and aftermarket competition. In Section 3, I consider the benchmark case of constant returns to scale and show that the one-monopoly-rent principle holds. In Section 4, I consider the case of increasing returns to scale in the aftermarket and two possible aftermarket configurations: perfect competition and monopoly. I prove that aftermarket power increases the degree of increasing dominance. Section 5 derives two implications of this result, one regarding long-run market shares, one regarding barriers to entry. Section 6 deals with social and consumer welfare. Finally, Section 7 concludes the paper.

2 Model[5]

Consider an industry with two sellers and an infinite series of overlapping consumers. In each period, one consumer is born and endowed with preferences for seller i's basic product. Sellers simultaneously set prices p_i for that product and the consumer chooses one of the sellers. Next, all consumers, old and new, purchase a complementary product in the aftermarket. I will not model in detail the nature of aftermarket competition. Rather, I assume

5. Cabral (2010) presents a more general framework of dynamic price competition that includes the present model as a particular case. The cost of a general framework is that few analytical results are possible.

that firm i receives a profit π_i, whereas a consumer attached to firm i earns a surplus λ_i (all consumers value the aftermarket product equally). Finally, at the end of the period one of the consumers dies, each with equal probability.

Throughout the paper, I assume that, in each period, there are 3 consumers in the aftermarket. This implies that, at the beginning of the period, there are 2 old consumers. Given symmetry, we have two possibilities: either both firms have the same installed base (1 consumer each) or one of the firms has a large installed base (2 consumers) whereas the other firm has a zero installed base.

I will be looking at Markov equilibria, where the state of the game is given by the firms' installed bases. For simplicity, if with some abuse of notation, I will denote by i the size of firm i's installed base. At the beginning of each period, we thus have $i + j = 2$.

I next study in greater detail the consumers' and the firms' choice problems.

■ **Consumer choice.** A newborn consumer is endowed with valuations ζ_i for firm i's basic product. I assume that $\xi_i \equiv \zeta_i - \zeta_j \sim U[-1/2, 1/2]$. Consider a new consumer's decision. In state i, the indifferent consumer has $\xi_i = x_i$, where the latter is given by

$$x_i - p_i + u_{i+1} = -p_j + u_{j+1}, \tag{1}$$

where p_i is firm i's price and u_i is the consumer's aftermarket value function, that is, the discounted value of the stream of payoff λ_i received while the consumer is alive (thus excluding both ζ_i and the price paid for the basic product). Specifically, in each period that a consumer is alive he receives an aftermarket payoff λ_i, where i is the size of the installed-base *in that period*.

The above problem looks very much like a Hotelling consumer decision (with firms located at $-1/2$ and $1/2$ and unit transport cost), except for the fact that u_{i+1} and u_{j+1} and endogenous values.

Firm i's demand is the probability of attracting the new consumer to its installed base. Since ξ_i is uniformly distributed in $[-1/2, 1/2]$, we have $F(\xi_i) = 1/2 + \xi_i$. Therefore, the probability that firm i attracts a new consumer to its installed base, q_i, is given by

$$\begin{aligned} q_i \;=\; \mathcal{P}(\xi_i > x_i) \;&=\; 1 - F(x_i) \;=\; \frac{1}{2} - x_i = \\ &=\; \frac{1}{2} - \Big(\big(p_i - p_j\big) - \big(u_{i+1} - u_{j+1}\big) \Big) \end{aligned} \tag{2}$$

where the last equality follows from (1). Finally, the consumer value functions are given by

$$u_i = \lambda_i + \delta \left(\frac{j}{3} q_i u_{i+1} + \left(\frac{j}{3} q_{j-1} + \frac{i-1}{3} q_{i-1} \right) u_i + \frac{i-1}{3} q_j u_{i-1} \right) \tag{3}$$

$i = 1, 2, 3$, $j = 3-i$. In words, a consumer who is attached to an installed base of size i receives λ_i in the current period. Beginning next period, four things may happen: a consumer from installed base i dies and the new consumer joins installed base j, in which case continuation value is u_{i-1}; a consumer from installed base j dies and the new consumer joins installed base i, in which case continuation value is u_{i+1}; and two events where death and birth take place in the same installed base, in which case continuation payoff is u_i.

■ **Firm's pricing decision.** Firm i's value function is given by

$$\begin{aligned} v_i = q_i \left(p_i + \pi_{i+1} + \delta \frac{j}{3} v_{i+1} + \delta \frac{i+1}{3} v_i \right) \\ + \left(1 - q_i\right) \left(\pi_i + \delta \frac{j+1}{3} v_i + \delta \frac{i}{3} v_{i-1} \right) \end{aligned} \tag{4}$$

where $i = 0, 1, 2$ and $j = 2 - i$. With probability q_i, firm i attracts the new consumer and receives p_i. This moves the aftermarket state to $i+1$, yielding a period payoff of π_{i+1}; following that, with probability $(i+1)/3$ firm i loses a consumer, in which case the state reverts back to i, whereas with probability $j/3$ firm j loses a consumer, in which case the state stays at $i+1$. With probability q_j, the rival firm makes the current sale. Firm i gets no revenues in the primary market. In the aftermarket, it gets π_i in the current period; following that, with probability $i/3$ network i loses a consumer, in which case the state drops to $i-1$, whereas with probability $(j+1)/3$ network j loses a consumer, in which case the state reverts back to i.

Equation (4) leads to the following first-order conditions for firm value maximization:

$$q_i + \frac{\partial q_i}{\partial p_i} \left(p_i + \pi_{i+1} - \pi_i + \delta \frac{j}{3} v_{i+1} + \delta \frac{i+1}{3} v_i - \delta \frac{j+1}{3} v_i - \delta \frac{i}{3} v_{i-1} \right) = 0$$

or simply

$$p_i = q_i - \left(\pi_{i+1} - \pi_i\right) - \delta \left(\frac{j}{3} v_{i+1} + \frac{i-j}{3} v_i - \frac{i}{3} v_{i-1} \right) \tag{5}$$

Finally, substituting (5) into (4) and simplifying, we get

$$v_i = q_i^2 + \pi_i + \delta \left(\frac{j+1}{3} \, v_i + \frac{i}{3} \, v_{i-1} \right) \tag{6}$$

■ **Equilibrium.** A Markov Nash equilibrium is a set of prices p_i and demands q_i for the basic product ($i = 0, 1, 2$), as well as a set of consumer value functions u_i ($i = 1, 2, 3$) and firm value functions v_i ($i = 0, 1, 2$), that satisfy equations (2) and (5) (quantities and prices, respectively), (3) and (6) (consumer and firm value functions, respectively).

The endogenous variables p, q, u, v (Roman letters) are parametric on the (exogenous) values of aftermarket profits and consumer surplus π, λ (Greek letters). I will next put a little more structure into these exogenous parameters.

■ **Aftermarket conditions.** In order to highlight the effects of market power as a transfer from buyer to seller, I assume that the aftermarket value created at each state is independent of seller power. Specifically, when one firm has an installed base of size i and its rival an installed base of size $j = 3 - i$, then total aftermarket welfare is given by

$$V_i \equiv \pi_i + \pi_j + i \, \lambda_i + j \, \lambda_j \tag{7}$$

My assumption is that, as aftermarket power conditions change, V_i remains constant. In other words, aftermarket power is simply a transfer from consumer surplus (λ_i) to firm profits (π_i). There are reasons for total surplus to be decreasing in market power (the usual Harberger triangle) or increasing in market power (see, for example, Carlton and Waldman, 2001). My assumption is intended to focus on the effects of dynamic competition on consumer and social welfare.

3 Constant returns to scale

In this section, I consider a benchmark case that essentially corresponds to results previously derived in the literature on aftermarkets. Suppose that

$$\begin{aligned} \lambda_i &= \lambda \\ \pi_i &= i \, \pi \end{aligned} \tag{8}$$

that is, we have constant returns to scale. My main result in this section is one of irrelevance, both irrelevance of dynamics and irrelevance (in the limit) of aftermarket power. Notice that (7) and (8) imply that $\pi + \lambda$ is a constant. Shifts in aftermarket power thus correspond to changes in π and λ that keep the sum constant, from the extreme when $\pi = 0$ to the extreme when $\lambda = 0$.

Proposition 1 *Under constant returns to scale, equilibrium price and demand are constant across states. Moreover, consumer welfare is invariant with respect to aftermarket power.*

A complete proof of this and the next results may be found in the appendix. A sketch of the proof is as follows. Suppose firm value is proportional to installed base (the proof derives this result rather than assume it). Then the first order condition (5) shows that the second and third terms are independent of i: the "prize" from capturing a new consumer, both in terms the current period profits and in terms of future profits, is the same for small and large firms. Since consumers do not care about firm size (they always get λ in the aftermarket), it follows that equilibrium price in the basic market is independent of firm size.

Given that price in the basic market is independent of firm size, the only factor of variation in the value function is aftermarket profits. But then the increased discounted value of aftermarket profits is exactly competed away by pricing in the basic market. In other words, the increase in firm profits from greater aftermarket power implies a higher "prize" for the firm that makes the current sale; and this prize is translated into lower prices in the basic market by the same amount. It follows that the current newborn consumer is indifferent with respect to the degree of market power.

To summarize, under constant returns to scale in the aftermarket consumers are indifferent to the degree of aftermarket power. In the next section, I compare this benchmark against the case of increasing returns to scale in the aftermarket. I show that, first, the dynamics are no longer trivial; and second, consumer and social welfare vary with aftermarket power in a nontrivial way.

4 Increasing dominance

In this and in the following sections, I consider the possibility of increasing returns to scale in the aftermarket, that is, the possibility that total surplus

Table 1: Aftermarket conditions: firm profit, π_i, and consumer surplus, λ_i, as a function of installed base, i, under two possible cases.

	π_i	λ_i
C: Aftermarket competition	0	$\omega + i\,\phi$
M: Aftermarket monopoly	$i\,(\omega + i\,\phi)$	0

increases more than proportionately with the size of the installed base. There are several instances when this is a reasonable assumption. For example, suppose that in each period the seller makes an investment which increases the value of the aftermarket product or service, and suppose the cost of such investment is a function of the quality increase but not of the number of consumers. Then, the greater the number of consumers, the greater the marginal gain from investment, and the greater the total value generated in the aftermarket. A second source of increasing returns is network effects. For example, videogame players get a greater value out of a game to the extent that they can play it with other players, and so consumer surplus is likely to be increasing in the size of the installed base.

Regarding aftermarket power, I consider two extreme cases: case C, when the aftermarket is competitive, and case M, when the aftermarket is monopolized. Profit and consumer surplus in each case are given in Table 1. In the competition scenario, seller profits are zero, whereas each consumer receives a surplus that is proportional to installed base size: $i\,\phi$. In the monopoly scenario, a seller with an installed base i earns $i\,\phi$ per consumer, yielding a total of $i^2\,\phi$, whereas consumers receive a surplus of zero. Note that $V_i \equiv \pi_i + \pi_j + i\,\lambda_i + j\,\lambda_j$ is the same in cases C and M.
My main result is that increasing returns lead to increasing dominance, the property whereby firms with larger installed bases are more likely to attract new customers. Moreover, aftermarket power increases the degree of increasing dominance. Let q_i^k be the probability that a seller with installed base i ($i = 0, 1, 2$) attracts the newborn consumer, assuming aftermarket conditions k ($k = C, M$).

Proposition 2 *The large seller is more likely to attract a new consumer, especially if sellers have aftermarket power:* $\frac{1}{2} < q_2^C < q_2^M$.

The proof of Proposition 2 is not particularly simple or elegant, but the result is fairly intuitive. Specifically, the intuition for $q_2^k > \frac{1}{2}$ is that increasing

returns in the aftermarket imply an *efficiency* or *joint profit* effect.[6] The idea is that firm value is a convex function of installed base size. This implies that a firm with an installed base of 2 has more to lose from dropping to 1 than a firm with an installed base of 0 has to gain from reaching 1. As a result, the large firm prices more aggressively and sells with greater probability. The intuition for $q_2^M > q_2^C$ is that an increase in aftermarket power increases the sellers' stakes in the aftermarket, that is, magnifies the size of the efficiency effect. This in turn results in a greater gap between the leader's and the follower's probability of attracting the newborn consumer.

5 Market concentration and barriers to entry

In this section, I derive two fairly straightforward implications of Proposition 2, both relating to the basic market: one regarding market concentration and one regarding barriers to entry.

Let μ_i^k be the stationary probability of being in state i ($i = 0, 1, 2$) given aftermarket conditions k ($k = C, M$). Symmetry implies that $\mu_0 = \mu_2$. Therefore, μ_1^k provides a sufficient statistic for the degree of basic market concentration: the greater μ_1^k is, the longer the system spends at the symmetric state, that is, the less concentrated the basic market is.

Proposition 3 *Aftermarket power implies basic market concentration:* $\mu_1^M < \mu_1^C$.

The idea is simple: death rates are independent of aftermarket conditions, whereas birth rates for the large firm are higher under aftermarket power (by Proposition 2). Together, these facts imply that the stationary distribution places greater weight on asymmetric states the greater the degree of aftermarket power.

Is an industry more attractive if sellers have aftermarket power? One might be tempted to say yes: more rents create a better prospect for an entrant. However, one must take into account the effect that aftermarket power has on basic market competition. In fact, for an entrant — that is, a firm that starts with an installed base of zero — all of the potential benefits from aftermarket power are competed away, and then some. In the

6. See Gilbert and Newbery (1982), Budd, Harris and Vickers (1993), Cabral and Riordan (1994), Athey and Schmutzler (2000).

end, an entrant is strictly worse off when there is aftermarket power. Let v_i^k be the value of a firm with installed base i given aftermarket conditions k ($k = C, M$). We then have the following result:

Proposition 4 *The value of a firm with no installed base is lower if sellers have aftermarket power:* $v_0^M < v_0^C$.

Proposition 4 follows naturally from Proposition 2. Increasing returns in the aftermarket makes firms more aggressive, especially large firms. This hurts small firms: while market power increases expected aftermarket profits, this gain is more than compensated by the loss from the rival's lower prices in the basic market.

A potential entrant compares the cost of entry to the expected benefit upon entry. Since an entrant starts with an installed base of zero, the expected benefit upon entry is given by v_0^k. For this reason, we may say that aftermarket power increases the size of the barriers to entry in the basic market.

6 Social welfare and consumer welfare

Increasing returns to scale create a situation of natural monopoly: social welfare is greater the more concentrated markets shares are. As a result, market forces that imply greater concentration also increase welfare.

Proposition 5 *There exists a* ϕ' *such that, if* $\phi < \phi'$*, then social welfare is strictly greater if sellers have aftermarket power.*

Proposition 3 implies that, under aftermarket power, asymmetric installed bases are more likely. As mentioned above, this in turn implies greater total welfare. The proof of Proposition 5 is not as trivial as it might seem because there is a countervailing effect on social welfare. To the extent that q_i is different from $\frac{1}{2}$, consumer "transportation" costs are greater than the minimum transportation costs. In other words, while the aftermarket component of social welfare is greater under aftermarket power, the primary market component is lower. In the proof, I show that the latter effect is dominated by the former. Specifically, I show that, around $\phi = 0$, both effects are of second order, but the effect on the aftermarket component of social welfare is strictly greater.

Table 2: Approximate values of equilibrium variables when $\delta = 1$ and ϕ is small.

	Aftermarket conditions	
	Competition	Monopoly
q_2	$\frac{1}{2} + \phi$	$\frac{1}{2} + 2\,\phi$
p_0	$\frac{1}{2} - 3\,\phi$	$\frac{1}{2} - 9\,\phi$
p_1	$\frac{1}{2} - 2\,\phi$	$\frac{1}{2} - 10\,\phi$
p_2	$\frac{1}{2} - \phi$	$\frac{1}{2} - 11\,\phi$
$\overline{\sum \lambda}$	$6\,\phi$	0
Consumer welfare	$\zeta' - \frac{1}{2} + 8\,\phi$	$\zeta' - \frac{1}{2} + 10\,\phi$

The inequality in Proposition 5 is strict. This implies that I can perturb my assumption of constant total aftermarket surplus in each state i. Suppose that total aftermarket surplus is ϵ higher in case C than in case M. If I make ϵ small enough (a tiny Harberger triangle), then I can find an open set of parameter values such that (a) in each state, social welfare is *lower* when there is aftermarket power; (b) in the steady state, social welfare is *greater* when there is aftermarket power. The justification for this apparently contradictory statement is that aftermarket power, while leading to a tiny loss in total surplus in each state, leads to a reallocation of steady state probabilities that places greater weight in states with strictly higher total surplus (and by more than ϵ).

I finally turn to one of my main results: the effect of aftermarket power on consumer welfare. Much of the previous literature on aftermarkets attempted to establish whether the injury to consumers resulting from aftermarket power is or is not significant. By contrast, I show that aftermarket power may actually *increase* consumer welfare.

Proposition 6 *There exist ϕ', δ' such that, if $\phi < \phi'$ and $\delta > \delta'$ then consumers are strictly better off with aftermarket power.*

Table 2 may be useful in understanding the effects of market power on equilibrium values, in particular the level of consumer welfare. If $\delta = 1$ and

$\phi = 0$, then consumer welfare is the same under aftermarket competition or monopoly (by Proposition 1). For $\delta = 1$ and small values of ϕ, I can approximate the values of the various endogenous variables by linear expansion around $\phi = 0$.

First notice that q_2 is greater under aftermarket power. This is consistent with Proposition 2. The idea is that, under market power, large firms have more to lose from not attracting a newborn consumer than small firms have to gain from attracting that same consumer. This leads large firms to price more aggressively and newborn consumers to choose large firms more likely.

Aftermarket power has two important effects on firm pricing in the basic market. First, prices are lower. The idea is that aftermarket power increases the prize from capturing an extra consumer, and basic market prices move accordingly. Second, whereas under aftermarket competition prices are increasing in the size of the installed base, under aftermarket monopoly prices are decreasing in the size of the installed base. The reason is that, under dynamic competition, there are two forces determining optimal price, which we may refer to sa the *harvesting* effect and the *investment* effect. The idea of the harvesting effect is that, to the extent a larger firm offers a better product in the eyes of the consumer, such firm prices higher accordingly. This is the main effect at work when the aftermarket is competitive. The idea of the investment effect is that, to the extent the value function is convex, a larger firm has more to gain from attracting the newborn consumer. This effect dominates when the aftermarket is monopolized.

Finally, we come to consumer welfare. There are two components to take into account: the aftermarket component and the basic market component. Under aftermarket competition, consumers expect a positive surplus in the aftermarket. On average, in the steady state, this is given by $6\,\phi$ (assuming for simplicity $\omega = 0$). Under aftermarket monopoly, consumers get zero in the aftermarket. For small ϕ, the basic market component is determined by prices (that is, product differentiation effects are of second order, as shown in the proof of Proposition 5). Under aftermarket competition, average price in the basic market is given by $\frac{1}{2} - 2\,\phi$, whereas under aftermarket monopoly we have $\frac{1}{2} - 10\,\phi$. This implies that the gain in basic market consumer welfare from aftermarket power, $8\,\phi$, more than compensates the loss in aftermarket consumer welfare from aftermarket power, $6\,\phi$.

To understand this result, it helps to think of price competition in the basic market as an auction, the object on the block being the newborn consumer's business. Suppose both firms have the same installed base. The

difference between winning and losing the auction is the difference between becoming a large firm and becoming a small firm. Aftermarket power increases the value of being a large firm; however, as shown in Proposition 4 it decreases the value of a small firm. The equilibrium value of a symmetric auction is equal to the value of the loser. So, Proposition 4 implies that the equilibrium value decreases with aftermarket power. For a small value of ϕ, most of this decrease in firm value corresponds to a transfer to consumers. In fact, as shown in the proof of Proposition 5, the first-order effect of aftermarket power on social welfare is zero.

7 Conclusion

Previous economic literature suggests that basic market competition partly compensates for aftermarket power. Some authors claim that consumers are considerably worse off when firms have aftermarket power, whereas other authors suggest consumers are nearly indifferent with respect to aftermarket conditions. In this paper I argue that, in the presence of increasing returns to scale in the aftermarket, consumers may actually be better off with a greater degree of aftermarket power. The idea is that the lure of future profits that increase more than proportionally with installed base size makes firms so much more aggressive that lower prices in the basic market more than compensate for higher prices in the aftermarket.

My analysis also shows that, in the presence of increasing returns to scale in the aftermarket, the dynamics of market shares in the basic market are no longer trivial: Large firms tend to attract new consumers with higher probability than small firms. Moreover, this increasing dominance effect is stronger the greater then degree of aftermarket power.

Propositions 2 through 6 correspond to strict inequalities. This implies that the results are not knife-edged: slightly perturbing the model does not change the sign of the main effects. This is important because, for the sake of exposition, I made a number of simplifying assumption. In particular, I assumed that changes in aftermarket power correspond to pure transfers from consumers to firms. More generally we would expect aftermarket power to imply some inefficiencies in the aftermarket (Harberger triangles). To the extent that demand elasticities are not very great, I would expect these inefficiencies to be of second order with respect to the gains implied by Propositions 5 and 6.

Appendix

Proof of Proposition 1: Since $\lambda_i = \lambda$, we have $u_i = u$. From (2), this implies

$$q_i = \frac{1}{2} - (p_i - p_j) \tag{9}$$

Taking the difference of (5) for $i = 2$ and $i = 0$, we have

$$p_2 - p_0 = q_2 - q_0 - \big(\pi_3 - \pi_2\big) + \big(\pi_1 - \pi_0\big) - \delta\,\frac{2}{3}\big(v_2 + v_0 - 2\,v_1\big)$$

Substituting (9) for q_i, (8) for π_i, and (6) for v_i, and simplifying, we get

$$p_2 - p_0 = -2\,\big(p_2 - p_0\big) - \frac{4\,\delta}{3-\delta}\,\big(p_2 - p_0\big)^2$$

The only solution such that $|p_2 - p_0| < \frac{1}{2}$ is $p_2 = p_0$. It follows that $q_2 = q_0 = \frac{1}{2}$. Moreover, by symmetry $q_1 = \frac{1}{2}$. Substituting in (5) and simplifying we get

$$p_i = p = \frac{1}{2} - \frac{3}{3 - 2\,\delta}\,\pi$$

Each consumers discounted utility from joining the network is given by

$$u = \lambda + \frac{2}{3}\,\delta\,u$$

where $\frac{2}{3}$ is the probability the consumer survives into the next period. Solving for u we get

$$u = \frac{\lambda}{1 - \frac{2}{3}\,\delta}$$

It follows that a consumer's net utility is given by

$$\frac{\lambda}{1 - \frac{2}{3}\,\delta} - \frac{1}{2} + \frac{3}{3 - 2\,\delta}\,\pi = -\frac{1}{2} + \frac{3}{3 - 2\,\delta}\,(\pi + \lambda)$$

It follows that any shift between π and λ that keeps the sum constant has no effect on consumer surplus. ■

Proof of Proposition 2: Define $q \equiv q_2$. This value of q summarizes the equilibrium, since $q_0 = 1 - q_2 = 1 - q$, and $q_1 = \frac{1}{2}$ by symmetry. The proof is

divided into three steps. First I solve for case C (competitive aftermarkets). Next I solve for case M (monopolized aftermarkets). Finally, I compare the values of q in each case.

□ **Case C:** $\lambda_i = i\,\phi$**,** $\pi_i = 0$**.** Substituting (3) for u_i and simplifying, we get

$$u_3 - u_1 = \frac{12\,\phi}{6 - (1 + 2\,q)\,\delta} \tag{10}$$

Substituting (5) for p_i, q for q_2, $1 - q$ for q_0, and 0 for π_i, and simplifying, we get

$$p_2 - p_0 = 2\,q - 1 - \delta\,\frac{2}{3}\big(v_2 + v_0 - 2\,v_1\big)$$

Substituting (6) for v_i, and simplifying, the above equation implies

$$p_2 - p_0 = 2\,q - 1 - \frac{\delta}{3 - \delta}\,\big(1 - 4\,q + 4\,q^2\big)$$

From (2), we know that

$$\big(p_2 - p_0\big) - \big(u_3 - u_1\big) = \frac{1}{2} - q \tag{11}$$

It follows that, by subtracting (10) from (11), we get

$$\frac{1}{2} - q = 2\,q - 1 - \frac{\delta}{3 - \delta}\,\big(1 - 4\,q + 4\,q^2\big) - \frac{12\,\phi}{6 - (1 + 2\,q)\,\delta} \tag{12}$$

Let ϕ^C be the (unique) solution of (12) with respect to ϕ. Computation establishes that

$$\frac{\partial\,\phi^C}{\partial\,q} = \frac{(1 - \delta\,q)\,(27 - 12\,\delta\,q) + \delta\,(3 - \delta)\,(1 - q) + \delta^2\,q}{6\,(3 - \delta)} \tag{13}$$

which, considering that $\delta \in (0, 1)$ and $q \in [0, 1]$, is positive. This implies that, in the relevant range of values of δ and q, the relation between ϕ and q is one-to-one. Hence, there exists a unique value of q implicitly given by (12).

□ **Case M:** $\pi_i = i^2\,\phi,\ \lambda_i = 0$**.** In this case, we clearly have $u_3 - u_1 = 0$. (A consumer expects a payoff of λ each period it is still alive, independently

of the size of the installed base.) Substituting (5) for p_i, (6) for v_i, q for q_2, $1-q$ for q_0, $i^2\,\phi$ for π_i, and simplifying, we get

$$p_2 - p_0 = 2\,q - 1 - \frac{12\,\phi}{3-\delta} - \frac{\delta}{3-\delta}\,\left(1 - 4\,q + 4\,q^2\right)$$

Substituting (11) for $p_2 - p_0$, and noting that $u_3 - u_1 = 0$, we get

$$\frac{1}{2} - q = 2\,q - 1 - \frac{12\,\phi}{3-\delta} - \frac{\delta}{3-\delta}\,\left(1 - 4\,q + 4\,q^2\right) \tag{14}$$

Let ϕ^M be the (unique) solution of (14) with respect to ϕ. Computation establishes that

$$\frac{\partial\,\phi^M}{\partial\,q} = \frac{1}{12}\big(9 + \delta - 8\,\delta\,q\big) \tag{15}$$

which, considering that $\delta \in (0,1)$ and $q \in [0,1]$, is positive. This implies that, in the relevant range of values of δ and q, the relation between ϕ and q is one-to-one. Hence, there exists a unique value of q implicitly given by (14).

□ **Relation between q^M and q^C.** The last step in the proof consists of comparing the equilibrium values of q in cases M and C, which I denote by q^M and q^C, respectively. Both q^M and q^C are strictly increasing in ϕ. Moreover, $\phi = 0$ implies that $q^M = q^C = \frac{1}{2}$ (by symmetry). It follows that $q^M > q^C$ if and only if $\Phi \equiv \phi^M - \phi^C > 0$. Solving $\frac{\partial\,\Phi}{\partial\,\delta} = 0$ with respect to q yields roots $1/\,2$ and

$$\frac{45 - 6\,\delta + \delta^2 \pm \sqrt{2025 - 1404\,\delta + 270\,\delta^2 - 12\,\delta^3 + \delta^4}}{2\,\delta\,(24 - 4\,\delta)}$$

Considering that $\delta \in (0,1)$, the latter two roots are greater than one or less than zero. It follows that, for $q \in (\frac{1}{2}, 1)$, the sign of $\frac{\partial\,\Phi}{\partial\,\delta}$ is the same as when $q = 1$. Computation establishes that

$$\left.\frac{\partial\,\Phi}{\partial\,\delta}\right|_{q=1} = -\frac{36 - 30\,\delta + 5\,\delta^2}{12\,(3-\delta)^2}$$

which is negative, given that $\delta \in (0,1)$. We conclude that a sufficient condition for $\Phi > 0$ when $\delta \in (0,1)$ is that $\Phi\,|_{\delta=1} > 0$. In fact, computation establishes that

$$\Phi\,|_{\delta=1} = \frac{1}{12}\,(2\,q - 1)\,(2 - q)\,(3 - 2\,q)$$

which is positive for all $q \in \left(\frac{1}{2}, 1\right)$. ■

Proof of Proposition 3: Let $M = m_{ik}$ be the Markov transition matrix across states $i, k = 0, 1, 2$. Let $[\mu_0, \mu_1, \mu_2]$ be the stationary distribution over states. Define $\mu = \mu_1$ and $q = q_2$. I next derive μ as a function of q. The first column of the Markov transition matrix is given by

$$\begin{aligned} m_{00} &= q + \frac{1}{3}\,(1-q) \\ m_{10} &= 1/\,6 \\ m_{20} &= 0 \end{aligned}$$

By definition of stationary state

$$\mu_0 = \sum_{k=0}^{2} m_{k0}\,\mu_k$$

Symmetry implies that $\mu_0 = \mu_2 = \frac{1}{2}\,(1-\mu)$. Substituting in the above expression, we have

$$\frac{1}{2}\,(1-\mu) = \frac{1}{2}\,(1-\mu)\left(q + \frac{1}{3}\,(1-q)\right) + \frac{1}{6}\,\mu$$

Solving for μ, we get

$$\mu = \frac{2-2\,q}{3-2\,q} \tag{16}$$

Straightforward derivation shows that μ is decreasing in q. The result then follows from Proposition 2. ■

Proof of Proposition 4: Solving (6) for $i = 0$, we get

$$v_0 = \frac{(1-q)^2}{1-\delta}$$

The result then follows from Proposition 2. ■

Proof of Proposition 5: Social welfare is given by two components: aftermarket total surplus and basic market total surplus. In terms of aftermarket

surplus, we have two possibilities. Either we are in more asymmetric split of installed bases ($i = 0, j = 3$ or $i = 3, j = 0$); or we are in a more symmetric split ($i = 1, j = 2$ or $i = 2, j = 1$). In the first case, total surplus is given by $3\,\omega + 0\,\phi + 3^2\,\phi = 3\,\omega + 9\,\phi$. In the second case, we have $3\,\omega + 1\,\phi + 2^2\,\phi = 3\,\omega + 5\,\phi$. So, the greater the asymmetry of installed bases, the greater social welfare.

The steady state probability of a more asymmetric aftermarket split of installed bases is given by $(1-\mu)\,q$. In words, the system must start from an asymmetric state, which happens with probability $1-\mu$; and the large firm must make the sale, which happens with probability q. We conclude that a sufficient statistic for steady-state social welfare *in the aftermraket* is

$$3\,\omega + (1-\mu)\,q\,9\,\phi + \big(1-(1-\mu)\,q\big)5\,\phi$$

The second component of social welfare is total surplus in the basic market. A sufficient statistic for this surplus is total transportation costs (or the negative of). Modulo a constant term, this is given by the extra transportation cost due to firms setting different prices. Specifically, at stage $i = 0$ or $i = 2$ we must take into account consumers with addresses between 0 and $q - \frac{1}{2}$, who now purchase from a firm that's located farther away. If $p_i = p_j$, these consumers would pay a transportation cost of $\frac{1}{2} - x$, where x is their address. Now they pay a transportation cost of $\frac{1}{2} + x$. The total increase in transportation costs is given by

$$\int_0^{q-\frac{1}{2}} 2\,x\,dx = \left(q - \frac{1}{2}\right)^2$$

This cost is incurred with probability $1-\mu$, which in the steady state is equal to $\frac{q}{3-2\,q}$.

Pulling the two components together, substituting (16) for μ, and simplifying, we have the following sufficient statistic of social welfare:

$$S = \frac{15-6\,q}{3-2\,q}\,\phi + \frac{q}{3-2\,q}\left(q - \frac{1}{2}\right)^2$$

Taking the total first derivative with respect to ϕ and recalling that $\phi = 0$ implies $q = \frac{1}{2}$, we obtain

$$\left.\frac{d\,S}{d\,\phi}\right|_{\phi=0} = 0$$

Taking the second total derivative, we get

$$\begin{aligned}\left.\frac{d^2\,S}{d\,\phi^2}\right|_{\phi=0} &= \left.\left(\frac{d}{d\,q}\left(\frac{15-6\,q}{3-2\,q}\right)-\frac{d^2}{d\,q^2}\left(\frac{q}{3-2\,q}\left(q-\frac{1}{2}\right)^2\right)\right)\frac{d\,q}{d\,\phi}\right|_{\phi=0}\\ &= \left.\frac{5}{2}\,\frac{d\,q}{d\,\phi}\right|_{\phi=0}\end{aligned}$$

Finally, since $\frac{d\,q^M}{d\,\phi} > \frac{d\,q^C}{d\,\phi}$ (by Proposition 2) the result follows. ■

Proof of Proposition 6: Let μ_i be the probability that, in the steady state, the system is at i. Let $\mu_1 = \mu$. By symmetry, $\mu_0 = \mu_2 = (1-\mu)/2$ and so $\mu_0 + \mu_2 = 1-\mu$. In terms of aftermarket states, we have the following possibilities: with probability $(\mu_0+\mu_2)\,q_2$, all consumers are in the same installed base; otherwise, there is a split, with two consumers with one installed base and one with the other installed base.

In terms of the price paid by the newborn consumer, we have the following possibilities: with probability $(\mu_0+\mu_2)\,q_2$, the consumer pays p_2; with probability $(\mu_0+\mu_2)\,q_0$, the consumer pays p_0; and with probability μ_1 the consumer pays p_1. Defining $q_2 = q$, we can compute consumer welfare in the steady state as follows:

$$\begin{aligned}C = \big((1-\mu)\,q\big)\big(3\,\lambda_3\big) + \big(\mu+(1-\mu)\,q\big)\big(\lambda_1+2\,\lambda_2\big) - \\ -(1-\mu)\,(1-q)\,p_0 - \mu\,p_1 - (1-\mu)\,q\,p_2\end{aligned} \tag{17}$$

Note that, at $\phi = 0$, $p_i = q = \mu = \frac{1}{2}$ and $\lambda_i = 0$. Moreover, substituting $q = \mu = \frac{1}{2}$ in (5) implies

$$p_0 + 2\,p_1 + p_2 = 2 - (\pi_3+\pi_2-\pi_1-\pi_0) - \delta\,\frac{4}{3}\,(v_2-v_0)$$

Using this, I can differentiate (17) to get

$$\begin{aligned}\left.\frac{d\,C}{d\,\phi}\right|_{\phi=0} = \frac{3}{4}\,\frac{d}{d\,\phi}\big(\lambda_1+2\,\lambda_2+\lambda_3\big) + \frac{1}{4}\,\frac{d}{d\,\phi}\big(\pi_3+\pi_2-\pi_1-\pi_0\big) + \\ + \frac{1}{3}\,\delta\,\frac{d}{d\,\phi}\big(v_2-v_0\big)\end{aligned} \tag{18}$$

□ Consider first case C: $\lambda_i = i\,\phi,\ \pi_i = 0$. Substituting $i\,\phi$ for λ_i and simplifying, we get

$$\frac{3}{4}\,\frac{d}{d\,\phi}\big(\lambda_1 + 2\,\lambda_2 + \lambda_3\big) = 6$$

Moreover,

$$\left.\frac{d\,(v_2 - v_0)}{d\,\phi}\right|_{\phi=0} = \frac{\partial\,(v_2 - v_0)}{\partial\,q}\,\left.\frac{d\,q}{d\,\phi}\right|_{q=\frac{1}{2}}$$

From (6), I determine that

$$\left.\frac{\partial\,(v_2 - v_0)}{\partial\,q}\right|_{q=\frac{1}{2}} = \frac{3\,(12 - 4\,\delta)}{2\,(9 - 9\,\delta + 2\,\delta^2)}$$

Differentiating (13) with respect to q, substituting $q = \frac{1}{2}$, and inverting, yields

$$\left.\frac{d\,q}{d\,\phi}\right|_{q=\frac{1}{2}} = \frac{2}{3 - \delta}$$

Substituting all of these expressions in (18), I finally get

$$\frac{d\,C}{d\,\phi} = \frac{2\,(27 - 25\,\delta + 6\,\delta^2)}{(3 - \delta)\,(3 - 2\,\delta)} \tag{19}$$

Notice that

$$\lim_{\delta\to 1}\,\frac{d\,C}{d\,\phi} = 8 \tag{20}$$

□ Consider first case M: $\pi_i = i^2\,\phi,\ \lambda_i = 0$. Substituting $i^2\,\phi$ for π_i, we get

$$\frac{1}{4}\,\frac{d}{d\,\phi}\big(\pi_3 + \pi_2 - \pi_1 - \pi_0\big) = 3$$

Regarding the second row in (18), I now must consider the fact that v_i depends on π_i, and so when computing the derivative with respect to ϕ, I must consider both the direct partial and the effect of ϕ through changes in q. The two partial derivatives are given by

$$\frac{\partial\,(v_2 - v_0)}{\partial\,\phi} = \frac{24 - 12\,\delta}{2\,(9 - 9\,\delta + 2\,\delta^2)}$$

$$\frac{\partial\,(v_2 - v_0)}{\partial\,q} = \frac{12 - 8\,\delta\,q}{2\,(9 - 9\,\delta + 2\,\delta^2)}$$

Differentiating (15) with respect to q, substituting $q = \frac{1}{2}$, and inverting, yields

$$\left.\frac{d\,q}{d\,\phi}\right|_{q=\frac{1}{2}} = \frac{4}{3 - \delta}$$

Substituting all of these expressions in (18), I finally get

$$\frac{d\,C}{d\,\phi} = \frac{27 - 7\,\delta}{(3 - \delta)\,(3 - 2\,\delta)} \tag{21}$$

Notice that

$$\lim_{\delta \to 1} \frac{d\,C}{d\,\phi} = 10 \tag{22}$$

Comparing (20) and (22), the result follows. ■

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