

The Informational Role of Prices*

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Abstract

We study the informational role of prices. To that end, we consider the framework of a dominant firm with a competitive fringe, which generalizes the monopoly framework (i.e., a dominant firm without a competitive fringe). When the competitive fringe is large enough, there exists a unique fully revealing equilibrium, in which the price conveys full information about the quality of the good to uninformed buyers. Deceiving the uninformed buyers by charging a high price and mimicking a high quality is not profitable when the competitive fringe is large enough. Since a higher price triggers more sales on the part of the competitive fringe, residual demand and thus profits are reduced. We also study the effect of asymmetric information and learning on the equilibrium outcomes. More uninformed buyers increases the price, reduces the quantity sold by the dominant firm, but increases the quantity sold by the competitive fringe.

Keywords: Asymmetric information, Dominant Firm with Fringe Competition, Informational externality, Learning, Monopoly, Quality, Signaling.

JEL Classifications: D21, D42, D82, D83, D84, L12, L15.

1 Introduction

In a world of asymmetric information, prices play an informational role. Indeed, traders rely on prices in order to obtain information about firms as much as tourists extract information from the prices to learn about the quality of the food in restaurants. The informational role of prices is important because it reduces the informational asymmetry among individuals. Several studies have provided conditions under which privately-held information by firms becomes public through prices, beginning with perfectly competitive markets (Kihlstrom and Mirman, 1975; Grossman, 1976, 1978; Grossman and Stiglitz, 1980) and continuing with imperfectly competitive markets (Wolinsky, 1983; Riordan, 1986; Bagwell and Riordan, 1991; Judd and Riordan, 1994; Daughety and Reinganum, 1995, 2005, 2007, 2008a,b; Janssen and Roy, 2010; Daher et al., 2012). In the case of perfectly competitive markets, the firms have no control over prices and thus have no ability to influence directly the amount of information conveyed by prices. However, in the case of imperfectly competitive markets, the conveyance of information through prices can be directly influenced by the firms. Specifically, since prices have some informational content, a firm selling a low-quality good may have an incentive to deceive uninformed buyers by charging a higher price, thus, mimicking a higher-quality firm. When such an incentive exists, buyers may not be able to extract information from the price and asymmetric information remains. The ability to manipulate information through prices arise as long as some firms (but not necessarily all) have market power.

While the literature on the informational role of prices has considered situations in which all firms have market power (e.g., monopoly), this is rarely the case. In many sectors, one firm has a greater market share relative to the remaining firms. In other words, one of the firms is dominant, e.g., Kodak for photographic film or IBM for the mainframe computer industry. In this paper, we study information flows when not all firms have market power. To that end, we embed asymmetric information and learning in a model of a dominant firm facing a competitive fringe. The dominant firm sets the price and the competitive fringe is a price-taker. Moreover, the dominant

firm has a cost advantage compared to the competitive fringe. Finally, all firms produce the same good, e.g., the dominant firm is an innovator and the competitive fringe are imitators.¹ The size of the competitive fringe is fixed, i.e., imitation is possible but not available to everybody so there is no unlimited entry. The quality of the good is known to all firms, but unknown to uninformed buyers who extract the information from prices. For comparability with the existing literature, we retain the linear demand in which the quality is related to the reservation or choke price (Bagwell and Riordan, 1991; Daughety and Reinganum, 1995, 2005, 2008a). Moreover, as in Bagwell and Riordan (1991), demand is assumed to be composed of informed and uninformed buyers. Unlike these previous studies, we assume that the unknown quality (i.e., the reservation price) is a continuum equal to the whole strictly positive real line.² We use a continuum to convey the idea that the uninformed buyers' prior beliefs about the unknown quality admit a rich array of possible levels of quality. Moreover, in our model, a continuum of quality on the real positive line yields a price strategy space which is the real positive line. This removes the need to specify out-of-equilibrium beliefs.

The literature has focused on the role of informed buyers in conveying information about quality through prices (Wolinsky, 1983; Riordan, 1986; Bagwell and Riordan, 1991). We study the role of the competitive fringe to guarantee existence of an equilibrium in which the price-setting firm signals quality to the uninformed buyers. We present two sets of results. We first characterize the unique fully revealing equilibrium for the class of price strategies that are continuous in the quality parameter. We show that the

¹We consider the standard framework of a dominant firm facing a competitive fringe with a cost disadvantage. The fringe firm knows the quality of the good. In our paper, the quality is the same across firms. Section 5 discusses the case of the dominant firm and the competitive fringe selling different and unknown qualities.

²The absence of an upper bound is merely for simplicity. Specifying an upper bound for the unknown quality (i.e., the reservation price) makes no difference in the analysis as we can also restrict possible prices between zero (not included) and the same upper bound.

competitive fringe is needed for the price to be informative.³ Our result on the necessity of the competitive fringe complements the result stated in Bagwell and Riordan (1991). Specifically, when the space of the unknown quality is restricted to two values, a large fraction of informed buyers is sufficient for the price to convey information about the quality of the good. We show that the condition on the size of the informed buyers is not sufficient when there is a rich array of possible levels of quality. In that case, the threat of competition on the part of a fringe enables the price to be informative. Indeed, suppose that there is no competitive fringe so that the dominant firm is a monopoly. In the absence of a competitive fringe, the monopolist of lower quality has an incentive to mimic a higher quality monopolist by charging a higher price, thereby sacrificing profit from the informed buyers, but yielding more profits since the uninformed buyers misinterpret the true quality.⁴ A large enough presence of a competitive fringe removes this incentive to deviate. While charging a higher price does yield more profit from the deceived uninformed buyers, it also triggers more sales on the part of the competitive fringe, thereby reducing demand, and, thus, profits of the dominant firm. With a large enough competitive fringe, a low-quality firm has no longer an incentive to deviate and mimic the price set by a high-quality firm.

We then study the effect of the informational externality on the equilibrium outcomes. The informational externality is due to the presence of uninformed buyers. Indeed, their learning activity has an effect on the dominant firm's profit through the updating rule embedded in demand. We show that the price set by the dominant firm is increasing in the fraction of uninformed buyers. However, a larger competitive fringe mitigates this effect on the price. Moreover, a change in the fraction of uninformed buyers changes

³Note that the uniqueness of our fully revealing equilibrium does not depend upon the correlation between the cost and the quality. In Bagwell and Riordan (1991), the correlation between the quality and the cost is necessary to eliminate a separating equilibrium that yields a price below the full information price. In our model, it is always the case that the equilibrium price increases due to asymmetric information and learning. Hence, our result holds when there is a correlation between quality and cost as in Bagwell and Riordan (1991), as well as when quality is unrelated to cost as in Judd and Riordan (1994).

⁴This is in contrast to Bagwell and Riordan (1991) which shows that a monopoly can credibly signal quality to the uninformed buyers when the space of the unknown quality is restricted to be two values.

the composition for the supply of the good. Specifically, more uninformed buyers reduces the amount sold by the dominant firm, but increases the amount sold by the competitive fringe. In other words, more uninformed buyers allows the competitive fringe to bear a higher cost in order to sell the good.

The paper is organized as follows. Section 2 presents the model. Section 3 provides the equilibrium and explains the role of the competitive fringe for existence of the equilibrium. Section 4 studies the effect of asymmetric information and learning through prices on the firms' behavior. Section 5 concludes and suggests avenues of research regarding the informational role of prices in different environments.

2 Model

In this section, we embed asymmetric information and learning in a model of a dominant firm facing a competitive fringe. We first present the model and state the assumptions. We then define the fully revealing equilibrium in which the price fully reveals quality to the uninformed buyers.

Consider a market for a good of quality $\theta \in \Theta \subset \mathbb{R}_+$ sold at price $P > 0$. The demand side is composed of informed and uninformed price-taking buyers. Informed buyers know θ and have demand $q(P, \theta)$. Uninformed buyers do not know θ , but infer it from observing the price. Specifically, upon observing P , the uninformed buyers' inference rule about quality is $\chi_u(P)$ where the subscript u refers to the group of *uninformed* buyers. Hence, the uninformed buyers' demand is $q(P, \chi_u(P))$. Normalizing the mass of buyers to one and letting $\lambda \in (0, 1)$ be the fraction of informed buyers, the market demand is

$$D(P, \theta, \chi_u(P)) = \lambda q(P, \theta) + (1 - \lambda)q(P, \chi_u(P)). \quad (1)$$

The supply side is composed of one dominant firm (d) and a competitive fringe (f). Both the dominant firm and the competitive fringe know the quality θ . The competitive fringe is an imitator and is able to produce and

sell the same good. Hence, there is one market price. The dominant firm has market power and sets the price, while the competitive fringe is a price-taker and chooses production. Specifically, given P , the competitive fringe sets production $x \geq 0$ so as to maximize profit $Px - C_f(\theta, x)$ where $C_f(\theta, x)$ is total cost, which yields the supply function $S_f(P, \theta)$. Given $S_f(P, \theta)$ and $\chi_u(P)$, the dominant firm chooses P so as to maximize profit

$$\pi = (P - c_d\theta) \max\{D(P, \theta, \chi_u(P)) - \varphi S_f(P, \theta), 0\}, \quad (2)$$

where $c_d\theta, c_d \in [0, 1)$ is the dominant firm's marginal cost and

$$\max\{D(P, \theta, \chi_u(P)) - \varphi S_f(P, \theta), 0\} \quad (3)$$

is the residual demand. From (2) or (3), the parameter $\varphi \in [0, 1]$ measures the size of the competitive fringe relative to the dominant firm.⁵ The model embeds the special case of monopoly, i.e., a dominant firm without a competitive fringe when $\varphi = 0$. Because imitation of the new product is difficult, the size of the competitive fringe is fixed and no other fringe firm can enter the market. In other words, our model concerns a situation of a dominant firm facing a competitive fringe in which no additional fringe firms can enter.

Definition 2.1 states the fully revealing equilibrium in which the dominant firm's price fully reveals quality to the uninformed buyers. The equilibrium consists of the dominant firm's price strategy as a function of quality, the uninformed buyers' inference rule as a function of price, and the competitive fringe's supply strategy as a function of price and quality. The last two equilibrium variables depend on the price because both the buyers and the competitive fringe are price-takers. Specifically, condition 1 defines the dominant firm's price strategy. The uninformed buyers' inference rule has an effect on the profits of the dominant firm through demand and thus imposes an informational externality on the dominant firm. Condition 2a states that the inference rule is the inverse of the dominant firm's price strategy, i.e., the price is fully revealing about the unknown quality. In other words, the

⁵Restricting the value of φ to $[0, 1]$ is a normalization similar to normalizing the mass of buyers to one.

uninformed buyers have rational expectations because the inference rule is consistent with the dominant firm's price strategy. Finally, Condition 2b defines the competitive fringe's supply strategy.

Definition 2.1. For $\lambda \in (0, 1)$, the tuple $\{P^*(\theta), \chi_u^*(P), S_f^*(P, \theta)\}$ is a fully revealing equilibrium if, for $\theta \in \Theta$,

1. Given $\chi_u^*(P)$ and $S_f^*(P, \theta)$, the dominant firm's price strategy satisfies

$$P^*(\theta) = \arg \max_{P > 0} \{(P - c_d \theta) \max\{D(P, \theta, \chi_u^*(P)) - \varphi S_f^*(P, \theta), 0\}\}. \quad (4)$$

2. Given $P^*(\theta)$,

- (a) The uninformed buyers' inference rule satisfies

$$\chi_u^*(P^*(\theta)) = \theta. \quad (5)$$

- (b) The fringe firm's supply strategy satisfies

$$S_f^*(P^*(\theta), \theta) = \arg \max_{x \geq 0} \{P^*(\theta)x - C_f(\theta, x)\}. \quad (6)$$

Having defined the fully revealing equilibrium, we next state the assumptions that hold for the remainder of the paper. In order to compare our results with those already established in the literature on monopoly signaling, we retain the linear demand in which the quality is related to the demand intercept, i.e., the reservation or choke price.⁶ Further, while the dominant firm and the competitive fringe produce the same good, we assume that the competitive fringe faces a higher cost of production.

⁶See Daughety and Reinganum (2008a) for a detailed discussion regarding the use of a linear demand in models in which the price reveals information about quality to uninformed buyers. The linear demand can be generated from a quadratic utility function or by aggregating unit demand functions of consumers with heterogeneous reservation prices. In Bagwell and Riordan (1991), quality can either be low or high. The demand for the high quality is linear, while the low quality product has a unit demand. In Daughety and Reinganum (2008a), the demand is $D(P, \theta) = (\alpha - (1 - \delta)\theta)/\beta - P/\beta$, where $\alpha, \beta, \delta > 0$ are known parameters and $\theta \in [\underline{\theta}, \bar{\theta}]$, $0 < \underline{\theta} < \bar{\theta}$, is the unknown parameter for which the price transmits information. As in our case, the demand intercept depends on the unknown parameter.

Assumption 2.2. For $\theta \in \Theta$, $q(P, \theta) = \max\{\theta - P, 0\}$ and $q(P, \chi_u(P)) = \max\{\chi_u(P) - P, 0\}$ so that

$$D(P, \theta, \chi_u(P)) = \lambda \max\{\theta - P, 0\} + (1 - \lambda) \max\{\chi_u(P) - P, 0\}. \quad (7)$$

Assumption 2.3. For $\theta \in \Theta$, $C_f(\theta, x) = c_f \theta x + x^2/2$ where $0 \leq c_d < c_f \leq 1$.

Given Assumption 2.3, Remark 2.4 provides the competitive fringe's supply strategy in a fully revealing equilibrium. The competitive fringe is a price-taker with a supply strategy that is independent of information flows. Thus, using (6), the supply strategy is derived independently of the remaining equilibrium variables.

Remark 2.4. Suppose that a fully revealing equilibrium exists. Then, for $\theta \in \Theta$, the competitive fringe's supply strategy is

$$S_f^*(P, \theta) = \max\{P - c_f \theta, 0\}. \quad (8)$$

Before proceeding with the characterization of the fully revealing equilibrium, we specify the space of the unknown quality. Because quality (and thus reservation price) may take on multiple values, we assume that the quality is a variable that is infinitely divisible on the positive real line. In particular, we make no arbitrary restriction on the lower bound of quality, which implies that the uninformed buyers' prior beliefs about quality has support on the full positive real line, i.e., a positive probability is assigned to any interval on the real positive line.⁷ In our model, a continuum of quality on the real positive line yields a price strategy space on the real positive line, which removes the need to specify out-of-equilibrium beliefs.

Assumption 2.5. Quality is defined on a continuum on the positive real line, i.e., $\Theta = (0, \infty)$.

Having presented the model and defined the equilibrium, we proceed with the analysis of the equilibrium. We first characterize the equilibrium by providing necessary conditions and showing that the presence of a competitive

⁷Let the p.d.f. ξ represent the uninformed buyers' prior beliefs. Then, for any nonempty $Z \subset \Theta = (0, \infty)$, the uninformed buyer's prior probability that $\theta \in Z$ is $\int_{z \in Z} \xi(z) dz > 0$.

fringe (i.e., $\varphi > 0$) is necessary for the existence of an equilibrium. We also provide conditions on the values of the parameters for existence, especially the size of the competitive fringe. We then discuss the effect of asymmetric information and learning among buyers on the behavior on the dominant firm by comparing the fully revealing equilibrium with the benchmark equilibrium of full information in which all buyers are informed, i.e., $\lambda = 1$.

3 Equilibrium

In this section, we characterize the unique fully revealing equilibrium. We first provide necessary conditions about the equilibrium (Propositions 3.1 and 3.2). We then turn to existence by showing that the absence of a competitive fringe (i.e., $\varphi = 0$) provides the dominant firm an incentive to deceive uninformed buyers, i.e., there is no fully revealing equilibrium (Proposition 3.3). Necessary and sufficient conditions on the size of the competitive fringe for the existence of the unique fully revealing equilibrium are provided and discussed (Proposition 3.4).

3.1 Characterization

Proposition 3.1 states that, when there are some informed buyers, there is a unique candidate for the fully revealing equilibrium for the class of price strategies that are continuous in the quality parameter. Moreover, in this candidate equilibrium, the dominant firm's price strategy is linear and strictly increasing in quality. Hence, in our model, Assumption 2.5 removes the need to specify any out-of-equilibrium beliefs, i.e., every price $P > 0$ is a possible outcome in equilibrium.

Proposition 3.1. *Suppose that a fully revealing equilibrium exists. Then, the equilibrium is unique. In equilibrium,*

1. $P^*(\theta) \in (c_d\theta, \theta)$ is linear and strictly increasing in $\theta \in (0, \infty)$.
2. $\lim_{\theta \rightarrow 0} P^*(\theta) = 0$.

Proof. See Appendix A. □

Proposition 3.2 provides the dominant firm's price strategy corresponding to the unique candidate for the fully revealing equilibrium.⁸ To simplify the discussion and provide a simple expressions for the equilibrium, we present the equilibrium in a special case in which the dominant firm's marginal cost is zero. We continue to assume that the competitive fringe has a cost disadvantage vis-à-vis the dominant firm, i.e., $c_d = 0$ and $c_f \in (0, 1]$. This restriction has no bearing on the analysis. Proposition A.1 in Appendix A provides the general characterization of the unique candidate for the fully revealing equilibrium, i.e., $0 \leq c_d < c_f \leq 1$. In general, for low values of the competitive fringe's cost, the dominant firm reveals quality by setting a price that induces the competitive fringe to sell the good (Statement 1). For high values of the cost, revelation occurs, but the competitive fringe does not sell the good (Statements 2 and 3).⁹ Since we consider a situation in which entry is not allowed, the dominant firm retains market power and sets the price above his marginal cost. In some cases, the dominant firm sets the price equal to the marginal cost of the competitive fringe.

⁸The uninformed buyers' inference rule is not stated because it is the inverse of the price strategy and the competitive fringe's supply strategy is stated in Remark 2.4.

⁹The effect of information flows on the dominant firm's ability to discourage the competitive fringe to supply is discussed in Section 4.

Proposition 3.2. *Suppose that a fully revealing equilibrium exists. If $c_d = 0$, then for $\theta > 0$,*

1. *When $c_f \in \left(0, \frac{2-\lambda}{2+\varphi}\right)$,*

$$P^*(\theta) = \frac{2-\lambda + \varphi c_f \theta}{2(1+\varphi)} > c_f \theta. \quad (9)$$

2. *When $c_f \in \left[\frac{2-\lambda}{2+\varphi}, \frac{2-\lambda}{2}\right]$,*

$$P^*(\theta) = c_f \theta. \quad (10)$$

3. *When $c_f \in \left(\frac{2-\lambda}{2}, 1\right]$,*

$$P^*(\theta) = \frac{2-\lambda}{2} \theta < c_f \theta. \quad (11)$$

Proof. See Appendix A. □

We now provide an intuitive derivation of the equilibrium stated in Proposition 3.2 by using the linearity of the inference rule. The proof of Proposition A.1 in Appendix A shows that the equilibrium is unique and thus there exists no equilibrium with a nonlinear price strategy.

From (7) and (8), residual demand faced by the dominant firm is

$$Q_R = \lambda \max\{\theta - P, 0\} + (1-\lambda) \max\{\chi_u(P) - P, 0\} - \varphi \max\{P - c_f \theta, 0\}. \quad (12)$$

Because the residual demand is kinked at $P = c_f \theta$, we solve the problem piecewise. Consider first the case in which the competitive fringe does not sell the good, i.e., $P \in (0, c_f \theta]$. Let $\chi_u^e(P) = P/\mathcal{A}^e$ be an arbitrary linear inference rule where $\mathcal{A}^e > 0$ is the uninformed buyers' *expected* parameter regarding the relationship between the price and the unknown quality. Given $\chi_u^e(P) = P/\mathcal{A}^e$ and using (4), the dominant firm's maximization problem is

$$\max_{P \in (0, c_f \theta)} P \cdot D(P, \theta, \chi_u^e(P)) \quad (13)$$

where, from (7), for $P \in (0, c_f\theta)$,

$$D(P, \theta, \chi_u^e(P)) = \lambda\theta - \left(1 - \frac{1-\lambda}{\mathcal{A}^e}\right) P. \quad (14)$$

Taking the first-order condition corresponding to (13) yields

$$\lambda\theta - 2 \left(1 - \frac{1-\lambda}{\mathcal{A}^e}\right) P = 0 \quad (15)$$

so that

$$P^e = \frac{\lambda\theta}{2 \left(1 - \frac{1-\lambda}{\mathcal{A}^e}\right)} \quad (16)$$

is the optimal price strategy for the dominant firm given the uninformed buyers' inference rule. In order to obtain a fully revealing equilibrium, the dominant firm's price strategy must be consistent (in the sense of rational expectations) with the uninformed buyers' inference rule, i.e., using the inverse of (16),

$$\chi_u^*(P) = \chi_u^e(P) \quad (17)$$

$$\frac{P}{\mathcal{A}^*} = \frac{2 \left(1 - \frac{1-\lambda}{\mathcal{A}^e}\right) P}{\lambda}, \quad (18)$$

such that $\mathcal{A}^* = \mathcal{A}^e$, which yields $\mathcal{A}^* = \frac{2-\lambda}{2} > 0$.¹⁰ Hence, $P^*(\theta) = \mathcal{A}^*\theta \in (0, c_f\theta]$ and the competitive fringe does not sell the good.

Consider next the case in which the competitive fringe sells the good, i.e., $P \in (c_f\theta, \theta)$. Let $\chi_u^e(P) = P/\mathcal{B}^e$ be an arbitrary linear inference rule where $\mathcal{B}^e > 0$ is the uninformed buyers' *expected* parameter regarding the relationship between the price and the unknown quality. Given $\chi_u^e(P) = P/\mathcal{B}^e$ and using (4), the dominant firm's maximization problem is

$$\max_{P \in (c_f\theta, (1+\varphi c_f)\theta)/(1+\varphi)} P \cdot D(P, \theta, \chi_u^e(P)) \quad (19)$$

¹⁰The second-order condition of (13) is satisfied since $\mathcal{A}^* > 1 - \lambda$.

where, from (7), for $P \in (c_f\theta, \theta)$,

$$D(P, \theta, \chi_u^e(P)) = (\lambda + \varphi c_f)\theta - \left(1 + \varphi - \frac{1 - \lambda}{B^e}\right) P. \quad (20)$$

Taking the first-order condition corresponding to (19) yields the optimal price strategy for the dominant firm given the uninformed buyers' inference rule,

$$P^e = \frac{(\lambda + \varphi c_f)\theta}{2 \left(1 + \varphi - \frac{1 - \lambda}{B^e}\right)}. \quad (21)$$

In order to obtain a fully revealing equilibrium, the dominant firm's price strategy must be consistent (in the sense of rational expectations) with the uninformed buyers' inference rule, i.e., using the inverse of (21),

$$\chi_u^*(P) = \chi_u^e(P) \quad (22)$$

$$\frac{P}{B^*} = \frac{2 \left(1 + \varphi - \frac{1 - \lambda}{B^e}\right) P}{\lambda + \varphi c_f}, \quad (23)$$

such that $B^* = B^e$, which yields $B^* = \frac{2 - \lambda + \varphi c_f}{2(1 + \varphi)} > 0$.¹¹ Hence, $P^*(\theta) = B^*\theta \in (c_f\theta, (1 + \varphi c_f)\theta/(1 + \varphi))$ implies that both the dominant firm and the competitive fringe sell the good.¹²

The two cases are mutually exclusive, i.e., $0 = c_d < B^* \leq A^* < (1 + \varphi c_f)/(1 + \varphi)$ where $B^* = \frac{2 - \lambda + \varphi c_f}{2(1 + \varphi)}$ and $A^* = \frac{2 - \lambda}{2}$.¹³ Hence, if $c_f \in \left(0, \frac{2 - \lambda}{2 + \varphi}\right)$, then the dominant firm's price strategy $P^*(\theta) = B^*\theta > c_f\theta$ is defined by (9), which induces the fringe firm to sell the good. If $c_f \in \left(\frac{2 - \lambda}{2}, 1\right]$, then the dominant firm's price strategy $P^*(\theta) = A^*\theta < c_f\theta$ is defined by (11), which induces the competitive fringe not to sell the good. Finally, for $c_f \in \left[\frac{2 - \lambda}{2 + \varphi}, \frac{2 - \lambda}{2}\right]$, the dominant firm sets the price at the kink of the demand, i.e., $P^*(\theta) = c_f\theta$ as defined in (10) so that again the competitive fringe does not sell either. In each case, the price fully reveals quality to the uninformed buyers. The

¹¹The second-order condition of (19) is satisfied since $B^* > (1 - \lambda)/(1 + \varphi)$.

¹²Specifically, $P^*(\theta) = B^*\theta > c_f\theta$ induces the competitive fringe to sell the good, while $P^*(\theta) < (1 + \varphi c_f)\theta/(1 + \varphi)$ means that residual demand is strictly positive, i.e., the dominant firm sells the good as well.

¹³For $\varphi \in (0, 1]$, $B^* < A^*$ while $B^* = A^*$ if $\varphi = 0$.

inference rule is the inverse of the price strategy, i.e., from Proposition 3.2,

$$\chi_u^*(P) = \begin{cases} \frac{2(1+\varphi)P}{2-\lambda+\varphi c_f}, & c_f \in \left(0, \frac{2-\lambda}{2+\varphi}\right) \\ \frac{P}{c_f}, & c_f \in \left[\frac{2-\lambda}{2+\varphi}, \frac{2-\lambda}{2}\right] \\ \frac{2P}{2-\lambda}, & c_f \in \left(\frac{2-\lambda}{2}, 1\right] \end{cases} . \quad (24)$$

3.2 Existence

Having characterized the only candidate for a fully revealing equilibrium, we turn next to existence by studying whether the price strategy in Proposition 3.2 is dominated by any other price strategy. We show that whether deviation from the candidate price strategy is profitable depends on the size of the competitive fringe. On the one hand, charging a price higher than the candidate price strategy yields more profit. Indeed, a higher price increases the purchases of the uninformed buyers who misinterpret the true quality because the inference rule consistent with the candidate equilibrium is increasing in price. On the other hand, a higher price triggers more sales on the part of the competitive fringe, thereby reducing residual demand, and, thus, profits of the dominant firm. When the competitive fringe is large enough, the cost of a reduced residual demand outweighs the benefit of deceiving the uninformed buyers. Hence, a large enough presence of a competitive fringe removes this incentive to deviate.

In order to understand the effect of a competitive fringe on existence, we begin by discussing the case in which there is no competitive fringe. Proposition 3.3 states that there is no fully revealing equilibrium without a competitive fringe. With no competitive fringe, the firm's price strategy is systematically dominated by prices above the reservation price that exclude informed buyers and deceive uninformed buyers. In this case, the firm (of any quality) has an incentive to mimic a higher quality firm by charging a price higher than the candidate price strategy. Hence, profits from the informed buyers, who reduce their purchases, are sacrificed while the profits from the uninformed buyers are increased. The uninformed buyers misinterpret the true quality because their inference rule is increasing in price. Proposition B.1

in Appendix B generalizes Proposition 3.3 to any cost $c_d \in [0, c_f)$.

Proposition 3.3. *Suppose that $c_d = 0$. If there is no competitive fringe (i.e., $\varphi = 0$), then there exists no fully revealing equilibrium.*

Proof. See Appendix B. □

We now provide an intuitive and graphical explanation of the nonexistence result stated in Proposition 3.3. Suppose that $\varphi = 0$. From Proposition 3.2, the firm's profit corresponding to the unique candidate for equilibrium is $\pi^*(\theta)|_{\varphi=0} = P^*(\theta)(\theta - P^*(\theta))$, $P^*(\theta) = (2 - \lambda)\theta/2 \in (0, \theta)$,¹⁴ or

$$\pi^*(\theta)|_{\varphi=0} = \frac{(2 - \lambda)\lambda\theta^2}{4}. \quad (25)$$

From Proposition 3.2, the uninformed buyers' updating rule is the inverse of the firm's price strategy, i.e., $\chi_u^*(P)|_{\varphi=0} = \frac{2P}{2-\lambda}$. Using (7), for $P > \theta$, demand evaluated at $\chi_u^*(P)|_{\varphi=0} = \frac{2P}{2-\lambda}$ is

$$D(P, \theta, \chi_u^*(P))|_{\varphi=0} = \frac{(1 - \lambda)\lambda P}{2 - \lambda}, \quad (26)$$

which is increasing in P due to the fact that for the uninformed buyers a higher price means higher quality. Hence, the firm's profit from deviating to $P > \theta$ is $\hat{\pi}(\theta, P)|_{\varphi=0} = (1 - \lambda)(\chi_u^*(P) - P)P$ or

$$\hat{\pi}(\theta, P)|_{\varphi=0} = \frac{(1 - \lambda)\lambda}{2 - \lambda} P^2. \quad (27)$$

The firm has an incentive to deviate from the price strategy $P^*(\theta) = (2 - \lambda)\theta/2 \in (0, \theta)$, the only possible equilibrium price strategy, as defined in Proposition 3.2 because from (25) and (27) $\hat{\pi}(\theta, P)|_{\varphi=0} > \pi^*(\theta)|_{\varphi=0}$ for $P > \frac{(2-\lambda)\theta}{2\sqrt{1-\lambda}}$.

Figure 1 illustrates the firm's systematic incentive to deviate when there is no competitive fringe. The solid line in Figure 1 depicts demand when there is no competitive fringe and the uninformed buyers's inference rule is

¹⁴In equilibrium, $\chi_u^*(P^*(\theta)) = \theta$.

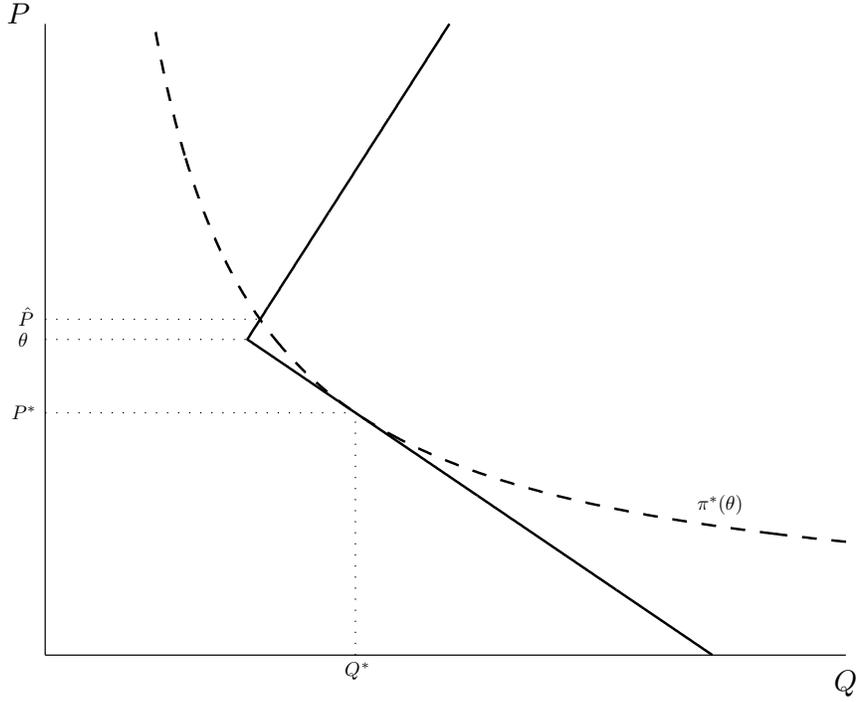


Figure 1: No Competitive Fringe, $\varphi = 0$

consistent with the candidate for equilibrium, i.e.,

$$Q^* = \lambda \max\{\theta - P, 0\} + (1 - \lambda) \max\{\chi_u^*(P) - P, 0\}, \quad (28)$$

where $\chi_u^*(P)|_{\varphi=0} = \frac{2P}{2-\lambda}$. For $P < \theta$, both informed and uninformed buyers purchase from the firm. For $P > \theta$, the informed buyers exit the market and the demand curve becomes upward-sloping because the informed buyers' demand is upward-sloping.¹⁵ The isoprofit curve (the dashed line) in Figure 1 is the locus of pairs $\{Q, P\}$ yielding equilibrium profits $\pi^*(\theta)|_{\varphi=0}$ as defined in (25). Hence, the isoprofit function is $P = \pi^*(\theta)|_{\varphi=0}/Q$. The point $\{Q^*, P^*\} = \left\{\frac{\lambda\theta}{2}, \frac{(2-\lambda)\theta}{2}\right\}$ is the solution in Proposition 3.2 evaluated

¹⁵Indeed, the informational externality due to the learning activity of the uninformed buyers establishes a positive relationship between the price and the quantity demanded by the uninformed buyers, i.e., $\chi_u^*(P) - P > 0$ is increasing in $P > 0$.

at $\varphi = 0$, which yields profits $\pi^*(\theta)|_{\varphi=0}$. Figure 1 shows that the absence of a competitive fringe always provides an incentive for the dominant firm to deviate from $\{Q^*, P^*\}$. Indeed, any prices above \hat{P} yield profits greater than $\pi^*(\theta)|_{\varphi=0}$ to the deviant dominant firm. By charging a higher price, the dominant firm sacrifices revenue from the informed buyers, but is able to deceive the uninformed buyers, making higher profits from them. Therefore, without a competitive fringe (i.e., $\varphi = 0$), the price strategy stated in Proposition 3.2 fails to yield a fully revealing equilibrium.

Having shown that the absence of a competitive fringe yields no equilibrium, we now discuss how large the competitive fringe must be to ensure existence of an equilibrium. In general, if the size of the competitive fringe is large enough, then there exists a fully revealing equilibrium in which the dominant firm's price strategy is defined in Proposition 3.2. Proposition 3.4 implies that for $\varphi \geq \underline{\varphi}^*$ the dominant firm has no incentive to deviate to any price above the reservation price. Note that, in Bagwell and Riordan (1991), when the space of the unknown quality is restricted to two values, a large fraction of informed buyers is sufficient for the price to convey information about the quality of the good. However, when there is a rich array of possible levels of quality, a large fraction of informed buyers does not prevent the dominant firm from pricing above the reservation price.

Proposition 3.4. *There is $\underline{\varphi}^* > 0$ such that for $\varphi \geq \underline{\varphi}^* \in (0, 1)$, a unique fully revealing equilibrium exists.*

Proof. See Appendix B. □

We now illustrate the importance of the part played by the competitive fringe to ensure the existence of an equilibrium. Specifically, we show graphically how the size of the competitive fringe removes the dominant firm's incentive for deviation by altering the slope of the residual demand above the reservation price. We first discuss the case in which the competitive fringe does not sell in equilibrium, i.e., $P^*(\theta) < c_f\theta$. We then consider the case in which the competitive fringe sells the good in equilibrium, i.e., $P^*(\theta) > c_f\theta$.

Suppose that the dominant firm sets the price below the competitive fringe's marginal cost, i.e., $P^*(\theta) < c_f\theta$. When the presence of the competi-

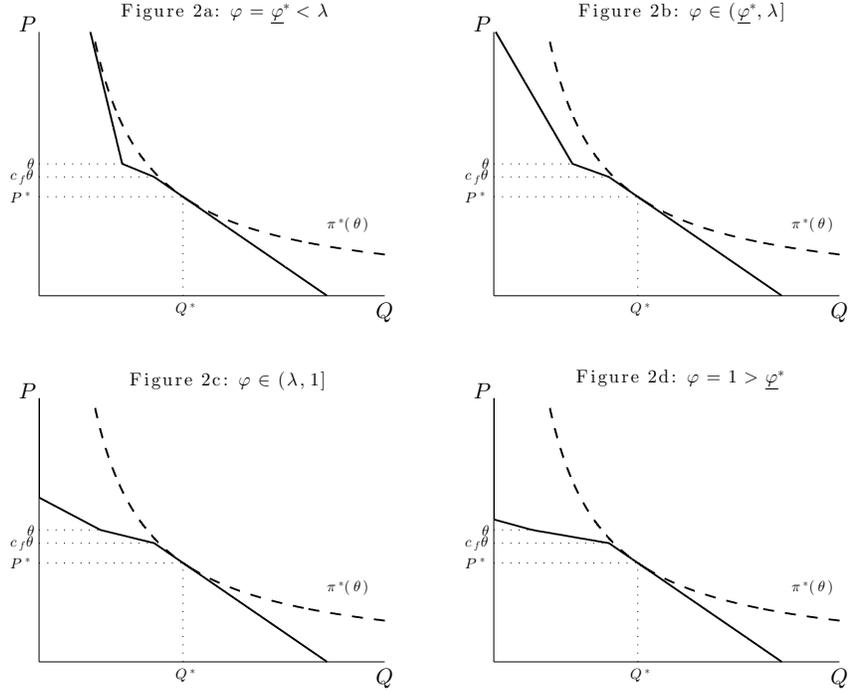


Figure 2: Dominant Firm with Strong Fringe Competition, $\varphi \in [\underline{\varphi}^*, 1]$

tive fringe is strong enough, (i.e., $\varphi \geq \underline{\varphi}^*$), the equilibrium residual demand is downward-sloping enough so as to remove any incentive for the dominant firm to deviate, as shown in Figure 2. Specifically, Figure 2 considers four cases for which the incentive to deviate is blocked. In each case, the solid line is the residual demand faced by the dominant firm evaluated at the inference rule that is consistent with the candidate for equilibrium, i.e.,

$$Q_R^* = \lambda \max\{\theta - P, 0\} + (1 - \lambda) \max\{\chi_u^*(P) - P, 0\} - \varphi \max\{P - c_f \theta, 0\}, \quad (29)$$

where $\chi_u^*(P) = \frac{2P}{2-\lambda}$ is the inverse function of the price strategy $P^*(\theta) = \frac{2-\lambda}{2}\theta$. As in Figure 1, the isoprofit curve represents the locus of pairs $\{Q, P\}$ yielding equilibrium profits $\pi^*(\theta)$. The point $\{Q^*, P^*\}$ is the candidate for equilibrium given in Proposition 3.2.

Graphically, for an equilibrium to exist, demand must never cross the isoprofit curve for prices above θ . All four cases depicted in Figure 2 illustrate the limits that the competitive fringe place on the dominant firm, i.e., the dominant firm cannot take advantage of the uninformed buyers' upward-sloping demand. In other words, the benefit of deceiving the uninformed buyers is reduced and is outweighed by the cost of facing competition on the part of a competitive fringe supplying the good. Specifically, Figure 2a presents the borderline case in which $\varphi = \underline{\varphi}^*$. In this case, the equilibrium residual demand is tangent to the isoprofit at two points. Deviating from the strategy $P^*(\theta) \in (0, \theta)$ to the other tangent point yields no improvement in profit for any θ . Figures 2b,c,d deal with an increasingly larger presence of the competitive fringe. The greater φ , the flatter the slope of the equilibrium residual demand above the reservation price, and, thus, the greater the cost of deviating from $P^*(\theta) \in (0, \theta)$.

Although the presence of a competitive fringe is necessary for blocking the incentive to deviate from $\{Q^*, P^*\}$, it is not sufficient even if the residual demand above the reservation price is downward-sloping. This is shown in Figure 3 where the benefit from deceiving the uninformed buyers is greater than the loss of profit due to the competitive fringe. In other words, there exist some prices $P > \hat{P}$ that provide an incentive for deviation by yielding profits higher than $\pi^*(\theta)$. Therefore, no equilibrium exists with a weak competitive fringe.

A strong enough competitive fringe is also necessary for existence of the equilibrium when the competitive fringe sells the good, i.e., $P^*(\theta) > c_f\theta$. To see this, consider Figures 4 and 5 depicting a situation in which the cost of the competitive fringe is low enough to sell the good. In Figure 4, when $\varphi < \underline{\varphi}^*$ the benefit from deceiving the uninformed buyers is greater than the loss of profit due to the competitive fringe. In other words, there exist some prices $P > \hat{P}$ that provide an incentive for deviation by yielding profits higher than $\pi^*(\theta)$. Thus, no equilibrium exists. Figure 5 depicts the same situation except that the competitive fringe is now large enough, thereby removing any incentive to deviate to $P > \theta$. The equilibrium residual demand never crosses the isoprofit curve and thus the dominant firm's price reveals quality

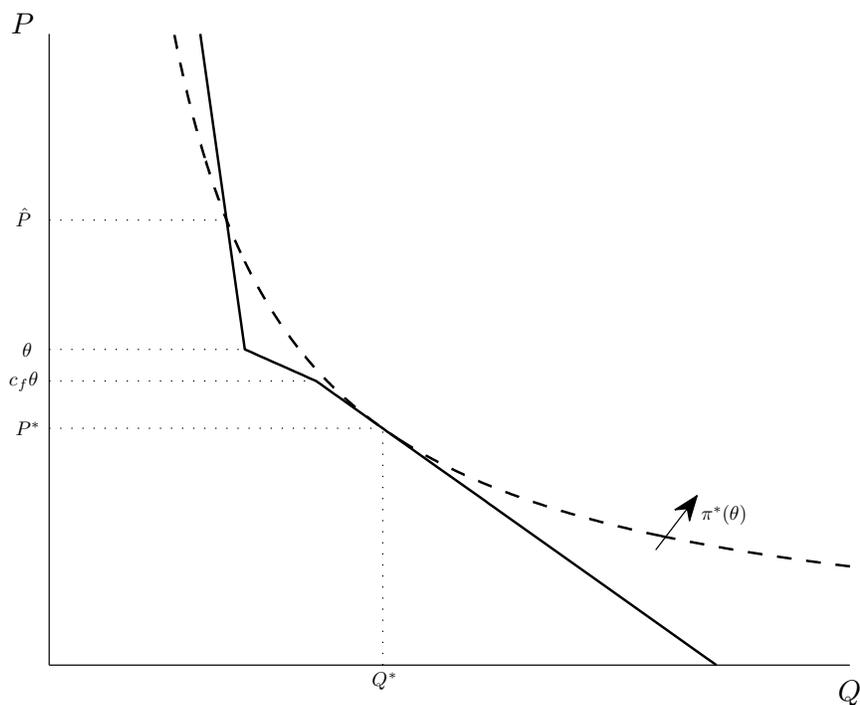


Figure 3: Weak Fringe Competition with Low Cost, i.e., $\varphi < \underline{\varphi}^*$ and $P^*(\theta) < c_f\theta$

and induces the competitive fringe to sell the good, i.e., in this case, an equilibrium exists.

4 The Effect of Informational Externality

In this section, we study the effect of the informational externality on the equilibrium price and quantities. As noted, there is an informational externality because the uninformed buyers's learning activity has an effect on the dominant firm's profit through the updating rule. To study the effect of the informational externality, we proceed in two ways. First, we consider how a change in the composition of buyers affects the equilibrium outcomes. Second, we compare the case of asymmetric information and learning (i.e.,

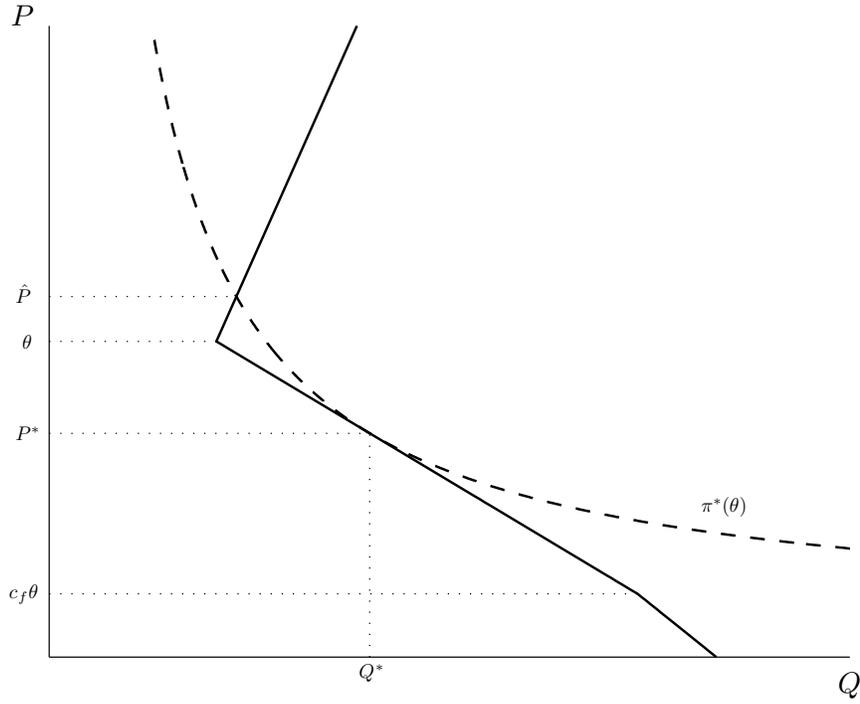


Figure 4: Weak Fringe Competition with Low Cost, i.e., $\varphi < \underline{\varphi}^*$ and $P^*(\theta) > c_f\theta$

$\lambda \in (0, 1)$) with the full-information case in which every buyer is informed (i.e., $\lambda = 1$). These approaches are complementary as the first explains how changes in the composition of buyers alter the fully revealing equilibrium, while the second compares the fully revealing equilibrium with the full-information optimal behavior. Remark 4.1 provides the dominant firm's optimal price strategy when every buyer is informed. The price strategy is derived by evaluating (9), (10), and (11) at $\lambda = 1$.¹⁶

Remark 4.1. *Suppose that every buyer is informed, i.e., $\lambda = 1$. If $c_d = 0$, then for $\theta > 0$,*

¹⁶Another approach is to solve the dominant firm's maximization problem when there are no uninformed buyers, i.e., $\max_{P>0} \{(P - c_d\theta) (\max\{\theta - P, 0\} - \varphi \max\{P - c_f\theta, 0\})\}$.

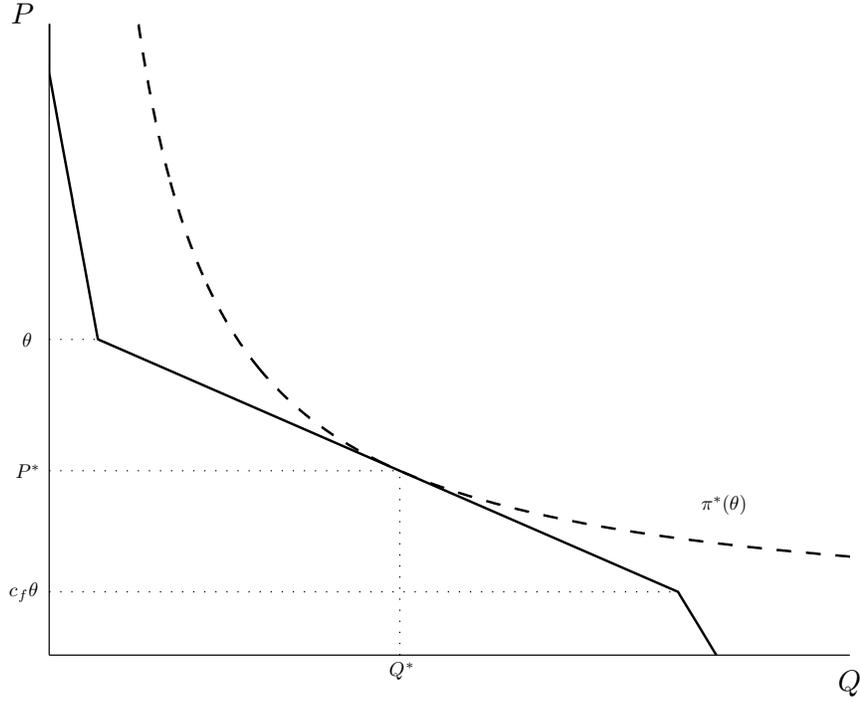


Figure 5: Strong Fringe Competition with Low Cost, i.e., $\varphi > \underline{\varphi}^*$ and $P^*(\theta) > c_f\theta$

$$P^{FI}(\theta) = \begin{cases} \frac{1+\varphi c_f}{2(1+\varphi)}\theta, & c_f \in \left(0, \frac{1}{2+\varphi}\right) \\ c_f\theta, & c_f \in \left[\frac{1}{2+\varphi}, \frac{1}{2}\right] \\ \frac{\theta}{2}, & c_f \in \left(\frac{1}{2}, 1\right] \end{cases} . \quad (30)$$

We first study the effect of the informational externality on the price strategy. Proposition 4.2 states that a decrease in the fraction of informed buyers induces the dominant firm to increase the price, except when the firm sets the price at the kink of the demand. Moreover, the impact of the informational externality is strongest when the competitive fringe does not sell the good. In other words, the active participation of the competitive fringe (through sales) mitigates the increase in the price due to an increase in the fraction of uninformed buyers.

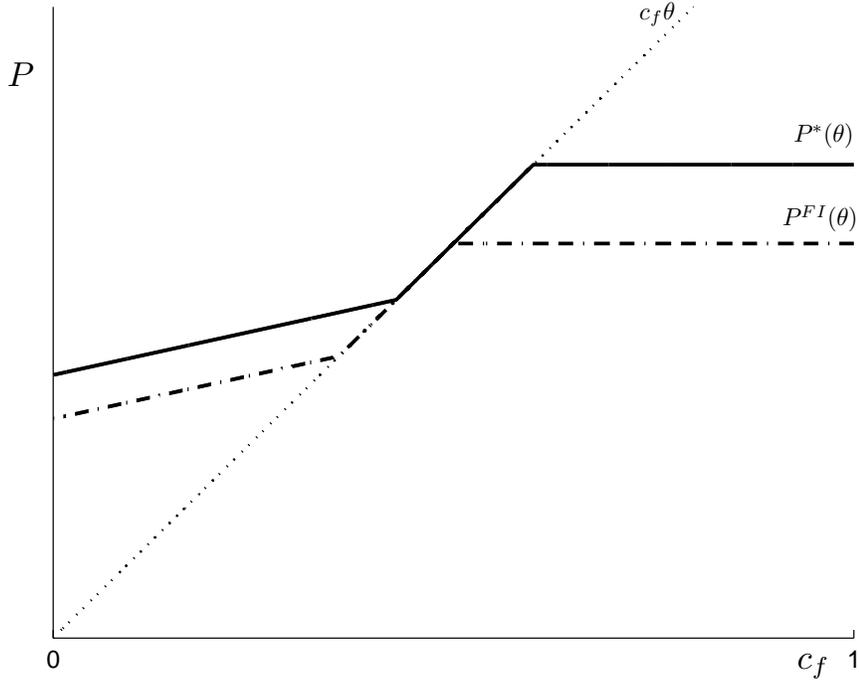


Figure 6: $P^*(\theta)|_{\lambda=0.8}$ vs. $P^{FI}(\theta)$

Proposition 4.2. From (9), (10), and (11),

$$\frac{\partial P^*(\theta)}{\partial \lambda} = \begin{cases} -\frac{\theta}{2(1+\varphi)}, & c_f \in \left(0, \frac{2-\lambda}{2+\varphi}\right) \\ 0, & c_f \in \left[\frac{2-\lambda}{2+\varphi}, \frac{2-\lambda}{2}\right] \\ -\frac{\theta}{2}, & c_f \in \left(\frac{2-\lambda}{2}, 1\right] \end{cases} . \quad (31)$$

Proposition 4.3 follows from, and complements Proposition 4.2 by stating that the full information optimal price strategy is below the fully revealing equilibrium price strategy. Except when the firm sets the price at the kink of demand, the fully revealing price is strictly above the full information price for any composition of demand.

Proposition 4.3. From (9), (10), (11), and (30), $P^*(\theta) \geq P^{FI}(\theta)$.

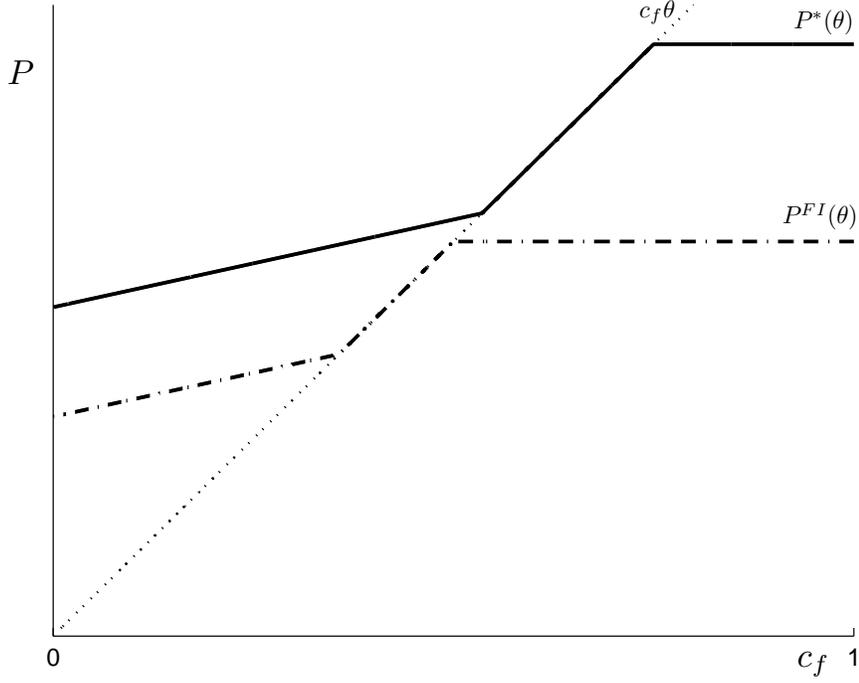


Figure 7: $P^*(\theta)|_{\lambda=0.5}$ vs. $P^{FI}(\theta)$

Figures 6 and 7 illustrate Proposition 4.3. In each figure, the solid line depicts the fully revealing equilibrium price and the dash-dot line refers to the full-information optimal price, both as a function of the competitive fringe's cost parameter c_f . Consistent with Proposition 3.2 and Remark 4.1, there are three cases represented by three different segments for each price strategy. For low values of c_f , the price set by the dominant firm is above the competitive fringe's marginal cost $c_f\theta$ (depicted by the dotted line), and, thus, the competitive fringe sells the good. For mid values of c_f , the price is equal to the competitive fringe's marginal cost. Finally, for high values of c_f , the price is strictly below the competitive fringe's marginal cost and only the dominant firm sells the good.

With a high fraction of informed buyers, the informational externality may have no effect, i.e., $P^*(\theta)|_{\lambda=0.8} = P^{FI}(\theta)$ in Figure 6. However, with

a lower fraction of informed buyers, the informational externality systematically increases the price, i.e., $P^*(\theta)|_{\lambda=0.5} > P^{FI}(\theta)$ in Figure 7. Hence, $P^*(\theta)$ and $P^{FI}(\theta)$ are farther apart with a decrease in the fraction of informed buyers.

The informational externality also has an effect on the quantities through the change in the price. Indeed, an increase in the fraction of uninformed buyers generally increases the price, which reduces the quantity supplied by the dominant firm but increases the quantity supplied by the competitive fringe. In other words, the dominant firm's quantity sold

$$D_R(P^*(\theta), \theta, \theta) = \begin{cases} \frac{\lambda + \varphi c_f}{2} \theta, & c_f \in \left(0, \frac{2-\lambda}{2+\varphi}\right) \\ \theta - c_f \theta, & c_f \in \left[\frac{2-\lambda}{2+\varphi}, \frac{2-\lambda}{2}\right] \\ \frac{\lambda \theta}{2}, & c_f \in \left(\frac{2-\lambda}{2}, 1\right] \end{cases} \quad (32)$$

is increasing in λ , except when the price is set at the kink of demand, while the competitive fringe's quantity sold

$$S_f^*(P^*(\theta), \theta) = \begin{cases} \frac{(2-\lambda-(2+\varphi)c_f)\theta}{2(1+\varphi)}, & c_f \in \left(0, \frac{2-\lambda}{2+\varphi}\right) \\ 0, & c_f \in \left[\frac{2-\lambda}{2+\varphi}, 1\right] \end{cases}. \quad (33)$$

is decreasing in λ . In addition to increasing the quantity supplied, more uninformed buyers (through a decrease in λ) increases the set of values for the cost parameter c_f for which the competitive fringe sells the good. Indeed, from (33), the competitive fringe sells the good if and only if $c_f \in \left(0, \frac{2-\lambda}{2+\varphi}\right)$ whose upper bound is decreasing in λ .

5 Final Remarks

The presence of a competitive fringe is necessary to enable the price-setting dominant firm to signal quality credibly. Specifically, when the competitive fringe is large enough, there exists a unique fully revealing equilibrium, in which the price conveys full information about the quality of the good to uninformed buyers. We also study the effect of asymmetric information and

learning on the equilibrium outcomes. More uninformed buyers increases the price, reduces the quantity sold by the dominant firm, but increases the quantity sold by the competitive fringe. In this paper, we have considered a situation in which there is only one good. It would be interesting to consider a richer model in which the dominant firm and the competitive fringe sell unknown but different levels of quality. While the threat of more sales on the part of the competitive fringe should enable the dominant firm to signal quality, the necessary strength of the competitive fringe in order to block any deviation should depend on the substitutability between the different goods offered by the dominant firm and the competitive fringe.

In order to compare our results with the literature, we have assumed a noiseless environment. Extending the study of the informational role of prices to a noisy environment would lessen the informational requirement of learning buyers about the structure of the market. It would also further our understanding of information flows in a more complex environment.¹⁷ Indeed, a noiseless environment separates two important but distinct effects of the informational externality.¹⁸ In a noiseless environment, the firm reacts to the informational externality, but has limited control over the flow of information. In other words, either the unknown parameter is not revealed and learning buyers revert to their prior beliefs, or it is fully revealed in equilibrium. However, in a noisy environment, the firm is able to affect more significantly the flow of information, i.e., the distribution of the price-signal depends on the firm's decision. In other words, the firm is able to take advantage of the noise by manipulating the beliefs of uninformed buyers.¹⁹

¹⁷Note that noise can also remove the need to specify out-of-equilibrium beliefs.

¹⁸The learning process of the uninformed buyers through the price influences profit, which constitutes an informational externality to the monopolist.

¹⁹This was originally done in Matthews and Mirman (1983) in a limit pricing model. Similarly, Judd and Riordan (1994) studies the informational role of the price set by a monopolist, which provides partial information about the quality of a new product. See also Mirman et al. (2013) for a recent study of noisy signaling in monopoly when the noise is embedded in demand.

A Candidate for Equilibrium

Proposition A.1 provides the unique candidate for a fully revealing equilibrium when $c_d \in [0, 1)$ and $c_f \in (c_d, 1]$. The uninformed buyers' inference rule is the inverse of the price strategy and the competitive fringe's supply strategy is stated in Remark 2.4. Evaluating the price strategy and the bounds at $c_d = 0$ in Proposition A.1 yields Proposition 3.2.

Proposition A.1. *Suppose that a fully revealing equilibrium exists. Then, for $\theta > 0$,*

1. When $c_f \in (c_d, \mathcal{B}^*)$,

$$P^*(\theta) = \mathcal{B}^*\theta > c_f\theta. \quad (34)$$

2. When $c_f \in [\mathcal{B}^*, \mathcal{A}^*]$,

$$P^*(\theta) = c_f\theta. \quad (35)$$

3. When $c_f \in (\mathcal{A}^*, 1]$,

$$P^*(\theta) = \mathcal{A}^*\theta < c_f\theta. \quad (36)$$

Here,

$$\mathcal{A}^* = \frac{2 - \lambda + c_d + \sqrt{(2 - \lambda + c_d)^2 - 8(1 - \lambda)c_d}}{4}, \quad (37)$$

$$\mathcal{B}^* = \frac{2 - \lambda + (1 + \varphi)c_d + \varphi c_f + \sqrt{(2 - \lambda + (1 + \varphi)c_d + \varphi c_f)^2 - 8(1 + \varphi)(1 - \lambda)c_d}}{4(1 + \varphi)}, \quad (38)$$

such that $c_d < \mathcal{B}^* \leq \mathcal{A}^*$.²⁰

Proof. We first provide the set of valid candidates for a fully revealing equilibrium. We then characterize the dominant firm's price strategy corresponding to the unique candidate for equilibrium.

1. **Set of Valid Candidates for Equilibrium.** For $\theta > 0$, $P^*(\theta) \in$

²⁰For $\varphi \in (0, 1]$, $\mathcal{B}^* < \mathcal{A}^*$ while $\mathcal{B}^* = \mathcal{A}^*$ if $\varphi = 0$.

$(c_d\theta, (1 + \varphi c_f)\theta/(1 + \varphi))$, which implies that $\lim_{\theta \rightarrow 0} P^*(\theta) = 0$.²¹ Posterior beliefs are the inverse of the price function. Hence, for $P > 0$, $\chi_u^*(P)$ is increasing in P with $\lim_{P \rightarrow 0} \chi_u^*(P) = 0$ and $\chi_u^*(P) > P$.

2. Characterization of Unique Candidate for Equilibrium. We now characterize the price strategy and the inference rule corresponding to the unique candidate for equilibrium. Two cases must be considered. The first one is the case in which the price strategy induces the competitive fringe not to sell the good, i.e., $P \in (c_d\theta, c_f\theta)$.²² The second one concerns the case in which the price strategy induces the competitive fringe to sell the good, i.e., $P \in (c_f\theta, \theta)$.

- (a) Consider first the case in which the competitive fringe does not sell the good.
- i. Using (4), (7) and (8), the dominant firm's maximization problem is

$$\max_{P \in (c_d\theta, c_f\theta)} (P - c_d\theta)(\lambda\theta + (1 - \lambda)\chi_u^*(P) - P), \quad (39)$$

where $\chi_u^*(P) > P$ for all $P > 0$. The first-order condition corresponding to (39) is

$$\lambda(\theta - P) + (1 - \lambda)(\chi_u^*(P) - P) + (P - c_d\theta) \left((1 - \lambda) \frac{d\chi_u^*(P)}{dP} - 1 \right) = 0. \quad (40)$$

In equilibrium, $P = P^*(\theta)$, $\chi_u^*(P^*(\theta)) = \theta$ and

$$\left. \frac{d\chi_u^*(P)}{dP} \right|_{P=P^*(\theta)} = \left(\frac{dP^*(\theta)}{d\theta} \right)^{-1}. \quad (41)$$

²¹Suppose to the contrary that $P^*(\theta') \notin (c_d\theta', (1 + \varphi c_f)\theta'/(1 + \varphi))$ for some $\theta' > 0$. Then, the dominant firm makes zero profits if either $P^*(\theta') \geq (1 + \varphi c_f)\theta'/(1 + \varphi)$ (since, from (7), the residual demand is zero) or $P^*(\theta') = c_d\theta'$, and makes negative profits if $P^*(\theta') \in (0, c_d\theta')$. Neither of these strategies are tenable because the dominant firm has an incentive to deviate to any price $P \in (c_d\theta', (1 + \varphi c_f)\theta'/(1 + \varphi))$ in order to obtain strictly positive profits from the informed buyers.

²²The case in which the dominant firm prices at the kink of the demand is discussed at the end of the proof.

Let $y \equiv P^*(\theta)$ and $y' \equiv \frac{dP^*(\theta)}{d\theta}$ so that (40) becomes

$$\theta - y + (y - c_d\theta)((1 - \lambda)/y' - 1) = 0, \quad (42)$$

which is a differential equation with the (limiting) initial condition $(y_0, \theta_0) = (0, 0)$. Rearranging (42) yields

$$y' = \frac{(1 - \lambda)(y - c_d\theta)}{2y - (1 + c_d)\theta}. \quad (43)$$

Given that $y > c_d\theta$ and $y' > 0$, it follows from (43) that $y > (1 + c_d)\theta/2 > c_d\theta$.

- ii. Next, we show that $P^*(\theta) = \mathcal{A}^*\theta$, \mathcal{A}^* defined by (37), is a solution to (42). Plugging $y = z\theta$ into (42) yields

$$\theta - z\theta + (z\theta - c_d\theta)((1 - \lambda)/z - 1) = 0. \quad (44)$$

Rearranging (44) yields the quadratic polynomial in z ,

$$2z^2 - (2 - \lambda + c_d)z + (1 - \lambda)c_d = 0. \quad (45)$$

Equation (45) has two positive roots and \mathcal{A}^* defined by (37) is the largest root. First, if $z = (1 + c_d)/2 > c_d$, then the left-hand side of (45) is strictly negative. Hence, the largest root of (45) is the only solution that satisfies $y > (1 + c_d)\theta/2$ and thus $y' > 0$. Second, if $z = 1 - \lambda$, then the left-hand side of (45) is strictly negative. Hence, the largest root is greater than $1 - \lambda$, and, thus, is the only solution that satisfies the second-order condition for the dominant firm's maximization problem.

- iii. We finally show that $y = \mathcal{A}^*\theta$, \mathcal{A}^* defined by (37), is the unique solution. Note that the right-hand side and the derivative of the right-hand side of (43) are both continuous for

$(\theta, y) \in S$, where

$$S = \{(\theta, y) : 2y > (1 + c_d)\theta, y > 0\}. \quad (46)$$

By the Fundamental Theorem of Differential Equation, there exists a unique solution $y = \phi(\theta)$ for any initial condition $(\theta_0, y_0) \in S$. However, our (limiting) initial condition $(0, 0) \notin S$. Therefore, we need to show as well that there is no other $y = \phi(\theta)$ with initial condition $(\theta_0, y_0) \in S \setminus (\theta, \mathcal{A}^*\theta)$ such that $\phi(0) = 0$, which satisfies (42). From (43),

$$\frac{dy'}{dy} = -\frac{(1 - \lambda)(1 - c_d)\theta}{(2y - (1 + c_d)\theta)^2} < 0, \quad (47)$$

for $(\theta, y) \in S$, which implies that any solution $y = \phi(\theta)$ above $y = \mathcal{A}^*\theta$ has a flatter slope and any solution $y = \phi(\theta)$ below $y = \mathcal{A}^*\theta$ has a steeper slope. Hence, no solution $y = \phi(\theta), (\theta, y) \in S \setminus (\theta, \mathcal{A}^*\theta)$ converges toward the origin.

iv. For $\mathcal{A}^* < c_f$, $P^*(\theta) = \mathcal{A}^*\theta$ and $\chi_u^*(P) = P/\mathcal{A}^*$ where \mathcal{A}^* is defined by (37).

(b) Consider next the case in which the fringe firm is active, i.e., $P^*(\theta) \in (c_f\theta, (1 + \varphi c_f)\theta/(1 + \varphi))$.

i. For $P \in (c_f\theta, (1 + \varphi c_d)\theta/(1 + \varphi))$, using (4), (7) and (8), the dominant firm's maximization problem is

$$\max_{P \in (c_f\theta, (1 + \varphi c_f)\theta/(1 + \varphi))} (P - c_d\theta)((\lambda + \varphi c_f)\theta + (1 - \lambda)\chi_u^*(P) - (1 + \varphi)P), \quad (48)$$

where $\chi_u^*(P) > P$ for all $P > 0$. The first-order condition corresponding to (48) is

$$(\lambda + \varphi c_f)\theta + (1 - \lambda)\chi_u^*(P) - (1 + \varphi)P + (P - c_d\theta) \left((1 - \lambda) \frac{d\chi_u^*(P)}{dP} - (1 + \varphi) \right) = 0. \quad (49)$$

In equilibrium, $P = P^*(\theta)$, $\chi_u^*(P^*(\theta)) = \theta$ and $\left. \frac{d\chi_u^*(P)}{dP} \right|_{P=P^*(\theta)} =$

$\left(\frac{dP^*(\theta)}{d\theta}\right)^{-1}$. Let $y \equiv P^*(\theta)$ and $y' \equiv \frac{dP^*(\theta)}{d\theta}$, so that (49) becomes

$$(1 + \varphi c_f)\theta - (1 + \varphi)y + (y - c_d\theta)((1 - \lambda)/y' - (1 + \varphi)) = 0, \quad (50)$$

which is a differential equation with the (limiting) initial condition $(y_0, \theta_0) = (0, 0)$. Rearranging (50) yields

$$y' = \frac{(1 - \lambda)(y - c_d\theta)}{2(1 + \varphi)y - (1 + (1 + \varphi)c_d + \varphi c_f)\theta}. \quad (51)$$

Given that $y > c_d\theta$ and $y' > 0$ it follows from (51) that $y > (1 + (1 + \varphi)c_d + \varphi c_f)\theta / (2(1 + \varphi)) > c_f\theta > 0$.

- ii. Next, we show that $P^*(\theta) = \mathcal{B}^*\theta$, \mathcal{B}^* defined by (38) is a solution to (50). Plugging $y = z\theta$ into (50) yields

$$(1 + \varphi c_f)\theta - (1 + \varphi)z\theta + (z\theta - c_d\theta)((1 - \lambda)/z - (1 + \varphi)) = 0. \quad (52)$$

Rearranging (52) yields the quadratic polynomial in z ,

$$2(1 + \varphi)z^2 - (2 - \lambda + (1 + \varphi)c_d + \varphi c_f)z + (1 - \lambda)c_d = 0. \quad (53)$$

Equation (52) has two positive roots and \mathcal{B}^* defined by (38) is the largest root. If $z = (1 + (1 + \varphi)c_d + \varphi c_f) / (2(1 + \varphi))$, then the left-hand side of (53) is strictly negative. Hence, the largest root of (53) is the only solution that satisfies $y > (1 + (1 + \varphi)c_d + \varphi c_f)\theta / (2(1 + \varphi))$ and thus $y' > 0$. Second, if $z = (1 - \lambda) / (1 + \varphi)$, then the left-hand side of (53) is strictly negative. Hence, the largest root is greater than $(1 - \lambda) / (1 + \varphi)$, and, thus, is the only solution that satisfies the second-order condition for the dominant firm's maximization problem. Finally, if $z = \mathcal{A}^*$, where \mathcal{A}^* is defined by (37), then the left-hand side of (53) is strictly positive. In addition, the derivative of the left-hand side of (53) evaluated at $\mathcal{B}^* = \mathcal{A}^*$

is strictly positive. This implies that $\mathcal{B}^* < \mathcal{A}^*$. Hence, $\mathcal{B}^* \in (\max\{(1 + (1 + \varphi)c_d + \varphi c_f)/(2(1 + \varphi)), (1 - \lambda)/(1 + \varphi)\}, \mathcal{A}^*)$.

- iii. We now show that $y = \mathcal{B}^*\theta$, \mathcal{B}^* defined by (38), is the unique solution. Note that the right-hand side and the derivative of the right-hand side of (51) are both continuous for $(\theta, y) \in S$, where

$$S = \{(\theta, y) : 2(1 + \varphi)y > (1 + (1 + \varphi)c_d + \varphi c_f)\theta, y > 0\}. \quad (54)$$

By the Fundamental Theorem of Differential Equation, there exists a unique solution $y = \phi(\theta)$ for any initial condition $(\theta_0, y_0) \in S$. However, our (limiting) initial condition $(0, 0) \notin S$. Therefore, we need to show as well that there is no $y = \phi(\theta)$ with initial condition $(\theta_0, y_0) \in S \setminus (\theta, \mathcal{B}^*\theta)$ such that $\phi(0) = 0$, which satisfies (50). From (51), for $(\theta, y) \in S$,

$$\frac{dy'}{dy} = -\frac{(1 - \lambda)(1 + \varphi c_f - (1 + \varphi)c_d)\theta}{(2(1 + \varphi)y - (1 + (1 + \varphi)c_d + \varphi c_f)\theta)^2} < 0, \quad (55)$$

with $1 + \varphi c_f - (1 + \varphi)c_d > 0$, which implies that any solution $y = \phi(\theta)$ above $P^*(\theta) = \mathcal{B}^*\theta$ has a flatter slope and any solution $y = \phi(\theta)$ below $P^*(\theta) = \mathcal{B}^*\theta$ has a steeper slope. Hence, no solution $y = \phi(\theta)$, $(\theta, y) \in S \setminus (\theta, \mathcal{B}^*\theta)$ converges toward the origin.

- iv. Hence, for $\mathcal{B}^* > c_f$, $P^*(\theta) = \mathcal{B}^*\theta$ and $\chi_u^*(P) = P/\mathcal{B}^*$ where \mathcal{B}^* is defined by (38).

3. For $c_f \in (c_d, \mathcal{A}^*]$, $P^*(\theta) = \max\{\mathcal{B}^*, c_f\}\theta$ and $\chi_u^*(P) = P/\max\{\mathcal{B}^*, c_f\}$.²³
 For $c_f \in (\mathcal{A}^*, 1]$, $P^*(\theta) = \mathcal{A}^*\theta$ and $\chi_u^*(P) = P/\mathcal{A}^*$. Hence, there is full revelation, i.e., for $\theta > 0$, $\chi_u^*(P^*(\theta)) = \theta$.

□

²³If $c_f \in [\mathcal{B}^*, \mathcal{A}^*]$, then the dominant firm sets the price at the kink, i.e., $P^*(\theta) = c_f\theta$.

B Existence

Proposition B.1 generalizes Proposition 3.3 to the case of $c_d \in [0, 1)$ and $c_f \in (c_d, 1]$.

Proposition B.1. *If there is no competitive fringe (i.e., $\varphi = 0$), then there exists no fully revealing equilibrium.*

Proof. Suppose that $\varphi = 0$. First, from Proposition A.1, the dominant firm's profit corresponding to the unique candidate for equilibrium is $\pi^*(\theta)|_{\varphi=0} = (P^*(\theta) - c_d\theta)(\theta - P^*(\theta))$, $P^*(\theta) = \mathcal{A}^*\theta \in (0, \theta)$,²⁴ or

$$\pi^*(\theta)|_{\varphi=0} = (\mathcal{A}^* - c_d)(1 - \mathcal{A}^*)\theta^2. \quad (56)$$

Second, from Proposition 3.2, the uninformed buyers' inference rule is the inverse of the dominant firm's price strategy, i.e., $\chi_u^*(P)|_{\varphi=0} = P/\mathcal{A}^*$. Using (7), for $P > \theta$, demand evaluated at $\chi_u^*(P)|_{\varphi=0} = P/\mathcal{A}^*$ is

$$D(P, \theta, \chi_u^*(P))|_{\varphi=0} = (1 - \lambda)(1/\mathcal{A}^* - 1)P, \quad (57)$$

which is increasing in P due to the fact that for the uninformed buyers a higher price means higher quality, i.e., $\mathcal{A}^* < 1$. Hence, the dominant firm's profit from deviating to $P > \theta$ is $\hat{\pi}(\theta, P)|_{\varphi=0} = (P - c_d\theta)(1 - \lambda)(\chi_u^*(P) - P)$ or

$$\hat{\pi}(\theta, P)|_{\varphi=0} = (P - c_d\theta)(1 - \lambda)(1/\mathcal{A}^* - 1)P. \quad (58)$$

From (56) and (58), the dominant firm has an incentive to deviate from the price strategy $P^*(\theta) = \mathcal{A}^*\theta$ and $\mathcal{A}^* \in (c_d, 1)$ as defined in Proposition A.1 because there exists $\hat{P} > 0$ such that for $P > \hat{P}$, $\hat{\pi}(\theta, P)|_{\varphi=0} > \pi^*(\theta)|_{\varphi=0}$ or

$$(1 - \lambda)(1/\mathcal{A}^* - 1)P^2 - (1 - \lambda)(1/\mathcal{A}^* - 1)c_d\theta P - (\mathcal{A}^* - c_d)(1 - \mathcal{A}^*)\theta^2 > 0 \quad (59)$$

for $P > \frac{(1-\lambda)(1/\mathcal{A}^*-1)c_d + \sqrt{(1-\lambda)^2(1/\mathcal{A}^*-1)^2c_d^2 + 4(1-\lambda)(1/\mathcal{A}^*-1)(\mathcal{A}^*-c_d)(1-\mathcal{A}^*)}}{2(1-\lambda)(1/\mathcal{A}^*-1)}\theta > 0$, which is true for all $\theta > 0$. \square

²⁴In equilibrium, $\chi_u^*(P^*(\theta)) = \theta$.

Proposition B.2 complements Proposition 3.4 by providing the exact conditions for the strength of the competitive fringe that blocks any deviations and thus proving existence.

Proposition B.2. *For $\theta > 0$, there is a unique fully revealing equilibrium if and only if $\varphi \in [\underline{\varphi}^*, 1]$ such that*

1. When $c_f \in (c_d, \mathcal{B}^*)$, $\underline{\varphi}^*$ is the smallest value of $\varphi \in (0, 1)$ that satisfies

$$\begin{aligned} & ((c_d - c_f)^2 - 4(\mathcal{B}^* - c_d)(c_f - \mathcal{B}^*)) (\mathcal{B}^*)^2 \varphi^2 \\ & + 2((1 - \lambda)c_f c_d + 2((1 - \lambda)c_f - (2 - \lambda)\mathcal{B}^*)(\mathcal{B}^* - c_d) - (1 - \lambda)c_d^2) (1 - \mathcal{B}^*) \mathcal{B}^* \varphi \\ & + (4\mathcal{B}^*(\mathcal{B}^* - c_d) + (1 - \lambda)c_d^2) (1 - \lambda) (1 - \mathcal{B}^*)^2 = 0. \end{aligned} \quad (60)$$

2. When $c_f \in [\mathcal{B}^*, \mathcal{A}^*]$, $\underline{\varphi}^*$ is the smallest value of $\varphi \in (0, 1)$ that satisfies

$$\begin{aligned} & (c_d - c_f)^2 c_f^2 \varphi^2 \\ & + (2(1 - \lambda)(c_f - c_d)c_d - 4c_f(c_f - c_d)) (1 - c_f)c_f \varphi \\ & + (4c_f(c_f - c_d) + (1 - \lambda)c_d^2) (1 - \lambda)(1 - c_f)^2 = 0. \end{aligned} \quad (61)$$

3. When $c_f \in (\mathcal{A}^*, 1]$, $\underline{\varphi}^*$ is the smallest value of φ that satisfies

$$\begin{aligned} & (c_d - c_f)^2 (\mathcal{A}^*)^2 \varphi^2 \\ & + (2(1 - \lambda)(c_f - c_d)c_d - 4\mathcal{A}^*(\mathcal{A}^* - c_d)) (1 - \mathcal{A}^*) \mathcal{A}^* \varphi \\ & + (4\mathcal{A}^*(\mathcal{A}^* - c_d) + (1 - \lambda)c_d^2) (1 - \lambda)(1 - \mathcal{A}^*)^2 = 0. \end{aligned} \quad (62)$$

Here,

$$\mathcal{A}^* = \frac{2 - \lambda + c_d + \sqrt{(2 - \lambda + c_d)^2 - 8(1 - \lambda)c_d}}{4}, \quad (63)$$

and

$$\mathcal{B}^* = \frac{2 - \lambda + (1 + \varphi)c_d + \varphi c_f + \sqrt{(2 - \lambda + (1 + \varphi)c_d + \varphi c_f)^2 - 8(1 + \varphi)(1 - \lambda)c_d}}{4(1 + \varphi)}. \quad (64)$$

Proof. We consider three cases.

1. Suppose that $c_f \in [\mathcal{A}^*, 1]$. Then, from Proposition A.1, the unique candidate for equilibrium is $P^*(\theta) = \mathcal{A}^*\theta < c_f\theta$. We now determine the condition on the strength of the competitive firm so that there is no incentive for the dominant firm to price above θ .²⁵ In other words, we characterize $\underline{\varphi}^*$ which is the minimum value of the strength of the competitive firm such that the dominant firm has no incentive to deviate from $P^*(\theta) = \mathcal{A}^*\theta$ to $P \geq \theta$. Graphically, $\underline{\varphi}^*$ is the level of strength of the fringe firm such that the equilibrium residual demand is tangent to the isoprofit yielding equilibrium profits $\pi^*(\theta) = (\mathcal{A}^* - c_d)(1 - \mathcal{A}^*)\theta^2$ above the reservation price θ . From (12), for $P > \theta$, the equilibrium residual demand is

$$P = \frac{\mathcal{A}^*\varphi c_f\theta}{\mathcal{A}^*\varphi - (1 - \lambda)(1 - \mathcal{A}^*)} - \frac{\mathcal{A}^*Q}{\mathcal{A}^*\varphi - (1 - \lambda)(1 - \mathcal{A}^*)}, \quad (65)$$

while the isoprofit curve is defined by

$$P = c_d\theta + \frac{\pi^*(\theta)}{Q}, \quad (66)$$

where $\pi^*(\theta) = (\mathcal{A}^* - c_d)(1 - \mathcal{A}^*)\theta^2$ is the equilibrium profits. Equating (65) and (66) defines the values of output for which the equilibrium residual demand and the isoprofit intersect, i.e.,

$$\frac{\mathcal{A}^*\varphi c_f\theta Q}{\mathcal{A}^*\varphi - (1 - \lambda)(1 - \mathcal{A}^*)} - \frac{\mathcal{A}^*Q^2}{\mathcal{A}^*\varphi - (1 - \lambda)(1 - \mathcal{A}^*)} = c_d\theta Q + \pi^*(\theta) \quad (67)$$

or

$$\frac{\mathcal{A}^*Q^2}{\mathcal{A}^*\varphi - (1 - \lambda)(1 - \mathcal{A}^*)} + \left(c_d - \frac{\mathcal{A}^*\varphi c_f}{\mathcal{A}^*\varphi - (1 - \lambda)(1 - \mathcal{A}^*)} \right) \theta Q + (\mathcal{A}^* - c_d)(1 - \mathcal{A}^*)\theta^2 = 0. \quad (68)$$

We want to find Q such that it is tangent to the isoprofit curve, i.e.,

²⁵Note that the dominant firm has no incentive to deviate to prices at or below the marginal cost $c_d\theta$ because such deviation yields zero or negative profits, respectively. Moreover, the dominant firm has no incentive to deviate to prices between $c_f\theta$ and θ , which yields lower profits due to a flatter demand curve.

the discriminant is zero, i.e.,

$$\left(c_d - \frac{\mathcal{A}^* \varphi c_f}{\mathcal{A}^* \varphi - (1 - \lambda)(1 - \mathcal{A}^*)}\right)^2 \theta^2 - \frac{4\mathcal{A}^*(\mathcal{A}^* - c_d)(1 - \mathcal{A}^*)\theta^2}{\mathcal{A}^* \varphi - (1 - \lambda)(1 - \mathcal{A}^*)} = 0 \quad (69)$$

or

$$\begin{aligned} & (c_d - c_f)^2 (\mathcal{A}^*)^2 \varphi^2 \\ & + (2(1 - \lambda)(c_f - c_d)c_d - 4\mathcal{A}^*(\mathcal{A}^* - c_d))(1 - \mathcal{A}^*)\mathcal{A}^* \varphi \\ & + (4\mathcal{A}^*(\mathcal{A}^* - c_d) + (1 - \lambda)c_d^2)(1 - \lambda)(1 - \mathcal{A}^*)^2 = 0, \end{aligned} \quad (70)$$

as in (62), where

$$\mathcal{A}^* = \frac{2 - \lambda + c_d + \sqrt{(2 - \lambda + c_d)^2 - 8(1 - \lambda)c_d}}{4}. \quad (71)$$

There are two roots, and $\underline{\varphi}^* \in (0, \lambda)$ is the smallest root of (70). There is thus no incentive for the dominant firm to deviate from $P^*(\theta) = \mathcal{A}^*\theta$ to some price $P > \theta$ as long as $\varphi \geq \underline{\varphi}^*$. The largest root of (70) is $\varphi = \lambda$. It follows that $\underline{\varphi} \in (0, \lambda)$ is the smallest root of (70). There is thus no incentive for the dominant firm to deviate from $P^*(\theta) = \mathcal{A}^*\theta$ to some price $P > \theta$ as long as $\varphi \geq \underline{\varphi}^*$.

2. Suppose that $c_f \in [\mathcal{B}^*, \mathcal{A}^*]$. Then, from Proposition A.1, the unique candidate for equilibrium is $P^*(\theta) = c_f\theta$. The derivation of the threshold is identical to the one for the case of $c_f \in [\mathcal{A}^*, 1]$. Hence, replacing \mathcal{A}^* by c_f in (62) or (70) yields (61).
3. Suppose that $c_f \in (c_d, \mathcal{B}^*)$. Then, from Proposition A.1, the unique candidate for equilibrium is $P^*(\theta) = \mathcal{B}^*\theta > c_f\theta$. The dominant firm has no incentive to deviate from the price strategy $P^*(\theta) = \mathcal{B}^*\theta$ as long as $\varphi \geq \underline{\varphi}^*$, where $\underline{\varphi}^*$ is the value of the strength of the fringe competition such that the equilibrium residual demand for prices above the reservation price θ is tangent to the isoprofit curve yielding equilibrium profits $\pi^*(\theta)$. Graphically, $\underline{\varphi}^*$ is the level of strength of the fringe firm such that the equilibrium residual demand is tangent to the isoprofit

yielding equilibrium profits $\pi^*(\theta) = (\mathcal{B}^* - c_d)(1 + \varphi c_f - (1 + \varphi)\mathcal{B}^*)\theta^2$ above the reservation point. From (12), for $P > \theta$, the equilibrium residual demand is

$$P = \frac{\mathcal{B}^* \varphi c_f \theta}{\mathcal{B}^* \varphi - (1 - \lambda)(1 - \mathcal{B}^*)} - \frac{\mathcal{B}^* Q}{\mathcal{B}^* \varphi - (1 - \lambda)(1 - \mathcal{B}^*)}, \quad (72)$$

while the isoprofit curve is defined by

$$P = c_d \theta + \frac{\pi^*(\theta)}{Q}, \quad (73)$$

where $\pi^*(\theta) = (\mathcal{B}^* - c_d)(1 + \varphi c_f - (1 + \varphi)\mathcal{B}^*)\theta^2$ is the equilibrium profits. Equating (72) and (73) defines the values of output for which the equilibrium residual demand and the isoprofit intersect, i.e.,

$$\frac{\mathcal{B}^* \varphi c_f \theta Q}{\mathcal{B}^* \varphi - (1 - \lambda)(1 - \mathcal{B}^*)} - \frac{\mathcal{B}^* Q^2}{\mathcal{B}^* \varphi - (1 - \lambda)(1 - \mathcal{B}^*)} = c_d \theta Q + \pi^*(\theta), \quad (74)$$

or

$$\begin{aligned} & \frac{\mathcal{B}^* Q^2}{\mathcal{B}^* \varphi - (1 - \lambda)(1 - \mathcal{B}^*)} + \left(c_d - \frac{\mathcal{B}^* \varphi c_f}{\mathcal{B}^* \varphi - (1 - \lambda)(1 - \mathcal{B}^*)} \right) \theta Q \\ & + (\mathcal{B}^* - c_d)(1 + \varphi c_f - (1 + \varphi)\mathcal{B}^*)\theta^2 = 0. \end{aligned} \quad (75)$$

The discriminant must be zero for a tangent point, i.e.,

$$\left(c_d - \frac{\mathcal{B}^* \varphi c_f}{\mathcal{B}^* \varphi - (1 - \lambda)(1 - \mathcal{B}^*)} \right)^2 = \frac{4\mathcal{B}^*(\mathcal{B}^* - c_d)(1 + \varphi c_f - (1 + \varphi)\mathcal{B}^*)}{\mathcal{B}^* \varphi - (1 - \lambda)(1 - \mathcal{B}^*)} \quad (76)$$

or

$$\begin{aligned} & ((c_d - c_f)^2 - 4(\mathcal{B}^* - c_d)(c_f - \mathcal{B}^*))(\mathcal{B}^*)^2 \varphi^2 \\ & + 2((1 - \lambda)c_f c_d + 2((1 - \lambda)c_f - (2 - \lambda)\mathcal{B}^*)(\mathcal{B}^* - c_d) - (1 - \lambda)c_d^2)(1 - \mathcal{B}^*)\mathcal{B}^* \varphi \\ & + (4\mathcal{B}^*(\mathcal{B}^* - c_d) + (1 - \lambda)c_d^2)(1 - \lambda)(1 - \mathcal{B}^*)^2 = 0, \end{aligned} \quad (77)$$

as in (60). Hence, $\underline{\varphi}^*$ is defined as the smallest value of φ such that (77) holds and $\mathcal{B}^* > c_f$.

□

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