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Monopoly Signaling: Non-Existence and Existence*

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Abstract

We study the signaling role of prices in monopoly. To that end, we consider a monopolist supplying a good whose quality is unknown to some buyers. When the good is potentially valueless (i.e., the lowest quality generates no demand), and demand is composed of both informed and uninformed buyers, there does not exist an equilibrium in which the price signals quality to the uninformed buyers (hereafter, a *signaling* equilibrium). By extending the monopoly model to the framework of a dominant firm with a competitive fringe, we show that the threat of competition can allow the monopolist to credibly signal quality. Without competition, the monopolist selling the valueless good always has an incentive to deceive the uninformed buyers by charging a higher price and mimicking a monopolist of higher quality, thereby preventing the price from conveying information. With competition (e.g., a dominant firm with a competitive fringe), deceiving the uninformed buyers becomes costly. Indeed, while charging a higher price yields more profit from the deceived uninformed buyers, it also triggers the entry of the competitive fringe, which reduces demand, and, thus, profits. When the fringe competition is large enough, the cost of facing competition outweighs the benefit of deceiving the uninformed buyers, which reestablishes informational content in the price.

Keywords: Asymmetric information, Monopoly, Dominant Firm with Fringe Competition, Learning, Quality, Signaling.
JEL Classifications: D21, D42, D82, D83, D84, L12, L15.

1 Introduction

In a world of asymmetric information, prices play an informative role. Indeed, traders rely on prices in order to obtain information about firms as much as tourists extract information from the prices to learn about the quality of the food in restaurants. The informative role of prices is important because it reduces the informational asymmetry among individuals. Several studies have already provided conditions under which privately-held information by firms becomes public through prices, beginning with perfectly competitive markets (Kihlstrom and Mirman, 1975; Grossman, 1976, 1978; Grossman and Stiglitz, 1980) and continuing with imperfectly competitive markets (Bagwell and Riordan, 1991; Judd and Riordan, 1994; Daughety and Reinganum, 1995, 2005, 2007, 2008a,b; Janssen and Roy, 2010). In the case of imperfectly competitive markets, the conveyance of information can be hindered by the firms' influence on prices. Specifically, if prices have some informational content, then a firm selling a low quality good may have an incentive to deceive uninformed buyers by charging a higher price, thus, mimicking a profitable firm. When such an incentive exists, the price may not be used as a signal and asymmetric information remains. The absence of any informational content in the price is particularly harmful to buyers when the good is potentially valueless, i.e., the good of lowest quality generates zero demand on the part of informed buyers. Indeed, if a valueless good is misinterpreted as valuable, then negative surplus is generated for the uninformed buyers.

It is the purpose of this paper to study the informational role of prices in imperfectly competitive markets when the good is potentially valueless. To that end, we embed signaling in the monopoly model in which the quality is known to the firm, but unknown to uninformed buyers who use the price as a signal. For comparability with the existing literature, we retain the linear demand in which the quality is related to the reservation or choke price (Bagwell and Riordan, 1991; Daughety and Reinganum, 1995, 2005, 2008a). Moreover, as in Bagwell and Riordan (1991), demand is assumed to be composed of informed and uninformed buyers. Unlike these previous studies, we assume that the unknown quality (i.e., the reservation price) is a continuum equal to the whole positive real line.[1]

We first show that the price set by a monopolist cannot play an informative role when the good is potentially valueless and demand is composed of both informed and uninformed buyers. The absence of an informational role of prices is due to the very nature of a pure monopoly, i.e., the firm has full control of the market. We then show that, when competition by a group price-taking firms is allowed, prices can play an informative role in equilibrium.

We now provide a more detailed discussion of our non-existence and existence results. When demand is composed of both informed and uninformed buyers, we show that there does not exist an equilibrium in which the price signals quality to the uninformed buyers (hereafter, a *signaling* equilibrium).[2] If there existed a signaling equilibrium with a potentially valueless good, the following two statements would be true. First, because the good is potentially valueless, the zero-quality monopolist would make zero profits. Second, due to the presence of informed buyers, a monopolist selling a positive quality would obtain strictly positive profits from the uninformed buyers. However, these two statements are incompatible in equilibrium. Indeed, there is always an incentive for the monopolist selling a valueless good to deceive the uninformed buyers, i.e., to deviate to the price charged by a positive-quality monopolist. Hence, the price cannot convey information in equilibrium.[3]

It should be noted that Bagwell and Riordan (1991) also shows that there is no separating equilibrium with a potentially valueless good. However, the reason for non-existence of a separating equilibrium in Bagwell and Riordan

[1] The zero lower bound allows for the possibility of a valueless good. The absence of an upper bound is merely for simplicity. Specifying an upper bound for the unknown quality (i.e., the reservation price) makes no difference in the analysis as we can also restrict possible prices between zero and the same upper bound.

[2] The terms *signaling* and *separating* are synonymous.

[3] We focus on the case of a demand with both informed and uninformed buyers. When all buyers are uninformed, there exists a signaling equilibrium in monopoly. However, when the good is potentially valueless, there is no trading in equilibrium. In other words, while the price is increasing in quality (and, thus, conveys information), the price is equal to the reservation price, which yields zero demand for any level of quality. Note that trading is possible when the good is not potentially valueless, i.e., the lowest quality does generate positive demand on the part of informed buyers. See Daughety and Reinganum (1995, 2005, 2008a).

(1991) is due to out-of-equilibrium beliefs.[4] Specifically, the only valid candidate for a separating equilibrium satisfying the intuitive criterion provides the high quality monopolist an incentive to deviate, i.e., to monopolize informed consumers by charging the full-information monopoly price (not on the equilibrium path), and, thus, lose the uninformed buyers. In our paper, we show that, without the problem presented by out-of-equilibrium beliefs, the signaling equilibrium nevertheless fails to exist. Here, the failure of informational conveyance through the price is not due to the incentive of a high quality firm to sacrifice uninformed buyers as in Bagwell and Riordan (1991). It is rather the opposite. It is in fact the low quality monopolist who has an incentive to deviate from the candidate for a separating equilibrium by mimicking the price set by a high quality monopolist.

We then show that existence of a signaling equilibrium can be reestablished when the firm is no longer a monopolist, but a dominant firm facing fringe competition. In other words, when the dominant firm does not have full control of its market (i.e., a monopoly), the reason for non-existence is circumvented, i.e., a low-quality firm has no longer an incentive to deviate and mimic the price set by a high-quality monopolist.

Specifically, we extend the monopoly model to a model of a dominant firm with a competitive fringe.[5] The dominant firm sets the price but faces a residual demand since the competitive fringe may also supply the good. The model embeds monopoly, i.e., a dominant firm without fringe competition. We show that if the competitive fringe is large enough, then the incentive for a low-quality dominant firm to deviate by charging a higher price, and, thus, mimicking a higher-quality dominant firm is blocked. In other words, the cost of facing competition outweighs the benefit of deceiving the uninformed buyers. Hence, with a competitive fringe, the price signals quality to the

[4] In Bagwell and Riordan (1991), the monopolist can either be of low or high quality. Since the space of the unknown quality is restricted to be two values, only two prices can be observed in a separating equilibrium, and, thus, out-of-equilibrium beliefs must be specified for prices that are not possible outcomes in equilibrium.

[5] We consider the standard framework of a dominant firm facing a competitive fringe with a cost disadvantage. The fringe firm knows the quality of the good. In our paper, the quality is the same across firms. Section 4 discusses the case of the dominant firm and the competitive fringe selling different and unknown qualities.

uninformed buyers.

Note that the incentive to deceive the uninformed buyers is due to the very nature of a monopoly. Indeed, the monopoly model precludes any sort of competition through entry no matter how high the price. Hence, in the absence of a competitive fringe, the monopolist of lower quality has an incentive to mimic a higher quality monopolist by charging a higher price, thereby sacrificing profit from the informed buyers, but yielding more profits since the uninformed buyers misinterpret the true quality. A large enough presence of a competitive fringe removes this incentive to deviate. Indeed, while charging a higher price does yield more profit from the deceived uninformed buyers, it also triggers the entry of the competitive fringe, thereby reducing demand, and, thus, profits of the dominant firm.

The paper is organized as follows. Section 2 considers signaling in monopoly. Section 3 applies the signaling analysis to the dominant firm-competitive fringe framework. Section 4 concludes and suggests avenues of research regarding the informational role of prices in different environments.

2 Monopoly Signaling

In this section, we embed signaling in the monopoly model. We first present the model and define the signaling equilibrium in which the price signals quality to uninformed buyers. We then present two results regarding the existence of a signaling equilibrium. First, when demand is composed of both informed and uninformed buyers, we show that there does not exist a signaling equilibrium. Second, when all buyers are uninformed, we show that there does exist a signaling equilibrium. However, it is an equilibrium with no trading.

The purpose of this section is two-fold. First, we provide an economic reason for the non-existence of a signaling equilibrium in pure monopoly. Second, the non-existence result along with its explanation serves as a motivation for the introduction of competition as a way to reestablish informational content in price.

2.1 Set Up

Consider a market for a good of quality $\theta \geq 0$ sold at price $P \geq 0$. Formally, $\theta \in \Theta = \mathbb{R}_+$.[6] The demand side is composed of informed and uninformed price-taking buyers. Informed buyers know θ and have demand $d^I(P,\theta)$, $d^I_1 < 0, d^I_2 > 0$. Uninformed buyers do not know θ, but infer it from observing the price. Specifically, upon observing P, the uninformed buyers' posterior beliefs about quality is $\chi(P)$, so that their demand is $d^U(P,\chi(P))$, $d^U_1 < 0, d^U_2 > 0$. Normalizing the mass of buyers to one and letting $\lambda \in [0,1)$ be the fraction of informed buyers, the market demand is

$$D(P,\theta,\chi(P)) = \lambda d^I(P,\theta) + (1-\lambda)d^U(P,\chi(P)). \tag{1}$$

The good is potentially valueless because $d^I(0,P) = 0$ for $P \geq 0$. Indeed, if $\theta = 0$, then no informed buyer wants to consume the good. Moreover, if uninformed buyers misinterpret a valueless good to be valuable, then negative surplus is generated. In order to compare our results with those already established in the literature (for the case in which the good is not potentially valueless, i.e., the lowest quality does generate positive demand regardless of buyers' beliefs), we retain the linear demand in which the quality is related to the demand intercept, i.e., the reservation or choke price.[7] See Bagwell and Riordan (1991), and Daughety and Reinganum (1995, 2005, 2007, 2008a,b).[8] Assumption 2.1 holds for the remainder of the paper.

Assumption 2.1. *For $\theta \geq 0$, $d^I(P,\theta) = \max\{\theta - P, 0\}$ and $d^U(P,\chi(P)) =$*

[6]Note that the zero lower bound along with the absence of an upper bound on θ removes the need to specify out-of-equilibrium beliefs. Indeed, we show that, when the signaling equilibrium exists, every price $P \geq 0$ is a possible outcome in equilibrium. See Section 3.

[7]See Daughety and Reinganum (2008a) for a detailed discussion regarding the use of a linear demand in signaling games. The linear demand can be generated from a quadratic utility function or by aggregating unit demand functions of consumers with heterogeneous reservation prices.

[8]In Bagwell and Riordan (1991), quality can either be low or high. The demand for the high quality is linear, while the low quality product has a unit demand. In Daughety and Reinganum (2008a), the demand is $Q^D = (\alpha - (1-\delta)\theta)/\beta - P/\beta$, where $\alpha, \beta, \delta > 0$ are known parameters and $\theta \in [\underline{\theta}, \overline{\theta}]$, $0 < \underline{\theta} < \overline{\theta}$, is the unknown parameter for which the price transmits information. As in our case, the demand intercept depends on the unknown parameter.

$\max\{\chi(P) - P, 0\}$ *so that*

$$D(P, \theta, \chi(P)) = \lambda \max\{\theta - P, 0\} + (1 - \lambda) \max\{\chi(P) - P, 0\}. \tag{2}$$

The supply side is composed of a monopolist who knows the quality θ and faces marginal cost $c_M\theta$, $c_M \in [0, 1)$. The objective of the monopolist is to choose P so as to maximize profit

$$\pi = (P - c_M\theta)D(P, \theta, \chi(P)). \tag{3}$$

Having presented the model, we now define the signaling equilibrium which consists of the price function $P^S(\theta)$ and the inference rule $\chi^S(P)$. The superscript S refers to *signaling*.

Definition 2.2. *The pair* $\{P^S(\theta), \chi^S(P)\}$ *is a signaling equilibrium if, for all* $\theta \geq 0$*,*

1. *Given* $\chi^S(P)$*,*

$$P^S(\theta) = \arg\max_{P \geq 0}(P - c_M\theta)D(P, \theta, \chi^S(P)). \tag{4}$$

2. *Given* $P^S(\theta)$*,*

$$\chi^S(P^S(\theta)) = \theta. \tag{5}$$

2.2 Existence and Trading

We now present two results regarding the existence of a signaling equilibrium for a monopoly selling a potentially valueless good. First, if some buyers are informed, then there does not exist a signaling equilibrium. Second, if all buyers are uninformed, then there exist a signaling equilibrium in which, while the price signals quality, the monopolist does not sell the good.

We begin with the nonexistence result stated in Proposition 2.3. If there existed a signaling equilibrium with a potentially valueless good, the following two statements would be true. First, because the good is potentially valueless, the zero-quality monopolist would make zero profits. Second, due to the presence of informed buyers, a monopolist selling a positive quality would obtain strictly positive profits from the uninformed buyers. However, these two statements are incompatible in equilibrium. Indeed, there is always an incentive for the monopolist selling a valueless good to deceive the uninformed buyers, i.e., to deviate to the price charged by a positive-quality monopolist. Hence, the price cannot convey information in equilibrium.

Proposition 2.3. *Suppose that Assumption 2.1 holds. If $\lambda \in (0,1)$, then there exists no signaling equilibrium.*

Proof. Suppose to the contrary that there exists a signaling equilibrium. In a signaling equilibrium, the zero-quality monopolist makes zero profits. Moreover, for $\theta > 0$, $P^S(\theta) \in (c_M\theta, \theta)$.[9] Hence, for $\theta > 0$,

$$(1-\lambda)(P^S(\theta) - c_M\theta)\max\{\chi^S(P^S(\theta)) - P^S(\theta), 0\} > 0, \tag{6}$$

where $\lambda \in (0,1)$ and $\chi^S(P^S(\theta)) = \theta$. Condition 6 implies that the zero-quality monopolist (generating zero profits) has an incentive to deviate to the price charged by a monopolist of positive quality in order to obtain strictly positive profit from the uninformed buyers instead of zero profit. Therefore, no signaling equilibrium exists. □

Propositions 2.4 and 2.5 show that removing informed buyers ensures the existence of the signaling equilibrium, but one that precludes trading. First, Proposition 2.4 states that there does not exist a signaling equilibrium with trading. The fact that the good is potentially valueless is again at the core of the nonexistence result. Indeed, if equilibrium demand is strictly positive for a positive quality, then the zero-quality monopolist has an incentive to deviate to the price charged by the monopolist of such positive quality. Such deviation yields strictly positive demand and, thus, strictly positive profits.

[9] Suppose rather that $P^S(\theta') \notin (c_M\theta', \theta')$ for some $\theta' > 0$. Then, the monopolist makes zero profits if either $P^S(\theta') \geq \theta'$ or $P^S(\theta') = c_M\theta'$, and negative profits if $P^S(\theta') < c_M\theta'$. Hence, the monopolist has an incentive to deviate to any price $P \in (c_M\theta', \theta')$ in order to obtain strictly positive profits from the informed buyers.

Proposition 2.4. *Suppose that Assumptions 2.1 holds. If $\lambda = 0$, then there is no signaling equilibrium in which, for $\theta > 0$,*

$$D(P^S(\theta), \theta, \chi^S(P^S(\theta))) > 0. \tag{7}$$

Proof. Suppose to the contrary that there is a signaling equilibrium in which trading occurs, i.e., $D(P^S(\theta), \theta, \chi^S(P^S(\theta))) > 0$ for $\theta > 0$. Since $\lambda = 0$, it follows that $D(P^S(\theta), \theta, \chi^S(P^S(\theta))) = \chi^S(P^S(\theta)) - P^S(\theta) = \theta - P^S(\theta) > 0$ so that $P^S(\theta) \in (0, \theta)$ for $\theta > 0$. Then, the zero-quality monopolist (generating zero profits) has an incentive to deviate to the price charged by the dominant firm of quality $\theta > 0$ in order to obtain strictly positive profits from the uninformed buyers. Therefore, there is no signaling equilibrium in which (7) holds. □

Next, Proposition 2.5 characterizes a continuum of signaling equilibrium in which there is no trading.

Proposition 2.5. *Suppose that Assumption 2.1 holds. If $\lambda = 0$, then there exists a continuum of signaling equilibrium points such that, for $\theta \geq 0$, $P^S(\theta) = \alpha\theta$ and $\chi^S(P) = P/\alpha$, $\alpha \geq 1$.*

Proof. Plugging $\chi^S(P) = P/\alpha$ into (4) yields the maximization problem

$$\max_{P \geq 0} (P - c_M\theta) \max\{P/\alpha - P, 0\}. \tag{8}$$

Since $\alpha \geq 1$ implies zero demand, $P^S(\theta) = \alpha\theta$, $\alpha \geq 1$, is a solution such that the posterior beliefs $\chi^S(P) = P/\alpha$ are consistent with Bayes' rule and the strategy of the monopolist, as in any signaling game. Indeed, given $P^S(\theta) = \alpha\theta$, $\chi^S(P^S(\theta)) = P^S(\theta)/\alpha = \theta$ for $\theta \geq 0$. That is, the pair $\{P^S(\theta), \chi^S(P)\} = \{\alpha\theta, P/\alpha\}$, $\alpha \geq 1$ is a signaling equilibrium. □

Note that the absence of informed buyers no longer restricts the set of valid candidates for a signaling equilibrium to be such that a positive-quality monopolist obtains strictly positive profits in equilibrium, i.e., the price strategy can be equal to or above the reservation price. Indeed, because demand is always zero in equilibrium, i.e., $D(P^S(\theta), \theta, \chi^S(P^S(\theta))) =$

$\max\{P^S(\theta)/\alpha - P^S(\theta), 0\} = 0$ for $\theta \geq 0$, it follows that equilibrium profit is always zero, regardless of quality.

3 Signaling of Dominant Firm with Competitive Fringe

Having presented the limitation of the signaling role of the price set by a monopolist, we now study the effect of competition (or the threat of entry) on information flows. To that end, we consider signaling when the firm is no longer a *monopolist*, but rather a *dominant firm* facing a fringe competition. The model embeds monopoly, i.e., a dominant firm without fringe competition.

We focus on the case in which demand is composed of both informed and uninformed buyers. For the case in which some buyers are informed, we have shown that there does not exist a signaling equilibrium in a monopoly. We now show that the addition of a competitive fringe may reestablish the existence of the equilibrium. Moreover, it is an equilibrium with trading.[10] Specifically, if the competitive fringe is large enough, then the incentive for the dominant firm to deviate by charging a higher price, and, thus, mimicking a higher quality is blocked. Hence, the price signals quality to the uninformed buyers.

Before proceeding with the formal analysis, we provide an informal discussion of the importance of a competitive fringe for the existence of a signaling equilibrium. In the absence of a competitive fringe, the monopolist of lower quality has an incentive to mimic a higher quality monopolist by charging a higher price, thereby sacrificing profit from the informed buyers, but yielding more profits since the uninformed buyers misinterpret the true quality. A large enough presence of a competitive fringe removes this incentive to deviate. Indeed, while charging a higher price does yield more profit from

[10] The case of no informed buyers (i.e., all buyers are uninformed) is relegated to Appendix A because the addition of a competitive fringe has no effect on the result stated in Proposition 2.4. Indeed, when there are no informed buyers, there exists no signaling equilibrium in which both the dominant firm and the fringe firm sell the good.

the deceived uninformed buyers, it also triggers the entry of the competitive fringe, thereby reducing demand, and, thus, profits of the dominant firm. In other words, if the fringe competition is large enough, then the cost of facing competition outweighs the benefit of deceiving the uninformed buyers.

Section 3 has three parts. We begin by extending the monopoly model to a model of a dominant firm facing a competitive fringe. We then turn to the issue of the existence of a signaling equilibrium. Unlike the monopoly, signaling in a dominant firm-competitive fringe framework is complicated by the fact that the dominant firm's price decision may induce entry of the fringe firm. In order to simplify the analysis, we first study a special case in which the fringe firm's marginal cost is equal to the reservation price. If the fringe competition is large enough, then there exists a signaling equilibrium with trading in which the dominant firm charges a price below the reservation price. Hence, in this special case, the fringe firm does not supply anything in equilibrium. Second, we consider the general case without restricting the fringe firm's marginal cost. We show that the existence of a signaling equilibrium with trading holds as long as the competitive fringe is large enough. Because the fringe firm's marginal cost is below the reservation price, it is possible that the dominant firm sets a price between the fringe firm's marginal cost and the reservation price, which induces entry of the fringe firm in equilibrium.

3.1 Set Up

The model of a dominant firm with a competitive fringe retains the demand and cost structure of the monopoly model. Both the dominant firm and the competitive fringe know the quality θ.[11] Given the price P and the marginal cost $c_F\theta$, $c_F \in [0, 1]$, the competitive fringe supplies $S(P, c_F\theta)$, $S_1 > 0, S_2 < 0$. The objective of the dominant firm is to choose P so as to maximize profit

$$\pi = (P - c_M\theta) \max\{D(P, \theta, \chi(P)) - \varphi S(P, c_F\theta), 0\}, \tag{9}$$

[11] The case in which the dominant firm and the competitive fringe sell unknown but different levels of quality is discussed in Section 4.

where $\max\{D(P, \theta, \chi(P)) - \varphi S(P, c_F\theta), 0\}$ is the residual demand. The parameter $\varphi \in [0, 1]$ measures the strength of the fringe firm.

The presence of fringe competition leads to a slight modification of the definition of a signaling equilibrium. Indeed, Definition 3.1 is analogous to Definition 2.2 except for the fact that the demand is replaced by the residual demand.

Definition 3.1. *The pair $\{P^S(\theta), \chi^S(P)\}$ is a signaling equilibrium if, for all $\theta \geq 0$,*

1. *Given $\chi^S(P)$,*

$$P^S(\theta) = \arg\max_{P \geq 0}(P - c_M\theta)\max\{D(P, \theta, \chi^S(P)) - \varphi S(P, c_F\theta), 0\}. \quad (10)$$

2. *Given $P^S(\theta)$,*

$$\chi^S(P^S(\theta)) = \theta. \quad (11)$$

In order to facilitate the analysis, we consider a linear fringe supply function, implicitly assuming that the maximization of the competitive fringe is $\max_{x \geq 0} Px - c_F\theta x - x^2/2$. Moreover, the dominant firm is assumed to have a cost advantage over the competitive fringe. Assumption 3.2 holds for the remainder of the paper.

Assumption 3.2. *For $\theta \geq 0$, the fringe supply is*

$$S(P, c_F\theta) = \max\{P - c_F\theta, 0\}, \quad (12)$$

$c_F \in [c_M, 1]$.

3.2 A Special Case

We begin with the special case in which the fringe firm's marginal cost is equal to the reservation price. This allows us to abstract from the issue of entry on the part of the fringe firm, and show clearly that the mere presence of the competitive fringe (or the threat of entry) can reestablish the existence of the signaling equilibrium with trading. Assumption 3.3 holds throughout Section 3.2.

Assumption 3.3. $c_F = 1$.

Proposition 3.4 establishes the necessary and sufficient condition for the strength of the competitive fringe (i.e., the lower bound on φ) for the existence of a signaling equilibrium with trading. Consistent with Proposition 2.3, there exists no signaling equilibrium with some informed buyers when $\varphi = 0$. Moreover, if the strength of the competitive fringe is equal to the fraction of informed buyers, (i.e., $\varphi = \lambda$), then the competitive fringe is large enough to ensure the existence of the signaling equilibrium.

Proposition 3.4. *Suppose that Assumptions 2.1, 3.2 and 3.3 hold. Then, there exists* $\underline{\varphi} \in (0, \lambda)$ *such that, for* $\lambda \in (0, 1)$*, there is a signaling equilibrium with trading if and only if* $\varphi \in [\underline{\varphi}, 1]$.

Next, Proposition 3.5 provides the equilibrium price and inference rule. In equilibrium, the dominant firm prices below the reservation price, and, thus, the fringe firm does not supply any of the good.

Proposition 3.5. *Suppose that Assumptions 2.1, 3.2 and 3.3 hold. For* $\lambda \in (0, 1)$ *and* $\varphi \in [\underline{\varphi}, 1]$, $\underline{\varphi} \in (0, \lambda)$, $P^S(\theta) = A\theta$, $\theta \geq 0$, *and* $\chi^S(P) = P/A$, *where*

$$A \equiv \frac{2 - \lambda + c_M + \sqrt{(2 - \lambda + c_M)^2 - 8(1 - \lambda)c_M}}{4}, \tag{13}$$

$A \in (\max\{(1 + c_M)/2, 1 - \lambda\}, 1)$.

The proof of Propositions 3.4 and 3.5 is relegated to Appendix B. Moreover, the exact value of $\underline{\varphi} \in (0, \lambda)$ is provided in the proof. Note that, when the monopolist faces no cost, the equilibrium price function, inference rule, and lower bound on the strength of the competitive fringe are easily characterized.

Corollary 3.6. *If* $c_M = 0$*, then, for* $\lambda \in (0, 1)$ *and* $\varphi \in [\lambda - \lambda^2, 1]$, $P^S(\theta) = (2 - \lambda)\theta/2$, $\theta \geq 0$, *and* $\chi^S(P) = 2P/(2 - \lambda)$.

Having characterized the signaling equilibrium, we now discuss the need for a competitive fringe. Specifically, we show that there is an incentive for a dominant firm of quality θ to deviate from the strategy $P^S(\theta) = A\theta$ when the competitive fringe is not large enough, i.e., $\varphi < \underline{\varphi}$. This is best seen graphically, as the condition on φ influences the slope of the equilibrium residual demand, which determines whether or not there is an incentive for the dominant firm to deviate from the strategy $P^S(\theta) = A\theta$.

In each of the following three figures, $c_F = 1$ and the equilibrium residual demand

$$Q = \lambda \max\{\theta - P, 0\} + (1-\lambda)\max\{\chi^S(P) - P, 0\} - \varphi \max\{P - c_F\theta, 0\}, \quad (14)$$

is depicted for different values of $\varphi \in [0,1]$.[12] Plugging $\chi^S(P) = P/A$ and $c_F = 1$ into (14) yields

$$Q = \begin{cases} \lambda\theta - \left(\lambda - (1-\lambda)\frac{1-A}{A}\right)P, & 0 \le P < \theta \\ \varphi\theta - \left(\varphi - (1-\lambda)\frac{1-A}{A}\right)P, & P \ge \theta \end{cases}, \quad (15)$$

$A \in (\max\{(1+c_M)/2, 1-\lambda\}, 1)$. Since $A > 1-\lambda$, the equilibrium residual demand is decreasing in P for $0 \le P < \theta$. However, for $P \ge \theta$, demand may have a positive slope depending on the value of φ.

First, consider the case of no fringe competition, i.e., $\varphi = 0$. The solid line in Figure 1 depicts (15) evaluated at $\varphi = 0$. For $P < \theta$, both informed and uninformed buyers purchase only from the dominant firm. Although the demand of the uninformed buyers is upward-sloping, the aggregate demand is downward-sloping for prices below the reservation price.[13] However, for $P > \theta$, the informed buyers exit the market and the demand curve becomes upward-sloping, due to the informed buyers' upward-sloping demand curve. The isoprofit curve in Figure 1 is the locus of pairs $\{Q, P\}$ yielding equilibrium profits $\pi^S(\theta)$.[14] The point $\{Q^S, P^S\} = \{(1-A)\theta, A\theta\}$ is the solution in

[12] To generate Figures 1, 2, and 3, we set $\theta = 1$, $c_M = 0.1$, and $\lambda = 0.5$.

[13] Indeed, signaling establishes a positive relationship between the price and the quantity demanded by the uninformed buyers, i.e., $\chi^S(P) - P > 0$ is increasing in $P \ge 0$.

[14] From Proposition 3.5, equilibrium profits are $\pi^S(\theta) = (P^S(\theta) - c_M\theta)(\chi^S(P^S(\theta)) - P^S(\theta)) = (A - c_M)(1-A)\theta^2$. Hence, the isoprofit function is $P = c_M\theta + \pi^S(\theta)/Q$.

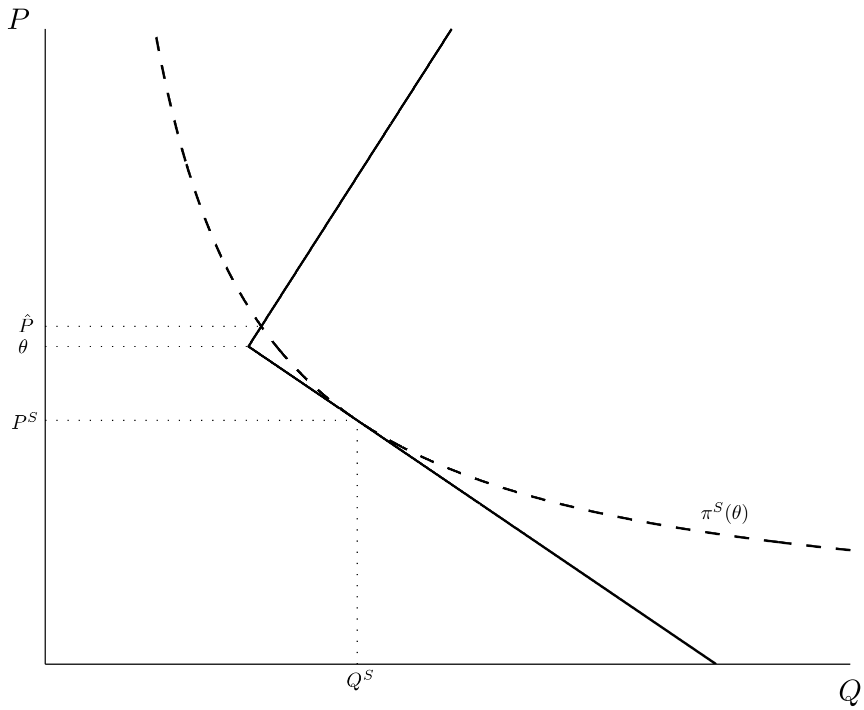

Figure 1: Dominant Firm with No Fringe Competition, $\varphi = 0$

Proposition 3.5, which yields profits $\pi^S(\theta)$. Figure 1 shows that the absence of a competitive fringe always provides an incentive for the dominant firm to deviate from $\{Q^S, P^S\}$. Indeed, any prices above $\hat{P}$ yield profits greater than $\pi^S(\theta)$ to the deviant dominant firm. By charging a higher price, the dominant firm sacrifices revenue from the informed buyers, but is able to deceive the uninformed buyers, making higher profits from them. Therefore, the outcome in Proposition 3.5 cannot constitute a signaling equilibrium without fringe competition.

The presence of a competitive fringe changes the slope of the equilibrium residual demand. Indeed, when there is enough presence of the competitive fringe (i.e., $\varphi \geq \underline{\varphi}$), the equilibrium residual demand is downward-sloping enough so as to remove any incentive for the dominant firm to deviate. In other words, the benefit of deceiving the uninformed buyers is reduced enough

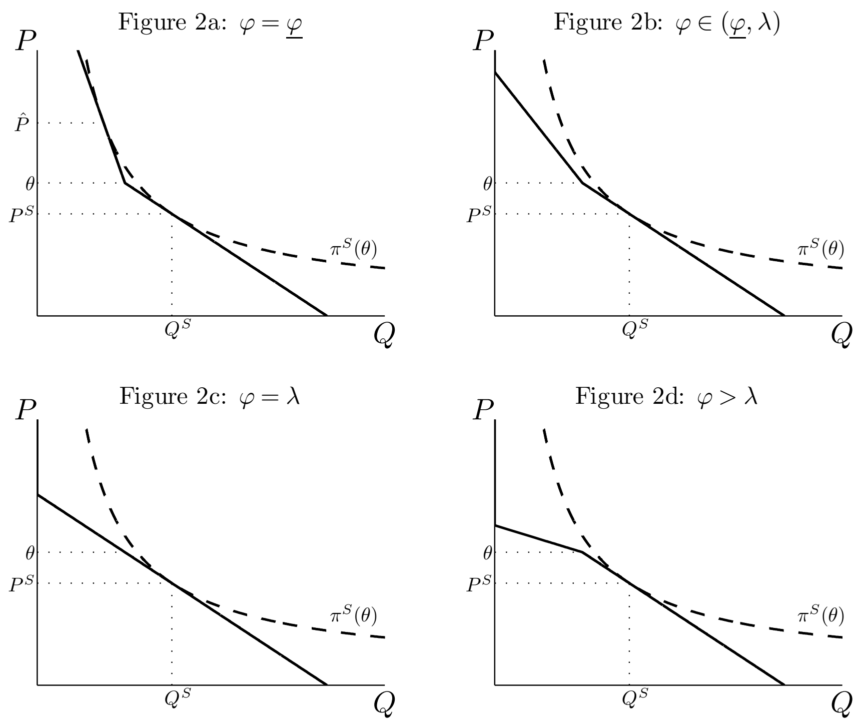

Figure 2: Dominant Firm with Strong Fringe Competition, $\varphi \in [\underline{\varphi}, 1]$

to be outweighed by the cost of facing competition. Graphically, for an equilibrium to exist, demand must never cross the isoprofit curve for prices above θ. Figure 2 considers four cases for which the incentive to deviate is blocked.[15] Figure 2a presents the borderline case in which $\varphi = \underline{\varphi}$. In this case, the equilibrium residual demand is tangent to the isoprofit at two points. Deviating from the strategy $P^S(\theta) = A\theta$ to the other tangent point $\hat{P}$ yields no improvement in profit for any θ. Figures 2b,c,d deal with an increasingly larger presence of the fringe competition. The greater φ, the flatter the slope of the equilibrium residual demand above the reservation price, and, thus, the greater the cost of deviating from $P^S(\theta) = A\theta$.

All four cases depicted in Figure 2 illustrate the limits that the fringe competition place on the dominant firm, i.e., the dominant firm cannot take

[15] For prices below θ, the equilibrium demand is identical in Figures 1 and 2.

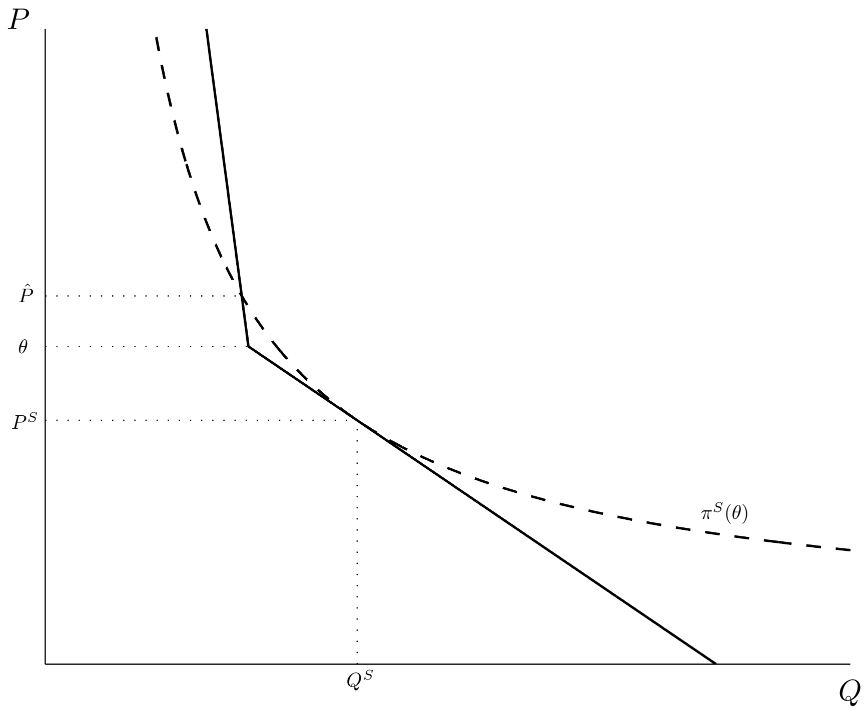

Figure 3: Dominant Firm with Weak Fringe Competition, $\varphi < \underline{\varphi}$

advantage of the uninformed buyers' upward-sloping demand, as in the case of the monopoly (i.e., a dominant firm without fringe competition). Rather, if the fringe is large enough, the dominant firm may only react to the learning activity of the uninformed buyers, and, thus, charges a price below θ in order to avoid the fringe competition.

While the presence of competitive fringe is necessary for blocking the incentive to deviate from $\{Q^S, P^S\} = \{(1-A)\theta, A\theta\}$, it is not sufficient as shown in Figure 3.[16] Indeed, when $\varphi \in (0, \underline{\varphi})$, the benefit from deceiving the uninformed buyers is greater than the loss of profit due to the competitive fringe. In other words, there exist some prices $P > \hat{P}$ that provide an incentive for deviation by yielding profits higher than $\pi^S(\theta)$. Therefore, no equilibrium exists with a weak competitive fringe.

[16]For prices below θ, the equilibrium demand is identical in Figures 1, 2, and 3.

3.3 The General Case

In Section 3.2, it is shown that a large enough competitive fringe reestablishes the existence of a signaling equilibrium with trading. While, in equilibrium, the fringe firm never enters because its marginal cost is equal to the reservation price, the threat of entry for prices above the reservation price is enough to block the dominant firm from deceiving the uninformed buyers. We conclude the analysis by generalizing the results of Section 3.2. Specifically, we now consider the general case in which the marginal cost of the fringe firm lies between the marginal cost of the dominant firm and the reservation price, i.e., $c_F \in [c_M, 1]$. We confirm that, in general, there exists a signaling equilibrium with trading as long as the competitive fringe is large enough. Unlike the equilibrium characterized in Section 3.2, the competitive fringe may also trade in equilibrium in the general case. We also provide conditions under which the dominant firm signals quality while, at the same time, facing competition from the fringe firm.

Propositions 3.7 and 3.8 generalize Propositions 3.4 and 3.5. Proposition 3.7 establishes the necessary and sufficient condition for the strength of the competitive fringe (i.e., the lower bound on φ) for the existence of a signaling equilibrium with trading. Proposition 3.8 provides the price and inference rule corresponding to the signaling equilibrium. The entry of the competitive fringe, in equilibrium, depends on the value of its marginal cost. Specifically, for low values of c_F, the dominant firm signals quality by setting a price that induces entry. For high values of c_F, signaling occurs, but competition is avoided.

Proposition 3.7. *Suppose that Assumptions 2.1 and 3.2 hold. Then, there exists* $\underline{\varphi} \in (0, \lambda)$ *such that, for* $\lambda \in (0, 1)$*, there is a signaling equilibrium with trading if and only if* $\varphi \in [\underline{\varphi}, 1]$.

Proposition 3.8. *Suppose that Assumptions 2.1 and 3.2 hold. For* $\lambda \in (0, 1)$

and $\varphi \in [\underline{\varphi}, 1]$, $\underline{\varphi} \in (0, \lambda)$,

$$P^S(\theta) = \begin{cases} B\theta, & c_F \in [c_M, B) \\ c_F\theta, & c_F \in [B, A] \\ A\theta, & c_F \in (A, 1] \end{cases}, \tag{16}$$

$\theta \geq 0$, *and*

$$\chi^S(P) = \begin{cases} P/B, & c_F \in [c_M, B) \\ P/c_F, & c_F \in [B, A] \\ P/A, & c_F \in (A, 1] \end{cases}. \tag{17}$$

Here, $B < A$ *such that*

$$A = \frac{2 - \lambda + c_M + \sqrt{(2 - \lambda + c_M)^2 - 8(1 - \lambda)c_M}}{4}, \tag{18}$$

such that $A \in (\max\{(1 + c_M)/2, 1 - \lambda\}, 1)$, *and*

$$B = \frac{2 - \lambda + (1 + \varphi)c_M + \varphi c_F + \sqrt{(2 - \lambda + (1 + \varphi)c_M + \varphi c_F)^2 - 8(1 + \varphi)(1 - \lambda)c_M}}{4(1 + \varphi)}, \tag{19}$$

such that $B \in (\max\{(1 + c_M + \varphi c_F)/(2(1 + \varphi)), (1 - \lambda)/(1 + \varphi)\}, A)$.

The proof of Propositions 3.7 and 3.8 is in Appendix B. The value of the threshold $\underline{\varphi} \in (0, \lambda)$ which depends on the inference rule is provided in the proof.

The graphical analysis is essentially the same as the one provided in Section 3.2. In each of the following figures, the equilibrium residual demand

$$Q = \lambda \max\{\theta - P, 0\} + (1 - \lambda) \max\{\chi^S(P) - P, 0\} - \varphi \max\{P - c_F\theta, 0\}, \tag{20}$$

is depicted for different values of $\varphi \in [0, 1]$ and $c_F \in [c_M, 1]$.[17] The isoprofit curve represents the locus of pairs $\{Q, P\}$ yielding equilibrium profits $\pi^S(\theta)$. The point $\{Q^S, P^S\}$ is the solution given in Proposition 3.8. In a signaling environment, (20) depends on the strategy of the dominant firm via the

[17]To generate Figures 4, and 5, we set $\theta = 1$, $c_M = 0.1$, and $\lambda = 0.5$. Moreover, $\varphi = 0.4$ so that the competitive fringe is strong and $c_F = \{0.2, 0.9\}$ for low vs. high cost.

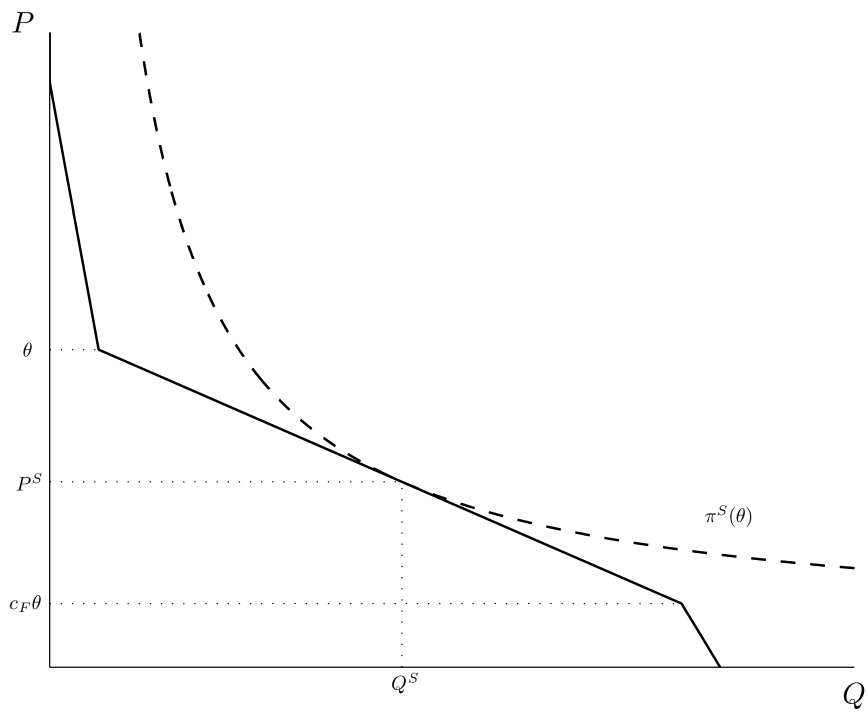

Figure 4: Strong Fringe Competition with Low Cost

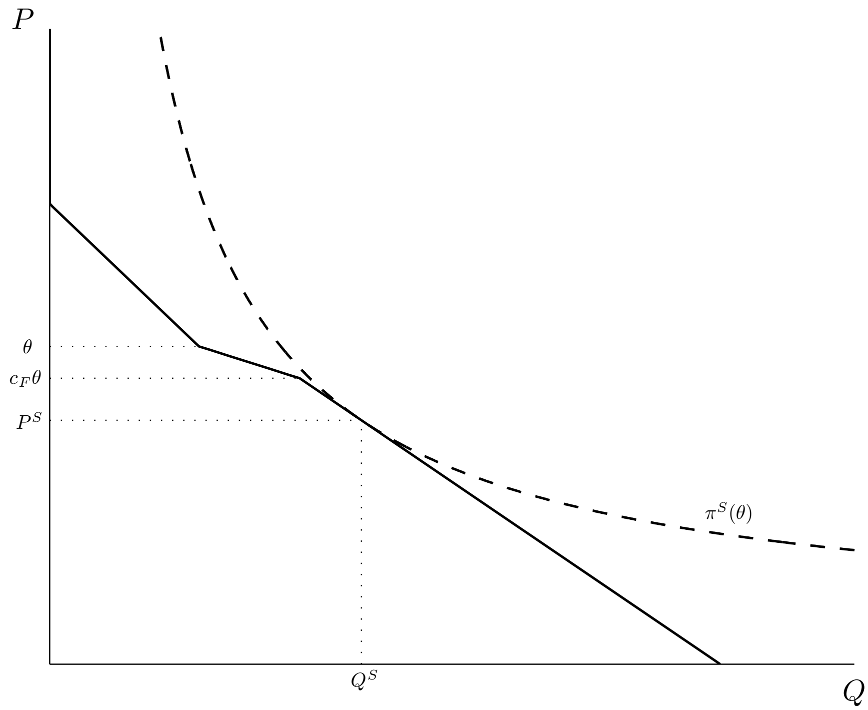

Figure 5: Strong Fringe Competition High Cost

equilibrium inference rule defined by (17).

Figures 4 and 5 illustrate the two possible outcomes in equilibrium for a large enough presence of the competitive fringe, i.e., $\varphi \geq \underline{\varphi}$. Specifically, in Figure 4, the dominant firm signals quality with a price that allows for entry of the fringe firm, i.e., the price is above the fringe firm's marginal cost $c_F\theta$. In Figure 5, the fringe firm has a high cost disadvantage with the dominant firm. Hence, in equilibrium, the dominant firm signals quality and avoids competition.

Note finally that Figures 4 and 5 represent situations in which the competitive fringe is large enough. As in the special case, the strength of the competitive fringe is key in blocking the incentive of the dominant firm to deviate from the strategy $P^S(\theta)$ as defined by (16). Since the need of a competitive fringe has already been discussed in Section 3.2, the discussion for the general case is relegated to Appendix C.

4 Final Remarks

A potentially valueless good limits the informational role of prices in monopoly. On the one hand, when there are both informed and uninformed buyers, the price cannot convey information. On the other hand, with only uninformed buyers, the price does convey information but without trading. The presence of competition is shown to enable the monopolist to credibly signal quality when demand is composed of informed and uninformed buyers. Because this paper focuses on the existence of a signaling equilibrium, the effect of signaling on the price and the profit of the dominant firm remains to be analyzed. Moreover, it would be interesting to consider a richer model in which the dominant firm and the competitive fringe sell unknown but different levels of quality. While the threat of competition should enable the dominant firm to signal quality, the necessary strength of the competitive fringe in order to block any deviation should depend on the substitutability between the different goods offered by the dominant firm and the fringe firm.

Finally, in order to compare our results with the literature, we have assumed a noiseless environment. Extending the study of the signaling role of prices to a noisy environment would lessen the informational requirement of learning buyers about the structure of the market. It would also further our understanding of information flows in a more complex environment. Indeed, a noiseless environment separates two important but distinct effects of the informational externality.[18] Specifically, in a noiseless environment, the firm reacts to the informational externality, but has limited control over the flow of information. In other words, either the unknown parameter is not revealed and learning buyers revert to their prior beliefs, or it is fully revealed in equilibrium. However, in a noisy environment, the firm is able to affect the flow of information, i.e., the distribution of the price-signal depends on the firm's decision. In other words, the firm is able to take advantage of the noise by

[18] The learning process of the uninformed buyers through the price influences profit, which constitutes an informational externality to the monopolist.

manipulating the beliefs of learning buyers.[19]

[19] This was originally done in Matthews and Mirman (1983) in a limit pricing model. Similarly, Judd and Riordan (1994) studies the signaling role of the price set by a monopolist, which provides partial information about the quality of a new product.

A The Need of Informed Buyers

In this section, we show that, when there are no informed buyers, there exists no signaling equilibrium in which both the dominant firm and the fringe firm sell the good. In particular, the absence of informed buyers preclude the residual demand to be strictly positive in equilibrium. Proposition A.1 complements and extends Proposition 2.4.

Proposition A.1. *Suppose that Assumptions 2.1 and 3.2 hold. If* $\lambda = 0$ *and* $\varphi \in (0, 1]$*, then there is no signaling equilibrium in which, for* $\theta > 0$*,*

$$D(P^S(\theta), \theta, \chi^S(P^S(\theta))) > S(P^S(\theta), c_F\theta) > 0. \tag{21}$$

Proof. Suppose to the contrary that there is a signaling equilibrium in which both the dominant firm and the competitive fringe trade, i.e., $D(P^S(\theta), \theta, \chi^S(P^S(\theta))) > S(P^S(\theta), c_F\theta) > 0$ for $\theta > 0$. Since $\lambda = 0$, it follows that $\chi^S(P^S(\theta)) - P^S(\theta) > \varphi P^S(\theta) - \varphi c_F\theta > 0$, so that $P^S(\theta) \in (c_F\theta, (1 + \varphi c_F)\theta/(1 + \varphi))$. Then, the zero-quality dominant firm (generating zero profits) has an incentive to deviate to the price charged by the dominant firm of quality $\theta > 0$ in order to obtain strictly positive profits from the uninformed buyers. Therefore, there is no signaling equilibrium in which (21) holds. □

B Proofs

Proof of Propositions 3.4 and 3.5. We first characterize the set of valid candidates for a signaling equilibrium. We then show that the price function and inference rule provided in Proposition 3.5 constitute a signaling equilibrium as long as $\varphi \in [\underline{\varphi}, 1]$ where $\underline{\varphi} \in (0, \lambda)$.

- **Valid Candidates for Equilibrium.** For $\theta > 0$, $P^S(\theta) \in (c_M\theta, \theta)$.[20] Hence, $P^S(0) = 0$. Moreover, since posterior beliefs are the inverse of the price function, $\chi^S(P)$ is increasing in P with $\chi^S(0) = 0$ and $\chi^S(P) > P$.

- **Characterization of Equilibrium.** We now characterize the equilibrium price function, and inference rule. Since the set of valid candidates for the equilibrium price function is $(c_M\theta, \theta)$, we restrict attention to $P \in (c_M\theta, \theta)$. We then show that there is no incentive for the dominant firm to deviate from $P^S(\theta) \in (c_M\theta, \theta)$ to any prices above θ as long as $\varphi \in [\underline{\varphi}, 1]$ where $\underline{\varphi} \in (0, \lambda)$.[21]

    - We first derive the first-order condition of the dominant firm. We then show that the solution for the price stated in Proposition 3.5 is the unique maximizer of the dominant firm.

        * Plugging (2), (12), and $c_F = 1$ into (10) yields the equivalent maximization problem

        $$\max_{P \in (c_M\theta, \theta)} (P - c_M\theta)(\lambda\theta + (1-\lambda)\chi^S(P) - P), \qquad (22)$$

          where $\chi^S(P) > P$ for all $P > 0$. The first-order condition

[20]Suppose rather that $P^S(\theta') \notin (c_M\theta', \theta')$ for some $\theta' > 0$. Then, the dominant firm makes zero profits if either $P^S(\theta') \geq \theta'$ or $P^S(\theta') = c_M\theta'$, and negative profits if $P^S(\theta') < c_M\theta'$. Hence, the dominant firm has an incentive to deviate to any price $P \in (c_M\theta', \theta')$ in order to obtain strictly positive profits from the informed buyers.

[21]Note that the dominant firm has no incentive to deviate to prices at or below the marginal cost $c_M\theta$ because such deviation yields zero or negative profits, respectively.

corresponding to (22) is

$$\lambda(\theta-P)+(1-\lambda)(\chi^S(P)-P)+(P-c_M\theta)\left((1-\lambda)\frac{\mathrm{d}\chi^S(P)}{\mathrm{d}P}-1\right)=0. \tag{23}$$

In equilibrium, $P = P^S(\theta)$, $\chi^S(P^S(\theta)) = \theta$ and

$$\left.\frac{\mathrm{d}\chi^S(P)}{\mathrm{d}P}\right|_{P=P^S(\theta)} = \left(\frac{\mathrm{d}P^S(\theta)}{\mathrm{d}\theta}\right)^{-1}. \tag{24}$$

Let $y \equiv P^S(\theta)$ and $y' \equiv \frac{\mathrm{d}P^S(\theta)}{\mathrm{d}\theta}$, so that (23) becomes

$$\theta - y + (y - c_M\theta)((1-\lambda)/y' - 1) = 0, \tag{25}$$

which is a differential equation with the initial condition $(y_0, \theta_0) = (0, 0)$. Rearranging (25) yields

$$y' = \frac{(1-\lambda)(y - c_M\theta)}{2y - (1+c_M)\theta}. \tag{26}$$

Given that $y' > 0$ for $\theta > 0$,, it follows from (26) that $y > (1+c_M)\theta/2$ for $\theta > 0$. In other words, $P^S(\theta) \in ((1+c_M)\theta/2, \theta)$ for $\theta > 0$.

∗ Next, we show that $P^S(\theta) = A\theta$, A defined by (13), is a solution to (25) with the initial condition $(\theta_0, y_0) = (0, 0)$. Plugging $y = A\theta$ into (25) yields

$$\theta - A\theta + (A\theta - c_M\theta)((1-\lambda)/A - 1) = 0, \tag{27}$$

which is true for $\theta = 0$. For $\theta > 0$, rearranging (27) yields

$$2A^2 - (2 - \lambda + c_M)A + (1-\lambda)c_M = 0. \tag{28}$$

Equation (28) has two positive roots. If $A = (1+c_M)/2$, then the left-hand side of (28) is negative. If $A = 1$, then the left-hand side of (28) is positive. It follows that the largest root of (28) is the only root that satisfies $y \in ((1+c_M)\theta/2, \theta)$ and, thus, $y' > 0$. Moreover, if $A = 1 - \lambda$, then the left-hand side of (28) is negative. It follows that the largest root is greater than $1-\lambda$, and, thus, is the only root that satisfies the second-order condition. Hence, $A \in (\max\{(1 + c_M)/2, 1 - \lambda\}, 1)$.

* We now show that $y = A\theta$, $A \in (\max\{(1 + c_M)/2, 1 - \lambda\}, 1)$ defined by (13), is unique. Note that the right-hand side and the derivative of the right-hand side of (26) are both continuous for $(\theta, y) \in S$, where

$$S = \{(\theta, y) : 2y > (1 + c_M)\theta, y > 0\}. \tag{29}$$

By the Fundamental Theorem of Differential Equation, there exists a unique solution $y = \phi(\theta)$ for any initial condition $(\theta_0, y_0) \in S$. However, our initial condition $(0, 0) \notin S$. Therefore, we need to show as well that there is no other $y = \phi(\theta)$ with initial condition $(\theta_0, y_0) \in S\setminus\{\theta, A\theta\}$ such that $\phi(0) = 0$, which satisfies (25). From (26),

$$\frac{\mathrm{d}y'}{\mathrm{d}y} = -\frac{(1-\lambda)(1-c_M)\theta}{(2y-(1+c_M)\theta)^2} < 0, \tag{30}$$

for $(\theta, y) \in S$, which implies that any solution $y = \phi(\theta)$ above $y = A\theta$ has a flatter slope and any solution $y = \phi(\theta)$ below $y = A\theta$ has a steeper slope. Hence, no solution $y = \phi(\theta), (\theta, y) \in S \setminus \{\theta, A\theta\}$ converges toward the origin.

– Having shown that, if there is a signaling equilibrium, then $P^S(\theta) = A\theta \in (c_M\theta, \theta)$, we now determine the condition on the strength of the competitive fringe so that there is no incentive for the dominant firm to price above θ. In other words, we characterize $\underline{\varphi}$ which is the minimum value of φ such that the dominant firm has no incentive to deviate from $P^S(\theta) = A\theta$ to $P \geq \theta$. Graphically, $\underline{\varphi}$ is the level of strength of the fringe firm such that the equilibrium residual demand is tangent to the isoprofit yielding equilibrium profits $\pi^S(\theta) = (A - c_M)(1 - A)\theta^2$ above the reservation price θ. See $\hat{P}$ in Figure 2a. From (15), for $P > \theta$, the equilibrium residual demand is

$$P = \frac{A\varphi\theta}{A\varphi - (1-\lambda)(1-A)} - \frac{AQ}{A\varphi - (1-\lambda)(1-A)}, \tag{31}$$

while the isoprofit curve is defined by

$$P = c_M\theta + \frac{\pi^S(\theta)}{Q}, \tag{32}$$

where $\pi^S(\theta) = (A - c_M)(1 - A)\theta^2$ is the equilibrium profits. Equating (31) and (32) defines the values of output for which the equilibrium residual demand and the isoprofit curve intersect, i.e.,

$$\frac{A\varphi\theta Q}{A\varphi - (1-\lambda)(1-A)} - \frac{AQ^2}{A\varphi - (1-\lambda)(1-A)} = c_M\theta Q + \pi^S(\theta) \tag{33}$$

or

$$\frac{AQ^2}{A\varphi - (1-\lambda)(1-A)} + \left(c_M - \frac{A\varphi}{A\varphi - (1-\lambda)(1-A)}\right)\theta Q + \pi^S(\theta) = 0. \tag{34}$$

Because we look for tangent points, the discriminant must be zero, i.e.,

$$\begin{aligned}(1-c_M)^2A^2\varphi^2 &+ 2((1-c_M)(1-\lambda)c_M - 2A(A-c_M))(1-A)A\varphi \\ &+ (4A(A-c_M) + (1-\lambda)c_M^2)(1-\lambda)(1-A)^2 = 0.\end{aligned} \tag{35}$$

The largest root of (35) is $\varphi = \lambda$, which refers to the tangent point $\{Q^S, P^S\}$ in Figures 1, 2, and 3. It follows that $\underline{\varphi} \in (0, \lambda)$ is the smallest root of (35). There is thus no incentive for the dominant firm to deviate from $P^S(\theta) = A\theta$ to some price $P > \theta$ as long as $\varphi \geq \underline{\varphi}$.

Hence, $P^S(\theta) = A\theta$, $A \in (\max\{(1 + c_M)/2, 1 - \lambda\}, 1)$ defined by (13), is the equilibrium price function. Moreover, the equilibrium inference rule is $\chi^S(P) = P/A$, which is consistent with Bayes' rule and the dominant firm's strategy, as in any signaling game, i.e., $\chi^S(P^S(\theta)) = \theta$ for all $\theta \geq 0$.

Proof of Propositions 3.7 and 3.8. The proof follows identical steps of the proof provided for Propositions 3.4 and 3.5 except that two cases need to be considered. The first one is the case in which the pricing strategy of the dominant firm does not lead to the entry of the fringe firm. The second one concerns the case in which the pricing strategy of the dominant firm does lead to the entry of the fringe firm.

We first characterize the set of valid candidates for a signaling equilibrium. We then show that the price function and inference rule provided in Proposition 3.8 constitute a signaling equilibrium as long as $\varphi \in [\underline{\varphi}, 1]$ where $\underline{\varphi} \in (0, \lambda)$. Because the proof is very similar to the previous one, we skip some details.

- **Valid Candidates for Equilibrium.** For $\theta > 0$, $P^S(\theta) \in (c_M\theta, (1 + \varphi c_M)\theta/(1 + \varphi))$.[22] Hence, $P^S(0) = 0$. Moreover, since posterior beliefs are the inverse of the price function, $\chi^S(P)$ is increasing in P with $\chi^S(0) = 0$ and $\chi^S(P) > P$.

- **Characterization of Equilibrium.** We now characterize the equilibrium price function, inference rule, and the lower bound for the strength of the competitive fringe.

  - Consider first the case in which the fringe firm is not active. i.e., $P^S(\theta) \in (c_M\theta, c_F\theta)$. We first characterize $P^S(\theta)$. We then show that there is no incentive for the firm to deviate from his price strategy.

[22] Suppose rather that $P^S(\theta') \notin (c_M\theta', (1 + \varphi c_M)\theta'/(1 + \varphi))$ for some $\theta' > 0$. Then, the dominant firm makes zero profits if either $P^S(\theta') \geq (1 + \varphi c_M)\theta'/(1 + \varphi)$ (because the residual demand is zero) or $P^S(\theta') = c_M\theta'$, and negative profits if $P^S(\theta') < c_M\theta'$. Hence, the dominant firm has an incentive to deviate to any price $P \in (c_M\theta', (1+\varphi c_M)\theta'/(1+\varphi))$ in order to obtain strictly positive profits from the informed buyers.

* The proof is analogous to the one provided for Propositions 3.4 and 3.5. Hence, the equilibrium price function is $P^S(\theta) = A\theta$ and $\chi^S(P) = P/A$, where A is defined by (18). The only additional requirement is that the price must be below the fringe firm's marginal cost, i.e., $P^S(\theta) < c_F\theta$ or $A < c_F$.
* Having shown that, if there is a signaling equilibrium without entry of the fringe firm, then $P^S(\theta) = A\theta$ with $A < c_F$, we now determine the condition on the strength of the competitive firm so that there is no incentive for the dominant firm to price above θ.[23] In other words, we characterize $\underline{\varphi}$ which is the minimum value of φ such that the dominant firm has no incentive to deviate from $P^S(\theta) = A\theta$ to $P \geq \theta$. Graphically, $\underline{\varphi}$ is the level of strength of the fringe firm such that the equilibrium residual demand is tangent to the isoprofit yielding equilibrium profits $\pi^S(\theta) = (A - c_M)(1 - A)\theta^2$ above the reservation price θ. For prices above θ, it is steeper than the demand segment in Figure 5 and flatter than the demand segment in Figure 7. From (20), for $P > \theta$, the equilibrium residual demand is

$$P = \frac{A\varphi c_F\theta}{A\varphi - (1-\lambda)(1-A)} - \frac{AQ}{A\varphi - (1-\lambda)(1-A)}, \quad (36)$$

while the isoprofit curve is defined by

$$P = c_M\theta + \frac{\pi^S(\theta)}{Q}, \quad (37)$$

where $\pi^S(\theta) = (A - c_M)(1 - A)\theta^2$ is the equilibrium profits. Equating (36) and (37) defines the values of output for which the equilibrium residual demand and the isoprofit intersect,

[23] Note that the dominant firm has no incentive to deviate to prices at or below the marginal cost $c_M\theta$ because such deviation yields zero or negative profits, respectively. Moreover, the dominant firm has no incentive to deviate to prices between $c_F\theta$ and θ, which yields lower profits due to a flatter demand curve. See Figure 5 by comparing the two demand segments for prices below θ.

i.e.,

$$\frac{A\varphi c_F\theta Q}{A\varphi-(1-\lambda)(1-A)}-\frac{AQ^2}{A\varphi-(1-\lambda)(1-A)}=c_M\theta Q+\pi^s(\theta) \quad (38)$$

or

$$\frac{AQ^2}{A\varphi-(1-\lambda)(1-A)}+\left(c_M-\frac{A\varphi c_F}{A\varphi-(1-\lambda)(1-A)}\right)\theta Q+\pi^s(\theta)=0. \quad (39)$$

Because we look for tangent points, the discriminant must be zero, i.e.,

$$(1-c_M)^2A^2c_F^2\varphi^2+(2(1-c_M)(1-\lambda)c_M-4A(A-c_M))(1-A)A\varphi \quad (40)$$
$$+(4A(A-c_M)+(1-\lambda)c_M^2)(1-\lambda)(1-A)^2=0.$$

Analogously to the previous proof, $\underline{\varphi}\in(0,\lambda)$ is the smallest root of (40). There is thus no incentive for the dominant firm to deviate from $P^S(\theta)=A\theta$ to some price $P>\theta$ as long as $\varphi\geq\underline{\varphi}$.

– Consider next the case in which the fringe firm is active, i.e., $P^S(\theta)\in(c_F\theta,(1+\varphi c_M)\theta/(1+\varphi))$. Plugging (2) and (12) into (10) yields the equivalent maximization problem

$$\max_{P\in(c_F\theta,(1+\varphi c_F)\theta/(1+\varphi))}(P-c_M\theta)((\lambda+\varphi c_F)\theta+(1-\lambda)\chi^S(P)-(1+\varphi)P), \quad (41)$$

where $\chi^S(P)>P$ for all $P>0$. The first-order condition corresponding to (41) is

$$(\lambda+\varphi c_F)\theta+(1-\lambda)\chi^S(P)-(1+\varphi)P+(P-c_M\theta)\left((1-\lambda)\frac{\mathrm{d}\chi^S(P)}{\mathrm{d}P}-(1+\varphi)\right)=0. \quad (42)$$

In equilibrium, $P=P^S(\theta)$, $\chi^S(P^S(\theta))=\theta$ and $\left.\frac{\mathrm{d}\chi^S(P)}{\mathrm{d}P}\right|_{P=P^S(\theta)}=$

$\left(\frac{\mathrm{d}P^S(\theta)}{\mathrm{d}\theta}\right)^{-1}$. Let $y \equiv P^S(\theta)$ and $y' \equiv \frac{\mathrm{d}P^S(\theta)}{\mathrm{d}\theta}$, so that (42) becomes

$$(\lambda+\varphi c_F)\theta-(1+\varphi)y+(y-c_M\theta)((1-\lambda)/y'-(1+\varphi))=0, \quad (43)$$

which is a differential equation with the initial condition $(y_0, \theta_0) = (0, 0)$. Rearranging (43) yields

$$y' = \frac{(1-\lambda)(y-c_M\theta)}{2(1+\varphi)y-(1+c_M+\varphi c_F)\theta}. \quad (44)$$

Given that $y' > 0$ for $\theta > 0$, it follows from (44) that $y > (1 + c_M + \varphi c_F)\theta/(2(1+\varphi)) > 0$ for $\theta > 0$. In other words, $P^S(\theta) \in ((1+c_M+\varphi c_F)\theta/(2(1+\varphi)), (1+\varphi c_M)\theta/(1+\varphi))$ for $\theta > 0$.

Next, we show that $P^S(\theta) = B\theta$, B defined by (19), is a solution to (43) with the initial condition is $(\theta_0, y_0) = (0, 0)$. Plugging $y = B\theta$ into (43) yields

$$(1+\varphi c_F)\theta-(1+\varphi)B\theta+(B\theta-c_M\theta)((1-\lambda)/B-(1+\varphi))=0, \quad (45)$$

which is true for $\theta = 0$. For $\theta > 0$, rearranging (45) yields

$$2(1+\varphi)B^2-(2-\lambda+(1+\varphi)c_M+\varphi c_F)B+(1-\lambda)c_M=0. \quad (46)$$

Equation (45) has two positive roots. If $B = (1+c_M+\varphi c_F)/(2(1+\varphi))$, then the left-hand side of (46) is negative. If $B = 1$, then the left-hand side of (46) is positive. It follows that the largest root of (46) is the only solution that satisfies $y \in ((1+c_M+\varphi c_F)\theta/(2(1+\varphi)), \theta)$ and, thus, $y' > 0$. Moreover, if $B = (1-\lambda)/(1+\varphi)$, then the left-hand side of (46) is negative. It follows that the largest root is greater than $(1-\lambda)/(1+\varphi)$, and, thus, is the only root that satisfies satisfies the second-order condition. Finally, if $B = A$, where A is defined by (18), then the left-hand side of (46) is positive. In addition, the derivative of the left-hand side of (46) evaluated at $B = A$ is positive. This implies that $B < A$. Hence,

$B \in (\max\{(1+c_M+\varphi c_F)/(2(1+\varphi)), (1-\lambda)/(1+\varphi)\}, A)$.

We now show that $y = B\theta$, $B \in (\max\{(1+c_M+\varphi c_F)/(2(1+\varphi)), (1-\lambda)/(1+\varphi)\}, A)$ defined by (19), is unique. Note that the right-hand side and the derivative of the right-hand side of (44) are both continuous for $(\theta, y) \in S$, where

$$S = \{(\theta, y) : 2(1+\varphi)y > (1+c_M+\varphi c_F)\theta, y > 0\}. \tag{47}$$

By the Fundamental Theorem of Differential Equation, there exists a unique solution $y = \phi(\theta)$ for any initial condition $(\theta_0, y_0) \in S$. However, our initial condition $(0,0) \notin S$. Therefore, we need to show as well that there is no $y = \phi(\theta)$ with initial condition $(\theta_0, y_0) \in S \setminus \{\theta, B\theta\}$ such that $\phi(0) = 0$, which satisfies (43). From (44),

$$\frac{\mathrm{d}y'}{\mathrm{d}y} = -\frac{(1-\lambda)(1+\varphi(c_F - c_M))\theta}{(2(1+\varphi)y - (1+c_M+\varphi c_F)\theta)^2} < 0, \tag{48}$$

for $(\theta, y) \in S$, which implies that any solution $y = \phi(\theta)$ above $P^S(\theta) = B\theta$ has a flatter slope and any solution $y = \phi(\theta)$ below $P^S(\theta) = B\theta$ has a steeper slope. Hence, no solution $y = \phi(\theta), (\theta, y) \in S \setminus \{\theta, B\theta\}$ converges toward the origin.

Having shown that, if there is a signaling equilibrium with entry of the fringe firm, then $P^S(\theta) = B\theta$ with $B > c_F$, we now determine the condition on the strength of the competitive firm so that there is no incentive for the dominant firm to price $P \notin (c_F\theta, (1+\varphi c_M)\theta/(1+\varphi))$. As before, the dominant firm has no incentive to deviate from the price strategy $P^S(\theta) = B\theta$ as long as $\varphi \geq \underline{\varphi}$, where $\underline{\varphi}$ is the value of the strength of the fringe competition such that the equilibrium residual demand for prices above the reservation price θ is tangent to the isoprofit curve yielding equilibrium profits $\pi^S(\theta)$. Graphically, $\underline{\varphi}$ is the level of strength of the fringe firm such that the equilibrium residual demand is tangent to the isoprofit yielding equilibrium profits

$\pi^S(\theta) = (B - c_M)(1 + \varphi c_F - (1 + \varphi)B)\theta^2$ above the reservation point. For prices above θ, it is steeper than the demand segment in Figure 4 and flatter than the demand segment in Figure 6. From (20), for $P > \theta$, the equilibrium residual demand is

$$P = \frac{B\varphi c_F \theta}{B\varphi - (1-\lambda)(1-B)} - \frac{BQ}{B\varphi - (1-\lambda)(1-B)}, \qquad (49)$$

while the isoprofit curve is defined by

$$P = c_M \theta + \frac{\pi^S(\theta)}{Q}, \qquad (50)$$

where $\pi^S(\theta) = (B - c_M)(1 + \varphi c_F - (1 + \varphi)B)\theta^2$ is the equilibrium profits. Equating (49) and (50) defines the values of output for which the equilibrium residual demand and the isoprofit intersect, i.e.,

$$\frac{B\varphi c_F \theta Q}{B\varphi - (1-\lambda)(1-B)} - \frac{BQ^2}{B\varphi - (1-\lambda)(1-B)} = c_M \theta Q + \pi^S(\theta) \qquad (51)$$

or

$$\frac{BQ^2}{B\varphi - (1-\lambda)(1-B)} + \left(c_M - \frac{B\varphi c_F}{B\varphi - (1-\lambda)(1-B)}\right)\theta Q + \pi^S(\theta) = 0. \qquad (52)$$

Because we look for tangent points, the discriminant must be zero, i.e.,

$$(1 - c_M)^2 B^2 c_F^2 \varphi^2 + (2(1 - c_M)(1-\lambda)c_M - 4B(B - c_M))(1 - B)B\varphi \qquad (53)$$

$$+ (4B(B - c_M) + (1-\lambda)c_M^2)(1-\lambda)(1-B)^2 = 0.$$

From, there exists $\underline{\varphi} \in (0, \lambda)$ such that (53) holds. There is thus no incentive for the dominant firm to deviate from $P^S(\theta) = B\theta$ to some price $P > \theta$ as long as $\varphi \geq \underline{\varphi}$.[24]

[24]Note that from (19), B depends on φ.

Hence, (16) defines the equilibrium price function.[25] Moreover, the equilibrium inference rule is defined by (17), which is consistent with Bayes' rule and the dominant firm's strategy, as in any signaling game, i.e., $\chi^S(P^S(\theta)) = \theta$ for all $\theta \geq 0$.

[25] If $c_F \in [B, A]$, then the dominant firm sets the price at the kink, i.e., $P^S(\theta) = c_F \theta$.

C The Need of Large Competitive Fringe

Figures 6 and 7 depict the same equilibrium residual demand as Figures 4 and 5 do, except for the fact that the fringe competition is weak, i.e., $\varphi < \underline{\varphi}$.[26] These two figures are thus analogous to Figure 1. A weak presence of the fringe firm provides the dominant firm an incentive to deviate by charging $P > \hat{P}$. Hence, signaling cannot occur in equilibrium.

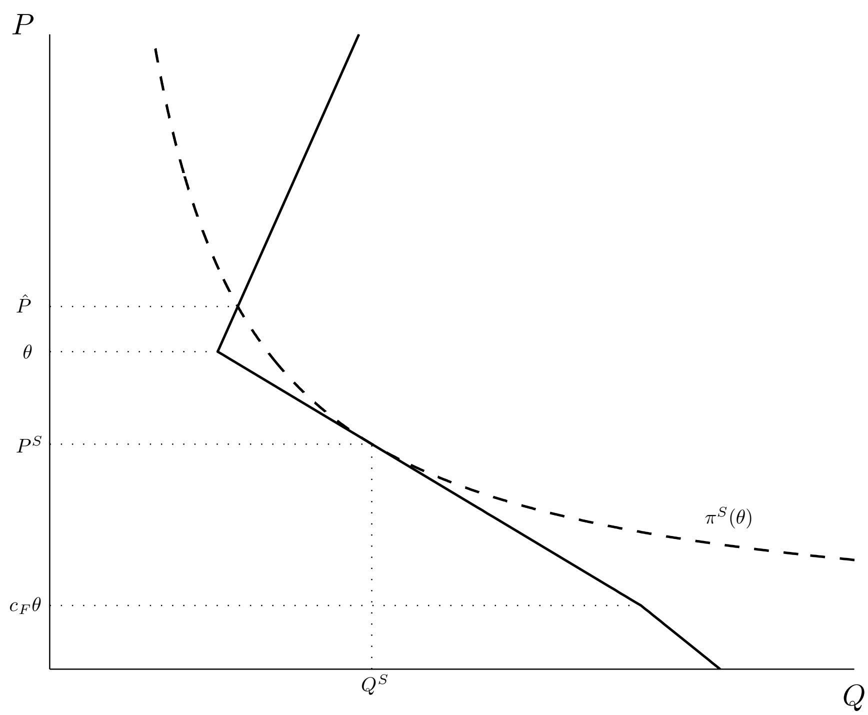

Figure 6: Weak Fringe Competition with Low Cost

[26]To generate Figures 6, and 7, we set $\theta = 1$, $c_M = 0.1$, and $\lambda = 0.5$. Moreover, $\varphi = 0.1$ so that the competitive fringe is weak and $c_F = \{0.2, 0.9\}$ for low vs. high cost.

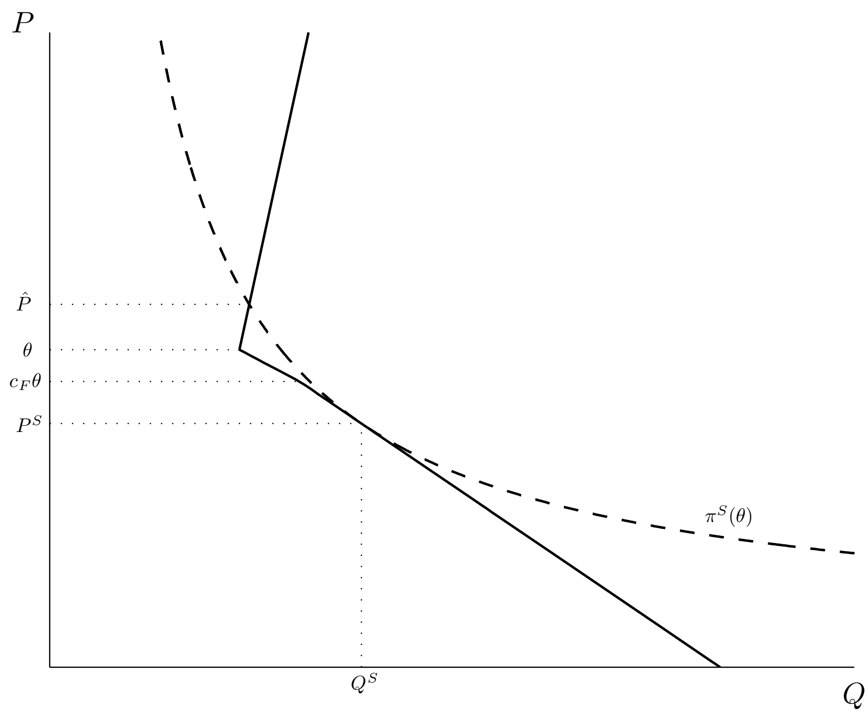

Figure 7: Weak Fringe Competition with High Cost

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