# Spanned and unspanned macro risk in the yield curve


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**Abstract**

This paper analyzes the predictive content of macroeconomic information for the yield curve of interest rates and excess bond returns within a dynamic factor model of yields and macroeconomic data. The model uses macroeconomic information for both extracting the yield curve factors and identifying the sources of unspanned risk. Estimation is performed using the Restricted EM algorithm and Kalman filter using US data from January 1970 to December 2009. Results show that: 1) the federal funds rate and money contain useful information only to extract the yield curve factors; 2) real variables are the primary source of unspanned risk; 3) nominal variables contain both information that is related to the yield curve and unspanned risk. The estimated factors explain up to 47% of the bond risk premium and have superior predictive ability than the Cochrane and Piazzesi (2005) and the Ludvingson and Ng (2009) factors jointly.

# 1 Introduction

Empirical evidence on yield curve modeling and forecasting suggests that augmenting the yield curve factors with macroeconomic indicators improves the predictive ability of yield curve models. On the other hand, recent evidence finds that factors with negligible impact on yields are the main drivers of bond risk premia. In this paper, we propose a joint model for the yield curve of interest rates and macroeconomic variables to identify the sources of unspanned macroeconomic risk and to investigate whether, and to which extent, macroeconomic information is useful for predicting both the yield curve and excess bond returns. The proposed macro-yields model is a state-space model that exploits the co-movements between yields and macroeconomic variables and that, at the same time, allows for unspanned macroeconomic risk, i.e. additional macroeconomic factors that have negligible effects on the cross-section of yields but that contain important information to forecast excess bond returns.

We model the linkages between the yield curve and the macroeconomic variables allowing yields and macroeconomic variables to be driven by the same sources of co-movements. This is in contrast with the macro-finance literature, see e.g. Ang and Piazzesi (2003), Ang, Piazzesi and Wei (2006) and Mönch (2008), where the interactions between yields and macroeconomic variables are modeled augmenting the yield curve factors with observable or latent macroeconomic factors. We deviate from this approach that assumes the macroeconomic factors are additional factors of the yield curve for three reasons. First, the idea behind factor models is parsimony and augmenting the number of factors goes against this notion, specially if the three yield curve factors already explain most of the variation of the yields. Second, the yield curve factors are highly correlated with measures of inflation and economic activity, see e.g. Diebold, Rudebusch and Aruoba (2006), therefore adding macroeconomic factors in the observation equation of the yields can be redundant. Third, our objective is to disentangle the variation in the macroeconomic variables that is common to the yield curve from the one that is unspanned by the cross-section of yields.

The macro-yields model proposed in this paper allows for any number of unspanned latent factors and does not require to specify a priori which macroeconomic variables are spanned by the yield curve nor which ones are unspanned. We use empirical evidence on the in-sample and out-of-sample performance of the model for both yields and excess bond returns to select the number of unspanned macroeconomic factors. Cochrane and Piazzesi (2005) find that a linear combination of forward rates is successful in explaining the bond risk premium, while principal components of the yield curve can account only for a small part of this predictability. In addition, Ludvigson and Ng (2009) show that macroeconomic variables are the main drivers of bond risk premia. Following these findings, Duffee (2011) builds a five-factor model with two unspanned factors, i.e. factors that cannot be inferred from the cross-section of yields but that have significant predictive content for excess returns. These hidden factors have opposite effects on bond risk premia and on expected future short rates. Joslin, Priebsch and Singleton (2010) build five factor model with two hidden macroeconomic factors, i.e. output growth and inflation. Both Duffee (2011) and Joslin et al. (2010) use an affine dinamic term structure model where explicit assumptions about the behavior of the time-varying risk premium have to be made and, in particular, about which factors drive the risk premium. In this paper, we combine the reduced form approach of Cochrane and Piazzesi (2005) and Ludvigson and Ng (2009) with a joint model for the yield curve and macroeconomic variables without, however, imposing any assumption on the behavior of the market price of risk.

The paper is organized as follows. Section 2 presents the proposed macro-yields model. Section 3 desribes the estimation procedure and the information criteria approach for model selection. Section 4 introduces the data and some preliminary empirical evidence. Empirical results about the estimated factors, the fit and forecast of the yields are contained in Section 5, while Section 5.1 contains results about the predictive regressions and prediction of excess bond returns.

## 2 The Macro-Yields Model

The macro-yields model is a dynamic factor model for the joint behavior of government bond yields and macroeconomic indicators. The yields with different maturities are driven by the Nelson and Siegel (1987) yield curve factors, while the macroeconomic indicators are driven both by the yield curve and a few macroeconomic-specific factors, the unspanned macro factors. All variables in the model have an autocorrelated idiosyncratic component, and the joint dynamic of the yield curve and macroeconomic factors follow a VAR(1). In what follows we detail on each of the points.

We assume that yields on bonds with different maturities are driven by three common factors and an idiosincratic component

$$y_t \ = \ a_y + \ \Gamma_{yy} \ F_t^y + v_t^y, \tag{1}$$

where $y_t$ is a $N_y \times 1$ vector of yields with $N_y$ different maturities at time $t$, $\Gamma_{yy}$ is a $N_y \times 3$ matrix of factor loadings, $F_t^y$ is a $3 \times 1$ vector of latent factors at time $t$ and $v_t^y$ is an $N_y \times 1$ vector of idiosincratic components. We identify the yield curve factors $F_t^y$ as the Nelson and Siegel (1987) factors imposing that

$$a_y = 0; \quad \Gamma_{yy}^{(i)} = \left[ 1 \ \ \frac{1-e^{-\lambda\tau_i}}{\lambda\tau_i} \ \ \frac{1-e^{-\lambda\tau_i}}{\lambda\tau_i} - e^{-\lambda\tau_i} \right], \tag{2}$$

where $\Gamma_{yy}^{(i)}$ is the $i$-th row of the matrix of factor loadings, $\lambda$ is a decay parameter of the factor loadings and $\tau_i$ denotes the maturity of the $i$-th bond. Diebold and Li (2006) show that this functional form of the factor loadings, implies that the three yield curve factors can be interpreted as the level, slope, and curvature of the yield curve. Indeed, the loading equal to one on the first factor, for all maturities, implies that an increase in this factor increases all yields equally, shifting the level of the yield curve. The loadings on the second factor are high for short maturities, decaying to zero for the long ones. Accordingly, an increase in the second factor increases the slope of the yield curve. Loadings on the third factor are zero for the shortest and the longest maturities, reaching the maximum for medium maturities. Therefore, an increase in this factor augments the curvature of the yield curve. Given these particular functional forms for the loadings on the three yield curve factors, one can thus disentangle movements in the term structure of interest rates into three factors which have a clear-cut interpretation. The parameter $\lambda$ governs the exponential decay rate: a small value of $\lambda$ can better fit the yield curve at long maturities, while large values can better fit it at short maturities. This parameter determines the maturity at which the loadings on the curvature factor reaches the maximum.

We further assume that macroeconomic variables are driven by the yield curve factors, a few macroeconomic-specific factors and an idiosyncratic component

$$x_t \;=\; a_x + \Gamma_{xy}\; F_t^y + \Gamma_{xx}\; F_t^x + v_t^x, \tag{3}$$

where $a_x$ is an $N_x \times 1$ vector of intercepts, $x_t$ is a $N_x \times 1$ vector of macroeconomic variables at time $t$, $\Gamma_{xy}$ is a $N_x \times 3$ matrix of factor loadings on the yield curve factors, $\Gamma_{xx}$ is a $N_x \times r$ matrix of factor loadings on the macro factors, $F_t^x$ is an $r \times 1$ vector of macroeconomic latent factors (normalized to have zero mean and unit variance) and $v_t^x$ is an $N_x \times 1$ vector of idiosincratic components.

We consider equation (1) and (3) in the unified framework of a macro-yields model as follows

$$\begin{pmatrix} y_t \\ x_t \end{pmatrix} \;=\; \begin{pmatrix} a_y \\ a_x \end{pmatrix} + \begin{bmatrix} \Gamma_{yy} & \Gamma_{yx} \\ \Gamma_{xy} & \Gamma_{xx} \end{bmatrix} \begin{pmatrix} F_t^y \\ F_t^x \end{pmatrix} + \begin{pmatrix} v_t^y \\ v_t^x \end{pmatrix}, \tag{4}$$

where $\Gamma_{yx} = 0$. By construction, yields only load on the yield curve factors $F_t^y$, while macroeconomic variables load on both the yield curve $F_t^y$ and the macro factors $F_t^x$. This allows $F_t^x$ to capture the source of co-movement in the macroeconomic variables that is not accounted by the yield curve factors, i.e. the unspanned macroeconomic risk.

The joint dynamics of the yield curve and the macroeconomic factors follow a VAR(1)

$$\begin{pmatrix} F_t^y \\ F_t^x \end{pmatrix} \;=\; \begin{pmatrix} \mu_y \\ \mu_x \end{pmatrix} + A \begin{pmatrix} F_{t-1}^y \\ F_{t-1}^x \end{pmatrix} + \begin{pmatrix} u_t^y \\ u_t^x \end{pmatrix}, \quad \begin{pmatrix} u_t^y \\ u_t^x \end{pmatrix} \sim N(0, Q) \tag{5}$$

where $Q$ is a diagonal matrix and $\mu_x = 0$. In addition, we assume that the idiosincratic components collected in $v_t = [v_t^y \quad v_t^x]'$ follow a univariate AR(1) process

$$v_t = B v_{t-1} + \xi_t, \quad \xi_t \sim N(0, R) \tag{6}$$

where $B$ and $R$ are diagonal matrices. Furthermore, the shocks to the idiosyncratic components of the individual variables, $\xi_t$, and the innovations driving the common factors, $u_t$, are assumed to be mutually independent.

We use of a dynamic factor for four reasons. First, due to the high level of co-movement of yields with different maturities, three yield curve factors can explain most of the variation in the yield curve, see e.g. Litterman and Scheinkman (1991). Second, macroeconomic variables are also characterized by a high degree co-movement and the bulk of their dynamics is explained a few common factors, see e.g. Sargent and Sims (1977), Stock and Watson (2002) and Giannone, Reichlin and Sala (2005). Third, using a factor model allows to use a large number of macroeconomic variables preserving the parsimony of the model. Alternatively, one could use a few selected macroeconomics indicators but this raises the problem of which indicators to use. Fourth, a common objection against empirical macro-finance models is that data revisions imply that the information set available to the econometrician is different from the information set available to investors. This could imply that estimates of the parameters governing the mutual interactions between macroeconomic and financial variables may be biased. However, data revision are typically series specific and hence have negligible effects when we extract the common factors, see e.g. Bernanke and Boivin (2003) and Giannone et al. (2005).

As shown in (2), we use restrictions on the factor loadings of the yields on the yield curve factors to identify the Nelson and Siegel (1987) factors. This choice is determined by the fact that empirically the Nelson-Siegel model fits the yield curve well and performs well in out-of-sample forecasting exercises, as shown by Diebold and Li (2006) and De Pooter, Ravazzolo and van Dijk (2007). Moreover, Joslin, Singleton and Zhu (2011) show that any linear combination of yields can serve as observable factors in a no-arbitrage model and Coroneo, Nyholm and Vidova-Koleva (2011) fail to reject the null that the Nelson and Siegel (1987) is statistically different from a gaussian affine term structure model. However, the approach proposed in the paper can also be used without imposing the Nelson and Siegel (1987) restrictions and just normalizing the yield curve factors to have zero mean and unit variance.

# 3 Estimation

The maximum likelihood estimators of the parameters of the macro-yields model are not available in closed form, as the yield curve and the macro factors are unobserved. One possibility is to maximize numerically the likelihood function but this is computationally demanding, due to the large number of parameters. For this reason, we estimate the macro-yields model by maximum likelihood, combining the Expectation Restricted Maximization (ERM) algorithm and the Kalman filter. This procedure allows us to consistently estimate the macro-yields model using a large number of variables and to successfully restrict the factor loadings to identify the yield curve factors.

The ERM algorithm is a generalization of the Expectation Maximization (EM) algorithm introduced by Shumway and Stoffer (1982) and derived in detail for dynamic factor models Ghahramani and Hinton (1996). Doz, Giannone and Reichlin (2006) show that the EM algorithm procedure makes maximum likelihood estimation of approximate factor models feasible for large cross sections and that consistency is guaranteed even when the hypothesis of orthogonality and absence of serial correlation of the idiosyncratic component are violated. This is particularly important in our case since, even if the macro-yields model has the form of an exact factor model, the assumption of orthogonal idiosyncratic elements is likely to be too restrictive. However, despite the fact that this procedure provides asymptotically valid estimates even when the assumption of absence of serial correlation in the idiosyncratic component assumption is violated, we explicitly model the idiosyncratic components as AR(1). This allows to exploit the persistency in the idiosyncratic component, improving the predictions of the model, see Stock and Watson (2002).

The macro-yields model in equations (4), (5) and (6) can be written in a compact form as

$$\begin{aligned} z_t &= a + \Gamma F_t + v_t, \\ F_t &= \mu + A F_{t-1} + u_t, \quad u_t \sim N(0, Q) \\ v_t &= B v_{t-1} + \xi_t, \qquad\quad \xi_t \sim N(0, R) \end{aligned}$$

where $Q$, $B$ and $R$ are diagonal matrices and $a = \begin{bmatrix} 0 \\ a_x \end{bmatrix}$, $\Gamma = \begin{bmatrix} \Gamma_{yy} & 0 \\ \Gamma_{xy} & \Gamma_{xx} \end{bmatrix}$, $\mu = \begin{bmatrix} \mu_y \\ 0 \end{bmatrix}$ and $\Gamma_{yy}$ satisfies the Nelson and Siegel (1987) restrictions in (2). Following Diebold and Li (2006), we fix the decay parameter $\lambda = 0.0609$ to the value that maximizes the loading on the curvature factor for the yields with maturity 30 months.[1]

[1] Using the Expectation Conditional Restricted Maximization (ECRM) algorithm is also possible to estimate $\lambda$.

To put the model in a state-space form, we augment the states with the idiosyncratic components and a constant as follows

$$\begin{aligned} z_t &= \Gamma^* F_t^* + v_t^*, \quad v_t^* \sim N(0, R^*) \\ F_t^* &= A^* F_{t-1}^* + u_t^*, \quad u_t^* \sim N(0, Q^*) \end{aligned}$$

where $\Gamma^* = \begin{bmatrix} \Gamma & a & I_n \end{bmatrix}$, $F_t^* = \begin{bmatrix} F_t \\ c_t \\ v_t \end{bmatrix}$, $A^* = \left[\begin{array}{cc|c|c} A & \mu & \dots & 0 \\ \hline \vdots & \iddots & 1 & \vdots \\ \hline 0 & \dots & \dots & B \end{array}\right]$, $u_t^* = \begin{bmatrix} u_t \\ \nu_t \\ \xi_t \end{bmatrix}$, $Q^* = \left[\begin{array}{c|c|c} Q & \dots & 0 \\ \hline \vdots & \varepsilon & \vdots \\ \hline 0 & \dots & R \end{array}\right]$ and $R = \varepsilon I_n$, with $\varepsilon$ a very small fixed coefficient and $c_t$ an additional state variable restricted to one.

The restrictions on the factor loadings $\Gamma^*$ and on the transition matrix $A^*$ can be written as

$$H_1 \ \text{vec}(\Gamma^*) = q_1, \qquad H_2 \ \text{vec}(A^*) = q_2, \tag{7}$$

where $H_1$ and $H_2$ are selection vectors, and $q_1$ and $q_2$ contain the restrictions.

We assume that $F_1^* \sim N(\pi_1, V_1)$ and define $y = [y_1, \dots, y_T]$ and $F^* = [F_1^*, \dots, F_T^*]$. Then denoting the parameters by $\theta = \{\Gamma^*, A^*, Q^*, \pi_1, V_1\}$, we can write the joint loglikelihood of $z_t$ and $F_t$, for $t = 1, \dots, T$, as

$$\begin{aligned} L(z, F^*; \theta) \quad = \quad & -\sum_{t=1}^{T} \left( \frac{1}{2} \left[ z_t - \Gamma^* F_t^* \right]' (R^*)^{-1} \left[ z_t - \Gamma^* F_t^* \right] \right) + \\ & -\frac{T}{2} \log |R^*| - \sum_{t=2}^{T} \left( \frac{1}{2} [F_t^* - A^* F_{t-1}^*]' (Q^*)^{-1} [F_t^* - A^* F_{t-1}^*] \right) + \\ & -\frac{T-1}{2} \log |Q^*| + \frac{1}{2} [F_1^* - \pi_1]' V^{-1} [F_1^* - \pi_1] + \\ & -\frac{1}{2} \log |V_1| - \frac{T(p+k)}{2} \log 2\pi + \lambda_1' \left( H_1 \, \text{vec}(\Gamma^*) - q_1 \right) + \lambda_2' \left( H_2 \, \text{vec}(A^*) - q_2 \right) \end{aligned}$$

where $\lambda_1$ contains the lagrangian multipliers associate with the constraints on the factor loadings $\Gamma^*$ and $\lambda_2$ contains the lagrangian multipliers associated with the constraints on the transition matrix $A^*$.

The ERM algorithm alternates Kalman filter extraction of the factors to the restricted maximization of the likelihood. In particular, at the $j$-th iteration the ERM algorithm performs two steps:

1. In the Expectation-step, we compute the expected log-likelihood conditional on the data and the estimates from the previous iteration, i.e.

$$\mathcal{L}(\theta) = E[L(z, F^*; \theta^{(j-1)}) | z]$$

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This algorithm allows to perform numerical maximization of the conditional likelihood with respect to $\lambda$, but, despite the increase in the computation burden, the results remain substantially unchanged.

which depends on three expectations

$$\begin{aligned}\hat{F}_t^* &\equiv E[F_t^*;\theta^{(j-1)}|z] \\ P_t &\equiv E[F_t^*(F_t^*)';\theta^{(j-1)}|z] \\ P_{t,t-1} &\equiv E[F_t^*(F_{t-1}^*)';\theta^{(j-1)}|z]\end{aligned}$$

These expectations can be computed, for given parameters of the model, using the Kalman filter.

2. In the Restricted Maximization-step, we update the parameters maximizing the expected log-likelihood with respect to $\theta$:

$$\theta^{(j)} = \arg\max_{\theta} \mathcal{L}(\theta)$$

This can be implemented taking the corresponding partial derivative of the expected log likelihood, setting to zero, and solving.

In practise, the estimation problem is reduced to a sequence of simple steps, each of which uses the Kalman smoother and two multivariate regressions. We initialize Nelson and Siegel (1987) factors using the two-steps OLS procedure introduced by Diebold and Li (2006). We then project the macroeconomic variables on the Nelson and Siegel (1987) factors and use the principal components of the residuals of this regression to initialize the macroeconomic factors. Initial values for $A^*$ and $Q^*$ are obtained estimating a VAR(1) on the initial factors.

### 3.1 Model Selection

The macro-yields model decomposes variations in yields and macroeconomic variables into yield curve factors and unspanned macroeconomic factors. The yield curve factors are identified as the Nelson and Siegel (1987) factors which have a clear interpretation as level, slope and curvature. However, the true number of unspanned macroeconomic factors is not known. We can select the optimal number of factors using an information criteria approach. The idea is to choose the number of factors that maximizes the general fit of the model using a penalty function to account for the loss in parsimony.

Bai and Ng (2002) derive information criteria to determine the number of factors in approximate factor models when the factors are estimated by principal components. They also show that their $IC_3$ information criterion can be applied to any consistent estimator of the factors provided that the penalty function is derived from the correct convergence rate. For the quasi-maximum likelihood estimator, Doz et al. (2006) show that it converges to the true value at a rate equal to

$$C_{NT}^{*2} = \min\left\{\sqrt{T}, \frac{N}{\log N}\right\} \tag{8}$$

where $N$ and $T$ denote the cross-section and the time dimension, respectively. Thus, a modified Bai and Ng (2002) information criterion that can be used to select the optimal number of factor

Table 1: Macroeconomic Variables

| Series N. | Mnemonic | Description | Transformation |
|---|---|---|---|
| 1 | AHE | Average Hourly Earnings: Total Private | 2 |
| 2 | CPI | Consumer Price Index: All Items | 2 |
| 3 | INC | Real Disposable Personal Income | 2 |
| 4 | FFR | Effective Federal Funds Rate | 0 |
| 5 | IP | Industrial Production Index | 2 |
| 6 | M1 | M1 Money Stock | 2 |
| 7 | Manf | ISM Manufacturing: PMI Composite Index (NAPM) | 0 |
| 8 | Paym | All Employees: Total nonfarm | 2 |
| 9 | PCE | Personal Consumption Expenditures | 2 |
| 10 | PPIc | Producer Price Index: Crude Materials | 2 |
| 11 | PPIf | Producer Price Index: Finished Goods | 2 |
| 12 | CU | Capacity Utilization: Total Industry | 1 |
| 13 | Un | Civilian Unemployment Rate | 1 |

This table lists the 13 macro variables used to estimate the macro-yields. Most series have been subjected to some transformation prior to the estimation, as reported in the last column of the table. The transformation codes are: 0 = no transformation, 1 = monthly growth rate and 3 = annual growth rate.

when estimation is performed by maximum likelihood is as follows

$$IC^*(s) = \log(V(s, \hat{F}^*_{(s)})) + s\, g(N,T), \quad g(N,T) = \frac{\log C^{*2}_{NT}}{C^{*2}_{NT}} \tag{9}$$

where $s$ denotes the number of factors, $\hat{F}^{(s)}$ are the estimated factors and $V(s, \hat{F}^*_{(s)})$ is the sum of squared idiosyncratic components (divided by NT) when $s$ factors are estimated. The penalty function $g(N,T)$ is a function of both $N$ and $T$ and depends on $C^{*2}_{NT}$, the convergence rate of the estimator, in our case given by (8).

# 4 Data and Empirical Evidence

We use monthly data spanning the period 1970:1-2009:12. The bond yield data are taken from the Fama-Bliss dataset available from the Center for Research in Securities Prices (CRSP) and contain observations on one- through five-year zero-coupon U.S. Treasury bond prices. The macroeconomic dataset consists of 13 macroeconomic variables, which include five inflation measures, six real variables, the federal funds rate and a money indicator. Table 1 contains a complete list of the macroeconomic variables along with the transformation applied to ensure stationarity. We use annual growth rates for all variables, except for capacity utilization and the unemployment rate (in monthly growth rates) and the federal funds rate and manufacturing index (in levels).

As preliminary analysis, we extract the Nelson and Siegel (1987) factors by ordinary least squares, as in Diebold and Li (2006). Table 2 reports the cumulative share of variance of the yields explained by the Nelson and Siegel (1987) level, slope and curvature factors. It is clear that the

Table 2: Yields and share of variance explained by the Nelson and Siegel factors

| Maturity | L | L+S | L+S+C |
|---|---|---|---|
| 12 | 0.59 | 0.85 | 1.00 |
| 24 | 0.63 | 0.79 | 0.99 |
| 36 | 0.68 | 0.79 | 1.00 |
| 48 | 0.72 | 0.79 | 0.99 |
| 60 | 0.76 | 0.82 | 1.00 |

This table lists the 5 maturities of government bond yields used for the estimation of the macro-yields model. The second to fourth columns provide, for each maturity, the cumulative shares of variance explained by the Nelson and Siegel (1987) level (L), slope (S), and curvature (C) factor, respectively.

Table 3: Correlations of macroeconomic variables with the Nelson and Siegel factors

| | L | S | C |
|---|---|---|---|
| AHE | 0.31 | 0.53 | 0.24 |
| CPI | 0.48 | 0.56 | 0.23 |
| INC | 0.08 | 0.05 | 0.33 |
| FFR | 0.78 | 0.64 | 0.43 |
| IP | 0.04 | 0.11 | 0.23 |
| M1 | 0.33 | -0.39 | -0.22 |
| Manf | -0.09 | -0.07 | 0.02 |
| Paym | 0.14 | 0.32 | 0.34 |
| PCE | 0.50 | 0.36 | 0.43 |
| PPIc | -0.04 | 0.34 | -0.07 |
| PPIf | 0.23 | 0.57 | 0.07 |
| CU | 0.01 | -0.19 | 0.01 |
| Un | -0.03 | 0.11 | -0.08 |

This table list the correlations of the 13 macro variables used in the macro-yields model with the estimated Nelson and Siegel (1987) Level (L), Slope (S) and Curvature (C) factors, respectively.

Table 4: Model Selection

| s | $IC^*(s)$ | $V(s, \hat{F}^*_{(s)})$ |
|---|---|---|
| 3 | 0.15 | 0.48 |
| 4 | 0.07 | 0.33 |
| 5 | 0.08 | 0.25 |
| 6 | 0.25 | 0.22 |
| 7 | 0.31 | 0.17 |
| 8 | 0.58 | 0.17 |

This table reports the information criterion $IC^*(s)$, as shown in (9) and (8), and the sum of the variance of the idiosyncratic components (divided by $NT$), $V(s, \hat{F}^*_{(s)})$, when $s$ factors are estimated.

Nelson and Siegel (1987) factors achieve an almost exact fit of the yields. The level explains most of the variation of yields, especially for long maturities. The level and the slope factors jointly explain about 80% of the variance of the yields, and adding the curvature factor we can explain almost 100% of the variance of the yields, leaving virtually no space for any other additional factor. We also compute the correlations of the extracted Nelson and Siegel (1987) factors with the macroeconomic variables. Results, displayed in Table 3, show that the federal funds rate, money, inflation and economic activity are highly correlated with the yield curve factors, as as also shown by Diebold et al. (2006), suggesting that the yield curve co-moves with the rest of the economy. This combined with the fact the the three Nelson and Siegel (1987) factors explain most of the variation in the yield curve, supports our choice of allowing the macroeconomic variables to be driven by the yield curve factors, instead of adding macroeconomic indicators, or macroeconomic factors, as additional factors in the observation equation of the yields.

# 5 Results

We estimate the macro-yields model in equations (4)–(6) by quasi-maximum likelihood as described in section 3 on the full sample of data, from January 1970 to December 2009.

As explained in Section 3.1, we need to select the number of unspanned factors. To this end, we estimate the macro-yields model allowing from three, i.e. only the Nelson and Siegel (1987) factors, up to a total of eight factors, where the first three factors are yield curve factors and the others are unspanned macro factors. Table 4 reports the information criterion, as shown in Equation (9), and the sum of the variance of the idiosyncratic components for each specification of the macro-yields model. The information criterion selects the model with the three Nelson and Siegel (1987) yield curve factors plus one unspanned factor, i.e. $s = 4$. This is also confirmed by the fact that the strongest reduction in the sum of the variances of the idiosyncratic components is obtained passing from the three to the four factors specification. However, while this model is only marginally

Table 5: Share of variance explained by the macro-yields model with 5 factors

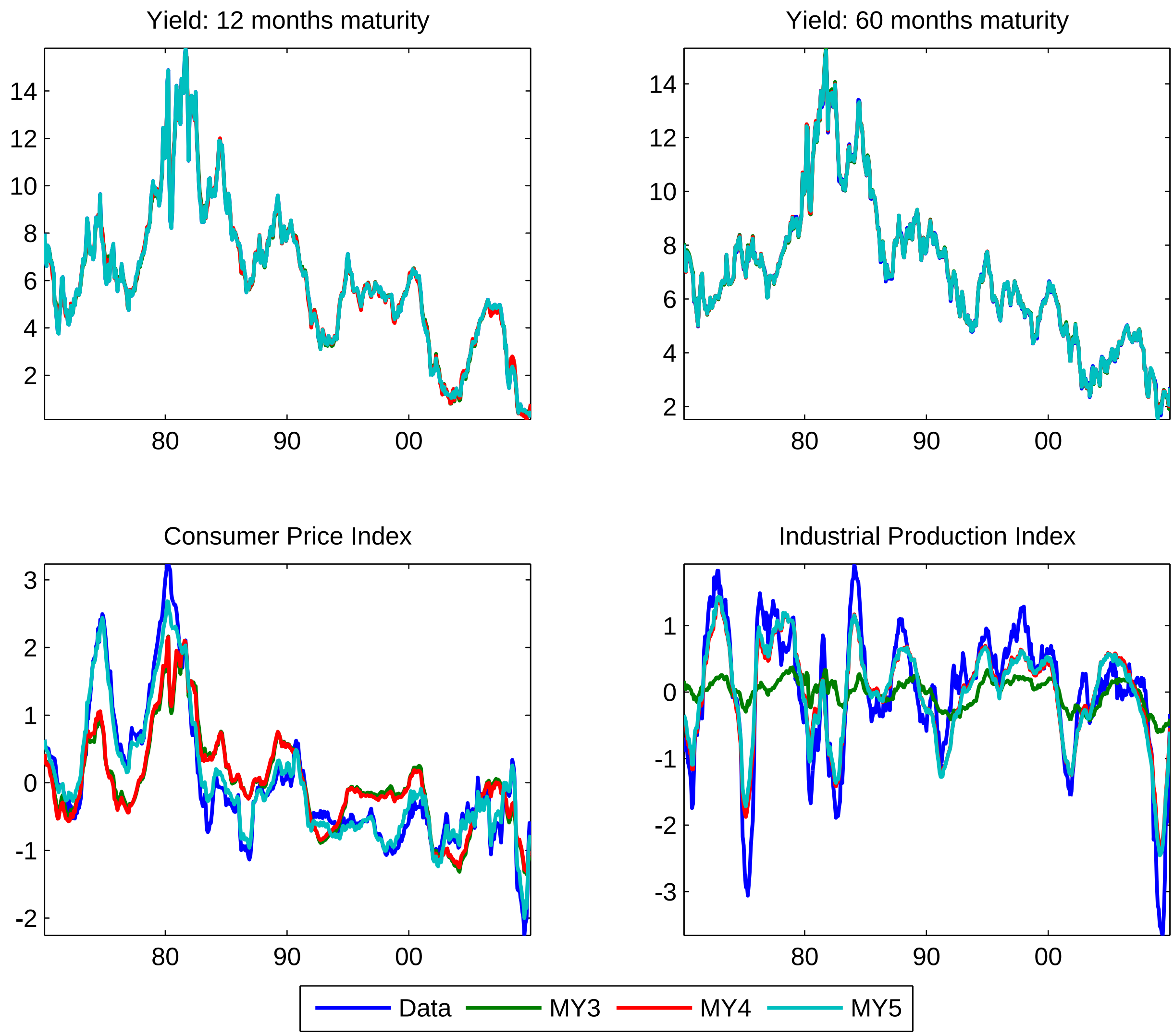
| Variable | $F^y$ | M1 | M2 | Total |
|---|---|---|---|---|
| y12 | 1.00 | 0.00 | 0.00 | 1.00 |
| y24 | 0.99 | 0.00 | 0.00 | 0.99 |
| y36 | 1.00 | 0.00 | 0.00 | 1.00 |
| y48 | 1.00 | 0.00 | 0.00 | 1.00 |
| y60 | 1.00 | 0.00 | 0.00 | 1.00 |
| AHE | 0.36 | 0.00 | 0.28 | 0.63 |
| CPI | 0.52 | 0.01 | 0.35 | 0.85 |
| INC | 0.07 | 0.25 | 0.02 | 0.35 |
| FFR | 0.97 | 0.00 | 0.00 | 0.97 |
| IP | 0.04 | 0.50 | 0.01 | 0.56 |
| M1 | 0.28 | 0.00 | 0.01 | 0.30 |
| Manf | 0.02 | 0.49 | 0.00 | 0.52 |
| Paym | 0.15 | 0.37 | 0.00 | 0.55 |
| PCE | 0.39 | 0.16 | 0.19 | 0.78 |
| PPIc | 0.13 | 0.15 | 0.19 | 0.53 |
| PPIf | 0.38 | 0.00 | 0.47 | 0.84 |
| CU | 0.04 | 0.20 | 0.02 | 0.23 |
| Un | 0.02 | 0.22 | 0.03 | 0.25 |

This table reports the share of variance of the yields and macro variables explained by the Nelson and Siegel (1987) factors (denoted by $F^y$) and the two unspanned macroeconomic factors (denoted by M1 and M2). The last column reports the total share of variance explained by the macro-yields model with 5 factors.

preferred to the model with five factors, the model with only the three Nelson and Siegel (1987) factors is clearly not able to capture the joint dynamic of the yields and macroeconomic variables. This indicates that, even if the macro variables are highly correlated with the yield curve factors, as shown in Table 3, we still need at least one additional factor to capture the co-movements in the macroeconomic variables that is unspanned by the yields.

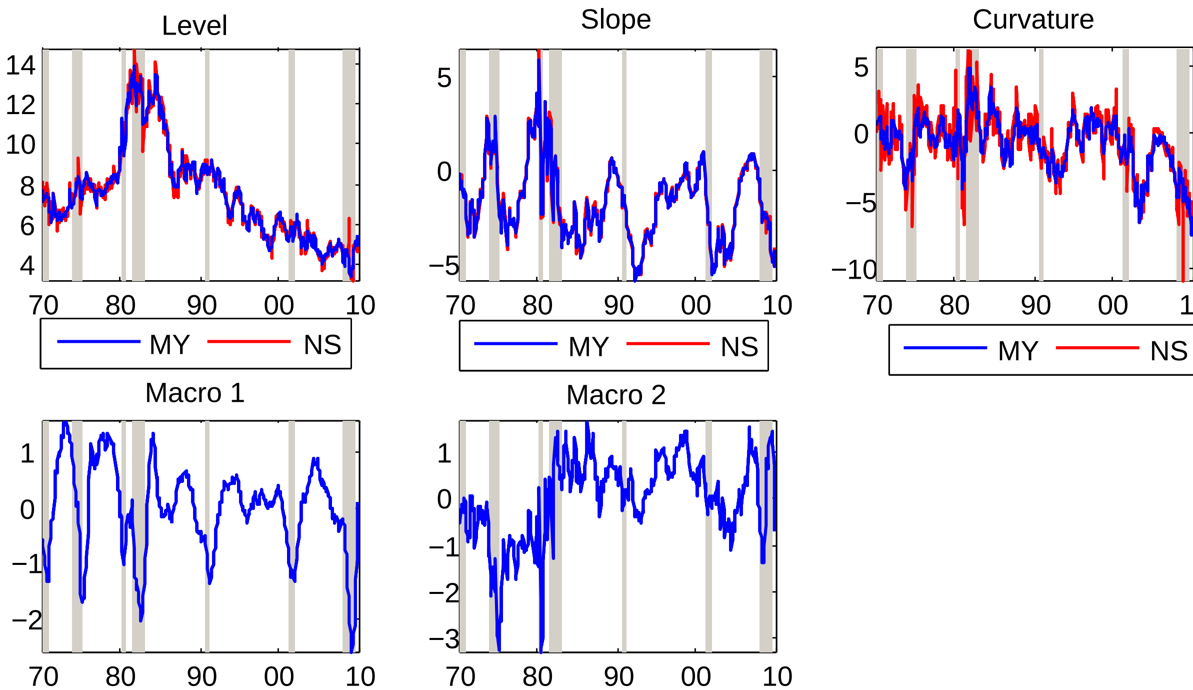
Figure 1 shows the in-sample fit of three specifications of the macro-yields model, i.e. with 3, 4 and 5 factors, for the yields with 12 and 60 months maturity, the consumer price index and the industrial production index. All the three specification of the macro-yields model provide a similar fit for the yields. This is due to the fact that the observation equation of the yields contains only the three Nelson and Siegel (1987) factors and, as shown in Table 2, they are able to capture well the yield curve dynamics. Results in Figure 1 also indicate that the fourth factor captures the dynamics of the industrial production index, while the fifth factor explains the dynamics of the consumer price index that are unspanned by the yield curve factors. This evidence that the fourth factor is a real factor and that the fifth factor is a nominal factor is confirmed by Table 5,

Figure 1: Macro-yields model in-sample fit: model selection




The figure displays the observed data in blue and the corresponding in-sample fit of the macro-yields model for different specifications of the model. The green line refers to the macro-yields model with only three Nelson and Siegel (1987) yield curve factors. The red line refers to the macro-yields model with four factors and the light blue line refers to the macro-yields model with five factors. The upper left plot refers to the yields with maturity 12 months, the upper right to the yields with maturity 60 months, the lower left to the consumer price index and the lower right to the industrial production index.

Figure 2: Macro-yields factors




The figure displays the estimated macro-yields factors (MY). The red lines in the top graphs refer to the Nelson and Siegel (1987) yield curve factors (NS) estimated by ordinary least squares as in Diebold and Li (2006). The grey-shaded areas indicate the recessions as defined by the NBER.

which reports the share of variance of the yields and macro variables explained by the macro-yields factors. The Nelson and Siegel (1987) factors explain most of the variance of the yields and the federal funds rate but also the bulk of the variance of price indices, nominal earnings, nominal consumption and money. The fourth factor captures the dynamics of industrial production and other real variables, while the fifth factor mainly explains the producer price index of finished gods and other nominal variables. Thus, Table 5 indicates that 1) the federal funds rate and money contain only useful information to extract the yield curve factors; 2) real variables are the primary source of unspanned risk; 3) nominal variables contain both information that is spanned by the yield curve and unspanned information.

Figure 2 displays the estimated macro-yields factors. The first three plots report the yield curve factors, while the last two refer to the unspanned factors. The estimated yield curve factors of the macro-yields model are highly correlated with the Nelson and Siegel (1987) factors. However, Figure 2 shows that there are some differences especially for the curvature and the level. This is due to the fact that, in the macro-yields model, the yield curve factors are common factors for the yield curve and the macroeconomic variables. In practice, we extract the yield curve factors from both yields and macroeconomic variables and impose the Nelson and Siegel (1987) restrictions on the factors loadings of the yields to identify them as yield curve factors. Thus the difference between the Nelson and Siegel (1987) factors and the first three macro-yields factors is due to the effect of the macroeconomic information. The second row of Figure 2 shows the unspanned macro factors. The first macro factor is a business cycle factor that starts to decrease at the beginning of the recessions and reaches the minimum at the end of the recessions. The second macro factor has an opposite behavior, it presents a trough at the beginning of the recession and then increases during the recession.

To evaluate the predictive ability of the macro-yields model, we generate out-of-sample iterative forecasts of the factors

$$E_t(F^*_{t+h}) \equiv \hat{F}^*_{t+h|t} = (\hat{A}^*_{|t})^h \hat{F}^*_{t|t},$$

where $h$ denotes the forecast horizon and $\hat{A}^*_{|t}$ is estimated using the information available till time $t$. We then compute out-of-sample forecasts of the yields given the projected factors

$$E_t(z_{t+h}) \equiv \hat{z}_{t+h|t} = \hat{\Gamma}^*_{|t} \hat{F}^*_{t+h|t}.$$

We forecast 1, 3, 6 and 12 steps ahead the yields estimating each model recursively using data from January 1970 until the time that the forecast is made, beginning in January 1985 to December 2009. We use two evaluation periods, from January 1985 to December 2009 and a smaller evaluation period from January 1985 to December 2003, which excludes the recent financial crisis and can be easy compared with previous works about the predictability of the yield curve, e.g. De Pooter et al. (2007).

To evaluate the prediction accuracy, we use the Mean Square Forecast Error (MSFE), i.e. the average square error in the evaluation period for the $h$-months ahead forecast of the yield with maturity $\tau_i$

$$MSFE_{t_0}^{t_1}(\tau_i, h, M) = \frac{1}{t_1 - t_0 + 1} \sum_{t=t_0}^{t_1} \left( \hat{y}^{(\tau_i)}_{t+h|t}(M) - y^{(\tau_i)}_{t+h} \right)^2, \tag{10}$$

where $t_0$ and $t_1$ denote, respectively, the start and the end of the evaluation period, $y^{(\tau_i)}_{t+h}$ is the realized yield with maturity $\tau_i$ at time $t+h$ and $\hat{y}^{(\tau_i)}_{t+h|t}(M)$ is the h-step ahead forecast of the yield with maturity $\tau_i$ from model $M$ using the information available up to $t$.

Forecast results for yields are usually expressed as relative performance with respect to the random walk, which is a naïve benchmark for yield curve forecasting very difficult to outperform given the high persistency of the yields. The random walk h-steps ahead prediction at time $t$ of the yield with maturity $\tau_i$ is

$$E_t(y^{(\tau_i)}_{t+h}) \equiv \hat{y}^{(\tau_i)}_{t+h|t} = y^{(\tau_i)}_t,$$

where the optimal predictor does not change regardless of the maturity of the yield and the forecast horizon. To measure the relative performance of the macro-yields model with respect to the random walk, we use the Relative MSFE computed as

$$RMSFE_{t_0}^{t_1}(\tau_i, h, M) = \frac{MSFE_{t_0}^{t_1}(\tau_i, h, M)}{MSFE_{t_0}^{t_1}(\tau_i, h, RW)}.$$

Table 5 reports the RMSFE with respect to the random walk for different specifications of the

Table 6: Out-of-sample predictive power for the yields

| | Evaluation 1985 - 2009 | | | | | Evaluation 1985 - 2003 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Maturity | 12 | 24 | 36 | 48 | 60 | 12 | 24 | 36 | 48 | 60 |
| Horizon | **Only Yield Model** | | | | | | | | | |
| 12 | 1.11 | 1.12 | 1.11 | 1.06 | 1.07 | 1.18 | 1.13 | 1.09 | 1.02 | 1.03 |
| 6 | **1.12** | **1.13** | **1.08** | **1.01** | **1.04** | 1.18 | 1.13 | 1.06 | 0.98 | 1.01 |
| 3 | **1.09** | **1.12** | **1.07** | **1.01** | **1.04** | 1.13 | 1.13 | 1.05 | **0.99** | **1.02** |
| 1 | **1.03** | **1.14** | 1.07 | 1.02 | 1.02 | **1.05** | 1.15 | 1.06 | **0.99** | 1.02 |
| Horizon | **Macro-Yield 3 factors** | | | | | | | | | |
| 12 | 1.51 | 1.47 | 1.47 | 1.44 | 1.46 | 1.43 | 1.31 | 1.25 | 1.19 | 1.21 |
| 6 | 1.41 | 1.34 | 1.28 | 1.24 | 1.28 | 1.31 | 1.19 | 1.11 | 1.06 | 1.09 |
| 3 | 1.36 | 1.26 | 1.18 | 1.15 | 1.20 | 1.24 | 1.16 | 1.07 | 1.04 | 1.09 |
| 1 | 1.43 | 1.21 | 1.10 | 1.09 | 1.14 | 1.33 | 1.17 | 1.05 | 1.03 | 1.11 |
| Horizon | **Macro-Yield 4 factors** | | | | | | | | | |
| 12 | 1.09 | 1.11 | 1.14 | 1.16 | 1.19 | **0.94** | **0.92** | **0.92** | **0.89** | **0.90** |
| 6 | 1.32 | 1.24 | 1.21 | 1.20 | 1.24 | **1.00** | **0.99** | **0.97** | **0.96** | **0.99** |
| 3 | 1.37 | 1.20 | 1.14 | 1.12 | 1.17 | **1.09** | **1.03** | **1.01** | 1.02 | 1.07 |
| 1 | 1.69 | 1.15 | **1.06** | 1.06 | 1.12 | 1.43 | **1.06** | **1.00** | 1.03 | 1.12 |
| Horizon | **Macro-Yield 5 factors** | | | | | | | | | |
| 12 | **1.01** | **1.01** | **1.02** | **1.01** | **1.01** | 1.05 | 1.00 | 0.97 | 0.92 | **0.90** |
| 6 | 1.42 | 1.33 | 1.28 | 1.25 | 1.25 | 1.31 | 1.20 | 1.15 | 1.12 | 1.10 |
| 3 | 1.42 | 1.26 | 1.19 | 1.15 | 1.15 | 1.35 | 1.19 | 1.12 | 1.10 | 1.09 |
| 1 | 1.41 | 1.16 | 1.07 | 1.02 | 1.00 | 1.43 | 1.12 | 1.03 | 1.01 | 0.99 |
| Horizon | **Macro-Yield 6 factors** | | | | | | | | | |
| 12 | 1.26 | 1.28 | 1.30 | 1.28 | 1.27 | 1.39 | 1.33 | 1.30 | 1.23 | 1.18 |
| 6 | 1.46 | 1.41 | 1.35 | 1.30 | 1.30 | 1.55 | 1.43 | 1.35 | 1.27 | 1.23 |
| 3 | 1.37 | 1.28 | 1.21 | 1.15 | 1.15 | 1.43 | 1.30 | 1.22 | 1.15 | 1.13 |
| 1 | 1.12 | 1.13 | 1.08 | **1.01** | **0.98** | 1.13 | 1.12 | 1.06 | 1.01 | **0.98** |

This table reports the Relative MSFE of different specifications of the macro-yields model with respect to the random walk. Bold values denote the smallest RMSFE for each maturity and forecast horizon.

macro-yields model for the two evaluation periods considered. We consider four specifications of the macro-yields models (with 3 up to 6 factors) and the only yields model (a restricted version of the macro-yields model where only the yields are used to extract the three yield curve factors and there are no additional macroeconomic factors). For both evaluation periods, results in Table 5 show that the only yield model is outperformed by the macro-yields model for the 12 months forecast horizons and that the macro-yields model with only the three yield curve factors is the worst performing model. This suggests that allowing for unspanned macroeconomic risk improves the out-of-sample predictive ability of the model. For the evaluation sample January 1985–December 2003, the macro-yields model with four factors outperforms the random walk, for medium–long horizons. However, for the longest evaluation sample, results indicate that during the recent financial crisis naive models, as the random walk, have outperformed more sophisticated models indicated a reduced predictability of the yields. The best performing model for middle horizons is the only yields model while for long horizons is the macro-yields model with five factors.[2] This suggests that the nominal factor, i.e. the fifth factor, has been important for capturing the dynamics of the yield curve during the recent financial crisis since, as shown in Figure 2, it provides a signal at the beginning of recessions.

## 5.1 Excess bond returns

We use the macro-yields model to analyze excess returns of government bond by the following simple transformation

$$rx_{t+12}^{(n)} = r_{t+12}^{(n)} - y_t^{(1)} = -(n-1)y_{t+12}^{(n-1)} + ny_t^{(n)} - y_t^{(1)} \tag{11}$$

where $rx_{t+12}^{(n)}$ is the excess return of a $n$-year bond and $r_{t+12}^{(n)}$ is the return of a $n$-year bond. We can rewrite Equation (11) in compact notation as

$$rx_{t+12} = \Pi_1 y_{t+12} + \Pi_2 y_t \tag{12}$$

where $\Pi_1 = \begin{bmatrix} D_{[-1:-K]} & 0_{[K\times 1]} \end{bmatrix}$, $\Pi_2 = \begin{bmatrix} 1_{[K\times 1]} & D_{[2:K+1]} \end{bmatrix}$, $D_{[-1:-K]}$ denotes a diagonal matrix with elements $-1, -2, \ldots, -K$ in the diagonal and $K+1$ denotes the total number of maturities.

The expectation hypothesis of interest rates states that excess bond returns should not be predictable with variables in the information set at time $t$. However, Cochrane and Piazzesi (2005) find that a linear combination of forward rates is successful in explaining the bond risk premium, while the first three principal components of the yield curve can account only for a small part of this predictability. To investigate whether the macro-yields factors have predictive ability for excess bond returns we run predictive regressions of one year excess bond returns on lagged macro-yields factors. Tables 7–8 reports result for the predictive regressions, i.e. estimates from OLS regressions of average excess bond returns on 12 months lagged factors. Results in Table 7 refer to the sample January 1970–December 2003 and are comparable with Cochrane and Piazzesi (2005) and Ludvigson and Ng (2009)[3], while Table 8 refers to the sample January 1970–December 2009

[2]Unreported results but available upon request show that the forecast performance for the evaluation period January 1985–December 2007 is similar to the one of the evaluation sample January 1985–December 2003.

[3]Notice that our sample period starts in 1970 while in Cochrane and Piazzesi (2005) and Ludvigson and Ng (2009)

which includes also the recent financial crisis. In addition to results for different specifications of the macro-yields model (with 3 to 6 factors) and for the only-yields model (a restricted version of the macro-yields model where only the yields are used to extract the three yield curve factors and there are no additional macroeconomic factors), we also report results for the Cochrane and Piazzesi (2005), the Ludvigson and Ng (2009)[4] and the Nelson and Siegel (1987) factors. For each regression, we report the regression coefficients, heteroskedasticity and serial correlation robust t-statistics, and adjusted $R^2$ statistic. We use use the Newey and West (1987) correction for serial correlation with 18 lags to compute the asymptotic standard errors. This correction is needed because the continuously compounded annual return has an MA(12) error structure under the null hypothesis that one-period returns are unpredictable. However, because the Newey and West (1987) correction down-weights higher-order autocorrelations, we follow Cochrane and Piazzesi (2005) and Ludvigson and Ng (2009) and use an 18-lag correction to ensure that the procedure fully corrects for the MA(12) error structure.

Table 7 confirms that the Cochrane and Piazzesi (2005) factors explain about a third of the variation in bond risk premia and that the Ludvigson and Ng (2009) factors have additional predictive ability over the Cochrane and Piazzesi (2005) factors. The Nelson and Siegel (1987) factors do not have any predictive ability above the Cochrane and Piazzesi (2005) factors, in line with the finding of Cochrane and Piazzesi (2005) that the first three principal components of yields do not have any predictive ability for the excess bond returns. The only-yields model has a similar performance than the Nelson and Siegel (1987) model, which suggests that using a state-space model with autocorrelated idiosyncratic components does not improve the predictive ability of the yield curve factors. However, the macro-yields model with three factors explains about a third of the variation on excess bond returns and, in particular, the slope and the curvature factors have predictive ability above the Cochrane and Piazzesi (2005) factor. The only difference between the macro-yields model with three yield curve factors and the only yields model is that the former uses both yields and macroeconomic information to extract the yield curve factors. Thus the fact that the macro-yields model with three factors has superior predictive ability for excess bond returns means that macroeconomic information helps to better extract the yield curve factors. However, the best performing model is the macro-yields model with four factors which explains 47% of the bond premium, higher than the Cochrane and Piazzesi (2005) and the Ludvigson and Ng (2009) factors jointly. If we use the full sample of data, from January 1970 to December 2009, we observe a decline in the predictability of excess bond returns but the conclusions are similar, except for the fact that the the predictive regressions using the macro-yield models with 4, 5 and 6 factors achieve a similar adjusted $R^2$.

To grasp further insight about the predictive ability of the macro-yields factors, we perform an out-of-sample forecast exercise of excess bond returns. The out-of-sample prediction of excess bond returns from the macro-yields model is as follows

$$E_t(rx_{t+12}) \equiv rx_{t+12|t} = \Pi_1(\hat{\Gamma}^*_{|t} F^*_{t+12|t}) + \Pi_2 y_t$$

it starts in 1964.

[4]The Ludvigson and Ng (2009) factors are downloaded from the personal webpage of Sydney Ludvigson and available only up to December 2003.

Table 7: Predictive Regression: January 1970–December 2003

| Factors | CP | L | S | C | M1 | M2 | M3 | LN1 | LN2 | $\overline{R}^2$ |
|---|---|---|---|---|---|---|---|---|---|---|
| CP | **1.00** | | | | | | | | | 0.31 |
| | (7.44) | | | | | | | | | |
| LN | | | | | | | | -0.05 | **1.03** | 0.22 |
| | | | | | | | | (-0.13) | (3.10) | |
| LN+CP | **0.93** | | | | | | | **0.80** | -0.01 | 0.41 |
| | (4.78) | | | | | | | (3.66) | (-0.03) | |
| NS | | 0.34 | **-0.80** | **0.52** | | | | | | 0.22 |
| | | (1.31) | (-3.35) | (3.49) | | | | | | |
| CP + NS | **1.00** | 0.00 | 0.00 | 0.00 | | | | | | 0.30 |
| | (5.02) | (0.00) | (0.00) | (0.00) | | | | | | |
| OY | | 0.23 | **-0.79** | **0.82** | | | | | | 0.25 |
| | | (0.89) | (-3.23) | (3.48) | | | | | | |
| CP + OY | **0.87** | -0.02 | -0.10 | 0.25 | | | | | | 0.31 |
| | (4.38) | (-0.09) | (-0.50) | (0.98) | | | | | | |
| MY3 | | 0.33 | **-1.15** | **0.82** | | | | | | 0.31 |
| | | (1.26) | (-3.86) | (2.26) | | | | | | |
| CP + MY3 | **0.65** | 0.05 | **-0.60** | **0.61** | | | | | | 0.36 |
| | (4.01) | (0.23) | (-1.72) | (1.64) | | | | | | |
| MY4 | | 0.19 | **-1.03** | **1.17** | **-1.42** | | | | | 0.47 |
| | | (0.79) | (-3.78) | (4.24) | (-3.41) | | | | | |
| CP +MY4 | **0.43** | 0.02 | **-0.68** | **0.99** | **-1.33** | | | | | 0.49 |
| | (3.58) | (0.11) | (-2.31) | (3.42) | (-3.14) | | | | | |
| MY5 | | 0.29 | **-0.77** | **0.76** | **-1.53** | **1.45** | | | | 0.44 |
| | | (1.33) | (-3.58) | (3.38) | (-4.16) | (2.71) | | | | |
| CP + MY5 | **0.59** | 0.10 | -0.31 | 0.41 | **-1.40** | **1.31** | | | | 0.47 |
| | (3.02) | (0.43) | (-1.27) | (1.50) | (-3.86) | (2.71) | | | | |
| MY6 | | 0.32 | **-0.74** | **0.69** | **-1.29** | **1.55** | -0.92 | | | 0.43 |
| | | (1.63) | (-3.47) | (3.06) | (-3.46) | (2.74) | (-1.56) | | | |
| CP + MY6 | **0.57** | 0.13 | -0.30 | 0.34 | **-1.17** | **1.42** | -0.78 | | | 0.46 |
| | (2.62) | (0.62) | (-1.22) | (1.21) | (-3.27) | (2.69) | (-1.34) | | | |

The table reports estimates from OLS regressions of average excess bond returns on lagged factors. The dependent variable is the average excess log return on Treasury bonds. The first column report the type of factors used as regressors. CP refers to the Cochrane and Piazzesi (2005) factor, LN refers to the Ludvigson and Ng (2009) factors, NS to the Nelson and Siegel (1987) factors estimated as in Diebold and Li (2006), OY refers to the only-yield model (a restricted version of the macro-yields model where only the yields are used to extract the three yield curve factors and there are no additional macroeconomic factor). MY3, MY4, MY5 and MY6 denote the macro-yields models with 3, 4, 5 and 6 factors. Newey and West (1987) corrected t-statistics have lag order 18 months and are reported in parentheses. Coefficients that are statistically significant at the 10% or better level are highlighted in bold. A constant is always included in the regression.

Table 8: Predictive Regression: January 1970–December 2009

| Factors | CP | L | S | C | M1 | M2 | M3 | $\overline{R}^2$ |
|---|---|---|---|---|---|---|---|---|
| CP | **1.00** | | | | | | | 0.22 |
| | (5.96) | | | | | | | |
| NS | | 0.22 | **-0.66** | **0.31** | | | | 0.14 |
| | | (0.92) | (-2.75) | (2.10) | | | | |
| CP + NS | **1.00** | 0.00 | 0.00 | 0.00 | | | | 0.21 |
| | (4.23) | (0.00) | (0.00) | (0.00) | | | | |
| OY | | 0.16 | **-0.66** | **0.45** | | | | 0.15 |
| | | (0.66) | (-2.78) | (2.13) | | | | |
| CP + OY | **0.96** | -0.01 | -0.03 | 0.06 | | | | 0.21 |
| | (4.25) | (-0.04) | (-0.15) | (0.26) | | | | |
| MY3 | | 0.39 | **-0.72** | -0.01 | | | | 0.16 |
| | | (1.47) | (-2.06) | (-0.02) | | | | |
| CP + MY3 | **0.86** | 0.15 | -0.14 | -0.17 | | | | 0.22 |
| | (5.17) | (0.62) | (-0.42) | (-0.50) | | | | |
| MY4 | | -0.05 | **-0.90** | **0.82** | **-1.91** | | | 0.40 |
| | | (-0.21) | (-3.61) | (2.91) | (-4.18) | | | |
| CP +MY4 | **0.45** | -0.15 | **-0.59** | **0.68** | **-1.85** | | | 0.42 |
| | (3.86) | (-0.66) | (-2.39) | (2.36) | (-4.03) | | | |
| MY5 | | 0.08 | **-0.66** | **0.41** | **-2.03** | **1.12** | | 0.38 |
| | | (0.38) | (-3.32) | (2.17) | (-6.16) | (1.95) | | |
| CP + MY5 | **0.65** | -0.05 | -0.23 | 0.17 | **-1.92** | **1.03** | | 0.41 |
| | (3.16) | (-0.23) | (-1.04) | (0.78) | (-5.90) | (1.97) | | |
| MY6 | | 0.18 | **-0.64** | **0.32** | **-1.92** | **1.40** | **-1.93** | 0.41 |
| | | (1.15) | (-3.34) | (1.82) | (-6.41) | (2.46) | (-2.62) | |
| CP + MY6 | **0.60** | 0.06 | -0.25 | 0.10 | **-1.80** | **1.30** | **-1.75** | 0.44 |
| | (2.71) | (0.35) | (-1.14) | (0.46) | (-6.74) | (2.46) | (-2.35) | |

The table reports estimates from OLS regressions of average excess bond returns on lagged factors. The dependent variable is the average excess log return on Treasury bonds. The first column report the type of factors used as regressors. CP refers to the Cochrane and Piazzesi (2005) factor, LN refers to the Ludvigson and Ng (2009) factors, NS to the Nelson and Siegel (1987) factors estimated as in Diebold and Li (2006), OY refers to the only-yield model (a restricted version of the macro-yields model where only the yields are used to extract the three yield curve factors and there are no additional macroeconomic factor). MY3, MY4, MY5 and MY6 denote the macro-yields models with 3, 4, 5 and 6 factors. Newey and West (1987) corrected t-statistics have lag order 18 months and are reported in parentheses. Coefficients that are statistically significant at the 10% or better level are highlighted in bold. A constant is always included in the regression.

Table 9: Out-of-sample predictive performance for excess returns

**Evaluation 1985-2009**

| n | CP | MY3 | MY4 | MY5 | MY6 | OY |
|---|---|---|---|---|---|---|
| 2 | 1.30 | 1.52 | 1.10 | **1.02** | 1.28 | 1.12 |
| 3 | 1.21 | 1.45 | 1.09 | **1.00** | 1.26 | 1.10 |
| 4 | 1.10 | 1.33 | 1.03 | **0.92** | 1.17 | 1.00 |
| 5 | 1.05 | 1.36 | 1.10 | **0.96** | 1.21 | 1.01 |

**Evaluation 1985-2003**

| n | CP | MY3 | MY4 | MY5 | MY6 | OY |
|---|---|---|---|---|---|---|
| 2 | 1.19 | 1.36 | **0.90** | 0.99 | 1.32 | 1.12 |
| 3 | 1.00 | 1.19 | **0.84** | 0.91 | 1.21 | 1.03 |
| 4 | 0.89 | 1.07 | **0.78** | 0.82 | 1.10 | 0.93 |
| 5 | 0.85 | 1.09 | **0.81** | 0.84 | 1.12 | 0.93 |

This table reports the Relative MSFE different specifications of the macro-yields model with respect to the constant expected returns benchmark where, apart from an MA(12) error term, excess returns are unforecastable as in the expectations hypothesis. Results refer to one-year-ahead out-of-sample forecast comparisons of n-period log excess bond returns, $rx^{(n)}_{t+12}$. CP refers to the Cochrane and Piazzesi (2005) model and OY to the only yields model. Bold values denote the smallest RMSFE for each maturity.

where $F^{*}_{t+12|t}$ is the 12-steps ahead forecasts made at time $t$ and $\hat{\Gamma}^{*}_{|t}$ is estimated using data up to time $t$.

We compare the out-of-sample forecasting performance of different specifications of the macro-yields models to the only-yields model and the Cochrane and Piazzesi (2005) factors. Table 9 contains the RMSFE of the selected models with respect to the constant expected returns benchmark where, apart from an MA(12) error term, excess returns are unforecastable as in the expectations hypothesis. Results are reported for two evaluation periods: from January 1985 to December 2003 and from January 1985 to December 2009. The RMSFE show that the macro-yields model is the best performing model. In particular, for the smaller evaluation sample, the macro-yields model with four factors is the best performing model outperforming the constant excess bond return benchmark for all the maturities. For the longest evaluation sample, we observe a general decline in predictability of excess bond returns with the macro-yields model with five factors outperforming all the other models and, for long maturities, also the benchmark. These results are in line with the predictive regressions of excess bond returns and also with the out-of-sample forecast performance, 12 steps ahead, of the macro-yields model for the yields.

## References


Ang, A., and M. Piazzesi (2003) 'A No-Arbitrage Vector Autoregression of Term Structure Dynamics with Macroeconomic and Latent Variables.' *Journal of Monetary Economics* 50(4), 745–787

Ang, A., M. Piazzesi, and M. Wei (2006) 'What Does the Yield Curve Tell us about GDP Growth.' *Journal of Econometrics* 131, 359–403

Bai, J., and S. Ng (2002) 'Determining the Number of Factors in Approximate Factor Models.' *Econometrica* 70(1), 191–221

Bernanke, B. S., and J. Boivin (2003) 'Monetary policy in a data-rich environment.' *Journal of Monetary Economics* 50(3), 525–546

Cochrane, J.H., and M. Piazzesi (2005) 'Bond risk premia.' *American Economic Review* pp. 138–160

Coroneo, L., K. Nyholm, and R. Vidova-Koleva (2011) 'How arbitrage-free is the nelson-siegel model?' *Journal of Empirical Finance*

De Pooter, M., F. Ravazzolo, and D. van Dijk (2007) 'Predicting the Term Structure of Interest Rates: Incorporating Parameter Uncertainty and Macroeconomic Information.' Tinbergen Institute Discussion Papers

Diebold, F. X., and C. Li (2006) 'Forecasting the term structure of government bond yields.' *Journal of Econometrics* 130, 337–364

Diebold, F.X., G. D. Rudebusch, and S. B. Aruoba (2006) 'The macroeconomy and the yield curve: A dynamic latent factor approach.' *Journal of Econometrics* 131, 309–338

Doz, C., D. Giannone, and L. Reichlin (2006) 'A quasi maximum likelihood approach for large approximate dynamic factor models.' C.E.P.R. Discussion Paper

Duffee, G.R. (2011) 'Information in (and not in) the term structure.' *Review of Financial Studies* 24(9), 2895–2934

Ghahramani, Z., and G.E. Hinton (1996) 'Parameter estimation for linear dynamical systems.' *University of Toronto technical report CRG-TR-96-2*

Giannone, D., L. Reichlin, and L. Sala (2005) 'Monetary Policy in Real Time.' *Nber Macroeconomics Annual 2004*

Joslin, S., K.J. Singleton, and H. Zhu (2011) 'A new perspective on gaussian dynamic term structure models.' *Review of Financial Studies* 24(3), 926

Joslin, S., M. Priebsch, and K.J. Singleton (2010) 'Risk premiums in dynamic term structure models with unspanned macro risks.' *Graduate School of Business Stanford University Working Paper* pp. 1–43

Litterman, R., and J. Scheinkman (1991) 'Common factors affecting bond returns.' *Journal of Fixed Income* 47, 129–1282

Ludvigson, S.C., and S. Ng (2009) 'Macro factors in bond risk premia.' *Review of Financial Studies* 22(12), 5027

Mönch, E. (2008) 'Forecasting the yield curve in a data-rich environment: A no-arbitrage factor-augmented var approach.' *Journal of Econometrics* 146(1), 26–43

Nelson, C.R., and A.F. Siegel (1987) 'Parsimonious modeling of yield curves.' *Journal of Business* 60, 473–89

Newey, W.K., and K.D. West (1987) 'A simple, positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix.' *Econometrica: Journal of the Econometric Society* pp. 703–708

Sargent, T.J., and C.A. Sims (1977) 'Business cycle modeling without pretending to have too much a priori economic theory.' *New Methods in Business Research, Federal Reserve Bank of Minneapolis, Minneapolis*

Shumway, R.H., and D.S. Stoffer (1982) 'An approach to time series smoothing and forecasting using the EM algorithm.' *Journal of Time Series Analysis* 3(4), 253–264

Stock, J.H., and M.W. Watson (2002) 'Macroeconomic forecasting using diffusion indexes.' *Journal of Business and Economic Statistics* 20(2), 147–162