Information Diversity and Market Efficiency Spirals

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Abstract

We analyze a model where the value of a traded security is affected by two different fundamentals, e.g., the quality of the firm's technology and the demand for its products, and where there are two groups of informed traders, each one informed about a different fundamental. We analyze the interaction between the informativeness of the price about the two fundamentals and characterize when it leads to attenuation and when it leads to amplification of shocks to market efficiency. Amplification occurs because the informativeness about one fundamental reduces the uncertainty in trading on information about the other fundamental and encourages traders to trade more aggressively on such information. This effect is dominant when the informativeness of the price is relatively balanced between the two fundamentals, which implies that economies with more diverse information – i.e., where the information is more evenly distributed between the two groups – will exhibit positive externalities and have higher levels of overall market efficiency. Finally, we endogenize the incentives for information production and show that the above effect leads to strategic complementarities in information production.

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1 Introduction

Does the presence of information in financial markets attract more traders to produce information and trade on it or does it keep them away from doing so? Knowing the answer to this question is crucial for our understanding of the workings of financial markets. If information in financial markets attracts the production and transmission of more information in the trading process, then shocks to market efficiency will be amplified, creating a spiral that leads to sharp changes in informativeness and efficiency. On the other hand, if information in financial markets deters traders from producing more information or from trading aggressively on the information that they have, then shocks to market efficiency will be attenuated, implying modest changes in informativeness and efficiency.

The seminal model of Grossman and Stiglitz (1980) provides an unambiguous answer to this question. In that model, when more informed traders are present in the market, others have a lower incentive to become informed and trade on their information. This is because, as more people are informed, the price provides more information, and so each trader has a lower incentive to produce information on his own, as he can get a good amount of information from the price. As a result, shocks to market efficiency are attenuated in equilibrium: The arrival of more information to the market deters others from becoming informed, and so the overall effect on price informativeness and market efficiency is modest.

But, in reality, things often seem to go in the opposite direction. As uncertainty about economic fundamentals heightens, traders tend to stay away from the market, avoiding trades even on issues related to their area of expertise. This was evident in the recent financial crisis, as markets froze, exhibiting very low trade volumes, following the increase in economic uncertainty that led market prices to be an unreliable source of information. The economic force behind this phenomenon is that risk averse traders want to avoid the risk in trading securities, whose underlying value is too uncertain.

For example, consider traders, who are informed about traditional (consumer and commercial) loans provided by a bank, that in normal times trade in the bank's stock based on their information about the quality of these loans. Suppose that, as in the recent crisis, a new dimension of uncertainty emerges, concerning the quality of the bank's investment in mortgage-backed securities. The lack of information in the market about these securities might deter the traders, who are informed about traditional loans, from trading, since there is a lot of risk involved in taking positions in stocks, whose value will be determined to a large extent by the quality of the mortgage-backed securities. Moreover, the fact that investors with information about loans leave the market might deter traders with information about mortgage-backed securities from trading in the market for the same reason: It is too risky for them to trade when the value of the stock is largely determined by something they do not understand and that is not reflected in the price. Hence, we get a self-reinforcing market efficiency spiral, by which less information on one dimension of uncertainty leads to less information on another dimension of uncertainty, and so on, ending with much lower levels of informativeness and efficiency. This can explain sudden market breakdowns as in the recent crisis.

More generally, there are many cases where firms face multiple dimensions of uncertainty, and, due to costs of information production and natural areas of expertise, different speculators specialize and trade on information along different dimensions. The value of technology firms is driven by the quality of their technology and by the demand for their products, and there are traders with technical expertise trading on information about technology, while there are traders with marketing expertise trading on information about the demand for firms' products. The value of multinational firms depends on the developments in various countries, and there are traders who understand the developments in a specific country and trade on this information, taking into account whether there are traders in the market trading on information about the developments in other countries. The idea that investors trade on one kind of information is featured in the literature on style investing (Chan, Chen, and Lakonishok, 2002; Barberis and Shleifer, 2003; Goetzmann and Brown, 2003). It is also related to behavioral arguments about investors being limited in the number of variables they can keep track of (Hong, Stein, and Yu, 2007; Brunnermeier and Oehmke, 2009).[1] Overall, we seek to explore the interaction between different types of investors trading on different types of information, and the extent to which this interaction leads to amplification

[1] This idea is akin to the notion of limited attention, which has been recently studied in the finance literature (e.g., Moscarini, 2004; Peng, 2005; Peng and Xiong, 2006; and Gabaix, Laibson, Moloche, and Weinberg, 2006).

or attenuation of market efficiency and price informativeness.

The model we study in this paper is an extension of Grossman and Stiglitz (1980) that considers two dimensions of uncertainty about the value of the traded security, say that the value of the security depends on the quality of the firm's technology and on the demand for the firm's products. We first consider an economy where traders are endowed with different types of information: Some traders are informed about the technology of the firm and others are informed about the demand for its products. Traders are risk averse, and trade in a market with uninformed traders and with noisy supply, as in Grossman and Stiglitz (1980). When they trade, they condition on the price of the security, and hence the information in the price benefits their trading decision.

We analyze the interaction between the two groups of informed traders in this model. Suppose that the size of the group of technology-informed traders increases, what is the effect on the informativeness of the price about the demand for the firm's products? There are two effects. First, the presence of more technology-informed traders creates an adverse selection effect that deters demand-informed traders from trading aggressively. This is because when they trade, they do so without having information that is available to an increased number of other traders in the market. This effect attenuates the impact of shocks to market efficiency: An increase in efficiency due to the increase in the number of traders who are informed on one dimension will cause traders who are informed on the other dimension to trade less aggressively, making the overall effect on market efficiency modest. Second, the presence of more technology-informed traders implies that more of their information will get into the price, reducing the overall uncertainty that demand-informed traders have to face concerning technology issues when they trade. This effect amplifies the impact of shocks to market efficiency: The presence of more technology-informed traders allows demand-informed traders to trade more aggressively on what they know without being exposed to risks they do not understand, and this increases efficiency further.[2] This is the basis for the market efficiency spiral that motivates our paper.

Interestingly, we find that in equilibrium the amplification effect dominates the atten-

[2]This is similar to a firm that specializes in one type of activity and hedges itself against other unrelated risks.

uation effect when the informativeness of the price is relatively balanced between the two dimensions of uncertainty. On the other hand, the attenuation effect dominates the amplification effect when the price is much more informative about one dimension than about the other. This has important implications for the optimal structure of information in the economy. Fixing the total mass of informed traders, an economy with a diverse information structure – i.e., where there is more balance between the two groups of informed traders – will exhibit a higher level of market efficiency than an economy with a concentrated information structure – i.e., where there are many more traders informed about one dimension than on the other. This is because the former economy benefits more from the amplification in informativeness as a result of the uncertainty reduction effect described above.

We then extend our analysis to allow traders to decide whether to become informed on one of the two dimensions of uncertainty.[3] We again analyze the interaction between the two types of information. Suppose that there are more technology-informed traders in the market, what will be the effect on the incentives of agents to acquire information about demand? On the one hand, the traditional Grossman-Stiglitz effect, reducing the incentive to produce information about demand when there are more technology-informed traders in the market, exists in our model. This is a strategic substitute that attenuates the effect on market efficiency. However, on the other hand, the uncertainty reduction effect mentioned above creates a strategic complementarity that amplifies the effect on market efficiency: Knowing that more technology information will go into the price, traders know they will face less uncertainty when trading on demand information, and hence have a stronger incentive to produce information about demand. We identify conditions under which this effect dominates, creating another form of market efficiency spiral that leads to amplification of shocks in equilibrium.

We finally compare our economy where the asset payoff has multiple dimensions of uncertainty and traders are informed of different dimensions, to the standard unidimensional economy, analyzed by Hellwig (1980) and Verrecchia (1982), where the asset payoff is modelled as a single random variable and traders are informed of the overall asset payoff. We

[3] We do not allow traders to acquire information in both dimensions, following the arguments above about traders having a comparative advantage in processing one kind of information or about the limited capacity that they have.

find that the standard result that the benefit of acquiring information decreases with the total size of the informed traders population in the unidimensional economy is not robust. In general, how traders respond to other traders becoming informed does not only depend on the overall size of informed group, but also depends on its composition. If traders are acquiring information about fundamentals that are weakly correlated, then acquiring information on different fundamentals tend to be a complement rather than a substitute. This result highlights the uniqueness of our analysis.

Several papers in the literature analyze models of financial markets, where the value of the traded security is affected by more than one fundamental, on which agents are informed. Froot, Scharfstein, and Stein (1992) show that short term investors tend to concentrate on collecting one type of information, because prices in the short-run will most likely reflect information that other people have. Goldman (2005) studies whether a two-division firm is better off when it is listed as one unit rather than when its two divisions are traded separately. Yuan (2005) demonstrates that benchmark securities allow heterogeneously informed investors to create trading strategies that are perfectly aligned with their signals about different fundamentals. Kondor (2012) shows that public announcement can create disagreement and increase trading volume when traders are informed of different fundamentals and have heterogeneous trading horizons.

More closely related is Lee (2010), which builds on Subrahmanyam and Titman (1999). Like us, Lee (2010) investigates the interactions between different types of informed traders in the same market and points out the possibility that learning can be a complement when informed traders pursue different trading strategies. In his paper, however, the complementarity comes from the fact that trades based on different types of information provide noise for each other in a market-order based model (Kyle, 1985). By contrast, our mechanism works through the uncertainty reduction effect (which is absent in Lee, 2010) due to the fact that information revealed in prices reduces the uncertainty faced by agents and encourages them to trade more aggressively and produce more information. Our analysis also goes beyond pointing out the learning complementarity, e.g., by showing that information diversity can have important implications for price informativeness and the cost of capital.

Another related literature analyzes models of trading in multiple securities, e.g., Admati

(1985) and Bernhardt and Taub (2008). Most closely related in this literature is the recent paper by Cespa and Foucault (2011). They study liquidity spillovers between *different securities* when speculators trading in one security may observe the price of the other traded security and learn information concerning the security they trade. In contrast, we show the amplification in efficiency and informativeness in a *one-security* setting and address different questions like the effect of information diversity on market efficiency, the effect of the cost of producing one type of information on the production of another type of information, etc.

Finally, our model contributes to the literature studying sources of strategic complementarities in financial markets, by highlighting a different mechanism by which such complementarities arise. Other papers in this literature include: Barlevy and Veronesi (2000), Ganguli and Yang (2009), Garcia and Strobl (2010), Goldstein, Ozdenoren, and Yuan (2010), Mele and Sangiorgi (2010), and Breon-Drish (2011).

The remainder of this paper is organized as follows. Section 2 presents the model and the characterization of the equilibrium. In Section 3, we analyze the interaction between the two types of information and provide a full characterization of when our model features amplification vs. attenuation of market efficiency. Section 4 studies the role of information diversity, showing that economies with more diverse information will feature greater market efficiency. In Section 5, we endogenize the information acquisition decision and analyze when it will exhibit strategic substitute vs. strategic complementarity. Section 6 checks the robustness of our results and highlights the uniqueness of our analysis, and Section 7 concludes.

2 The Model

2.1 Setup

We study an environment similar to Grossman and Stiglitz (1980). There are two assets traded in the financial market: one riskless asset (bond) and one risky asset (stock). The bond is in unlimited supply; its payoff is 1, and its price is normalized to 1. The stock has a total supply of 1 unit and has a price of $\tilde{p}$, which is determined endogenously in the financial market, and its payoff $\tilde{v}$ is given by:

$$\tilde{v} = \tilde{v}_1 + \tilde{v}_2. \tag{1}$$

As we see in (1), the payoff of the stock is composed of two ingredients, $\tilde{v}_1$ and $\tilde{v}_2$, sometimes referred to as fundamentals, which are independent and identically distributed (i.i.d.) according do a normal distribution function: $\tilde{v}_i \sim N(0, 1/\rho)$ $(i = 1, 2)$, where ρ represents the common prior precision of $\tilde{v}_1$ and $\tilde{v}_2$. The idea is that there are two dimensions of uncertainty about the payoff from the stock, captured by the variables, $\tilde{v}_1$ and $\tilde{v}_2$, and, as we will discuss below in more detail, they are potentially observable to different traders.

There are three types of rational traders (of a total mass of 1) trading the bond and the stock in the financial market: (1) $\tilde{v}_1$-informed traders (of mass $\lambda_1 > 0$), who observe the realization of the first component $\tilde{v}_1$ of the stock payoff. (2) $\tilde{v}_2$-informed traders (of mass $\lambda_2 > 0$), who observe the realization of the second component $\tilde{v}_2$ of the stock payoff. (3) Uninformed traders (of mass $\lambda_u = 1 - \lambda_1 - \lambda_2 \geq 0$), who do not observe any information. While we currently assume that the masses of informed traders, λ_1 and λ_2, are exogenous, we endogenize them later in Section 5, where we analyze learning complementarities. All three types of traders condition their trades on the stock price $\tilde{p}$. Their utility from consumption C is given by the usual constant-absolute-risk-aversion (CARA) function, $-e^{-\gamma C}$, where γ is the risk-aversion parameter. Finally, to prevent fully revealing prices, we assume that there are noise traders who trade a random amount $\tilde{x} \sim N(0, 1/\chi)$ (with $\chi > 0$) of the stock, which is independent of the realizations of $\tilde{v}_1$ and $\tilde{v}_2$.

As mentioned above, $\tilde{v}_1$ and $\tilde{v}_2$ represent two dimensions of uncertainty about the traded firm. For example, one dimension can be the demand for the firm's products and the other one can be the quality of the firm's technology. Then, different traders may be exposed to different types of information, depending on their expertise and background. For example, people with technical background may know more about the technology of the firm, while consumers may have assessments about the demand for the firm's products. Also, retail investors may be exposed to information about aspects of the firm which are different than those that institutional investors are exposed to.

For the sake of tractability, we have assumed that the two ingredients $\tilde{v}_1$ and $\tilde{v}_2$ are mutually independent and that $\tilde{v}_i$-informed traders perfectly observe $\tilde{v}_i$. In Section 6 we remove these two assumptions and show that our main results go through. We have also assumed that $\tilde{v}_1$ and $\tilde{v}_2$ are identically distributed, as this assumption gives us a natural aspect of information diversity based on the proportion of traders informed about each fundamental (which is formalized by equation (22)). If instead the two fundamentals are not identically distributed, we have not found a tractable way to implement a notion of information diversity, because it is difficult to control the impact of the total amount of information while we vary parameters to examine the impact of information diversity.

2.2 Equilibrium Definition

The equilibrium concept that we use is the rational expectations equilibrium (REE), as in Grossman and Stiglitz (1980). In equilibrium, traders trade to maximize their expected utility given their information set, where $\tilde{v}_i$-informed traders know $\tilde{v}_i$ and $\tilde{p}$ $(i = 1, 2)$, and uninformed traders know only $\tilde{p}$. The price $\tilde{p}$ is determined, in turn, by the market-clearing condition, whereby the sum of demands from the three types of rational traders and the noise traders is equal to the supply of the stock 1. As in most of the literature, we consider a linear equilibrium, where the price $\tilde{p}$ linearly depends on the signals $\tilde{v}_1$ and $\tilde{v}_2$ and the noisy trading $\tilde{x}$:

$$\tilde{p} = \alpha_0 + \alpha_1 \tilde{v}_1 + \alpha_2 \tilde{v}_2 + \alpha_x \tilde{x}. \tag{2}$$

The coefficients α_0, α_1, α_2, and α_x will be endogenously determined.

2.3 Equilibrium Characterization

We start by deriving the demands of the different traders. The $\tilde{v}_i$-informed traders have information set $\mathcal{F}_i = \{\tilde{p}, \tilde{v}_i\}$. The CARA-normal setup indicates that their demand function is:

$$D_i(\tilde{p}, \tilde{v}_i) = \frac{E\left(\tilde{v}|\mathcal{F}_i\right) - \tilde{p}}{\gamma Var\left(\tilde{v}|\mathcal{F}_i\right)} = \frac{\tilde{v}_i + E(\tilde{v}_j|\mathcal{F}_i) - \tilde{p}}{\gamma Var(\tilde{v}_j|\mathcal{F}_i)}, \tag{3}$$

where the second equality follows from equation (1). Given the price function (2), the information set $\mathcal{F}_i$ is equivalent to the following signal in predicting $\tilde{v}_j$:

$$\begin{aligned} \tilde{s}_{j|i} &\equiv \frac{\tilde{p}-\alpha_0-\alpha_i\tilde{v}_i}{\alpha_j} &(4)\\ &= \tilde{v}_j+(\alpha_x/\alpha_j)\,\tilde{x}. &(5)\end{aligned}$$

By Bayes' rule, we have,

$$\begin{aligned} E(\tilde{v}_j|\mathcal{F}_i) &= E(\tilde{v}_j|\tilde{s}_{j|i}) = \frac{(\alpha_j/\alpha_x)^2\,\chi\tilde{s}_{j|i}}{\rho+(\alpha_j/\alpha_x)^2\,\chi}, &(6)\\ Var(\tilde{v}_j|\mathcal{F}_i) &= Var(\tilde{v}_j|\tilde{s}_{j|i}) = [\rho+(\alpha_j/\alpha_x)^2\,\chi]^{-1}. &(7)\end{aligned}$$

Plugging the above expressions into equation (3), we can express the demand function of $\tilde{v}_i$-informed traders as follows:

$$D_i(\tilde{p},\tilde{v}_i)=\frac{\tilde{v}_i+E(\tilde{v}_j|\tilde{s}_{j|i})-\tilde{p}}{\gamma Var(\tilde{v}_j|\tilde{s}_{j|i})}=\frac{\tilde{v}_i+\frac{(\alpha_j/\alpha_x)^2\chi\tilde{s}_{j|i}}{\rho+(\alpha_j/\alpha_x)^2\chi}-\tilde{p}}{\gamma[\rho+(\alpha_j/\alpha_x)^2\,\chi]^{-1}}, \quad (8)$$

for $i=1,2$.

The uninformed traders only observe the price $\tilde{p}$ and their demand function is:

$$D_u(\tilde{p})=\frac{E\left(\tilde{v}|\tilde{p}\right)-\tilde{p}}{\gamma Var\left(\tilde{v}|\tilde{p}\right)}. \quad (9)$$

The price $\tilde{p}$ is equivalent to the following signal in predicting the total payoff $\tilde{v}$:

$$\tilde{s}_u\equiv\frac{\tilde{p}-\alpha_0}{\alpha_x}=\frac{\alpha_1}{\alpha_x}\tilde{v}_1+\frac{\alpha_2}{\alpha_x}\tilde{v}_2+\tilde{x}. \quad (10)$$

Applying Bayes' rule, we have:

$$\begin{aligned} E\left(\tilde{v}|\tilde{p}\right) &= \beta_{\tilde{v},\tilde{p}}\tilde{s}_u, &(11)\\ Var\left(\tilde{v}|\tilde{p}\right) &= \frac{(\alpha_1/\alpha_x-\alpha_2/\alpha_x)^2+2\rho\chi^{-1}}{(\alpha_1/\alpha_x)^2\,\rho+(\alpha_2/\alpha_x)^2\,\rho+\rho^2\chi^{-1}}, &(12)\end{aligned}$$

where

$$\beta_{\tilde{v},\tilde{p}} = \frac{(\alpha_1/\alpha_x) + (\alpha_2/\alpha_x)}{(\alpha_1/\alpha_x)^2 + (\alpha_2/\alpha_x)^2 + \rho\chi^{-1}}. \tag{13}$$

The equilibrium price is determined by the market-clearing condition for the risky asset:

$$\lambda_1 D_1(\tilde{p}, \tilde{v}_1) + \lambda_2 D_2(\tilde{p}, \tilde{v}_2) + \lambda_u D_u(\tilde{p}) + \tilde{x} = 1. \tag{14}$$

Plugging the expressions for $D_i(\tilde{p}, \tilde{v}_i)$ and $D_u(\tilde{p})$ (equations (8) and (9)) into the above market-clearing condition, we can solve for the price $\tilde{p}$ as a function of the variables $\tilde{v}_1$, $\tilde{v}_2$ and $\tilde{x}$. Then, comparing coefficients with those in the conjectured price function (equation (2)), we get the following proposition that characterizes the linear rational-expectations equilibrium. The proof of all propositions are in the appendix.

Proposition 1 *For any $\lambda_1 > 0$ and $\lambda_2 > 0$, there exists a unique linear rational-expectations equilibrium, in which*

$$\tilde{p} = \alpha_0 + \alpha_1 \tilde{v}_1 + \alpha_2 \tilde{v}_2 + \alpha_x \tilde{x}.$$

The coefficients $\alpha_0 < 0$, $\alpha_1 > 0$, $\alpha_2 > 0$, and $\alpha_x > 0$ are given as a function of the exogenous parameters of the model in the proof in the appendix.

3 Trading Incentives and the Interaction between the Two Types of Information

3.1 Trading Incentives and Price Informativeness

Going back to Grossman and Stiglitz (1980), price informativeness is often defined as the ratio between the sensitivity of the price to fundamental shocks and the sensitivity of the price to noise. In our model, there are two types of fundamentals, $\tilde{v}_1$ and $\tilde{v}_2$, and we would like to examine the informativeness of the price about each one of them and the interaction between the two. Hence, we define $I_1 \equiv \frac{\alpha_1}{\alpha_x}$ and $I_2 \equiv \frac{\alpha_2}{\alpha_x}$ as the price informativeness about signals $\tilde{v}_1$ and $\tilde{v}_2$, respectively.[4]

[4] Our definition of price informativeness I_i about $\tilde{v}_i$ is in the same spirit as Grossman and Stiglitz (1980) who intepret price informativeness as the extent to which variations of the price come from fundamental

We are also interested in a measure of overall informativeness, i.e., how much the price reflects the total stock payoff $\tilde{v}$, which also measures the market efficiency. As in Brunnermeier (2005), we define this as $I_v \equiv \frac{1}{Var(\tilde{v}|\tilde{p})}$, i.e., the reciprocal of the variance of $\tilde{v}$ conditional on the price. This corresponds to how much residual uncertainty is faced by the uninformed traders after conditioning on the price. By equation (12) and the definitions $I_i = \frac{\alpha_i}{\alpha_x}$ $(i = 1, 2)$, we can see how the overall price informativeness measure I_v depends on the other two informativeness measures I_1 and I_2:

$$I_v = \frac{1}{Var\left(\tilde{v}|\tilde{p}\right)} = \frac{I_1^2\rho + I_2^2\rho + \rho^2\chi^{-1}}{\left(I_1 - I_2\right)^2 + 2\rho\chi^{-1}}. \tag{15}$$

Note that I_1 and I_2 positively affect the overall price informativeness I_v. That is, $\frac{\partial I_v}{\partial I_1} > 0$ and $\frac{\partial I_v}{\partial I_2} > 0$. This result is intuitive, as one would expect that the price system will reveal more information about the total asset payoff $(\tilde{v}_1 + \tilde{v}_2)$ when the variations of the price come more from either component of the asset payoff than from the noisy asset trading.[5]

In equilibrium, price informativeness is determined by the trading incentives of the informed traders, that is, by the extent to which a change in their signal affects the amount that they trade, which is represented by $\frac{\partial D_i(\tilde{p},\tilde{v}_i)}{\partial \tilde{v}_i}$ for the $\tilde{v}_i$-informed traders. Intuitively, this is because prices reflect information precisely as a result of informed traders trading on their private signals. To see this formally, consider the following experiment. A unit increase in $\tilde{v}_i$ will cause each of these traders to buy $\frac{\partial D_i(\tilde{p},\tilde{v}_i)}{\partial \tilde{v}_i}$ more stocks, and so as a group $\tilde{v}_i$-informed traders will buy $\lambda_i \frac{\partial D_i(\tilde{p},\tilde{v}_i)}{\partial \tilde{v}_i}$ more stocks. If it happens that the noise traders supply the same number of extra shares, then the price will not change. That is, changing $\tilde{v}_i$ by one unit has the same price impact as changing $\tilde{x}$ by $\lambda_i \frac{\partial D_i(\tilde{p},\tilde{v}_i)}{\partial \tilde{v}_i}$ units, and as a result, the price informativeness about $\tilde{v}_i$ is given by:

$$I_i = \frac{\alpha_i}{\alpha_x} = \lambda_i \frac{\partial D_i(\tilde{p}, \tilde{v}_i)}{\partial \tilde{v}_i}, \text{ for } i = 1, 2. \tag{16}$$

$\tilde{v}_i$ rather than from the noise $\tilde{x}$. Strictly speaking, a full notion of inofmrativness might be the precision of $\tilde{v}_i$ conditional on the price $\tilde{p}$, $\frac{1}{Var(\tilde{v}_i|\tilde{p})} = \rho\left(1 + \frac{I_i^2}{I_j^2+\rho\chi^{-1}}\right)$. But our measures I_1 and I_2 are analytically more tractable and they give us building blocks we can work with toward those variables of final interest, for example the price informativeness about the overall payoff $\tilde{v} = \tilde{v}_1 + \tilde{v}_2$ and the cost of capital.

[5] Note, however, that the result is not trivial due to the interactions between I_1 and I_2 in determining I_v. Hence, establishing that $\frac{\partial I_v}{\partial I_1} > 0$ and $\frac{\partial I_v}{\partial I_2} > 0$ requires us to rely on the relation between I_1 and I_2 specified in equation (18). For details, see equations (57) and (58) in the appendix.

3.2 The Interaction between the Two Types of Information

Using equation (3) for the demands of the two types of informed speculators, we can see how these demands are affected by changes in the signals, and how this in turn determines price informativeness:

$$
\begin{aligned}
I_i &= \lambda_i \frac{\partial D_i(\tilde{p}, \tilde{v}_i)}{\partial \tilde{v}_i} = \lambda_i \left(\frac{1}{\gamma} \frac{1}{Var(\tilde{v}_j|\mathcal{F}_i)} + \frac{1}{\gamma} \frac{\partial}{\partial \tilde{v}_i} \frac{E(\tilde{v}_j|\mathcal{F}_i)}{Var(\tilde{v}_j|\mathcal{F}_i)} \right) \qquad (17) \\
&= \lambda_i \left(\gamma^{-1}(\rho + \chi I_j^2) - \gamma^{-1} \chi I_i I_j \right), \qquad (18)
\end{aligned}
$$

where the last equality follows from equations (4)-(7).

An increase in the signal $\tilde{v}_i$ has two effects on the trading of a $\tilde{v}_i$-informed trader, as can be seen in equation (17). First, as we can see in the first term in the parentheses, he is going to demand more of the asset due to the direct effect that $\tilde{v}_i$ has on the payoff of the asset. Note that this effect gets smaller when there is a higher residual variance about $\tilde{v}_j$, since a higher variance implies that the trader takes more risk when holding a position in the asset, and this makes him trade less aggressively. Second, as we can see in the second term in the parentheses, he is going to demand less of the asset due to the indirect effect via the expectation that he has about $\tilde{v}_j$: Holding the price constant, an increase in $\tilde{v}_i$ implies a lower expectation about $\tilde{v}_j$, and so reduces the trader's demand of the asset.

Equation (18) also demonstrates the interaction between the informativeness of the price about $\tilde{v}_i$ and the informativeness of the price about $\tilde{v}_j$. Consider equation (18) for the $\tilde{v}_i$-informed traders. An increase in the informativeness of the price about $\tilde{v}_j$, I_j, has two effects on their trading incentives. First, it reduces the residual uncertainty they face when they trade, and hence makes them trade more aggressively on their information about $\tilde{v}_i$. We label this channel the "uncertainty reduction effect". It is represented in the first term in the parentheses. Second, it makes them use the price more to infer information about $\tilde{v}_j$, and so trade less aggressively on their information about $\tilde{v}_i$. (An increase in $\tilde{v}_i$, holding the price constant, provides now stronger indication that $\tilde{v}_j$ is low, and this makes the traders rely less on their private information when trading.) We label this channel the "adverse selection effect". It is represented in the second term in the parentheses.

Rearranging terms in (18), we get two response functions that jointly determine I_1 and

I_2 in equilibrium:

$$I_i = h_i\left(I_j; \lambda_i, \rho, \gamma, \chi\right) \equiv \frac{\lambda_i(\rho + \chi I_j^2)}{\gamma + \lambda_i \chi I_j}, \text{ for } i, j = 1, 2 \text{ and } j \neq i. \tag{19}$$

As we can see, due to the two effects mentioned above ("uncertainty reduction effect" and "adverse selection effect"), the two response functions may have either a positive or a negative slope. Analyzing (19), we can see that $h_i\left(I_j; \cdot\right)$ is decreasing in I_j when I_j is low, and then increasing when I_j is high. That is, the adverse selection effect dominates for low levels of informativeness and the uncertainty reduction effect dominates for high levels of informativeness. As we show in the next subsection, whether we get amplification or attenuation of shocks to market efficiency depends on the slopes of the two response functions in equilibrium.

3.3 Market Efficiency Multipliers

Let Q be one of the five exogenous parameters $(\lambda_1, \lambda_2, \rho, \gamma, \chi)$ that determine the price informativeness measures (I_1 and I_2) in the system defined in (19).[6] The following proposition characterizes the effect of Q on I_i, separating between the direct effect and the indirect effect due to the feedback loop between the two types of information. It identifies when the direct effect of Q on I_i is amplified and when it is attenuated due to the interdependence between the two price informativeness measures.

Proposition 2 *(i) The effect of an exogenous parameter Q on the informativeness I_i of the price about $\tilde{v}_i$ is given by:*

$$\underbrace{\frac{dI_i}{dQ}}_{\textit{total effect}} = \mathcal{M} \underbrace{\left(\frac{\partial h_i}{\partial Q} + \frac{\partial h_i}{\partial I_j}\frac{\partial h_j}{\partial Q}\right)}_{\textit{direct effect}}, \tag{20}$$

where the term $\left(\frac{\partial h_i}{\partial Q} + \frac{\partial h_i}{\partial I_j}\frac{\partial h_j}{\partial Q}\right)$ captures the "direct effect" of changing Q on I_i and the

[6] Note that we can rewrite (19) as $I_i \equiv \frac{\lambda_i(\rho/\chi + I_j^2)}{\gamma/\chi + \lambda_i I_j}$. So, the three parameters (γ, ρ, χ) affect (19) only through $(\rho/\chi, \gamma/\chi)$, meaning that (γ, ρ, χ) only have two degrees of freedom in determining the system (19). Actually, we can normalize any one of (γ, ρ, χ) to 1 without affecting the equilibrium price informativeness measures.

coefficient $\mathcal{M}$ is a "multiplier" given by:

$$\mathcal{M} = \left(1 - \frac{\partial h_1}{\partial I_2}\frac{\partial h_2}{\partial I_1}\right)^{-1} > 0. \tag{21}$$

(ii) Suppose that $\lambda_1 > 0$ and $\lambda_2 > 0$. Then, (a) when $\frac{1}{2} < \frac{I_1}{I_2} < 2$, $\mathcal{M} > 1$, and so the effect of Q on I_i is amplified in equilibrium; (b) when $\frac{I_1}{I_2} < \frac{1}{2}$ or when $\frac{I_1}{I_2} > 2$, $0 < \mathcal{M} < 1$, and so the effect of Q on I_i is attenuated in equilibrium; and (c) when $\frac{I_1}{I_2} = \frac{1}{2}$ or when $\frac{I_1}{I_2} = 2$, $\mathcal{M} = 1$.

The direct effect of a shock in a parameter Q on price informativeness I_i can be thought of as the initial impact before the feedback effect between the two informativeness measures is considered. It is given by $\left(\frac{\partial h_i}{\partial Q} + \frac{\partial h_i}{\partial I_j}\frac{\partial h_j}{\partial Q}\right)$. The feedback loop then creates a multiplier that either amplifies or attenuates the direct effect. It is given by $\mathcal{M} = \left(1 - \frac{\partial h_1}{\partial I_2}\frac{\partial h_2}{\partial I_1}\right)^{-1}$, which, as the proof of the proposition shows, is strictly positive. Whether the direct effect is attenuated or amplified in equilibrium then depends on whether $\mathcal{M}$ is smaller than or bigger than 1, which depends on the signs of the cross derivatives $\frac{\partial h_1}{\partial I_2}$ and $\frac{\partial h_2}{\partial I_1}$. The proof of the proposition shows that it is impossible to have both cross derivatives negative in equilibrium (i.e., $\frac{\partial h_1}{\partial I_2} < 0$ and $\frac{\partial h_2}{\partial I_1} < 0$ at the point where the two response functions in (19) intersect). Hence, either one of them is negative and one is positive, in which case $\mathcal{M} < 1$ and the direct effect of a shock on price informativeness is attenuated in equilibrium, or both of them are positive, in which case $\mathcal{M} > 1$ and the direct effect of a shock on price informativeness is amplified in equilibrium.

The result shows that when I_1 and I_2 are very far from each other (specifically, when $\frac{I_1}{I_2} < \frac{1}{2}$ or when $\frac{I_1}{I_2} > 2$) – which will be true when λ_1 and λ_2 are very far from each other – the uncertainty reduction effect dominates for one response function, while the adverse selection effect dominates for the other. As a result, the interaction between the two price informativeness measures tends to attenuate the initial shock to the economy; that is, $0 < \mathcal{M} < 1$. In contrast, when I_1 and I_2 are close to each other (specifically, when $\frac{1}{2} < \frac{I_1}{I_2} < 2$) – which will be true when λ_1 and λ_2 are close to each other – the uncertainty reduction effect dominates for both response functions. As a result, the two price informativeness measures reinforce each other and any exogenous shock affecting one price informativeness measure will lead to a "market efficiency spiral", whereby a shock directly improving one type of price informativeness will propagate to the other type, which will in turn feed back on the original one, amplifying the effect of the initial shock and improving both types of price informativeness, as well as the total market efficiency. In this case, we have $\mathcal{M} > 1$.[7]

Figure 1 uses a numerical example to illustrate this spiral reaction to an increase in λ_1. The figure depicts the two h functions in a case where they intersect at the upward sloping parts (i.e., the uncertainty reduction effect is the dominant effect for both of them). In the figure, we set the parameters at $\rho = 50$, $\chi = 50$ and $\gamma = 3$. Suppose that initially $\lambda_1 = \lambda_2 = 0.05$, so that the equilibrium is at Point E and $I_1 = I_2 = 0.83$. Now suppose that λ_1 increases to 0.1. Then, function h_1 moves upward to h_1', and the equilibrium moves to point E' ($I_1 = 1.28$ and $I_2 = 1.07$). We can see how the effect of the increase in the amount of informed trading on $\tilde{v}_1$ is amplified in equilibrium along the route $E \to A \to B \to C ... \to E'$. That is, when there are more informed traders who know $\tilde{v}_1$, the price is automatically more informative about $\tilde{v}_1$. Then, traders who know $\tilde{v}_2$ trade more aggressively on their information because they face less uncertainty (and the uncertainty reduction effect dominates the adverse selection effect), and so the price is more informative about $\tilde{v}_2$ as well. Now traders trade more aggressively on $\tilde{v}_1$ as well, which increases the price informativeness on $\tilde{v}_1$ further. The amplification chain continues on and on till it converges to much higher levels of informativeness on both $\tilde{v}_1$ and $\tilde{v}_2$.

[FIGURE 1 ABOUT HERE]

[7] We note that the competition between the adverse selection effect and the uncertainty reduction effect is also responsible for the uniqueness of the financial market equilibrium described by Proposition 1. Specifically, the uncertainty reduction effect increases the possibility of multiple equilibria through its self-reinforcing feature, while the adverse selection effect decreases it. In our economy, both effects are present so that the strength of self-fulfilling expectations is weak and there are no multiple equilibria. In a recent study by Cespa and Foucault (2011) who consider a two-security setting, they indeed find the possibility of multiple linear equilibria, because in their economy, informed traders only specialize in one market and there is no adverse selection effect to counterbalance the uncertainty reduction effect, thereby causing the strength of self-fulfilling expectations to be particularly strong.

4 The Impact of Information Diversity

In the previous section, we showed that the interaction between the two types of information can either amplify or attenuate shocks to market efficiency in our model, and that whether we obtain amplification or attenuation depends on the difference between the two types of informativeness. A natural implication of this result is that the diversity of information in the model should have important implications for overall market efficiency and, as a result, for the cost of capital of the traded firm. In this section, we explore this implication formally.

4.1 Price Informativeness

In our model, there are two types of informed agents. The mass of traders informed about $\tilde{v}_1$ is λ_1 and the mass of traders informed about $\tilde{v}_2$ is λ_2. We would like to analyze the effect of the difference between λ_1 and λ_2 on the informativeness of the price in our model, while keeping the total size of the informed-traders population fixed. For this purpose, we set $\lambda_1 + \lambda_2 = \Lambda$ (where Λ is a constant), and define the following measure of information diversity:

$$\Delta \equiv 1 - \frac{|\lambda_1 - \lambda_2|}{\Lambda} \in [0, 1] \,. \tag{22}$$

A higher Δ means that the two groups of informed traders are closer in size, and so the total amount of information is more equally distributed between the two types of informed traders; hence, there is more diversity of information in the economy. (By this logic, a situation with less diversity is one where most people know the same thing and so Δ is low.)

Following Proposition 2, we know that when Δ is close to 0, either λ_1 or λ_2 is close to 0, and so I_1 and I_2 are far from each other, implying that the market efficiency multiplier $\mathcal{M}$ is smaller than 1. On the other hand, when Δ is close to 1, we know that λ_1 is close to λ_2, and so I_1 and I_2 are close to each other, and by Proposition 2, the market efficiency multiplier $\mathcal{M}$ is greater than 1.

This link between Δ and $\mathcal{M}$ has important implications for market efficiency. Specifically, the following proposition shows that as information diversity increases, the overall price informativeness I_v goes up.

Proposition 3 *Information diversity increases the total price informativeness regarding $\tilde{v}$, that is, $\frac{dI_v}{d\Delta} > 0$.*

To understand this result, compare the following two economies with the same total mass of informed traders $\lambda_1 + \lambda_2 = \Lambda$, but with levels of diversity at the two ends of the spectrum: (1) Economy I, where $\lambda_1 = \Lambda - \varepsilon \approx \Lambda$, $\lambda_2 = \varepsilon \approx 0$, and $\Delta \approx 0$ (a "concentrated" economy); and (2) Economy II, where $\lambda_1 = \lambda_2 = \frac{\Lambda}{2}$ and $\Delta = 1$ (a "diverse" economy). These two economies can be obtained by injecting a total mass $(\Lambda - 2\varepsilon)$ of informed traders to an initial economy where there is almost no information (i.e., $\lambda_1 = \lambda_2 = \varepsilon \approx 0$) along two different paths. To obtain Economy I, we only add traders who are informed about $\tilde{v}_1$, while keeping the mass of agents informed about $\tilde{v}_2$ close to 0. In contrast, to obtain Economy II, we simultaneously add traders informed about $\tilde{v}_1$ and traders informed about $\tilde{v}_2$.

Adding informed traders along both paths improves market efficiency, since $\frac{\partial I_v}{\partial \lambda_i} > 0$ for $i = 1, 2$. However, the impact of the new information is different on the two different paths leading to the two economies because of the market-efficiency-multiplier effect identified by Proposition 2. Along the path to obtain Economy I, the multiplier $\mathcal{M}$ is smaller than 1, and thus the impact of the new added information is attenuated, while along the path to obtain Economy II, the multiplier $\mathcal{M}$ is greater than 1, and the impact of the new added information is amplified. As a result, the total impact of the added mass $(\Lambda - 2\varepsilon)$ of informed traders on market efficiency I_v is larger in Economy II than in Economy I.

Overall, market efficiency is higher in our model when there is more diversity of information, or when there is more balance between the amount of information available on different dimensions. This is because the effect of adding more informed agents on market efficiency is greater when the two levels of informativeness are relatively close to each other, as then the uncertainty reduction effect dominates, and we get a self-reinforcing spiral, by which more information encourages more aggressive trading, increasing informativeness further.

Figure 2 demonstrates graphically the effect of information diversity in our model for the parameter values $\rho = \chi = 50$, $\gamma = 3$ and $\Lambda = 0.1$. In Panel (a), we see that the market efficiency multiplier $\mathcal{M}$ is below 1 for low levels of diversity and above 1 for high levels of diversity. Panel (b) confirms the result of Proposition 3 that the overall price informativeness

I_v increases in information diversity (as a result of the multiplier effect in Panel (a)).

[FIGURE 2 ABOUT HERE]

4.2 Cost of Capital

One important implication of price informativeness is its effect on the cost of capital. We define the cost of capital as follows (see, e.g., Easley and O'Hara, 2004):

$$CC = E\left(\tilde{v} - \tilde{p}\right). \tag{23}$$

That is, the cost of capital is the expected difference between the cash flow generated by the security and its price, which is due to the risk taken by the traders who hold the security. By Proposition 1, we have

$$CC = -\alpha_0 = \frac{\gamma}{A_{p0}}, \tag{24}$$

where A_{p0} is the average trading aggressiveness across all traders, and is given by equation (46) in part A of the appendix as follows:

$$\begin{aligned} A_{p0} &= \frac{\lambda_1}{Var\left(\tilde{v}|\mathcal{F}_1\right)} + \frac{\lambda_2}{Var\left(\tilde{v}|\mathcal{F}_2\right)} + \frac{\lambda_u}{Var\left(\tilde{v}|\tilde{p}\right)} && (25)\\ &= \lambda_1\left(\rho + I_2^2\chi\right) + \lambda_2\left(\rho + I_1^2\chi\right) + \left(1-\Lambda\right)I_v. && (26)\end{aligned}$$

The right hand side of equation (25) captures the average trading aggressiveness of the different kinds of traders, which is a function of the residual risk they have to bear. Combining (25) with (24), we can see that the cost of capital increases in the risk that traders are exposed to per unit of the security: $Var\left(\tilde{v}|\mathcal{F}_1\right)$ for $\tilde{v}_1$-informed traders, $Var\left(\tilde{v}|\mathcal{F}_2\right)$ for $\tilde{v}_2$-informed traders, and $Var\left(\tilde{v}|\tilde{p}\right)$ for uninformed traders.

The following proposition shows that information diversity lowers the cost of capital:

Proposition 4 *Information diversity lowers the cost of capital, that is,* $\frac{dCC}{d\Delta} < 0$.

There are two effects leading to the result in Proposition 4. First, greater information diversity causes uninformed traders to bear less risk and trade more aggressively (as captured by the term $(1 - \Lambda) I_v$ in equation (26)), because, as we showed in the previous subsection, information diversity increases the total price informativeness I_v regarding $\tilde{v}$. This raises prices and lowers the cost of capital. Second, the two groups of informed traders, in aggregate, also bear less risk and trade more aggressively (as captured by the term $\lambda_1 (\rho + I_2^2 \chi) + \lambda_2 (\rho + I_1^2 \chi)$ in equation (26)). To understand this, suppose, without loss of generality, that there are more $\tilde{v}_1$-informed traders than $\tilde{v}_2$-informed traders, so that in general, the price system reflects more information regarding $\tilde{v}_1$ than $\tilde{v}_2$ (i.e., $I_1 > I_2$), and thus $\tilde{v}_2$-informed traders are exposed to less risk when they trade than $\tilde{v}_1$-informed traders (i.e., $(\rho + I_1^2 \chi) > (\rho + I_2^2 \chi)$). Increasing information diversity is equivalent to replacing some $\tilde{v}_1$-informed traders with $\tilde{v}_2$-informed traders, and as a result, the average risk that informed traders are exposed to decreases. This leads to an increase in price and a reduction in the cost of capital. Panel (c) of Figure 2 demonstrates the effect of information diversity on the cost of capital.

5 Endogenous Information Acquisition

So far, we assumed that the proportions of agents who are informed about the two fundamentals $\tilde{v}_1$ and $\tilde{v}_2$ – λ_1 and λ_2, respectively – were exogenous. We now endogenize these parameters and examine how they are determined in light of the incentives to become informed in our model. The new result that we get relative to the literature is that sometimes there will be a dominant strategic complementarity effect, whereby the increase in the mass of agents acquiring information on one fundamental will lead more agents to acquire information about the other fundamental. This is because of the uncertainty reduction effect identified earlier.

5.1 Information Acquisition in Equilibrium

We assume that there are two groups of traders of sizes $\overline{\lambda}_1, \overline{\lambda}_2 > 0$, $\overline{\lambda}_1 + \overline{\lambda}_2 = 1$. Traders in $\overline{\lambda}_1$ can acquire the signal $\tilde{v}_1$ at cost $c_1 > 0$, and traders in $\overline{\lambda}_2$ can acquire the signal $\tilde{v}_2$ at cost $c_2 > 0$. Traders who choose to acquire $\tilde{v}_1$ ($\tilde{v}_2$) become part of the λ_1 (λ_2) group in the trading model described in previous sections, while those who choose not to acquire information become part of the λ_u group.

Note that, for simplicity, we assume that any trader has an opportunity to become informed only about one of the two signals. This can be viewed as an extreme case of asymmetric expertise in information acquisition. That is, there are two groups of traders, and each group can only gather a particular type of information at a finite cost. This seems particularly reasonable when speculators acquire and trade on inside information. But, even without inside information, it is reasonable to think of different traders having potential access to different pieces of news based on their different expertise. Moreover, our setting can be justified based on limited ability to process information, e.g., Brunnermeier and Oehmke (2009) argue that investors may be subject to computational limitations or limited in the number of variables that they can keep track of. This idea is akin to the notion of limited attention, which has been recently studied in the finance literature (e.g., Moscarini, 2004; Peng, 2005; Peng and Xiong, 2006; and Gabaix, Laibson, Moloche, and Weinberg, 2006).

Using arguments similar to those in Grossman and Stiglitz (1980), we can show that for given fractions (λ_1, λ_2) of informed traders (and hence given values of I_1 and I_2, which are determined by the system in (19)), the expected net benefit from purchasing information $\tilde{v}_i$ to a potential purchaser is:

$$B_i\left(I_1, I_2\right) = \log\left(\frac{Var\left(\tilde{v}|\tilde{p}\right)}{Var(\tilde{v}|\mathcal{F}_i)}\right) - 2\gamma c_i = \log\left(\rho + I_j^2\chi\right) - \log\left(I_v\right) - 2\gamma c_i, \text{ for } i, j = 1, 2 \text{ and } j \neq i. \tag{27}$$

Intuitively, the net benefit from acquiring information is increasing in the quality of information available to the trader after purchasing the signal $\tilde{v}_i$ (given by $\rho + I_j^2\chi$), decreasing in the quality of information available to the trader by only observing the price (given by I_v), and decreasing in the cost of information production.

We focus on interior solutions, where $0 < \lambda_1^* < \overline{\lambda}_1$ and $0 < \lambda_2^* < \overline{\lambda}_2$.[8] Hence, conditions $B_1\left(I_1^*, I_2^*\right) = 0$ and $B_2\left(I_1^*, I_2^*\right) = 0$ characterize the equilibrium price informativeness measures I_1^* and I_2^*, such that speculators are indifferent between acquiring information and not

[8]We assume that parameters are such that interior solutions exist. This implies that the cost of information acquisition is neither too high nor too low. Here and in the rest of the section, we add a superscript * to all endogenous variables to emphasize that the information structure is endogenous.

acquiring information. These equilibrium price informativeness measures, in turn, characterize the equilibrium fractions $(\lambda_1^*, \lambda_2^*)$ of informed traders through the system (19). While we cannot prove analytically that the equilibrium in the information-acquisition stage is unique, in simulation analysis, the equilibrium always appears to be unique.

5.2 Strategic Complementarities vs. Strategic Substitutes

We now analyze the strategic interactions among traders producing information. In particular, we show that learning the two independent pieces of information $\tilde{v}_i$ and $\tilde{v}_j$ can be complementary, in the sense that an increase in the mass of agents acquiring information on one fundamental will increase the incentive of agents to acquire information about the other fundamental.

Specifically, by equation (27), an increase in the population λ_j of $\tilde{v}_j$-informed traders has two offsetting effects on the benefit B_i of acquiring signal $\tilde{v}_i$, which are reflected by the two terms, $\log\left(\rho + I_j^2\chi\right)$ and $\log\left(I_v\right)$. First, an increase in λ_j will improve the informativeness I_j of the price about signal $\tilde{v}_j$ (as $\frac{dI_j}{d\lambda_j} > 0$ by Proposition 2). This increased I_j directly reduces the remaining uncertainty of a $\tilde{v}_i$-informed trader, which allows him to trade more aggressively and increases his welfare due to the uncertainty reduction effect identified in earlier sections. This increases the benefit of becoming $\tilde{v}_i$-informed, which is reflected by the term $\log\left(\rho + I_j^2\chi\right)$ in equation (27). Second, an increase in λ_j also causes the price to be more informative about the total cash flow $\tilde{v}$ (as $\frac{dI_v}{d\lambda_j} > 0$ by Proposition 2 and equation (15)). This reduces the incentive of uninformed traders to become informed about $\tilde{v}_i$, which is part of $\tilde{v}$, as they can now gain more information about $\tilde{v}$ from the price. This effect is the standard Grossman-Stiglitz substitution effect, whereby having more informed traders, reduces the incentive to become informed. This negative effect is reflected by the term $\log\left(I_v\right)$ in equation (27).

When the positive uncertainty reduction effect dominates the negative Grossman-Stiglitz effect, an increase in λ_j, the size of $\tilde{v}_j$-informed traders, can increase B_i, the benefit of acquiring signal $\tilde{v}_i$, thereby causing acquiring the two types of information to be complementary. We can show that this is true when $I_j > I_i$; that is, $\frac{\partial B_i}{\partial \lambda_j} > 0$ if and only if $I_j > I_i$. Intuitively, when $I_j > I_i$, the price $\tilde{p}$ is mainly a public signal about the component $\tilde{v}_j$. As a result, acquiring $\tilde{v}_i$ will be particularly profitable when used in combination with $\tilde{p}$, a public signal about $\tilde{v}_j$, in the trading stage (i.e., a strong uncertainty reduction effect); and at the same time, the knowledge of $\tilde{v}_i$ will not be passed on to the price very much (i.e., a weak Grossman-Stiglitz effect), since the price is mainly informative about $\tilde{v}_j$.

Interestingly, learning the same information is still a strategic substitute, i.e., $\frac{\partial B_i}{\partial \lambda_i} < 0$: An increase in the population λ_i of $\tilde{v}_i$-informed traders will always decrease the benefit B_i of acquiring signal $\tilde{v}_i$. This is because the uncertainty reduction effect discussed above operates through the price informativeness I_j about the other component $\tilde{v}_j$, while increasing λ_i mainly increases I_i.[9] Thus, the complementarity effect in acquiring different information is not present in the traditional unidimensional Grossman-Stiglitz framework, and can only be uncovered by considering the two-dimension framework in our paper. We summarize the above discussion in the following proposition.

Proposition 5 *Acquiring information on the same fundamental is a strategic substitute: As more traders become informed of $\tilde{v}_i$, the benefit B_i of acquiring $\tilde{v}_i$ decreases; that is, $\frac{\partial B_i}{\partial \lambda_i} < 0$. Acquiring information on different fundamentals can be a strategic substitute or a complement: As more traders become informed of $\tilde{v}_i$, the benefit of acquiring $\tilde{v}_j$ can decrease or increase, and $\frac{\partial B_i}{\partial \lambda_j} > 0$ if and only if $I_j > I_i$.*

5.3 The Impact of Information Acquisition Cost

In this subsection, we will conduct comparative-statics analysis, examining the impact of changing the cost c_i of acquiring information $\tilde{v}_i$ on the equilibrium fractions $(\lambda_1^*, \lambda_2^*)$ of informed traders, on market efficiency I_v^*, and on the cost of capital CC^* in the overall equilibrium. The comparative-statics analysis is based on the equilibrium conditions $B_1(I_1^*, I_2^*) = 0$ and $B_2(I_1^*, I_2^*) = 0$ in the information-acquisition stage and on the system in (19) charac-

[9] Formally, we can argue that the uncertainty reduction effect generated by an increase in the size of traders informed of the same information either works in the same direction as the Grossman-Stiglitz effect or is small in magnitude. Specifically, the uncertainty reduction effect is captured by log $\left(\rho + I_j^2 \chi\right)$ in equation (27) and its strength is related to $\frac{dI_j}{d\lambda_i}$. By Propostion 2, we have $\frac{dI_j}{d\lambda_i} = \frac{\partial h_j}{\partial I_i}\frac{dI_i}{d\lambda_i}$, which can be positive or negative, depending on the sign of $\frac{\partial h_j}{\partial I_i}$. If $\frac{\partial h_j}{\partial I_i} < 0$, then $\frac{dI_j}{d\lambda_i} < 0$ and the uncertainty reduction effect works in the same direction as the Grossman-Stiglitz effect. If $\frac{\partial h_j}{\partial I_i} > 0$, we have $\frac{\partial h_j}{\partial I_i} < 1$ by equation (52), and hence the effect of λ_i on I_j is smaller that its effect on I_i; therefore, the uncertainty reduction effect is limited.

terizing price informativeness measures in the trading stage. The cost of information c_i represents a measure of the easiness of acquiring information on one fundamental $\tilde{v}_i$: A proliferation of sources of information about the firm (say, abundant disclosure, large analyst/media coverage, and advanced communication technologies) leads to easier access to information and corresponds to a low value of c_i (e.g., Fishman and Hagerty, 1989; Kim and Verrecchia, 1994). Our results depend on the strategic interactions between traders producing information on the two different dimensions, as we discussed above. We now turn to describe these effects and the comparative-statics results that they generate.

As we will show in Proposition 6, a decrease in the cost c_i of acquiring signal $\tilde{v}_i$ increases the equilibrium size λ_i^* of the population of $\tilde{v}_i$-informed traders. This is intuitive since a lower c_i corresponds to a higher net benefit of knowing $\tilde{v}_i$. More interestingly, the complementarity effect emphasized in Proposition 5 implies that a decrease in c_i, the cost of acquiring $\tilde{v}_i$, can increase the equilibrium size λ_j^* of the population of $\tilde{v}_j$-informed traders, too. As a result, λ_i^* and λ_j^* move in the same direction in response to the change in c_i, which is another form of a positive market efficiency spiral. Proposition 6 shows that $\frac{d\lambda_j^*}{dc_i} < 0$ if and only if $c_i < c_j$. Specifically, when $c_i < c_j$, we have $I_i^* > I_j^*$, which, according to Proposition 5, implies $\frac{\partial B_j}{\partial \lambda_i} > 0$, so that the increased λ_i^* (due to the decrease in c_i) increases the incentive of traders to acquire $\tilde{v}_j$, leading to a higher equilibrium size λ_j^* of the population of $\tilde{v}_j$-informed traders.

The proposition also shows that a decrease in the cost c_i of acquiring signal $\tilde{v}_i$ always leads the price to reveal more about the fundamental of the underlying asset, and as a result, the total price informativeness measure I_v^* increases. Finally, the equilibrium cost of capital CC^* is given by equations (24) and (26) as follows:

$$CC^* = \frac{\gamma}{A_{p0}^*},$$

where

$$\begin{aligned} A_{p0}^* &= \lambda_1^* \left(\rho + I_2^{*2}\chi\right) + \lambda_2^* \left(\rho + I_1^{*2}\chi\right) + \left(1 - \lambda_1^* - \lambda_2^*\right) I_v^* \\ &= \left[\lambda_1^* \left(\rho + I_2^{*2}\chi - I_v^*\right) + \lambda_2^* \left(\rho + I_1^{*2}\chi - I_v^*\right)\right] + I_v^*. \end{aligned} \tag{28}$$

Clearly, the increased I_v^* caused by the decrease in c_i tends to decrease CC^*. We can also show that the bracket term in equation (28), which captures the willingness to bear risk by all informed traders, increases. Thus, a decrease in c_i will decrease the equilibrium cost of capital CC^*. This suggests that disclosure, reducing the cost of information acquisition by speculators improves market efficiency and reduces the cost of capital.

The proposition is stated as follows:

Proposition 6 *A decrease in the cost c_i of acquiring information $\tilde{v}_i$*
(i) increases the equilibrium size λ_i^* of $\tilde{v}_i$-informed traders (i.e., $\frac{d\lambda_i^*}{dc_i} < 0$);
(ii) increases the equilibrium size λ_j^* of $\tilde{v}_j$-informed traders if and only if $c_i < c_j$ (i.e., $\frac{d\lambda_j^*}{dc_i} < 0$ if and only if $c_i < c_j$);
(iii) increases the overall price informativeness (i.e., $\frac{dI_v^*}{dc_i} < 0$); and
(iv) decreases the equilibrium cost of capital CC^* (i.e., $\frac{dCC^*}{dc_i} > 0$).

Figure 3 plots λ_1^* (Panel (a)), λ_2^* (Panel (b)), and I_1^* and I_2^* (Panel (c)) as functions of $\frac{1}{c_1}$ for the same numerical example as in Figure 1 with $c_2 = 0.1$ and $c_1 \in [0.05, 0.13]$. We can see that, for low values of c_1, we have $I_1^* > I_2^*$, and so in equilibrium learning the two types of information is complementary to each other, as λ_1^* and λ_2^* both become larger when c_1 decreases. In contrast, for high values of c_1, the Grossman-Stiglitz substitution effect dominates and information choices are strategic substitutes.

[FIGURE 3 ABOUT HERE]

6 An Extension: Correlated Payoff Ingredients and Noisy Signals

In this section, we analyze an extended economy where the two ingredients of the asset payoff are correlated and where informed traders receive noisy signals about the two ingredients. The analysis serves two purposes. First, it demonstrates the robustness of our main results – that information diversity improves overall price informativeness and reduces the cost of capital and that there are strategic complementarities at the stage of learning – by showing that they hold in this extended economy. Second, it highlights the importance of multiple dimensions of fundamentals in driving our results, as the extended economy nests both the traditional unidimensional model analyzed by Hellwig (1980) and Verrecchia (1982) and the new two-dimensional economy analyzed by this paper, thereby allowing a direct comparison between them to sharpen the distinctions.

6.1 Setup

The asset payoff is still given by the sum of two ingredients: $\tilde{v} = \tilde{v}_1 + \tilde{v}_2$. To accommodate the correlation between $\tilde{v}$ and $\tilde{v}_2$, we assume that they are generated by the following loading structure:

$$\tilde{v}_1 = \sqrt{\phi}\tilde{\theta} + \sqrt{1-\phi}\tilde{\eta}_1, \tag{29}$$

$$\tilde{v}_2 = \sqrt{\phi}\tilde{\theta} + \sqrt{1-\phi}\tilde{\eta}_2, \tag{30}$$

where $\tilde{\theta} \sim N(0, 1/\rho)$ is the common factor affecting both ingredients and where $\tilde{\eta}_i \sim N(0, 1/\rho)$ is an idiosyncratic factor. We assume that $(\tilde{\theta}, \tilde{\eta}_1, \tilde{\eta}_2)$ are mutually independent. Thus, parameter $\phi \in [0, 1]$ is the correlation coefficient between $\tilde{v}_1$ and $\tilde{v}_2$.

As in the our baseline model analyzed in previous sections, we assume that a continuum $[0, 1]$ of traders can acquire signals regarding $\tilde{v}_1$ and $\tilde{v}_2$ at costs $c_1 > 0$ and $c_2 > 0$. Still, each trader can only acquire one signal, and let λ_i denote the size of traders who decide to become informed of $\tilde{v}_i$. But now we do not assume that those informed traders observe $\tilde{v}_i$ perfectly. Instead, an informed trader j receives a noisy signal about $\tilde{v}_i$:

$$\tilde{s}_i^j = \tilde{v}_i + \tilde{\varepsilon}_i^j, \text{ with } \tilde{\varepsilon}_i^j \sim N(0, 1/n), \ n > 0, \tag{31}$$

for $j \in [0, \lambda_i]$ and $i = 1, 2$. Let $\tilde{x} \sim N(0, \chi^{-1})$ be the noise traders' demand for the risky asset. We assume that $(\tilde{\theta}, \tilde{\eta}_1, \tilde{\eta}_2, \{\tilde{\varepsilon}_i^j\}_{i,j}, \tilde{x})$ are mutually independent. Our baseline model corresponds to the special case of $\phi = 0$ and $n = \infty$ (i.e., the noise-to-signal ratio $\frac{\rho}{n} = 0$), and the traditional unidimensional model analyzed by Hellwig (1980) corresponds to the special case of $\phi = 1$ and $n \in (0, \infty)$. Thus, this structure is general enough to serve our two purposes mentioned above – demonstrating the robustness of our results and highlighting the uniqueness of our analysis.

The price $\tilde{p}$ still linearly depends on the signals $\tilde{v}_1$ and $\tilde{v}_2$ and the noisy trading $\tilde{x}$, given by equation (2). We still use $I_i = \frac{\alpha_i}{\alpha_x}$ to measure the price informativeness regarding $\tilde{v}_i$, and use $I_v = \frac{1}{Var(\tilde{v}|\tilde{p})}$ to measure the overall price informativeness regarding $\tilde{v} = \tilde{v}_1 + \tilde{v}_2$.

6.2 Equilibrium Characterization

The CARA-normal feature implies that the demand of investor j informed of $\tilde{v}_i$ is:

$$D_i(\tilde{p}, \tilde{s}_i^j) = \frac{E(\tilde{v}|\tilde{p}, \tilde{s}_i^j) - \tilde{p}}{\gamma Var(\tilde{v}|\tilde{p}, \tilde{s}_i^j)}. \tag{32}$$

Applying Bayes' rule yields:

$$E(\tilde{v}|\tilde{p}, \tilde{s}_i^j) = \beta_i^{(1)} \frac{\tilde{p} - \alpha_0}{\alpha_x} + \beta_i^{(2)} \tilde{s}_1^j, \tag{33}$$

$$Var(\tilde{v}|\tilde{p}, \tilde{s}_i^j) = \frac{(1-\phi)(\rho/\chi) + 2(\rho/\chi)(\rho/n) + (1-\phi)(\rho/n)(I_1 - I_2)^2}{\left(1-\phi^2\right) I_j^2 + (\rho/\chi)(1 + (\rho/n)) + (\rho/n)(I_1^2 + I_2^2 + 2\phi I_1 I_2)} (1+\phi)\rho^{-1}, \tag{34}$$

with

$$\beta_i^{(1)} = \frac{(1+\phi)\left[(\rho/n) I_i + ((\rho/n) + 1 - \phi) I_j\right]}{\left(1-\phi^2\right) I_j^2 + (\rho/\chi)(1 + (\rho/n)) + (\rho/n)(I_1^2 + I_2^2 + 2\phi I_1 I_2)}, \tag{35}$$

$$\beta_i^{(2)} = \frac{(1+\phi)\left[(\rho/\chi) + (1-\phi) I_j^2 - (1-\phi) I_1 I_2\right]}{\left(1-\phi^2\right) I_j^2 + (\rho/\chi)(1 + (\rho/n)) + (\rho/n)(I_1^2 + I_2^2 + 2\phi I_1 I_2)}. \tag{36}$$

The system determining the price informativeness measures I_1 and I_2 in the financial market is:

$$I_i = \int_0^{\lambda_i} \frac{\partial E\left(\tilde{v}|\tilde{p}, \tilde{s}_i^j\right)}{\partial \tilde{s}_i^j} dj = \frac{\lambda_i \beta_i^{(2)}}{\gamma Var(\tilde{v}|\tilde{p}, \tilde{s}_1^j)}, \text{ for } i = 1, 2. \tag{37}$$

Equations (34), (36) and (37) jointly determine the equilibrium I_1 and I_2. Bayes' rule also implies that the overall price informativeness is:

$$I_v = \frac{1}{Var(\tilde{v}|\tilde{p})} = \frac{(\rho/\chi) + I_1^2 + I_2^2 + 2\phi I_1 I_2}{\left[2(\rho/\chi) + (1-\phi)(I_1 - I_2)^2\right](1+\phi)\rho^{-1}}. \tag{38}$$

The cost of capital is:

$$CC = E(\tilde{v} - \tilde{p}) = \frac{\gamma}{\lambda_1 Var^{-1}(\tilde{v}|\tilde{p}, \tilde{s}_1^j) + \lambda_2 Var^{-1}(\tilde{v}|\tilde{p}, \tilde{s}_2^j) + (1 - \lambda_1 - \lambda_2) I_v}. \tag{39}$$

Going back to the information acquisition stage, the learning benefit (certainty equivalent multiplied by 2γ) of observing signal $\tilde{s}_i^j$ is:

$$B_i(I_1, I_2) = \log\left[\frac{Var(\tilde{v}|\tilde{p})}{Var(\tilde{v}|\tilde{p}, \tilde{s}_i^j)}\right] - 2\gamma c_i. \tag{40}$$

The equilibrium fractions λ_1^* and λ_2^* of informed traders are obtained in two steps: first, set $B_i(I_1^*, I_2^*) = 0$ (for $i = 1, 2$) to solve I_1^* and I_2^* for the overall equilibrium; and second, use the system defining I_1 and I_2 at the trading stage, that is, equations (34), (36) and (37), to back out λ_1^* and λ_2^*.

6.3 Robustness of Our Results

We now examine our two main results: (i) at the trading stage, information diversity increases overall price informativeness and decreases the cost of capital; and (ii) at the learning stage, there is strategic complementarity in information acquisition.

The information diversity Δ is still defined by equation (22):

$$\Delta \equiv 1 - \frac{|\lambda_1 - \lambda_2|}{\Lambda} \in [0, 1],$$

where Λ is a fixed total size of the informed traders population. The complexity of the equations defining the equilibrium I_1 and I_2 precludes simple analytical analysis. Instead we use numerical analysis to examine the implications of information diversity and present the result in Figure 4. We have checked a wide range of parameters and found that the results hold for all parameter configurations.

[FIGURE 4 ABOUT HERE]

In Figure 4, we choose parameter values similar to those in Figure 2. That is, we set

$\rho = \chi = 50$, $\gamma = 3$ and $\Lambda = 0.1$. In Panel (a), we set $\phi = 0.5$ and $n = \infty$ (i.e., the noise-to-signal ratio $\frac{\rho}{n}$ is 0) to isolate the impact of the correlation ϕ between the two ingredients $\tilde{v}_1$ and $\tilde{v}_2$. In Panel (b), we set $\phi = 0$ and $n = 250$ (i.e., the noise-to-signal ratio $\frac{\rho}{n}$ is 0.2) to isolate the impact of the noise in the signal $\tilde{s}_i^j$. We find that, consistent with Propositions 3 and 4 in Section 4, increasing Δ increases I_v and decreases CC in both panels.

We use Figures 5 and 6 to investigate the implications of ϕ and n for learning complementarities, respectively. In both figures, we conduct an exercise similar to Figure 3 to examine Proposition 6 – we fix c_2 and decrease c_1, and check whether there is a region for which λ_1^* and λ_2^* move in the same direction in equilibrium.[10] Since λ_1^* always decreases with c_1, we only plot λ_2^* against (c_2/c_1), and a U-shaped λ_2^* indicates the existence of learning complementarities.

[FIGURES 5 AND 6 ABOUT HERE]

Specifically, in both figures, we set $\rho = \chi = 50$ and $\gamma = 3$. Then, in each panel, we set c_2 at a value $\bar{c}$ such that when $c_1 = c_2 = \bar{c}$, the equilibrium fractions of informed traders are $\lambda_1^* = \lambda_2^* = 0.05$. We see that, consistent with Panel (b) of Figure 3, for all economies in Figures 5 and 6, when c_1 is sufficiently low, so that the equilibrium I_1^* is sufficiently large, λ_2^* increases with (c_2/c_1) and hence moves in the same direction as λ_1^*. Figure 5 also suggests the turning point of (c_2/c_1) increases with ϕ. This is because when ϕ is high, the Grossman-Stiglitz effect is strong: The high correlation between $\tilde{v}_1$ and $\tilde{v}_2$ implies that much information contained in signal $\tilde{s}_2^j$ can be learned from observing prices, which aggregate many new acquired $\tilde{s}_1^j$ signals due to the drop in c_1.

6.4 Two-Dimensional Economy versus Hellwig (1980)

In this last subsection, we will compare our economy with the unidimensional economy studied by Hellwig (1980) and Verrecchia (1982) to highlight the role of multiple dimensions of fundamentals in delivering our results. As we mentioned before, when we choose $\phi = 1$ and

[10] Note that Proposition 6 is an equilibrium result of complementarities in information acquisition described in Proposition 5. So, Figures 5 and 6 are a stronger test of learning complementarities. We will examine Proposition 5 more closely in the next subsection when we compare our economy with the standard unidimensional Hellwig (1980) economy.

$n \in (0, \infty)$, the two ingredients of asset payoff reduce to one common factor and informed traders acquire a noisy signal about this common factor representing the overall asset payoff. In this case, using the equations characterizing the equilibrium, we can analytically compute the price informativeness measures, the cost of capital, and the benefit of acquiring signals, which all depend on the total size Λ of the informed traders population. In particular, unlike Propositions 3-5 in previous sections, now information diversity Δ does not influence the overall price informativeness I_v and the cost of capital ($\frac{dI_v}{d\Delta} = \frac{dCC}{d\Delta} = 0$), and acquiring information is always a substitute ($\frac{\partial B_i}{\partial \lambda_i} = \frac{\partial B_i}{\partial \lambda_j} < 0$), which suggests that our results are driven by the fact that informed traders are informed of non-perfectly correlated components of asset payoff (i.e., $\phi \neq 1$).[11] We summarize these results in the following proposition.

Proposition 7 ***[Hellwig (1980)]*** *Suppose* $\phi = 1$ *and* $n \in (0, \infty)$. *Then, (i)* $I_1 = \frac{\lambda_1 n}{2\gamma}$, $I_2 = \frac{\lambda_2 n}{2\gamma}$, $I_v = \frac{\rho}{4} + \left(\frac{\Lambda n}{4\gamma}\right)^2 \chi$, $CC = \frac{4\gamma}{\rho + \Lambda n + \left(\frac{\Lambda n}{2\gamma}\right)^2 \chi}$, *and hence* $\frac{dI_v}{d\Delta} = \frac{dCC}{d\Delta} = 0$; *(ii)* $B_i(\lambda_1, \lambda_2) = \log\left[\frac{n}{\rho + \left(\frac{\Lambda n}{2\gamma}\right)^2 \chi} + 1\right] - 2\gamma c_i$, *and hence* $\frac{\partial B_i}{\partial \lambda_i} = \frac{\partial B_i}{\partial \lambda_j} < 0$, *for* $i = 1, 2$ *and* $i \neq j$.

Figures 7 and 8 further use numerical examples to compare our economy and the Hellwig (1980) economy. In both figures, Panel (a) corresponds to the one-dimensional economy studied by Hellwig (1980): $\phi = 1$. Panel (b) corresponds to a two-dimensional economy where the two payoff ingredients are non-perfectly correlated: $\phi = 0.5$. In both economies, we assume that informed traders observe noisy signals about the asset payoff by setting $n = 250$ (i.e., the noise-to-signal ratio $\frac{\rho}{n} = 0.2$). The other parameter values in both panels are: $\rho = \chi = 50$ and $\gamma = 3$. In Figure 7, Panel (a) illustrates Proposition 7 by showing that the overall price informativeness I_v and the cost of capital CC do not vary with information diversity Δ, while Panel (b) confirms Propositions 3 and 4 in the two-dimensional economy

[11] An alternative view is that the Hellwig (1980) economy is the limiting case of our two-dimensional economy and our results hold trivially. Specifically, since all traders are effectively informed of only common factor, then (i) in effect, there are no distinctions between λ_1 and λ_2, and the variable of information diversity Δ effectively takes only one value of 0, and thus the overall price informativeness I_v and the cost of capital CC take only one value accordingly, which is consistent with Propositions 3 and 4 trivially (as there are no variations of Δ), and (ii) for the same reason, $\frac{\partial B_i}{\partial \lambda_j}$ effectively captures the strategic substitute in acquiring information on the same fundamental, which is consistent with Proposition 5, too. But both views highlight the fact that our results can only be developed in a two-dimensional economy like the one analyzed in this paper.

by showing that I_v increases with Δ and that CC decreases with Δ.

[FIGURES 7 AND 8 ABOUT HERE]

In Figure 8, for both the Hellwig economy and the two-dimensional economy, we conduct two exercises to examine learning complementarities, which correspond respectively to Propositions 5 and 6. First, to examine Proposition 5 which directly describes learning complementarities through the impact of the number of informed traders on the benefit of acquiring information, we draw the simplex $\{(\lambda_1, \lambda_2) \in \mathbb{R}_+^2 : \lambda_1 + \lambda_2 \leq 1\}$ – which is the whole range of (λ_1, λ_2) – and use "+" to indicate the complementarity region for which the benefit B_2 of acquiring signal about $\tilde{v}_2$ increases with the size λ_1 of $\tilde{v}_1$-informed group (i.e., $\frac{\partial B_2}{\partial \lambda_1} > 0$). We see that there is no such a complementarity region in the Hellwig economy, while the size of the complementarity region is nontrivial in the two-dimensional economy. In addition, we find that $\frac{\partial B_2}{\partial \lambda_1}$ is more likely to be positive in the two-dimensional economy when λ_1 is relatively large relative to λ_2, which is consistent with Proposition 5 which says that $\frac{\partial B_2}{\partial \lambda_1} > 0$ if and only if $I_1 > I_2$ in the baseline model.

Second, we follow Figure 3 and draw the equilibrium λ_2^* against (c_2/c_1) to examine Proposition 6 which specifies the implications of changing the cost of information in the presence of learning complementarities. Specifically, we fix c_2 at a constant $\bar{c}$ (such that $\lambda_1^* = \lambda_2^* = 0.05$ when $c_1 = c_2 = \bar{c}$) and decrease c_1, and then plot λ_2^* against $(c_2/c_1) = (\bar{c}/c_1)$. Then, a U-shaped λ_2^* indicates the existence of learning complementarities, since the right upward-sloping branch is a region for which λ_1^* and λ_2^* move in the same direction in response to a change in c_1. We see that, in the two-dimensional economy, λ_2^* indeed first decreases and then increases with (c_2/c_1), which is consistent with Proposition 6. In contrast, in the Hellwig economy, λ_2^* generally decreases with (c_2/c_1): When $(c_2/c_1) < 1$, we have $c_1 > c_2 = \bar{c}$, and all informed traders acquire information about $\tilde{v}_2$ (which is the overall payoff $\tilde{v}$ too) since the cost of acquiring $\tilde{v}_2$ is cheaper than that of acquiring $\tilde{v}_1$ (which is $\tilde{v}$ too), and the size λ_2^* is maintained at $\Lambda^* = 0.1$; when $(c_2/c_1) > 1$, then $c_2 = \bar{c} > c_1$ and thus no one will acquire $\tilde{v}_2$ and hence $\lambda_2^* = 0$; when $(c_2/c_1) = 1$, then $c_1 = c_2 = \bar{c}$, the exact size λ_2^* is indeterminate, and it can take any value in $[0, \Lambda^*]$.

Now the central message at the learning stage should be clear. The standard result that the benefit of acquiring information decreases with the total size of the informed traders population in the unidimensional Hellwig (1980) economy is not robust to perturbations in the correlation of different ingredients of the asset payoff. How traders respond to other traders becoming informed does not only depend on the total size of the informed group, but also on its composition. If traders are acquiring information about fundamentals that are weakly correlated, then acquiring information on different fundamentals tend to be a complement instead of a substitute, as the price will reveal more information that is unknown even after traders become informed and hence that is particularly useful in reducing uncertainty when combined with their own acquired information in a particular dimension.

7 Conclusion

Our paper addresses a fundamental question in the study of market efficiency and price informativeness: Does information in financial markets attract or deter the production and transmission of more information? Extending the seminal Grossman and Stiglitz (1980) model to include two dimensions of uncertainty in the value of the traded asset, we uncover a rich set of interactions, providing new insights into this question.

When the price becomes more informative about one fundamental, traders informed about another fundamental may reduce their trading aggressiveness due to an adverse selection effect, or increase it due to an uncertainty reduction effect. We show that the latter effect dominates in equilibrium if and only if the levels of informativeness about the two fundamentals are sufficiently close to each other, implying that greater diversity of information in the economy generates amplification of informativeness and hence greater equilibrium market efficiency. We also show that the uncertainty reduction effect can generate strategic complementarities in information production, leading to another form of a market efficiency spiral.

The paper highlights the importance of information diversity in determining the level of market efficiency. Future empirical research can test the predictions of the paper by linking the diversity of trading styles to measures of market efficiency and price informativeness.

Future theoretical research can extend the model in various directions, such as including more dimensions of uncertainty and allowing traders to be informed in various dimensions, to improve our understanding of the effect of information structure on the efficiency of financial markets.

Appendix: Proofs

A. Proof of Proposition 1

Using the expressions for $D_i(\tilde{p}, \tilde{v}_i)$ and $D_u(\tilde{p})$ (given by equations (8) and (9)) and the definition of $\tilde{s}_{j|i}$ (given by equations (4)), we can rewrite the market clearing condition (equation (14)) as follows:

$$\begin{aligned}
& \lambda_1 \left[\rho + (\alpha_2/\alpha_x)^2 \chi - (\alpha_2/\alpha_x)^2 \chi/\alpha_2\right] \tilde{p} \\
& +\lambda_2 \left[\rho + (\alpha_1/\alpha_x)^2 \chi - (\alpha_1/\alpha_x)^2 \chi/\alpha_1\right] \tilde{p} + \lambda_u \frac{\tilde{p} - E(\tilde{v}|\tilde{p})}{Var(\tilde{v}|\tilde{p})} \\
= \; & \lambda_1 (\alpha_2/\alpha_x)^2 \chi \frac{-\alpha_0}{\alpha_2} + \lambda_2 (\alpha_1/\alpha_x)^2 \chi \frac{-\alpha_0}{\alpha_1} - \gamma \\
& +\lambda_1 \left[\rho + (\alpha_2/\alpha_x)^2 \chi - (\alpha_1/\alpha_x)(\alpha_2/\alpha_x)\chi\right] \tilde{v}_1 \\
& +\lambda_2 \left[\rho + (\alpha_1/\alpha_x)^2 \chi - (\alpha_1/\alpha_x)(\alpha_2/\alpha_x)\chi\right] \tilde{v}_2 + \gamma \tilde{x}. \qquad (41)
\end{aligned}$$

Note that the left-hand side of the above equation is only related to $\tilde{p}$, while the right-hand side is only related to $\tilde{v}_1$, $\tilde{v}_2$, and $\tilde{x}$. Hence, based on (41) and (2), we form the following system of two equations in terms of two unknowns, (α_1/α_x) and (α_2/α_x):

$$(\alpha_1/\alpha_x) = \lambda_1 \left[\rho + (\alpha_2/\alpha_x)^2 \chi - (\alpha_1/\alpha_x)(\alpha_2/\alpha_x)\chi\right] \gamma^{-1}, \qquad (42)$$

$$(\alpha_2/\alpha_x) = \lambda_2 \left[\rho + (\alpha_1/\alpha_x)^2 \chi - (\alpha_1/\alpha_x)(\alpha_2/\alpha_x)\chi\right] \gamma^{-1}. \qquad (43)$$

By equation (43), we can express (α_2/α_x) in terms of (α_1/α_x) as follows (this is equation (19) for $i = 2$ in the main text):

$$\frac{\alpha_2}{\alpha_x} = \frac{\lambda_2 \left[\rho + \chi (\alpha_1/\alpha_x)^2\right]}{\left[\gamma + \lambda_2 \chi (\alpha_1/\alpha_x)\right]}. \qquad (44)$$

Then, plugging the above expression into equation (42), we have the following cubic polynomial in (α_1/α_x):

$$\begin{aligned}
& \left(\gamma\lambda_1\lambda_2\chi^2 + \gamma\lambda_2^2\chi^2\right)(\alpha_1/\alpha_x)^3 + 2\left(\gamma^2\lambda_2\chi - \lambda_1\lambda_2^2\rho\chi^2\right)(\alpha_1/\alpha_x)^2 \\
& + \left(\gamma\lambda_1\lambda_2\rho\chi - 2\gamma\lambda_1\lambda_2\rho\chi + \gamma^3\right)(\alpha_1/\alpha_x) + \left(-\lambda_1\lambda_2^2\rho^2\chi - \gamma^2\lambda_1\rho\right) \\
= \; & 0. \qquad (45)
\end{aligned}$$

The discriminant of the polynomial is

$$-\chi^2\lambda_1\lambda_2\left(\gamma^2+\rho\chi\lambda_2^2\right)^2\left(\begin{array}{c}4\gamma^6+24\gamma^4\chi\lambda_1\lambda_2\rho+32\chi^3\lambda_1^3\lambda_2^3\rho^3\\+\gamma^2\chi\lambda_1\lambda_2\left(26\lambda_1^2\rho^2+\left(\lambda_1\rho-3\lambda_2\rho\right)^2+18\lambda_2^2\rho^2\right)\end{array}\right)<0.$$

Thus, equation (45) has only one real solution for (α_1/α_x). In addition, this solution is positive, because when $\lambda_1>0$ and $\lambda_2>0$, the left-hand side of equation (45) is negative at $(\alpha_1/\alpha_x)=0$ and becomes positive as (α_1/α_x) is sufficiently large. By equation (44), we also know that the equilibrium coefficient ratio $\frac{\alpha_2}{\alpha_x}$ is positive.

Using equations (9)-(13) to express out $\frac{\tilde{p}-E(\tilde{v}|\tilde{p})}{Var(\tilde{v}|\tilde{p})}$ in equation (41) delivers

$$\left(A_{p0}-A_{px}\frac{1}{\alpha_x}\right)\tilde{p}=A_1\tilde{v}_1+A_2\tilde{v}_2+\gamma\tilde{x}+\left(-A_{01}\frac{\alpha_0}{\alpha_1}-A_{02}\frac{\alpha_0}{\alpha_2}-A_{0x}\frac{\alpha_0}{\alpha_x}-\gamma\right)$$

where the coefficients of A's are known positive values – which are determined by (α_1/α_x) and (α_2/α_x) – defined as follows:

$$A_{p0} = \frac{\lambda_1}{Var\left(\tilde{v}|\mathcal{F}_1\right)}+\frac{\lambda_2}{Var\left(\tilde{v}|\mathcal{F}_2\right)}+\frac{\lambda_u}{Var\left(\tilde{v}|\tilde{p}\right)}, \tag{46}$$

$$A_{px} = \lambda_1\left(\alpha_2/\alpha_x\right)\chi+\lambda_2\left(\alpha_1/\alpha_x\right)\chi+\frac{\lambda_u\beta_{\tilde{v},\tilde{p}}}{Var\left(\tilde{v}|\tilde{p}\right)}, \tag{47}$$

$$A_1=\gamma\left(\alpha_1/\alpha_x\right), A_2=\gamma\left(\alpha_2/\alpha_x\right),$$

$$A_{01}=\lambda_2\left(\alpha_1/\alpha_x\right)^2\chi, A_{02}=\lambda_1\left(\alpha_2/\alpha_x\right)^2\chi, A_{0x}=\lambda_u\frac{\beta_{\tilde{v},\tilde{p}}}{Var\left(\tilde{v}|\tilde{p}\right)}.$$

Thus, we can solve for α_x:

$$\alpha_x=\frac{\gamma}{A_{p0}-A_{px}\frac{1}{\alpha_x}}\Rightarrow\alpha_x=\frac{\gamma+A_{px}}{A_{p0}}>0. \tag{48}$$

Combining the known ratios (α_1/α_x) and (α_2/α_x) with the value of α_x gives the values of α_1 and α_2, which are positive. Once we know α_1, α_2 and α_x, then we can solve α_0 using

$$\alpha_0=\frac{-A_{01}\frac{\alpha_0}{\alpha_1}-A_{02}\frac{\alpha_0}{\alpha_2}-A_{0x}\frac{\alpha_0}{\alpha_x}-\gamma}{A_{p0}-A_{px}\frac{1}{\alpha_x}}\Rightarrow\alpha_0=-\frac{\gamma}{\left(A_{p0}-A_{px}\frac{1}{\alpha_x}\right)+A_{01}\frac{1}{\alpha_1}+A_{02}\frac{1}{\alpha_2}+A_{0x}\frac{1}{\alpha_x}}.$$

We can further use the solved expressions of A_{01}, A_{02}, A_{0x} and α_x to simplify the denominator of the above expression of α_0 and show that

$$\alpha_0=-\frac{\gamma}{A_{p0}}<0.$$

This in turn implies that the cost of capital is

$$CC=E\left(\tilde{v}-\tilde{p}\right)=-\alpha_0=\frac{\gamma}{A_{p0}}. \tag{49}$$

QED.

B. Proof of Proposition 2

Taking total differentiation of equation (19) (for $i = 1, 2$) with respect to Q implies:

$$\frac{dI_1}{dQ} = \frac{\partial h_1}{\partial Q} + \frac{\partial h_1}{\partial I_2}\frac{dI_2}{dQ} \text{ and } \frac{dI_2}{dQ} = \frac{\partial h_2}{\partial Q} + \frac{\partial h_2}{\partial I_1}\frac{dI_1}{dQ}.$$

Solving for $\frac{dI_1}{dQ}$ and $\frac{dI_2}{dQ}$ delivers

$$\frac{dI_1}{dQ} = \frac{\frac{\partial h_1}{\partial Q} + \frac{\partial h_1}{\partial I_2}\frac{\partial h_2}{\partial Q}}{1 - \frac{\partial h_1}{\partial I_2}\frac{\partial h_2}{\partial I_1}} \text{ and } \frac{dI_2}{dQ} = \frac{\frac{\partial h_2}{\partial Q} + \frac{\partial h_2}{\partial I_1}\frac{\partial h_1}{\partial Q}}{1 - \frac{\partial h_1}{\partial I_2}\frac{\partial h_2}{\partial I_1}},$$

which is equation (20).

Next, we examine the sign and magnitude of $\mathcal{M} = \left(1 - \frac{\partial h_1}{\partial I_2}\frac{\partial h_2}{\partial I_1}\right)^{-1}$. By equation (19), direct computation shows

$$\frac{\partial h_i}{\partial I_j} = 1 - \frac{\gamma^2 + \chi\rho\lambda_i^2}{(\gamma + \chi\lambda_i I_j)^2}. \tag{50}$$

Using equations (42)-(43), we can express λ_i in terms of I_1 and I_2 as follows:

$$\lambda_i = \frac{\gamma I_i}{\rho + I_j^2\chi - I_1 I_2\chi}. \tag{51}$$

Plugging the above expression into equation (50) yields:

$$\frac{\partial h_i}{\partial I_j} = \frac{(2I_j - I_i)\,\chi I_i}{\rho + \chi I_j^2}. \tag{52}$$

Thus, we have

$$\begin{aligned}
\mathcal{M}^{-1} &= 1 - \frac{\partial h_1}{\partial I_2}\frac{\partial h_2}{\partial I_1} \\
&= 1 - \frac{(2I_2 - I_1)\,\chi I_1}{\rho + \chi I_2^2}\frac{(2I_1 - I_2)\,\chi I_2}{\rho + \chi I_1^2} \\
&= \frac{(\rho + 2\chi I_1 I_2)\left(\rho + \chi\,(I_1 - I_2)^2\right)}{\left(\rho + \chi I_2^2\right)\left(\rho + \chi I_1^2\right)} > 0.
\end{aligned}$$

That is, $\mathcal{M} > 0$.

Whether $M > 1$ depends on whether $\frac{\partial h_1}{\partial I_2}$ and $\frac{\partial h_2}{\partial I_1}$ have the same sign. Specifically, we have three cases.

Case 1. If $\frac{\partial h_i}{\partial I_j} = 0$ for some i, then $\mathcal{M} = 1$. By equation (52), this will be true if and only if

$$\frac{\partial h_i}{\partial I_j} = \frac{(2I_j - I_i)\,\chi I_i}{\rho + \chi I_j^2} = 0 \Rightarrow \frac{I_i}{I_j} = 2.$$

Case 2. If $\frac{\partial h_i}{\partial I_j} > 0$ for $i = 1, 2$, then $\mathcal{M} > 1$. By equation (52), this will be true if and only if

$$\frac{\partial h_i}{\partial I_j} = \frac{(2I_j - I_i)\,\chi I_i}{\rho + \chi I_j^2} > 0 \Rightarrow \frac{I_i}{I_j} < 2, \forall i \Rightarrow \frac{1}{2} < \frac{I_1}{I_2} < 2.$$

Case 3. If $\frac{\partial h_i}{\partial I_j} > 0$ and $\frac{\partial h_j}{\partial I_i} < 0$, then $0 < \mathcal{M} < 1$. This will be true if and only if $\frac{I_j}{I_i} > 2$, i.e., $\frac{I_1}{I_2} > 2$ or $\frac{I_1}{I_2} < \frac{1}{2}$.

Note that it is not possible to have both $\frac{\partial h_1}{\partial I_2} < 0$ and $\frac{\partial h_2}{\partial I_1} < 0$, because these two inequalities combine to imply $2 < \frac{I_j}{I_i} < \frac{1}{2}$, which is impossible. QED.

C. Proof of Proposition 3

Without loss of generality, we assume that $\lambda_1 > \lambda_2$, and as a result, increasing diversity Δ while fixing Λ is equivalent to decreasing λ_1 and increasing λ_2. Formally, we have

$$\lambda_1 + \lambda_2 = \Lambda \text{ and } \Delta = 1 - \frac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2} = \frac{2\lambda_2}{\lambda_1 + \lambda_2}.$$

Then, taking total differentiation of the above system, yields:

$$\begin{aligned} d\lambda_1 + d\lambda_2 &= d\Lambda = 0, \\ d\left(\frac{2\lambda_2}{\lambda_1 + \lambda_2}\right) &= 2\left(-\frac{\lambda_2}{(\lambda_1 + \lambda_2)^2} d\lambda_1 + \frac{(\lambda_1 + \lambda_2) - \lambda_2}{(\lambda_1 + \lambda_2)^2} d\lambda_2\right) = d\Delta, \end{aligned}$$

which implies:

$$\frac{d\lambda_1}{d\Delta} = -\frac{\Lambda}{2} \text{ and } \frac{d\lambda_2}{d\Delta} = \frac{\Lambda}{2}, \tag{53}$$

and

$$\frac{\partial h_1}{\partial Q} = \frac{\partial h_1}{\partial \Delta} = -\frac{\partial h_1}{\partial \lambda_1}\frac{\Lambda}{2} \text{ and } \frac{\partial h_2}{\partial Q} = \frac{\partial h_2}{\partial \Delta} = \frac{\partial h_2}{\partial \lambda_2}\frac{\Lambda}{2}, \tag{54}$$

in equation (20).

Now, we analyze the impact of Δ on the total price informativeness I_v. By equations (20) and (54), we have

$$\begin{aligned} \frac{dI_1}{d\Delta} &= \mathcal{M}\left(\frac{\partial h_1}{\partial \Delta} + \frac{\partial h_1}{\partial I_2}\frac{\partial h_2}{\partial \Delta}\right) \\ &= \mathcal{M}\left(-\frac{\partial h_1}{\partial \lambda_1} + \frac{\partial h_1}{\partial I_2}\frac{\partial h_2}{\partial \lambda_2}\right)\frac{\Lambda}{2}. \end{aligned}$$

Then, using equations (50) and (51), we have

$$\frac{dI_1}{d\Delta} = \mathcal{M}\frac{\Lambda}{2}\gamma\left[-\frac{\rho + \chi I_2^2}{\left(\gamma + \chi\frac{\gamma I_1}{\rho + I_2^2\chi - I_1 I_2 \chi} I_2\right)^2} + \frac{(2I_2 - I_1)\chi I_1}{\rho + \chi I_2^2}\frac{\rho + \chi I_1^2}{\left(\gamma + \chi\frac{\gamma I_2}{\rho + I_1^2\chi - I_1 I_2 \chi} I_1\right)^2}\right]. \tag{55}$$

Similarly, we can compute:

$$\frac{dI_2}{d\Delta} = \mathcal{M}\frac{\Lambda}{2}\gamma\left[\frac{\rho + \chi I_1^2}{\left(\gamma + \chi\frac{\gamma I_2}{\rho + I_1^2\chi - I_1 I_2 \chi} I_1\right)^2} - \frac{(2I_1 - I_2)\chi I_2}{\rho + \chi I_1^2}\frac{\rho + \chi I_2^2}{\left(\gamma + \chi\frac{\gamma I_1}{\rho + I_2^2\chi - I_1 I_2 \chi} I_2\right)^2}\right]. \tag{56}$$

Recall that by equation (15), we have:

$$I_v = \frac{1}{Var(\tilde{v}|\tilde{p})} = \frac{I_1^2\rho + I_2^2\rho + \rho^2\chi^{-1}}{(I_1 - I_2)^2 + 2\rho\chi^{-1}}.$$

Direct computation shows:

$$\frac{\partial I_v}{\partial I_1} = \frac{2\chi\rho(I_1 + I_2)\left(\rho + \chi I_2^2 - \chi I_1 I_2\right)}{\left(2\rho + \chi I_1^2 + \chi I_2^2 - 2\chi I_1 I_2\right)^2} > 0, \tag{57}$$

$$\frac{\partial I_v}{\partial I_2} = \frac{2\chi\rho(I_1 + I_2)\left(\rho + \chi I_1^2 - \chi I_1 I_2\right)}{\left(2\rho + \chi I_1^2 + \chi I_2^2 - 2\chi I_1 I_2\right)^2} > 0, \tag{58}$$

where the inequalities follow from equations (42)-(43), namely, $(\rho + I_j^2\chi - I_1 I_2\chi) = \frac{\gamma I_i}{\lambda_i} > 0$.

By chain rule:

$$\frac{dI_v}{d\Delta} = \frac{\partial I_v}{\partial I_1}\frac{dI_1}{d\Delta} + \frac{\partial I_v}{\partial I_2}\frac{dI_2}{d\Delta}.$$

Plugging equations (55)-(58) into the above equation delivers:

$$\frac{dI_v}{d\Delta} = \mathcal{M}\frac{\gamma\Lambda}{2}\frac{2\chi^2\rho\left(I_1^2 - I_2^2\right)(I_1 + I_2)}{\left(2\rho + \chi I_1^2 + \chi I_2^2 - 2\chi I_1 I_2\right)^2}\frac{\rho^3 + \chi\rho^2(I_1 + I_2)^2 + 3\chi^2\rho I_1 I_2 (I_1 - I_2)^2 + \chi^3 I_1 I_2 (I_1 - I_2)^4}{\gamma^2\left(\rho + \chi I_2^2\right)\left(\rho + \chi I_1^2\right)} > 0,$$

because $I_1 > I_2$ by $\lambda_1 > \lambda_2$. QED.

D. Proof of Proposition 4

We build on the proof of the previous proposition. By equations (25) and (24), to show $\frac{dCC}{d\Delta} < 0$, it suffices to show $\frac{dA_{p0}}{d\Delta} > 0$. By equations (26) and (53), we have

$$\begin{aligned}\frac{dA_{p0}}{d\Delta} &= \frac{d\lambda_1}{d\Delta}\left(\rho + \chi I_2^2\right) + \lambda_1\left(\chi 2 I_2\frac{dI_2}{d\Delta}\right) + \frac{\partial\lambda_2}{\partial\Delta}\left(\rho + \chi I_1^2\right) + \lambda_2\left(\chi 2 I_1\frac{dI_1}{d\Delta}\right) + (1 - \Lambda)\frac{dI_v}{d\Delta} \\ &= \frac{\Lambda}{2}\chi\left(I_1^2 - I_2^2\right) + (1 - \Lambda)\frac{dI_v}{d\Delta} + 2\chi\left(\lambda_1 I_2\frac{dI_2}{d\Delta} + \lambda_2 I_1\frac{dI_1}{d\Delta}\right).\end{aligned}$$

Since $\frac{\Lambda}{2}\chi\left(I_1^2 - I_2^2\right) > 0$ and $(1 - \Lambda)\frac{dI_v}{d\Delta} > 0$, to show $\frac{dA_{p0}}{d\Delta} > 0$, it suffices to show

$$\left(\lambda_1 I_2\frac{dI_2}{d\Delta} + \lambda_2 I_1\frac{dI_1}{d\Delta}\right) > 0,$$

which is true, because equations (51), (55) and (56) imply:

$$\begin{aligned}&\lambda_1 I_2\frac{dI_2}{d\Delta} + \lambda_2 I_1\frac{dI_1}{d\Delta} \\ = \;&\mathcal{M}\frac{\Lambda}{2}\chi I_1 I_2\left(I_1^2 - I_2^2\right)\frac{\rho^3 + \chi\rho^2(I_1 + I_2)^2 + 3\chi^2\rho I_1 I_2(I_1 - I_2)^2 + \chi^3 I_1 I_2(I_1 - I_2)^4}{\left(\rho + \chi I_2^2\right)\left(\rho + \chi I_1^2\right)\left(\rho + \chi I_2^2 - \chi I_1 I_2\right)\left(\rho + \chi I_1^2 - \chi I_1 I_2\right)} > 0.\end{aligned}$$

QED.

E. Proof of Proposition 5

Let us first prove $\frac{\partial B_i}{\partial \lambda_i} < 0$. By Proposition 2, we have:

$$\frac{dI_i}{d\lambda_i} = \mathcal{M}\frac{\gamma(\rho+\chi I_j^2)}{(\gamma+\lambda_i\chi I_j)^2} > 0 \text{ and } \frac{dI_j}{d\lambda_i} = \frac{(2I_i - I_j)\,\chi I_j}{\rho+\chi I_i^2}\frac{dI_i}{d\lambda_i}.$$

By equation (27) and the above expression of $\frac{dI_j}{d\lambda_i}$, we have:

$$\begin{aligned}
\frac{\partial B_i}{\partial \lambda_i} &= \frac{2I_j\chi}{\rho+I_j^2\chi}\frac{dI_j}{d\lambda_i} - \frac{1}{I_v}\left(\frac{\partial I_v}{\partial I_i}\frac{dI_i}{d\lambda_i} + \frac{\partial I_v}{\partial I_j}\frac{dI_j}{d\lambda_i}\right) \\
&= \left[\left(\frac{2I_j\chi}{\rho+I_j^2\chi} - \frac{1}{I_v}\frac{\partial I_v}{\partial I_j}\right)\frac{(2I_i-I_j)\,\chi I_j}{\rho+\chi I_i^2} - \frac{1}{I_v}\frac{\partial I_v}{\partial I_i}\right]\frac{dI_i}{d\lambda_i}.
\end{aligned}$$

Then, using equations (15), (57) and (58), we can show the bracket term in the above equation is negative:

$$\begin{aligned}
&\left(\frac{2I_j\chi}{\rho+I_j^2\chi} - \frac{1}{I_v}\frac{\partial I_v}{\partial I_j}\right)\frac{(2I_i-I_j)\,\chi I_j}{\rho+\chi I_i^2} - \frac{1}{I_v}\frac{\partial I_v}{\partial I_i} \\
= & -\frac{2\chi\left(\rho+\chi I_j^2-\chi I_1 I_2\right)\left[\begin{array}{c}\rho^2 I_1+\rho^2 I_2+\chi\rho I_1^3+\chi\rho I_2^3+\chi\rho I_j\,(I_1-I_2)^2 \\ +2\chi\rho I_i^2 I_j+\chi^2 I_j\left(2I_i^2+I_j^2\right)(I_1-I_2)^2\end{array}\right]}{\left(\rho+\chi I_1^2\right)\left(\rho+\chi I_2^2\right)\left(\rho+\chi I_1^2+\chi I_2^2\right)\left(2\rho+\chi I_1^2+\chi I_2^2-2\chi I_1 I_2\right)} < 0,
\end{aligned}$$

where we have used the fact of $\left(\rho+\chi I_j^2-\chi I_1 I_2\right) = \frac{\gamma I_i}{\lambda_i} > 0$. Thus, $\frac{\partial B_i}{\partial \lambda_i} < 0$.

Next, we prove $\frac{\partial B_i}{\partial \lambda_j} > 0$ if and only if $I_j > I_i$. Following the similar argument as above, we can show:

$$\begin{aligned}
\frac{\partial B_i}{\partial \lambda_j} &= \left[\frac{2I_j\chi}{\rho+I_j^2\chi} - \frac{1}{I_v}\frac{\partial I_v}{\partial I_j} - \frac{1}{I_v}\frac{\partial I_v}{\partial I_i}\frac{(2I_j-I_i)\,\chi I_i}{\rho+\chi I_j^2}\right]\frac{dI_j}{d\lambda_j} \\
&= \frac{2\chi\,(I_j-I_i)\left(\rho+\chi I_j^2-\chi I_1 I_2\right)^2}{\left(\rho+\chi I_j^2\right)\left(2\rho+\chi I_1^2+\chi I_2^2-2\chi I_1 I_2\right)\left(\rho+\chi I_1^2+\chi I_2^2\right)}\frac{dI_j}{d\lambda_j}.
\end{aligned}$$

Since $\frac{dI_j}{d\lambda_j} > 0$, we have $\frac{\partial B_i}{\partial \lambda_j} > 0$ if and only if $I_j > I_i$. QED.

F. Proof of Proposition 6

Setting $B_i\,(I_1, I_2) = 0$ for $i = 1, 2$ delivers the following system:

$$\left\{\begin{array}{l}\log\left(\rho+I_2^{*2}\chi\right) - \log\left(I_v^*\right) = 2\gamma c_1, \\ \log\left(\rho+I_1^{*2}\chi\right) - \log\left(I_v^*\right) = 2\gamma c_2.\end{array}\right. \tag{59}$$

Suppose we decrease c_1. Applying the implicit function theorem to the above system delivers:

$$\begin{cases} -\frac{1}{I_v^*}\frac{\partial I_v^*}{\partial I_1^*}\frac{dI_1^*}{dc_1} + \left(-\frac{1}{I_v^*}\frac{\partial I_v^*}{\partial I_2^*} + \frac{2I_2^*\chi}{\rho+I_2^{*2}\chi}\right)\frac{dI_2^*}{dc_1} = 2\gamma, \\ \left(-\frac{1}{I_v^*}\frac{\partial I_v^*}{\partial I_1^*} + \frac{2I_1^*\chi}{\rho+I_1^{*2}\chi}\right)\frac{dI_1^*}{dc_1} - \frac{1}{I_v^*}\frac{\partial I_v^*}{\partial I_2^*}\frac{dI_2^*}{dc_1} = 0. \end{cases}$$

Thus,

$$\frac{dI_1^*}{dc_1} = -\frac{2\gamma}{D}\frac{1}{I_v^*}\frac{\partial I_v^*}{\partial I_2^*} \text{ and } \frac{dI_2^*}{dc_1} = -\frac{2\gamma}{D}\left(-\frac{1}{I_v^*}\frac{\partial I_v^*}{\partial I_1^*} + \frac{2I_1^*\chi}{\rho+I_1^{*2}\chi}\right), \tag{60}$$

where

$$D = \frac{4\chi^2\left(I_1^{*2}+I_2^{*2}\right)\left(\rho+\chi I_2^{*2}-\chi I_1^*I_2^*\right)\left(\rho+\chi I_1^{*2}-\chi I_1^*I_2^*\right)}{\left(\rho+\chi I_2^{*2}\right)\left(\rho+\chi I_1^{*2}\right)\left(\rho+\chi I_1^{*2}+\chi I_2^{*2}\right)\left(2\rho+\chi I_1^{*2}+\chi I_2^{*2}-2\chi I_1^*I_2^*\right)} > 0.$$

Impact on λ_1^* and λ_2^*. By equation (51),

$$\frac{d\lambda_1^*}{dc_1} = \gamma\frac{\left(\rho+I_2^{*2}\chi\right)\frac{dI_1^*}{dc_1} + \left(I_1^*-2I_2^*\right)I_1^*\chi\frac{dI_2^*}{dc_1}}{\left(\rho+I_2^{*2}\chi-I_1^*I_2^*\chi\right)^2}.$$

Then, plugging equation (60), (15), (57) and (58) into the above expression yields:

$$\frac{d\lambda_1^*}{dc_1} = -\frac{4\gamma^2\chi\left(\rho+\chi I_1^{*2}-\chi I_1^*I_2^*\right)}{D\left(\rho+I_2^{*2}\chi-I_1^*I_2^*\chi\right)^2}\frac{\begin{bmatrix}\rho^2I_1^*+\rho^2I_2^*+\chi\rho I_1^{*3}+\chi\rho I_2^{*3}+2\chi\rho I_1^*I_2^{*2} \\ +\chi I_1^*\left(I_1^*-I_2^*\right)^2\left(\rho+\chi I_1^{*2}+2\chi I_2^{*2}\right)\end{bmatrix}}{\left(\rho+\chi I_1^{*2}\right)\left(\rho+\chi I_1^{*2}+\chi I_2^{*2}\right)\left(2\rho+\chi I_1^{*2}+\chi I_2^{*2}-2\chi I_1^*I_2^*\right)} < 0.$$

Similarly, we can compute:

$$\begin{aligned} \frac{d\lambda_2^*}{dc_1} &= \gamma\frac{I_2^*\left(I_2^*-2I_1^*\right)\chi\frac{dI_1^*}{dc_1} + \left(\rho+I_1^{*2}\chi\right)\frac{dI_2^*}{dc_1}}{\left(\rho+I_1^{*2}\chi-I_1^*I_2^*\chi\right)^2} \\ &= -\frac{4\gamma^2\chi\left(I_1^*-I_2^*\right)}{D\left(\rho+I_1^{*2}\chi-I_1^*I_2^*\chi\right)^2}\frac{\left(\rho+\chi I_1^{*2}-\chi I_1^*I_2^*\right)^2}{\left(\rho+\chi I_1^{*2}+\chi I_2^{*2}\right)\left(2\rho+\chi I_1^{*2}+\chi I_2^{*2}-2\chi I_1^*I_2^*\right)}. \end{aligned}$$

Thus, $\frac{d\lambda_2^*}{dc_1} < 0$ if and only if $I_1^* > I_2^*$.

Finally, we have

$$I_1^* > I_2^* \Leftrightarrow c_1 < c_2,$$

because the system of determining I_1^* and I_2^* (equation (59)) implies that

$$\log\left(\rho+I_2^{*2}\chi\right) - \log\left(\rho+I_1^{*2}\chi\right) = 2\gamma\left(c_1-c_2\right).$$

Therefore. $\frac{d\lambda_2^*}{dc_1} < 0$ if and only if $c_1 < c_2$.

Impact on I_v^*. By the chain rule,

$$\frac{dI_v^*}{dc_1} = \frac{\partial I_v^*}{\partial I_1^*}\frac{dI_1^*}{dc_1} + \frac{\partial I_v^*}{\partial I_2^*}\frac{dI_2^*}{dc_1}.$$

Plugging equations (57), (58) and (60) into the above equation delivers:

$$\frac{dI_v^*}{dc_1} = -\frac{2\gamma}{D}\frac{\partial I_v^*}{\partial I_2^*}\frac{2I_1^*\chi}{\rho + I_1^{*2}\chi} < 0.$$

Impact on CC^*. Given $\frac{dI_v^*}{dc_1} < 0$, to show $\frac{dCC^*}{dc_1} > 0$, it suffices to show that in equation (28) the term $\left[\lambda_1^*\left(\rho + I_2^{*2}\chi - I_v^*\right) + \lambda_2^*\left(\rho + I_1^{*2}\chi - I_v^*\right)\right]$ decreases with c_1. By equations (15) and (51), we have:

$$\lambda_1^*\left(\rho + I_2^{*2}\chi - I_v^*\right) + \lambda_2^*\left(\rho + I_1^{*2}\chi - I_v^*\right) = \gamma\rho\frac{I_1^* + I_2^*}{2\rho + \chi I_1^{*2} + \chi I_2^{*2} - 2\chi I_1^* I_2^*}.$$

Direct computation shows:

$$\begin{aligned}
&\frac{d}{dc_1}\left[\gamma\rho\frac{I_1^* + I_2^*}{2\rho + \chi I_1^{*2} + \chi I_2^{*2} - 2\chi I_1^* I_2^*}\right] \\
= \;& \gamma\rho\frac{\left(2\rho - \chi I_1^{*2} + 3\chi I_2^{*2} - 2\chi I_1^* I_2^*\right)\frac{dI_1^*}{dc_1} + \left(2\rho + 3\chi I_1^{*2} - \chi I_2^{*2} - 2\chi I_1^* I_2^*\right)\frac{dI_2^*}{dc_1}}{\left(2\rho + \chi I_1^{*2} + \chi I_2^{*2} - 2\chi I_1^* I_2^*\right)^2}.
\end{aligned}$$

Plugging equations (57), (58) and (60) into the above equation shows:

$$\begin{aligned}
&\frac{d}{dc_1}\left[\lambda_1^*\left(\rho + I_2^{*2}\chi - I_v^*\right) + \lambda_2^*\left(\rho + I_1^{*2}\chi - I_v^*\right)\right] \\
= \;& -\frac{4\gamma^2\rho\chi\left(\rho I_1^* + \chi I_1^{*3} + \chi I_2^{*3} + 2\chi I_1^* I_2^{*2} + I_1^*\left(\rho + \chi I_1^{*2} - \chi I_1^* I_2^*\right)\right)\left(\rho + \chi I_1^{*2} - \chi I_1^* I_2^*\right)}{D\left(2\rho + \chi I_1^{*2} + \chi I_2^{*2} - 2\chi I_1^* I_2^*\right)^2\left(\rho + \chi I_1^{*2}\right)\left(\rho + \chi I_1^{*2} + \chi I_2^{*2}\right)} < 0.
\end{aligned}$$

QED.

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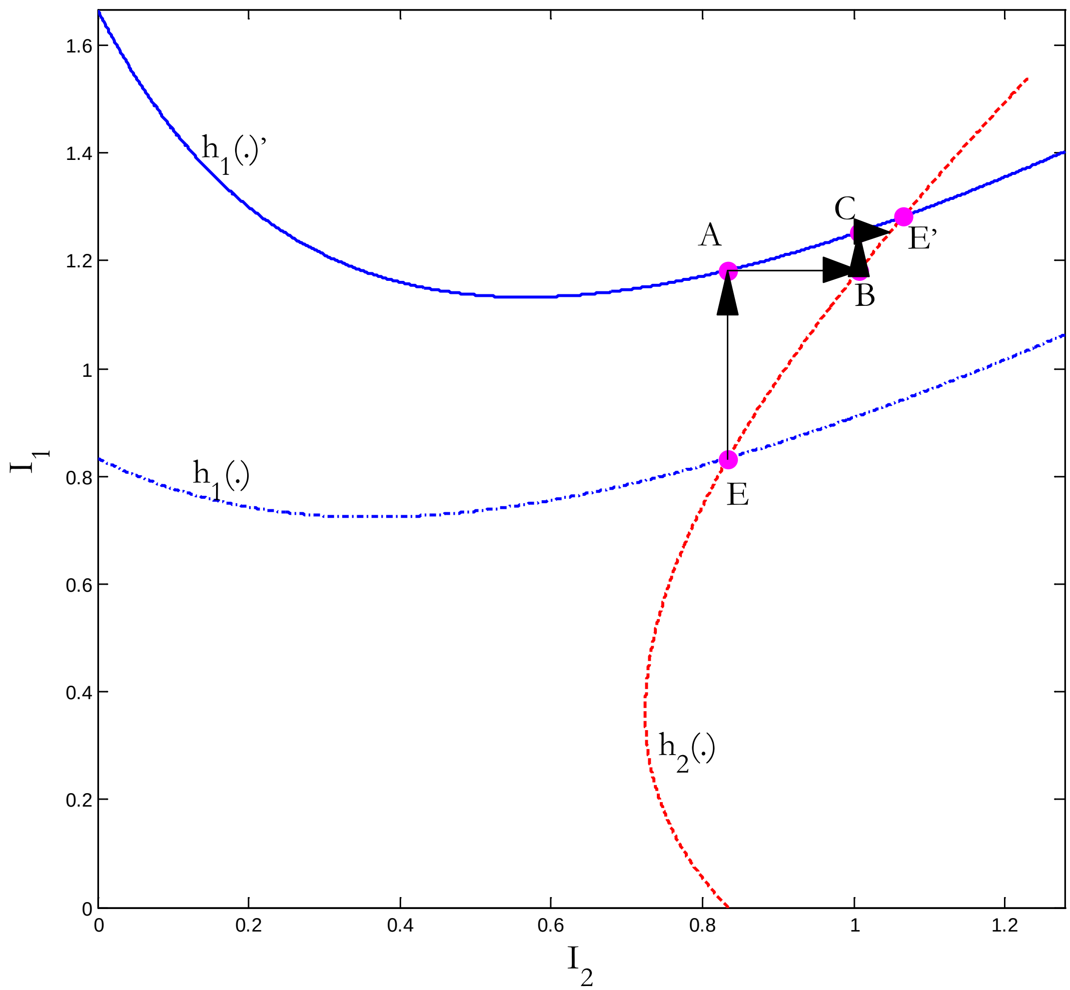
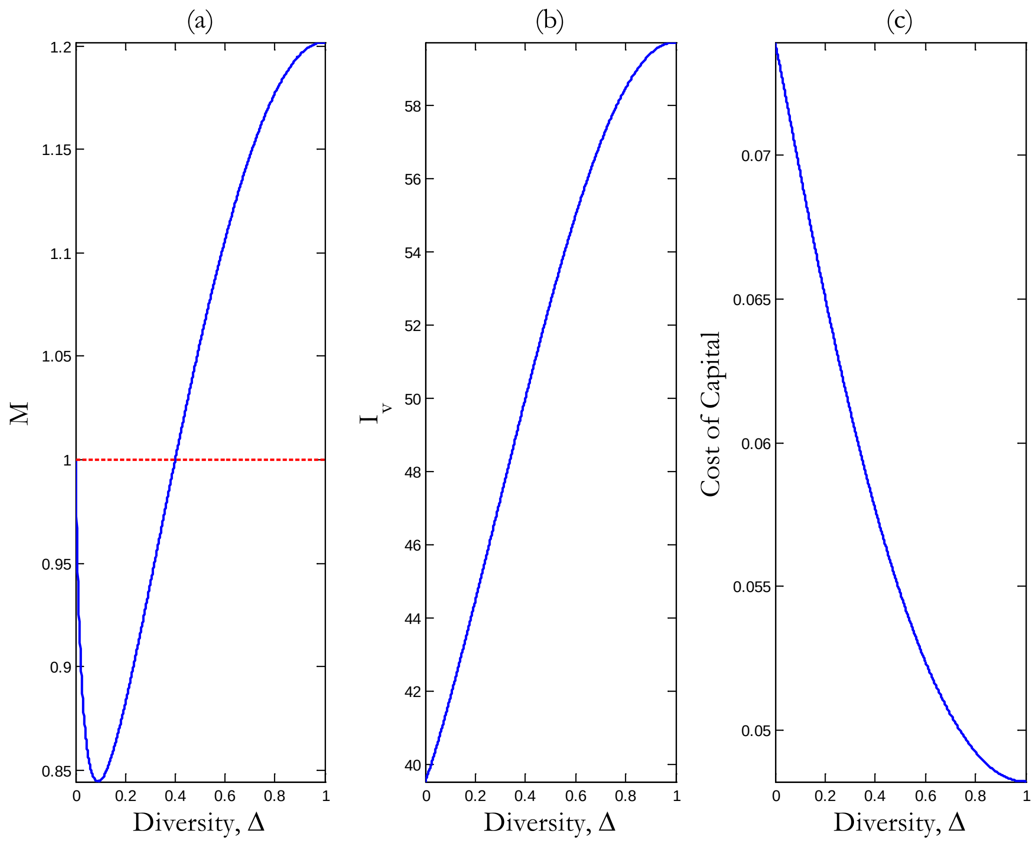
Figure 1: Market Efficiency Spirals



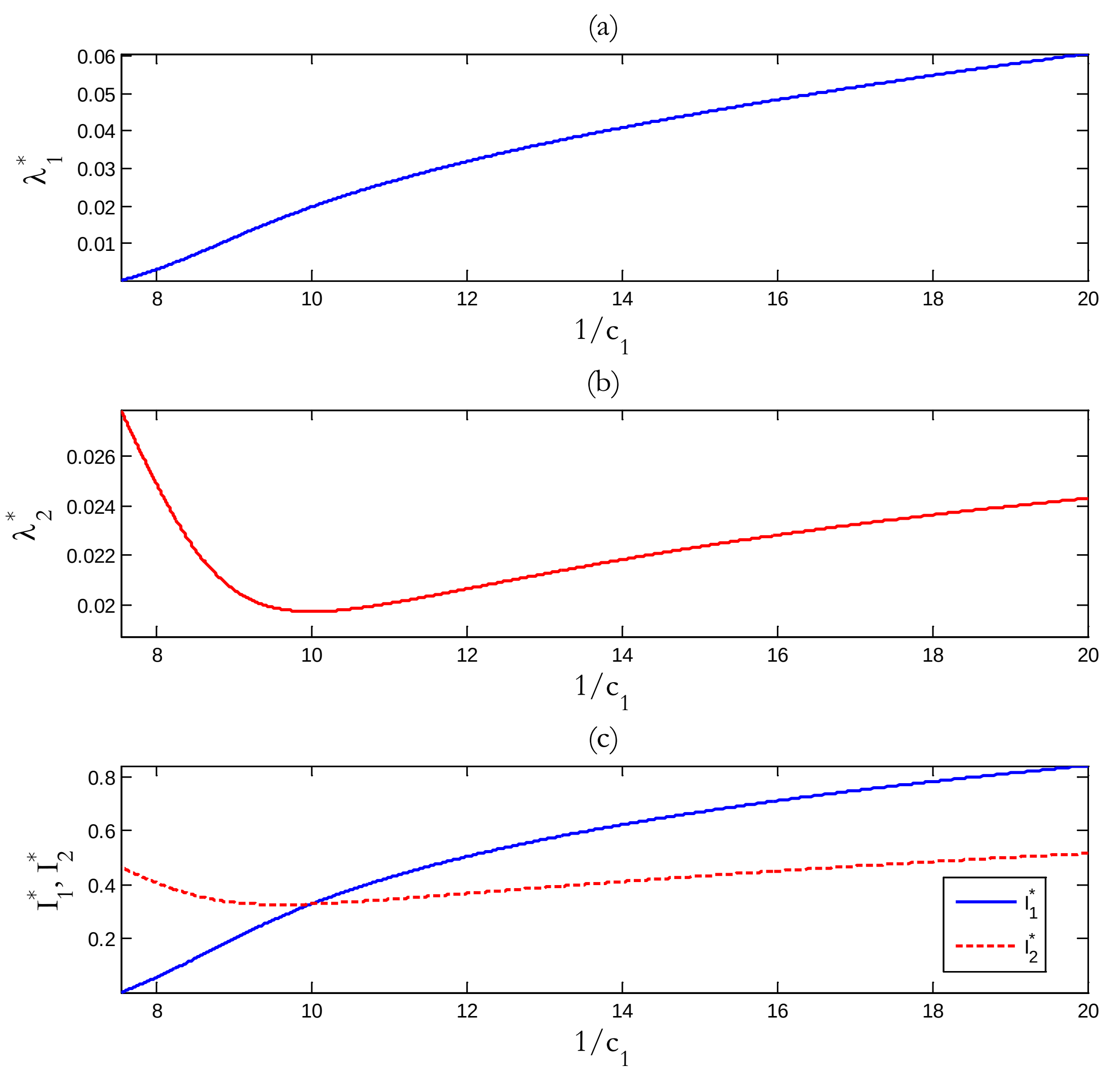
This figure plots the functions h_1 and h_2 whose intersections determine the equilibrium values of I_1 and I_2. Parameter λ_1 increases from 0.05 to 0.1. Other relevant parameter values are: $\rho = \chi = 50$ and $\gamma = 3$.

Figure 2: Impact of Diversity (Δ)



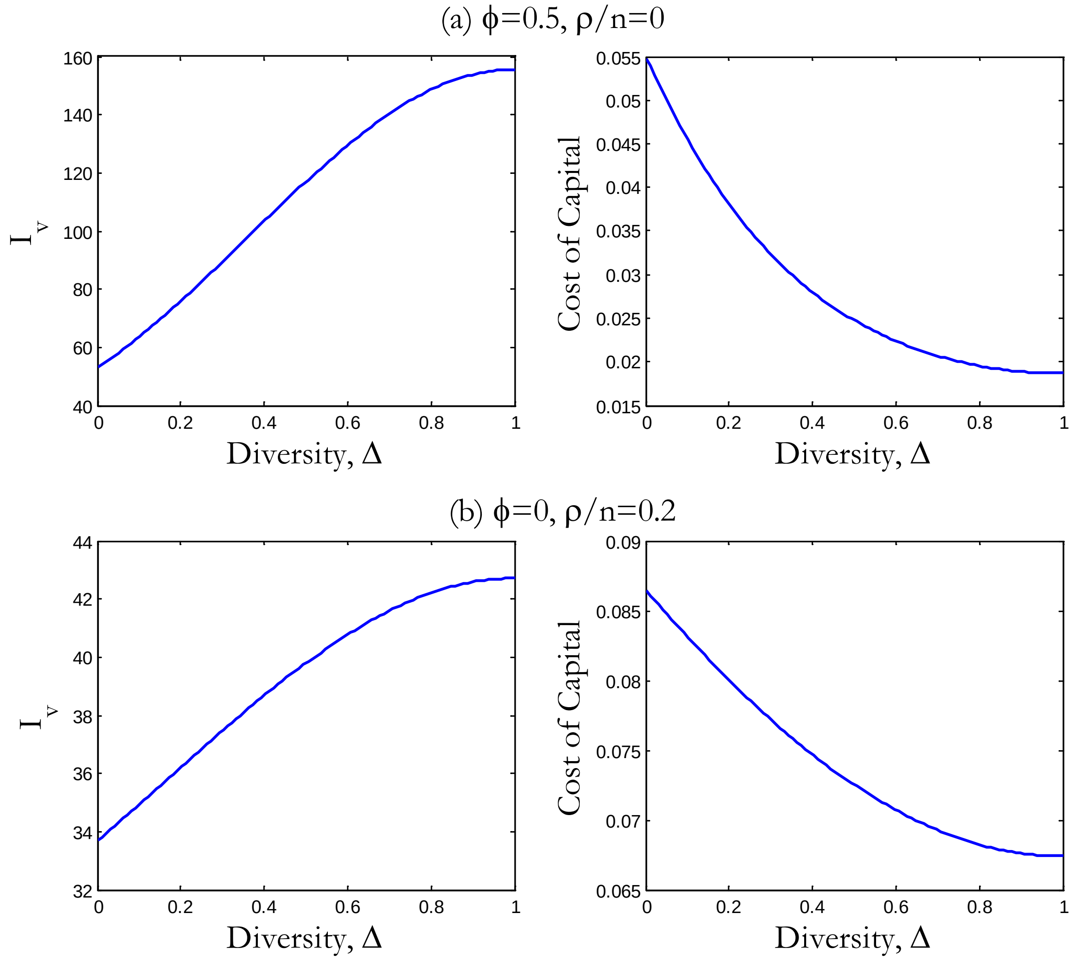
This figure plots the impact of information diversity on market efficiency multiplier, price informativeness regarding total cash flow, and cost of capital. The parameter values are $\rho = \chi = 50$, $\gamma = 3$ and $\Lambda = 0.1$.

Figure 3: Learning Complement vs. Substitute



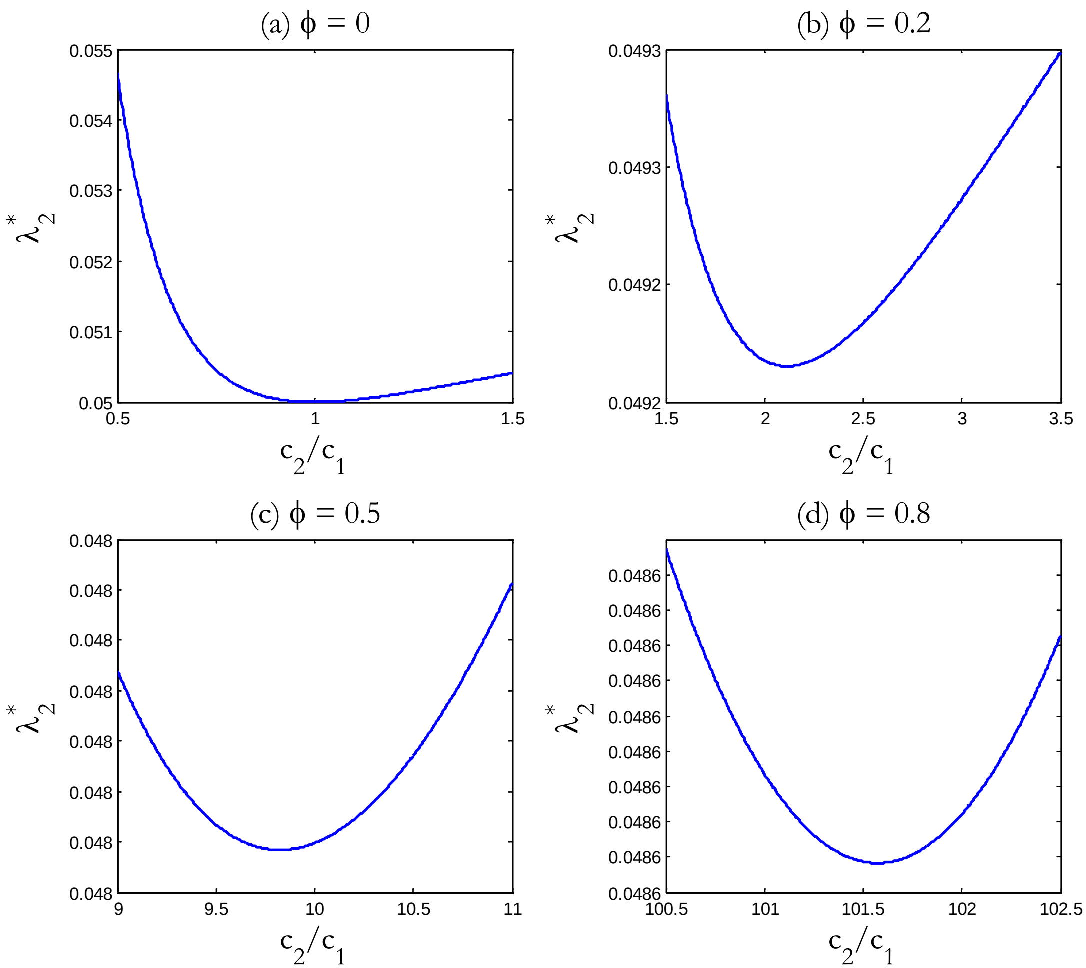
This figure plots the impact of the cost c_1 of acquiring $\tilde{v}_1$ on the endogenous fractions of informed traders and the price informativeness measures. When the two fractions λ_1^* and λ_2^* of informed traders move in the same direction, there exist learning complementarities; otherwise, information acquisition is a substitute. The parameter values are: $\rho = \chi = 50$, $\gamma = 3$, and $c_2 = 0.1$. The range of c_1 is [0.05, 0.13].

Figure 4: Impact of Diversity (Δ) in Extended Economies



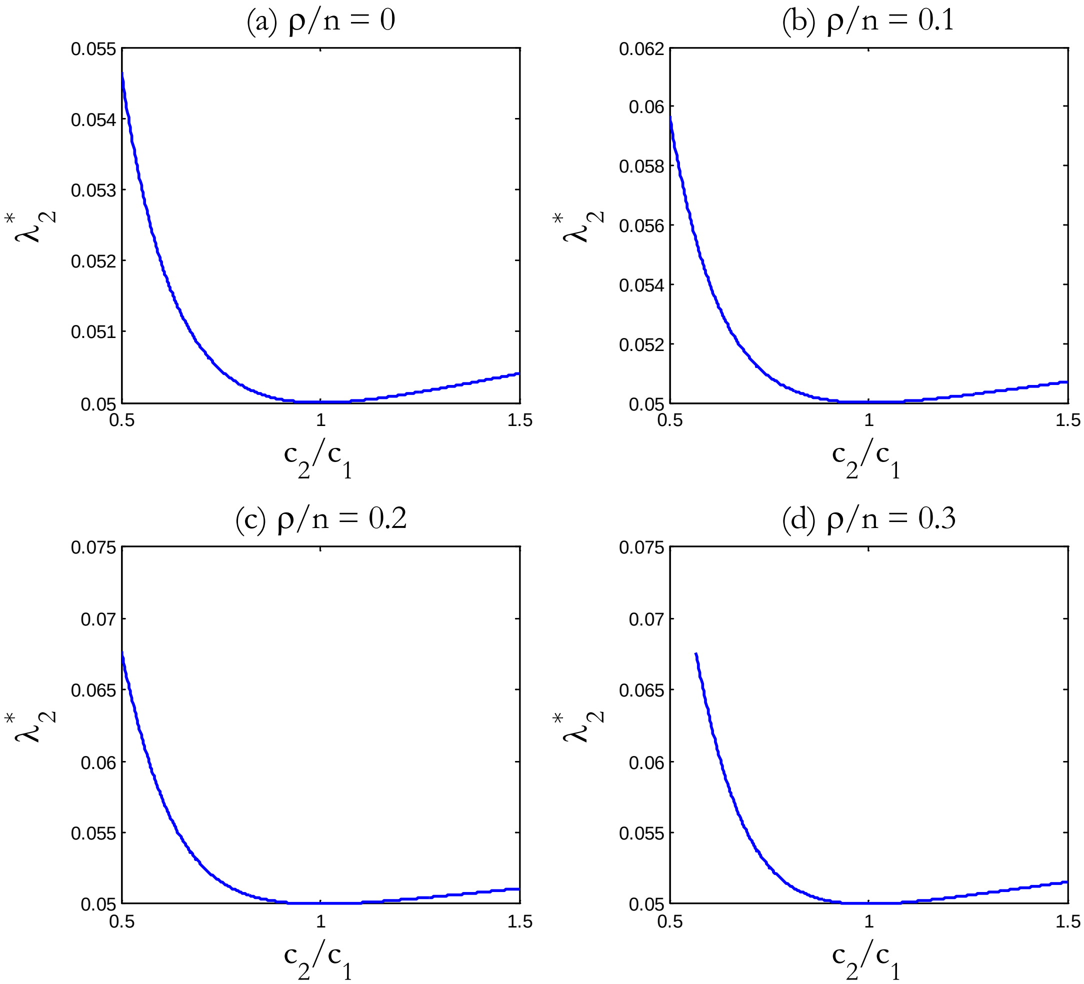
This figure plots the impact of information diversity on price informativeness regarding overall cash flow and the cost of capital in the extended model described in Section 6, where the two payoff ingredients are correlated and where informed traders observe noisy signals about the two ingredients. The parameter values are $\rho = \chi = 50$, $\gamma = 3$ and $\Lambda = 0.1$.

Figure 5: Impact of Cash Flow Correlation (ϕ) on Learning Complementarties



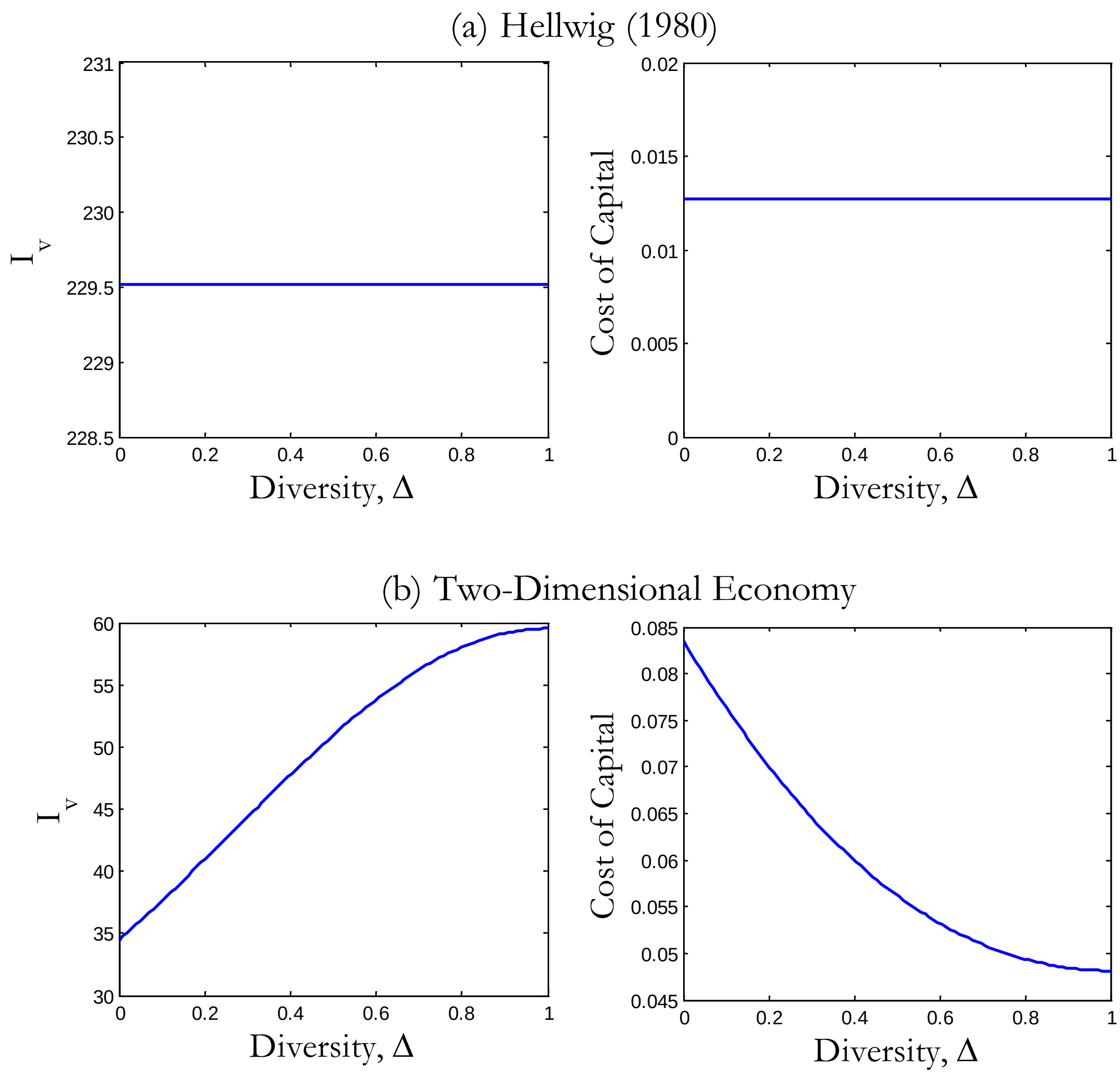
This figure plots the impact of the cash flow correlation parameter ϕ on the response of λ_2^* to the decrease in c_1 in the extended model described in Section 6, where the two payoff ingredients are correlated and where informed traders observe noisy signals about the two ingredients. For each panel, c_2 is fixed at a value $\bar{c}$, which is chosen to make $\lambda_1^* = \lambda_2^* = 0.05$ when $c_1 = c_2 = \bar{c}$. The other parameter values are: $\rho = \chi = 50$ and $\gamma = 3$.

Figure 6: Impact of Noise-to-Signal Ratio (ρ/n) on Learning Complementarties



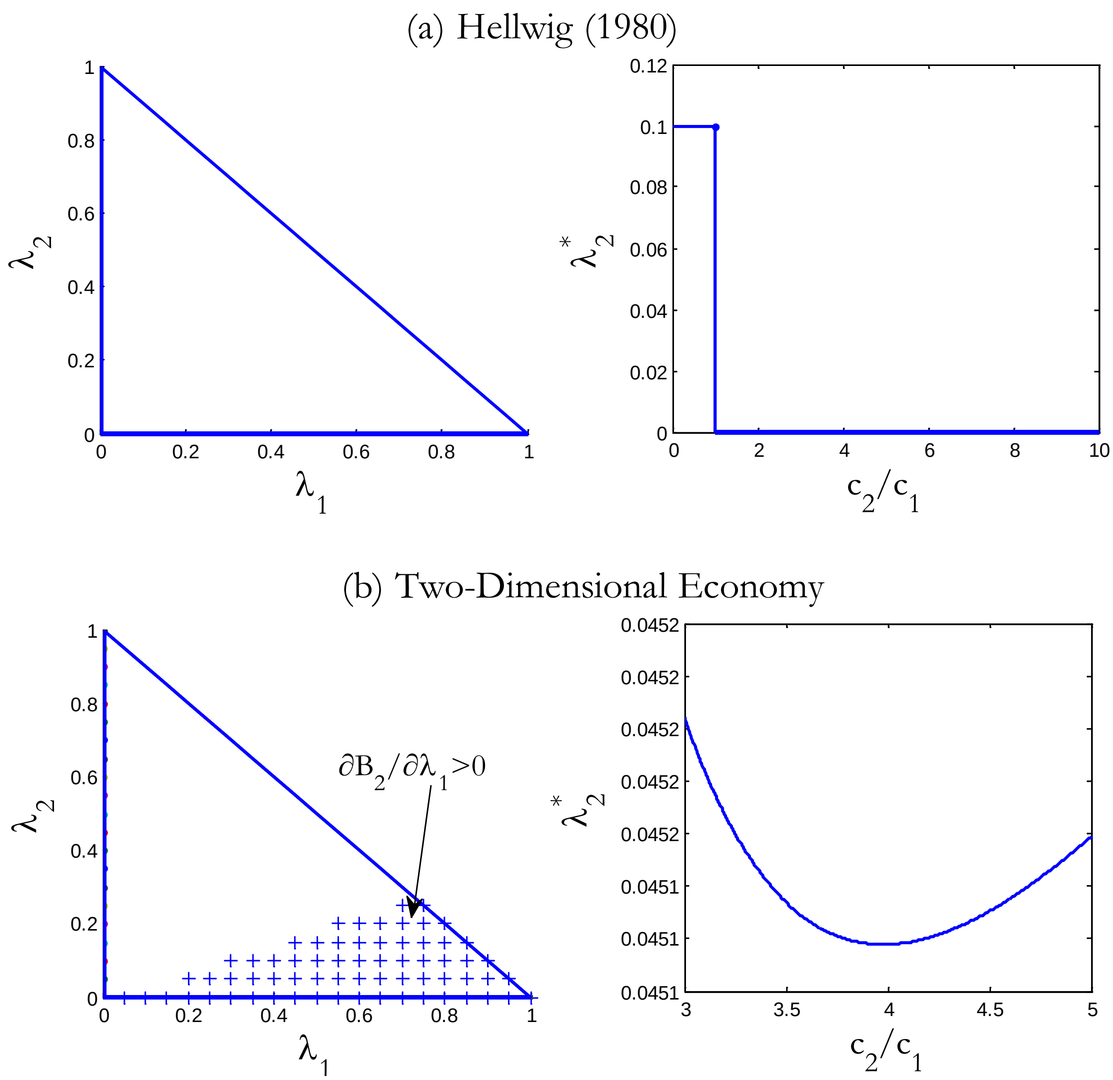
This figure plots the impact of the noise-to-signal ratio (ρ/n) of the acquired signals on the response of λ_2^* to the decrease in c_1 in the extended model described in Section 6, where the two payoff ingredients are correlated and where informed traders observe noisy signals about the two ingredients. For each panel, c_2 is fixed at a value $\bar{c}$, which is chosen to make $\lambda_1^* = \lambda_2^* = 0.05$ when $c_1 = c_2 = \bar{c}$. The other parameter values are: $\rho = \chi = 50$ and $\gamma = 3$.

Figure 7: Two-Dimensional Economy vs. Hellwig (1980): Impact of Diversity (Δ)



This figure plots the impact of information diversity on price informativeness regarding total cash flow and the cost of capital in two different economies. Panel (a) corresponds to the one-dimensional economy studied by Hellwig (1980); that is, the two payoff ingredients are perfectly correlated and informed traders observe noisy signals about the two ingredients (ϕ=1 and n=250). Panel (b) corresponds to a two-dimensional economy studied by Section 6; that is, the two payoff ingredients are non-perfectly correlated and where informed traders observe noisy signals about the two ingredients (ϕ=0.5 and n=250). The other parameter values in both panels are $\rho = \chi = 50$, $\gamma = 3$ and $\Lambda = 0.1$.

Figure 8: Two-Dimensional Economy vs. Hellwig (1980): Learning Complementarties



This figure plots the possibility of learning complementarities in two economies. Panel (a) corresponds to the one-dimensional economy studied by Hellwig (1980); that is, the two payoff ingredients are perfectly correlated and informed traders observe noisy signals about the two ingredients (ϕ=1 and n=250). Panel (b) corresponds to a two-dimensional economy studied by Section 6; that is, the two payoff ingredients are non-perfectly correlated and where informed traders observe noisy signals about the two ingredients (ϕ=0.5 and n=250). In both panels, on the left we use "+" to indicate the region for which the benefit B_2 of acquiring signal about $\tilde{v}_2$ increases with the size λ_1 of $\tilde{v}_1$-informed traders (i.e., $\frac{\partial B_2}{\partial \lambda_1} > 0$). On the right we plot the response of equilibrium λ_2^* to the decrease in c_1, while c_2 is fixed at a constant $\bar{c}$, which is chosen to make $\lambda_1^* = \lambda_2^* = 0.05$ when $c_1 = c_2 = \bar{c}$. The other parameter values in both panels are: $\rho = \chi = 50$ and $\gamma = 3$.