Swiss Finance Institute
Research Paper Series
N°18-04

Time-Varying Risk Premia in Large International Equity Markets

Ines CHAIEB
University of Geneva and Swiss Finance Institute

Hugues LANGLOIS
HEC Paris

Olivier SCAILLET
University of Geneva and Swiss Finance Institute

Time-Varying Risk Premia in Large International Equity Markets

Ines Chaieb[a], Hugues Langlois[b] and Olivier Scaillet[a]*

June 2018

Abstract

We use an estimation methodology tailored for large unbalanced panels of individual stock returns to address key economic questions about the factor structure, pricing performance of factor models, and time-variations in factor risk premia in international equity markets. We estimate factor models with time-varying factor exposures and risk premia at the individual stock level using $62,320$ stocks in 46 countries over the 1985-2018 period. We consider market, size, value, momentum, profitability, and investment factors aggregated at the country, regional, and world level. We find that adding an excess country market factor to world or regional factors is sufficient to capture the factor structure for both developed and emerging markets. We do not reject asset pricing restriction tests for multifactor models in 74% to 91% of countries. Value and momentum premia show more variability over time and across countries than profitability and investment premia. The excess country market premium is statistically significant in many developed and emerging markets but economically larger in emerging markets.

JEL Classification: C12, C13, C23, C51, C52 , G12, G15.

Keywords: large panel, approximate factor model, risk premium, international asset pricing, market integration.

[a] University of Geneva and Swiss Finance Institute, [b] HEC Paris.

*Acknowledgements: We thank Vihang Errunza, Thierry Foucault, Patrick Gagliardini, Lukas Kremens (discussant), Sebastien Laurent, Per Mykland, Rogier Quaedvlieg, Ioanid Rosu, Christophe Speanjers, Raman Uppal, Dacheng Xiu, Paolo Zaffaroni (discussant), and seminar participants at the University of Chicago, Northwestern University, HEC Paris, 2017 SFI Research Days, Boston University, 2017 Big Data Workshop at PUC University, Computational and Financial Econometrics 2017 Conference, SGF 2018, Frontiers of Factor Investing 2018 Conference, TSE Financial Econometrics Conference, QFFE 2018, SoFiE 2018, HEC Liege, and EDHEC for helpful comments. We are grateful for a grant from Inquire Europe. Langlois is grateful for financial support from the Investissements d'Avenir Labex (ANR-11-IDEX-0003/Labex Ecodec/ANR-11-LABX-0047). Please address correspondence to langlois@hec.fr.

1 Introduction

Understanding and measuring the determinants of expected returns in international equity markets is crucial to form optimal global investment portfolios, evaluate the performance of global equity managers, and obtain the cost of capital for global firms. Compared to a domestic setting, a key determinant is market integration.

Much of the previous work studying risk premia (i.e., expected excess returns) and market integration in international markets uses highly aggregated test assets, such as country portfolios, industry portfolios, or style portfolios. However, asset pricing tests and estimation of risk premia can differ when using individual stocks instead of aggregated test portfolios because of a potential aggregation bias.[1] This problem is particularly severe in a dynamic setting because aggregating assets into portfolios may induce misspecification in the dynamics of factor exposures.[2] In addition, testing asset pricing restrictions and recovering factor risk premia also crucially depend on not omitting important factors.

Motivated by these concerns, we consider the following questions: Do results of asset pricing tests in international equity markets change when we use individual stocks instead of test portfolios? How many factors and what are the key factors needed to explain the cross-section of international expected stock returns? How do factor risk premia estimated with individual stocks vary over time and across countries?

Methodology preview. We address these questions by estimating at the individual stock level international factor models with time-varying factor exposures and risk premia using a large unbalanced panel of

[1]Avramov and Chordia (2006) show that anomalies from conditional factor models differ a lot when considering single securities instead of portfolios. Ang, Liu, and Schwarz (2017a) argue that we loose a lot of efficiency when only considering portfolios as base assets, instead of individual stocks, to estimate equity risk premia in models with time-invariant coefficients. Lewellen et al. (2010) advocate working with a large number of assets instead a small number of portfolios exhibiting a tight factor structure. Previous international studies using style-portfolios as test assets run global- or regional-level tests to ensure sufficiently high number of stocks in portfolios and enough dispersion in expected returns (see, for example, Fama and French, 1998, 2012, 2017). Studies at the country level use at most ten portfolios sorted on one firm characteristic or at most nine portfolios if they use a three-way size and book-to-market sort as the low number of stocks prevents a five-way sort as used in the U.S. market (see, for example, Fama and French, 1998; Hou et al., 2011; Griffin et al., 2003). Cattaneo, Crump, Farrell, and Schaumburg (2017) argue that an appropriate choice of the number of portfolios is key for drawing valid empirical conclusions and that we need more portfolios than currently considered in the literature.

[2]See Appendix H of the supplemental material of Gagliardini et al. (2016) for theoretical arguments and empirical evidence for the U.S. market. Style-sorted portfolios result in stable betas which mask part of the time-variation in the risk premia.

62,320 stocks from 46 countries over the period 1985 to 2018.

We use international arbitrage pricing theory in a multi-period economy (Hansen and Richard, 1987) with a flexible stochastic representation in which stock excess returns vary with their exposures to multiple risk factors. Our model accommodates for cross-correlations in stock returns created by the conversion of local returns to a common numeraire currency, the U.S. dollar in our empirical application. Our estimation method is based on two-pass regressions (Fama and MacBeth, 1973; Black et al., 1972) for individual stock returns and uses the bias correction of Gagliardini et al. (2016) (GOS) to correct for the Error-in-Variable problem coming from the estimation of betas in the first pass regressions in large unbalanced panels.

Our objective is not to propose a new factor model, but to assess the ability of leading factor models to price individual international stocks. We consider several candidate factor models, including the CAPM, the Carhart (1997) four-factor model with market, size, value, and momentum factors (Fama and French, 2012), the five-factor model with market, size, value, profitability, and investment factors (Fama and French, 2017), and a model related to the q-factor model of Hou et al. (2015) with market, size, profitability, and investment factors.[3] See, among others, Hou et al. (2011), Asness et al. (2013), Titman et al. (2013) and Watanabe et al. (2013) for the role of size, value, momentum, profitability, and investment in international stock returns.

We build a set of factors for each of the 46 countries and then construct regional and global factors by aggregating country factors. We consider three regions of developed markets (North America, Developed Europe, and Asia Pacific) and three regions of emerging markets (Latin America, Middle East and Africa, and Emerging Asia). We show that market, size, value, momentum, profitability, and investment factors deliver positive average returns in almost all regions over our sample period. For textbook treatment of factor investing and investment issues in emerging markets, see Ang (2014) and Karolyi (2015), respectively.

Main empirical results. We make three main empirical contributions. We first determine which factors are needed to explain the comovements between individual stock returns. Underlying our international no-

[3]The non-market factors are additional sources of systematic risk in the IAPT. See, for example, Liew and Vassalou (2000), Cooper and Priestley (2011), Cooper et al. (2017), for evidence supporting risk-based interpretation of size and value, investment, and value and momentum, respectively, in the U.S. and international markets. But there exist alternative views to the risk-based explanation of the non-market factors (see, among others, Lakonishok et al., 1994). Kozak et al. (2018) argue that the success of a factor model does not discriminate between rational and behavioral explanations.

arbitrage model is the assumption that idiosyncratic shocks are weakly cross-sectionally correlated. Strong residual cross-correlations could be generated by a missing equity factor, a missing currency factor, or both. A missing currency factor may come from exchange rate shocks simultaneously affecting many stocks when local-currency returns are converted to U.S. dollar returns. Most importantly, a remaining factor structure in residuals invalidates the estimation of risk premia and inference on asset pricing restrictions.

To assess the different factor models, we apply the diagnostic criterion of Gagliardini et al. (2017) that determines if large unbalanced panel errors are weakly cross-sectionally correlated. We find omitted factors in the errors for models with regional factors, including those with profitability and investment factors. This is the case for both emerging (EMs) and developed markets (DMs). However, adding the excess country market factor—defined as the spread between the country market and the regional market—is sufficient to capture the factor structure in the large cross-section of individual stock returns of all DMs and almost all EMs.[4] For example, a four-factor model with regional factors augmented with the excess country market factor captures the factor structure in stock returns for 100% of DMs and 92% of EMs. In contrast, the same model without the excess country market factor captures the factor structure of only 64% of DMs and 17% of EMs. We find similar results using world instead of regional factors.

Our results differ from the standard approach in the literature in which different sets of factors are compared based on their pricing errors. For instance, Griffin (2002) finds that the Fama-French three-factor model with country market, size, and value factors produces lower pricing errors than models with global versions of these factors. We proceed instead in two steps. First, we verify that a set of factors is correctly specified; there are no remaining factors in the model errors. We find that we need only the excess country market factor in addition to regional or world factors to explain comovements across individual stocks. We refer to factor models augmented with the excess country market factor as mixed factor models. Then, we proceed to the second step which consists of estimating these factor risk premia and testing whether the asset pricing restrictions hold.

Results for asset pricing tests in international equity markets change when we use individual stocks

[4] Past studies assess market segmentation by testing whether a country market factor orthogonal to the world market factor has explanatory power beyond the world market factor in time-series regressions, see, for example, Stehle (1977), Solnik (1974b), Errunza and Losq (1985). Errunza and Losq (1985) provide a theoretical foundation for using an orthogonalized country market factor. Karolyi and Wu (2018) find strong support for including a related factor in multifactor models.

instead of test portfolios. Our second main empirical contribution is to show that the asset pricing restriction tests are not rejected in a large proportion of countries. For example, the mixed regional CAPM is not rejected in 72% of countries.[5] The mixed regional four-factor, five-factor, and q-factor models fare even better. The hypothesis that individual stock alphas are jointly equal to zero is not rejected for 91% of countries for the mixed four-factor model, 87% for the mixed five-factor model, and 74% for the mixed q-factor model.

These results contrast with asset pricing tests for international stocks that rely on aggregated test portfolios. For example, Fama and French (2017) add profitability and investment factors to explain average stock returns in DMs. Using the theoretical insight of international asset pricing models with partial segmentation, Karolyi and Wu (2018) add global and local externality factors to explain average stock returns in DMs and EMs. While in both cases the new factors significantly add explanatory power to the three-factor model, the null hypotheses that portfolio alphas are jointly equal to zero are more often than not rejected. In contrast, using individual stocks we find that the null hypotheses are most often not rejected. One notable exception is found in Hou et al. (2011) who do not reject a model with foreign and local market, momentum, and cash flow-to-price factors. Our results differ in that we obtain low rejection rates for world and regional models augmented with only the excess country market factor, not non-market factors.

Alternatively, Giglio and Xiu (2017) propose a method for estimating factor risk premia when some factors are omitted. Our approach differs in that we first make sure that all relevant factors have been specified for an approximate time-varying factor structure (no omitted factors remain in the errors), and then estimate the factor risk premia for time-varying specifications and large unbalanced panels of individual stocks.

Next, we provide a comprehensive examination of how factor risk premia estimated with individual stocks vary over time and across countries. This is important because most empirical studies so far have focused on risk premia estimated from test portfolios, which hinders the estimation of the risk premium dynamics. To the best of our knowledge, this is the first paper to document time-varying risk premia estimated from such a large panel of individual international stock returns.[6]

[5] The asset pricing restriction test does not suffer from low power since we work with a large cross-sectional dimension, see the Monte Carlo results in GOS.

[6] Tests of conditional international asset pricing models with country portfolios such as Harvey (1991), Chan et al. (1992), Ferson

We report two important stylized facts in the time series of factor risk premia over time and across countries. First, the excess country market factor risk premium is statistically significant but economically small compared to the world market or the regional market risk premium in DMs. Hence, the excess country market factor is required to explain comovements in stock returns and is significantly priced, but it carries on average a small risk premium. In contrast, the excess country market factors for EMs have larger risk premia relative to world or regional market factors.

Second, we find that non-market factors carry significant risk premia, but premia for value and momentum are more volatile across countries and over time than profitability and investment premia. The cross-country median average value (momentum) premium is 2.31% (1.45%) with an inter-quartile range of -3.57% to 5.27% (-3.17% to 8.06%).[7] In contrast, the profitability and investment premia estimated from the mixed q-factor model show less variations over both time and cross-sectional dimensions. The cross-country median average profitability (investment) premium is 1.55% (2.57%) with an inter-quartile range of -1.29% to 3.41% (0.05% to 4.48%). Those new empirical findings about the heterogeneity in factor premia between and within DMs and EMs are made possible through the use of individual stocks and time-varying specifications.

Our results are important for characterizing global and regional market integration. The necessary and sufficient condition for market integration is that risk premia on a common set of risk factors that drive returns in all countries are equal and each country-specific factor has a zero risk premium. We find that the excess country market factor is priced in world and regional models not only for EMs but also for DMs, which provides evidence of segmentation along both world and regional lines.[8]

Data quality is especially important when testing asset pricing models at the individual stock level. Here we do not rely on value-weighted test portfolios in which aggregation attenuates data errors frequently found in international stock databases. We conduct a comprehensive comparative analysis of international stock databases by comparing individual stock data in each country using data from Thomson Reuters Datastream

and Harvey (1993), Dumas and Solnik (1995), De Santis and Gerard (1997), Carrieri et al. (2007), and Carrieri et al. (2013), among many others, find significant time variation in the prices of the risk factors.

[7]The large negative correlation between the value and momentum factors could explain the high volatility of their risk premia.

[8]Ideally, we would like to estimate models in which all country market factors are included. The curse of dimensionality precludes the estimation of such a model.

and S&P Compustat Global. In Appendix 3, we build a comprehensive list of filters and data corrections to use with the S&P Compustat Global database that supplement those used in the literature.

Related literature. Our paper contributes to the theoretical and empirical literatures on international APT models (IAPT). On the theoretical side, Solnik (1983) shows how to generalize the APT to an international setting for time-invariant models if exchange rates are explained by the same factor structure as stock returns and idiosyncratic shocks are cross-sectionally independent. Ikeda (1991) does not assume that exchange risk is spanned by the factors, and shows how to derive the no-arbitrage restriction by hedging exchange rate risk in continuous time.[9] Our model is set in discrete time and uses a flexible factor structure with weak cross-sectional dependence in the residuals potentially induced by currency conversion of stock returns.

We use and extend the methodology of GOS which is inspired by macro-econometrics and forecasting methods that extract cross-sectional and time-series information simultaneously from large unbalanced panels.[10] Their arbitrage pricing theory (APT) model also inspires our reliance on individual stocks returns. In this setting, approximate factor structures with nondiagonal error covariance matrices answer the potential empirical mismatch of exact factor structures with diagonal error covariance matrices underlying the original APT of Ross (1976). Under weak cross-sectional dependence among idiosyncratic shocks, such approximate factor models generate no-arbitrage restrictions in large economies when the number of assets grows. Using this theoretical framework, we empirically investigate the pricing of individual stocks in international markets. See also Uppal and Zaffaroni (2016) for a recent use of the APT in a portfolio choice context.

On the empirical side, early studies testing an international APT model with constant factor exposures and risk premia using data from the 1970-1980s found mixed results on market integration. Cho, Eun, and Senbet (1986) find evidence of market segmentation in ten DMs while Gultekin, Gultekin, and Penati (1989) show that the U.S. and Japanese markets are integrated after the liberalization of capital controls.

[9] Exchange risk hedging by holding locally riskless bonds is also proposed in utility-based International Asset Pricing Models (IAPM). See, for example, Solnik (1974a), Sercu (1980), Stulz (1981), and Adler and Dumas (1983).

[10] See e.g. Stock and Watson (2002a), Stock and Watson (2002b), Bai (2003), Bai (2009), Bai and Ng (2002), Bai and Ng (2006), Forni et al. (2000), Forni et al. (2004), Forni et al. (2005), Pesaran (2006). Ludvigson and Ng (2007, 2009) exemplify this promising route when studying bond risk premia. Connor, Hagmann, and Linton (2012) show that large cross-sections exploit data more efficiently in a semiparametric characteristic-based factor model of stock returns.

While these studies use a small set of individual stocks, we provide evidence of market segmentation in many DMs using the whole cross-section of stocks. Evidence of market segmentation for DMs using aggregated test portfolios also vary depending on the model and sample period used. Using test portfolios sorted by size, Korajczyk and Viallet (1989) show that multifactor APT models outperform the CAPM in both domestic and international forms, and domestic APT models outperform international ones pointing to market segmentation for major DMs. Using an APT for currencies, bonds, and stock returns in ten European countries, Sentana (2002) finds that country-specific risks are priced, thus providing evidence of market segmentation. In contrast, test results based on international equilibrium asset pricing models find evidence of market integration among DMs (see, for example, De Santis and Gerard, 1997; Carrieri et al., 2013). Our paper provides new results on market integration using a large unbalanced panel of international individual stock returns for a period spanning more than three decades.[11]

Our work also contributes to the debate on whether global, regional, or country factors perform better for international stocks. In her review of the international equity pricing literature, Lewis (2011) states that (p.443): "returns depend upon more than a single factor and that at least some of these additional factors depend upon local sources of risk" (see also Karolyi and Stulz, 2003, for a review of the literature).[12] Several papers show that regional factors perform better than global factors (see Bekaert et al., 2009; Fama and French, 2012, 2017). Stehle (1977), Griffin (2002), and Hou et al. (2011) further show that adding country factors results in lower pricing errors.[13] Whereas the latter two papers consider multiple country factors, our findings indicate that adding the excess country market factor to world or regional factors is

[11] Various authors have examined the issue of international capital market integration and suggested different measures. Bekaert and Harvey (1995) use regime switching statistical models to examine the dynamics of world market integration. Carrieri et al. (2007) and Carrieri et al. (2013) estimate a conditional version of the asset pricing model of Errunza and Losq (1985) and obtain a time-varying measure of integration. Eun and Lee (2010) document a mean-variance convergence in the risk-return distance among 17 international stock markets. Bekaert et al. (2011) suggest a new measure based on earnings multiples and show that DMs has been integrated since 1993 while EMs remain segmented. Pukthuanthong and Roll (2009) use the R-squared of linear factor models as a measure of integration. Eiling and Gerard (2015) measure integration as the fraction of total risk due to global factors. Errunza and Miller (2000) examine changes in equity valuations at the firm level following the introduction of depositary receipts and show a significant reduction in the cost of capital.

[12] Explicit barriers to investment such as those modeled in Errunza and Losq (1985) or differences in information across markets as modeled in Dumas et al. (2017) explain why returns are priced with global and local factors.

[13] Eun, Lai, de Roon, and Zhang (2010) examine these issues from a portfolio perspective.

sufficient to fully explain the factor structure in stock returns. Finally, Rouwenhorst (1999) finds that similar factors explain the cross-section of average returns in EMs than in DMs. We investigate the performance of the same type of risk factors for pricing individuals stocks in both DMs and EMs.

Layout. We present the theoretical model in Section 2, provide our empirical methodology in Section 3, describe our data in Section 4, and discuss the model diagnostics and estimation results in Section 5. Section 6 concludes. Appendix 1 and Appendix 2 summarize the key inference tools used to get our numerical and graphical results. Appendix 3 details the construction of the equity database.

2 A multi-period international APT

In this section, we provide a multi-period international APT with currency risk and varying degree of market segmentation. We start by describing the factor structure for excess returns. We then combine this factor structure with the absence of arbitrage opportunities to obtain asset pricing restrictions. We work in a multi-period economy under an approximate factor structure with a continuum of assets as in GOS, and refer to their proof for asset pricing results as well as a detailed discussion on the use of a continuum.

2.1 A time-varying factor model for stock returns with currency risk

We consider C countries. In each country $c \in \{1, ..., C\}$, we use the index $\gamma \in [0, 1]$ to designate an asset belonging to a continuum of assets on an interval normalized to $[0, 1]$ without loss of generality. We assume that each country has its own currency and we use the U.S. dollar (USD) as the numeraire currency. The return in USD at time t on asset γ in country c in excess of the U.S. risk-free rate, $r_{c,t}(\gamma)$, follows the factor structure:

$$r_{c,t}(\gamma) = a_{c,t}(\gamma) + b_{c,t}(\gamma)' f_{c,t} + \varepsilon_{c,t}(\gamma). \tag{1}$$

In Equation (1), $a_{c,t}(\gamma)$ and $b_{c,t}(\gamma)$ are a time-varying intercept and time-varying exposures to K systematic factors $f_{c,t}$.

Both the intercept $a_{c,t}(\gamma)$ and factor loadings $b_{c,t}(\gamma)$ are $\mathcal{F}_{t-1}$-measurable, where $\mathcal{F}_{t-1}$ is the information available to all investors at time $t - 1$.[14] The error terms have mean zero, $E[\varepsilon_{c,t}(\gamma)|\mathcal{F}_{t-1}] = 0$, and are

[14]GOS impose some non-degeneracy conditions on $a_{c,t}(\gamma)$ and $b_t(c, \gamma)$, which prevent that only a few assets load on a factor.

uncorrelated with the factors, $Cov[\varepsilon_{c,t}(\gamma), f_{c,t}|\mathcal{F}_{t-1}] = 0$. These conditions allow to identify $a_{c,t}(\gamma)$ and $b_{c,t}(\gamma)$ as time-varying regression coefficients.

Our factor model in Equation (1) applies to international asset returns converted to a common currency. Under this assumption, the factor model also applies to foreign risk-free bonds which are risky assets when measured in USD. Hence, we implicitly assume that currency returns follow the same factor structure,

$$r_{c,t}(\gamma_s) = a_{c,t}(\gamma_s) + b_{c,t}(\gamma_s)' f_{c,t} + \varepsilon_{c,t}(\gamma_s), \tag{2}$$

where $r_{c,t}(\gamma_s)$ is the excess return on country c currency γ_s in units of the numeraire currency (USD).[15]

Our factor structure (1)-(2) is similar to Solnik (1983) in that we impose a factor structure on returns converted to a numeraire currency. However, our model differs from his in two important aspects. First, he uses a common set of factors for all international stocks which is appropriate for the case of integrated markets. In contrast, our set of systematic factors $f_{c,t}$ can be specific to country c or common across several countries. We further discuss the issue of market integration in Section 2.2 below. The second difference is that idiosyncratic shocks $\varepsilon_{c,t}(\gamma)$ in the Solnik (1983) model are cross-sectionally independent. In our model, we explicitly consider the impact of currency conversion on correlations across stocks since we do not impose a priori an exact factor structure (diagonal covariance matrix of the errors).[16]

There are two ways in which currency risk may impact the correlation structure of U.S.-denominated stock returns. First, stock returns in one currency converted to USD are all impacted by the currency-specific shock $\varepsilon_{c,t}(\gamma_s)$, which may result in higher cross-correlation for securities in this country. Second, currency-specific shocks $\varepsilon_{c,t}(\gamma_s)$ can be correlated across currencies if there exists currency-specific factors (i.e., $\varepsilon_{c,t}(\gamma_s)$ follows a factor structure). In such case, this currency-specific factor structure results in higher correlation between countries. In both cases, the correlations induced by currency conversion within and across blocks of securities can invalidate the estimation of risk premia. There can also be other sources of correlation between stock returns besides currencies such as industry effects.

To handle potential correlations across idiosyncratic shocks, we impose an approximate factor structure (as opposed to an exact factor structure) for model (1) in each country c. Precisely, for any sequence $(\gamma_{i,c})$

Other regularity conditions are assumed for the theory and inference procedures to work (see GOS for details).

[15] In Equation (2), the terms involving the risk-free rate of country c are absorbed by the terms on the right-hand side.

[16] See also the discussion in Ikeda (1991).

in $[0,1]$, for $i = 1, ..., n_c$, let $\Sigma_{\varepsilon_{c,t,n_c}}$ denote the $n_c \times n_c$ conditional variance-covariance matrix of the error vector $[\varepsilon_{c,t}(\gamma_{1,c}), ..., \varepsilon_{c,t}(\gamma_{n_c,c})]'$ conditional on $\mathcal{F}_{t-1}$. We assume that there exists a set such that the ratio of the largest eigenvalue of $\Sigma_{\varepsilon_{c,t,n_c}}$ to n_c converges to 0 in L^2 as n_c grows. The validity of this assumption is also sufficient if we want to estimate risk premia for an integrated world market with all countries or for an integrated region with all countries in that region.[17]

Chamberlain and Rothschild (1983) (p. 1294) use a sequence of variance-covariance matrices for the error terms that have uniformly bounded eigenvalues. Our assumption is weaker and generalizes previous international APTs to a more flexible setup and realistic market structure. In particular, we allow for block cross-correlations between idiosyncratic shocks that may be induced by currency conversion. In Section 5.1, we empirically examine which set of candidate risk factors captures the factor structure in excess stock returns denominated in USD. Indirectly, this allows us to check the assumption that currency returns follow the same factor structure and that no systematic equity or currency factor is missing, in which case the errors are weakly cross-sectionally dependent.

2.2 Asset pricing restrictions

We now combine the approximate factor structure introduced in the last section with the absence of asymptotic arbitrage opportunities to obtain asset pricing restrictions.

Following GOS, we rule out asymptotic arbitrage opportunities to obtain a restriction on the intercept $a_{c,t}(\gamma)$, namely, there exists a unique $\mathcal{F}_{t-1}$-measurable K-by-1 vector $\nu_{c,t}$ such that

$$a_{c,t}(\gamma) = b_{c,t}(\gamma)'\nu_{c,t}, \tag{3}$$

for almost all $\gamma \in [0,1]$ in country c. If there does not exist such a vector $\nu_{c,t}$, there are arbitrage opportunities in country c. Equivalently, we can rewrite the asset pricing restriction (3) as the usual linear link

[17] Our assumption on the covariance matrix of idiosyncratic shocks in country c translates into a similar result for the covariance matrix of all idiosyncratic shocks across the C countries. Indeed, the largest eigenvalue of a positive semi-definite matrix is less than or equal to the sum of the largest eigenvalue associated to each diagonal block. This is true without the need for the off-diagonal blocks to be zeros, i.e., without assuming zero correlation between countries. Hence, the largest eigenvalue of the conditional covariance matrix of all idiosyncratic shocks $\varepsilon_{c,t}$ divided by the total number of stocks $n = \sum_{c=1}^{C} n_c$ also converges to 0 in L^2 as n grows.

between conditional expected excess returns and conditional factor risk premia:

$$E\left[r_{c,t}(\gamma)|\mathcal{F}_{t-1}\right] = b_{c,t}(\gamma)'\lambda_{c,t}, \tag{4}$$

where $\lambda_{c,t} = \nu_{c,t} + E\left[f_{c,t}|\mathcal{F}_{t-1}\right]$ is the vector of the conditional factor risk premia in country c.

In the CAPM, we have $K = 1$ and $\nu_{c,t} = 0$. More generally, in a multifactor model in which factors are excess returns on tradable portfolios, we have $\nu_{c,t,k} = 0$, for $k = 1, ..., K$. In the empirical section below, we use as factors long-short portfolios built to capture the size, value, momentum, profitability, and investment effects. As these factors imply buying and short-selling a large amount of securities and their returns do not reflect transaction and short-selling costs, they may not be tradable, especially in less developed markets.[18] Therefore, we test both the asset pricing restrictions, $a_{c,t}(\gamma) = b_{c,t}(\gamma)'\nu_{c,t}$, and the asset pricing restrictions with tradable factors, $a_{c,t}(\gamma) = 0$. The latter corresponds to the usual testing procedure of Gibbons et al. (1989) but in a large unbalanced panel structure.

Following the empirical methodology developed in GOS which we extend in Section 3, the restriction (3) is testable with large equity datasets and large sample sizes. Therefore, we are not affected by the Shanken (1982) critique, namely the problem that finiteness of the sum of squared pricing errors for a given countable economy, $\sum_{i=1}^{\infty}\left(a_{c,t}(\gamma_i) - b_{c,t}(\gamma_i)'\nu_{c,t}\right)^2$, cannot be tested empirically.

We distinguish between the importance of a country-level factor and market segmentation. A country-level factor, say the German stock market, may drive German stock returns through Equation (1), but this is not a sufficient condition for market segmentation. Nor is a common set of factors f_t driving stock returns in Switzerland and Germany a sufficient condition for these two markets to be integrated.[19] The necessary and sufficient condition for market integration is that risk premia on a common set of risk factors that drive returns in both countries are equal and each country-level factor has zero risk premium.

In our empirical tests, we use the asset pricing restrictions (3) to test different factor model specifications. We investigate models with a country-specific set of risk factors, $f_{c,t}$, and models with a common set of factors, f_t, across countries.

[18] Using spanning tests, Bekaert and Urias (1996) and De Roon et al. (2001) show that diversification benefits of emerging markets disappear when accounting for short sale constraints and transaction costs in those markets.

[19] A similar argument is made in Cho, Eun, and Senbet (1986) and Heston et al. (1995), among others.

3 Empirical methodology

This section describes our estimation methodology. We first discuss how we parameterize time-varying factor exposures and risk premia. Next, we discuss how we deal with the unbalanced nature of our panel of stock returns. Finally, we use a two-pass approach (Fama and MacBeth, 1973; Black et al., 1972) building on Equations (1) and (3) to estimate risk premia.

The conditioning information $\mathcal{F}_{t-1}$ contains $Z_{c,t-1}$ and $Z_{c,t-1}(\gamma), c = 1, ..., C$. The p-by-1 vector of lagged instruments $Z_{c,t-1}$ is common to all stocks of country c and may include a constant, past observations of the factors, and some additional variables such as macroeconomic and financial variables common to all countries or country-specific. The q-by-1 vector of lagged instruments $Z_{c,t-1}(\gamma)$ is specific to stock γ in country c, and may include past observations of firm characteristics.

To obtain a workable version of the conditional IAPT from the previous section, we use linear specifications. First, the vector of factor loadings is a linear function of lagged instruments $Z_{c,t-1}$ (Shanken, 1990; Ferson and Harvey, 1991; Dumas and Solnik, 1995) and $Z_{c,t-1}(\gamma)$ (Avramov and Chordia, 2006),

$$b_{c,t}(\gamma) = B_c(\gamma)Z_{c,t-1} + C_c(\gamma)Z_{c,t-1}(\gamma), \tag{5}$$

where $B_c(\gamma)$ is a K-by-p matrix and $C_c(\gamma)$ is a K-by-q matrix.[20]

Second, the vector of risk premia is a linear function of lagged instruments $Z_{c,t-1}$ (Dumas and Solnik, 1995; Cochrane, 1996; Jagannathan and Wang, 1996),

$$\lambda_{c,t} = \Lambda_c Z_{c,t-1}, \tag{6}$$

where Λ_c is a K-by-p matrix. Finally, the conditional expectation of the factors $f_{c,t}$ given the information $\mathcal{F}_{t-1}$ is,

$$E[f_{c,t}|\mathcal{F}_{t-1}] = F_c Z_{c,t-1} \tag{7}$$

where F_c is a K-by-p matrix.

[20]See also Ang et al. (2017b) for an application to the analysis of mutual fund performance.

3.1 Choice of instruments

We use the lagged world dividend yield, DY_{t-1}, and a country lagged dividend yield, $DY_{c,t-1}$, as common instruments in addition to a constant. Hence, $Z_{c,t-1} = (1, DY_{t-1}, DY_{c,t-1})'$ in our empirical application.[21] To ensure that conditional expectations of world factors in Equation (7) are equal across countries, we set the elements of F_c corresponding to their loading on the country dividend yield to zero. In the interest of parsimony, we impose that both factor loadings $b_{c,t}(\gamma)$ in Equation (5) and the vector $\nu_{c,t}$ in Equation (3) do not load on the global instrument DY_{t-1}.[22] In models where we use regional instead of world factors, DY_{t-1} is the regional dividend yield. The country, world, and regional dividend yields are standardized.

In asset pricing tests with test portfolios, it is the *composition* of the test assets that varies over time as stocks with similar characteristics are assembled into different portfolios. In such case, we can expect that estimating betas using either the full sample or rolling windows would adequately capture each test portfolio factor loadings. For example, a size sorted portfolio with small capitalization firms will consistently have a positive loading on a size factor over time.

When we test asset pricing models using individual stocks, the composition of the test assets is fixed (i.e., one stock) and it is their *characteristics* that vary over time. As a firm evolves and its stock characteristics change over time, we cannot expect its betas to be constant and estimating betas over rolling windows would necessarily lag its true time-varying factor exposures as time-invariant OLS estimates average recent and more distant exposures.

Therefore, as stock-specific instruments, we use the cross-sectional ranks of the size, value, momentum, profitability, and investment characteristics depending on which factors are included in the model.[23] This

[21] Several studies show that the dividend yield help predict time variation in international equity returns and use it as an instrumental variable (see, for example, Harvey, 1991; Ferson and Harvey, 1991, 1993). Other variables such as the term spread and the default spread have also some predictive power for equity returns and are used as instruments in past studies.

[22] The vector of conditional risk premia involves both the conditional expectation of the factors, via the coefficients matrix F_c, and the process $\nu_{c,t}$. The restriction on $\nu_{c,t}$ is equivalent to restricting the element of F_c for the loading of the global factor on the global instrument to be equal to its corresponding element in Λ_c in Equation (6). Therefore, we assume that the global factor risk premium depends on DY_{t-1} only through its conditional expectation.

[23] The only exception is the CAPM for which we use the size and value characteristics. Connor et al. (2012) also use the corresponding firm characteristic to model the beta of a given factor, i.e., the firm size for the beta of the size factor, etc. Among others, Shanken (1990), Fama and French (1997), Avramov and Chordia (2006) and GOS assume factor loading to vary with firm

method, which differs from the empirical strategy of GOS, has several advantages. First, we directly capture the time-varying exposure of single stocks to factors. For example, if the market capitalization of a stock suddenly decreases, it becomes part of the long leg of the size factor and its cross-sectional size rank gets smaller. This instrument will accordingly capture its increased exposure to the size factor. Another advantage of using cross-sectional characteristic ranks instead of the characteristic is that it attenuates the impact of data errors in individual stock characteristics (see Freyberger et al., 2017, for a similar choice).[24]

3.2 First pass regressions

To ease notation, we present below the estimation methodology for the time-invariant case with no instruments. We describe in details the time-varying case with instruments in Appendix 1. To ensure that cross-sectional limits exist and are invariant to reordering of the assets, we introduce a sampling scheme as in GOS. Observable assets are random draws $i = 1, ..., n_c$ from an underlying population (Andrews, 2005). By random sampling, we get a standard random coefficient model (see, for example, Hsiao, 2003, Chapter 6).

We use the simplifying notation $r_{i,c,t} = r_t(\gamma_{i,c})$, $a_{i,c} = a(\gamma_{i,c})$, $b_{i,c} = b\,(\gamma_{i,c})$, and $\varepsilon_{i,c,t} = \varepsilon_t(\gamma_{i,c})$ from which we get the compact formulation

$$r_{i,c,t} = \beta'_{i,c} x_{c,t} + \varepsilon_{i,c,t}, \tag{8}$$

with $\beta_{i,c} = (a_{i,c}, b'_{i,c})'$ and $x_{c,t} = (1, f'_{c,t})'$.

We account for the unbalanced nature of the panel through a collection of indicator variables: we define $I_{i,c,t} = 1$ if the return of asset i in country c is observable at date t, and 0 otherwise (Connor and Korajczyk, 1987). The first pass consists in computing time-series OLS estimators

$$\hat{\beta}_{i,c} = \hat{Q}^{-1}_{x,i,c} \frac{1}{T_{i,c}} \sum_t I_{i,c,t} x_{c,t} r_{i,c,t},$$

for all stocks $i = 1, ..., n_c$, where $\hat{Q}_{x,i,c} = \dfrac{1}{T_{i,c}} \displaystyle\sum_t I_{i,c,t} x_{c,t} x'_{c,t}$ and $T_{i,c} = \displaystyle\sum_t I_{i,c,t}$.

---

characteristics.

[24] Asness et al. (2017) and Kozak et al. (2017) also use normalized ranks of stock characteristics.

The random sample size $T_{i,c}$ for stock i in country c can be small, and the inversion of matrix $\hat{Q}_{x,i,c}$ can be numerically unstable, possibly yielding unreliable estimates of $\beta_{i,c}$. To address this problem, we trim the cross-section of stocks. First, we keep only stocks with at least five years of monthly returns, $T_{i,c} \geq 60$. Second, we keep only stocks for which the time-series regression is not too badly conditioned. As a criterion we use the condition number which is the square root of the ratio of the largest eigenvalue to the smallest eigenvalue of $\hat{Q}_{x,i,c}$, $CN\left(\hat{Q}_{x,i,c}\right) = \sqrt{eig_{\max}\left(\hat{Q}_{x,i,c}\right)/eig_{\min}\left(\hat{Q}_{x,i,c}\right)}$. A too large value of $CN\left(\hat{Q}_{x,i,c}\right)$ indicates multicollinearity problems and ill-conditioning (Belsley et al., 2004; Greene, 2008). In our empirical tests, we use a threshold of 50. We then define the indicator variable $\mathbf{1}^{\chi}_{i,c}$ which takes a value of one if stock i is kept and zero otherwise.

Given our choice of instruments, we encounter many cases of multicollinearity. Consider for example a stock that remains among the largest stocks in its market during the sample period. Then its size cross-sectional rank, $Z^{size}_{i,c,t-1}$, is relatively constant and the interaction terms $Z^{size}_{i,c,t-1} f_{c,t}$ are highly correlated with $f_{c,t}$ for all factors in the regression. We therefore extend the empirical strategy of GOS to address this problem by parsimoniously restricting the regressor matrix for each stock. First, we compute the correlations between each regressor. Whenever a regressor pair has a correlation higher than 0.85 in absolute value, we keep only the regressor that has the highest correlation in absolute value with the stock returns. We repeat this procedure with a lower correlation threshold until the condition number is below 50 or the correlation threshold is no longer positive. We have found that this cleaning procedure avoids throwing away many stocks and that no regressors are systematically removed.

3.3 Diagnostic criterion for the choice of factors

To empirically assess whether a factor model successfully captures systematic risk, we use the diagnostic tool of Gagliardini et al. (2017) (GOS2) that checks whether there remains a common factor structure in idiosyncratic shocks $\varepsilon_{i,c,t}$. Specifically, we compute the T_c-by-T_c matrix Υ

$$\Upsilon = \sum_{i=1}^{n_c} \mathbf{1}^{\chi}_{i,c} \bar{\varepsilon}_{i,c} \bar{\varepsilon}'_{i,c},$$

where $\bar{\varepsilon}_{i,c}$ is a T_c-by-one vector of standardized residuals $\bar{\varepsilon}_{i,c,t} = \frac{I_{i,c,t}\varepsilon_{i,c,t}}{\sqrt{\frac{1}{T_c}\sum_{t=1}^{T_c} I_{i,c,t}\varepsilon^2_{i,c,t}}}$.

The diagnostic criterion is given by

$$\zeta = eig_{\max}\left(\frac{\Upsilon}{n_c T_c}\right) - g\left(n_c, T_c\right), \tag{9}$$

where $g\left(n_c, T_c\right) = -P\kappa ln\left(\kappa\right)$ is a penalty term with $\kappa = \frac{\left(\sqrt{n_c}+\sqrt{T_c}\right)^2}{n_c T_c}$ and P is a data-driven constant.[25] If there are no factors in the residuals, the maximum eigenvalues of the scaled matrix Υ on the right hand side of Equation (9) goes to zero at a faster rate than the penalty term as n_c and T_c increase. If there remains at least one factor in the residuals, then the maximum eigenvalue remains large and positive. Hence, GOS2 use a negative value of ζ as a criterion to conclude that there does not remain any factor structure in the residuals, and that we achieve weak cross-section dependence for the errors.[26]

3.4 Second pass regression

The second pass consists in computing a cross-sectional estimator of ν_c by regressing the $\hat{a}_{i,c}$s on the $\hat{b}_{i,c}$s keeping only the non-trimmed assets. We use a Weighted Least Squares (WLS) approach,

$$\hat{\nu}_c^{WLS} = \hat{Q}_b^{-1}\frac{1}{n_c}\sum_i \hat{w}_{i,c}\hat{b}_{i,c}\hat{a}_{i,c}, \tag{10}$$

where $\hat{Q}_b = \frac{1}{n_c}\sum_i \hat{w}_{i,c}\hat{b}_{i,c}\hat{b}_{i,c}'$ and $\hat{w}_{i,c} = \mathbf{1}_{i,c}^{\chi}\hat{v}_{i,c}^{-1}$ are the weights.

The terms $v_{i,c} = \tau_{i,c}c_{\nu_c}' Q_x^{-1} S_{ii,c} Q_x^{-1} c_{\nu_c}$ are the asymptotic variances of the standardized errors $\sqrt{T}\left(\hat{a}_{i,c} - \hat{b}_{i,c}'\nu_c\right)$ in the cross-sectional regression for large T, where $\tau_{i,c} = E[I_{i,c,t}|\gamma_{i,c}]^{-1}$, $c_{\nu_c} = (1, -\nu_c')'$, $Q_x = E\left[x_{c,t}x_{c,t}'\right]$, and $S_{ii,c} = E\left[\varepsilon_{i,c,t}^2 x_{c,t}x_{c,t}'|\gamma_{i,c}\right]$.

To operationalize this WLS approach, we first estimate $\hat{\nu}^{OLS}$ by OLS using unit weights $\hat{w}_{i,c} = 1$. We then use the estimates $\tau_{i,c} = \frac{T_c}{T_{i,c}}$, $c_{\hat{\nu}_c} = (1, -\hat{\nu}_c^{OLS'})'$, $\hat{S}_{ii,c} = \frac{1}{T_{i,c}}\sum_t I_{i,c,t}\hat{\varepsilon}_{i,c,t}^2 x_{c,t}x_{c,t}'$, and $\hat{\varepsilon}_{i,c,t} = r_{i,c,t} - \hat{\beta}_{i,c}'x_{c,t}$ to estimate $\hat{\nu}^{WLS}$ by WLS.[27]

[25] We use a simulation-based method to select P, see Appendix 7 in GOS2 for details and Monte Carlo results for unbalanced panels.

[26] GOS2 extend the well-known procedures of Bai and Ng (2002) and Bai and Ng (2006) to unbalanced panels and estimated errors instead of the true ones. See also Onatski (2010) and Ahn and Horenstein (2013).

[27] In their additional empirical results, GOS show that a value-weighting scheme does not change point estimate values but can increase confidence intervals due to a precision loss.

The distribution of $\hat{\nu}_c^{WLS}$ is

$$\sqrt{n_c T_c}\left(\hat{\nu}_c^{WLS} - \frac{1}{T_c}\hat{B}_{\nu_c} - \nu_c\right) \Rightarrow N\left(0, \Sigma_{\nu_c}\right), \tag{11}$$

where the presence of the bias term $\hat{B}_{\nu_c}$ comes from the well-known Error-In-Variable problem, i.e., factor exposures are estimated with errors in the first step time-series regressions. We report the expressions for the bias term $\hat{B}_{\nu_c}$ and the estimation methodology for the covariance matrix Σ_{ν_c} in Appendix 1.

Using Equation (3), the estimator of the risk premia vector can be written as,

$$\hat{\lambda}_c = \hat{\nu}_c^{Unbiased} + \frac{1}{T}\sum_t f_{c,t}, \tag{12}$$

where $\hat{\nu}_c^{Unbiased} = \hat{\nu}_c^{WLS} - \frac{1}{T_c}\hat{B}_{\nu_c}$ is the unbiased estimator of ν_c.

3.5 Testing for asset pricing restrictions

To evaluate the asset pricing restrictions, we compute the weighted sum of squared residuals (SSR) of the second-pass cross-sectional regression $\hat{Q}_e = \dfrac{1}{n_c}\sum_i \hat{e}_i'\hat{w}_i\hat{e}_i$, with $\hat{e}_i = \hat{a}_{i,c} - \hat{b}_{i,c}'\hat{\nu}_c^{Unbiased}$.

Under the null hypothesis that the asset pricing restrictions hold, $a_{i,c} = b_{i,c}\nu_c$, the expected value for the mean squared pricing errors, $E[Q_e]$, is 0. Under the null the test statistic

$$\tilde{\Sigma}_e^{-1/2} T_c \sqrt{n_c}\left(\hat{Q}_e - \frac{1}{T_c}\right) \sim N(0,1), \tag{13}$$

has a standard normal distribution. The covariance matrix $\tilde{\Sigma}_e$ is given in Appendix 1. We can test the asset pricing restrictions with tradable factors, $a_{i,c} = 0$, by replacing $\hat{\nu}_c^{Unbiased}$ by a vector of 0 in these expressions.

4 Data

Our empirical analyses require individual stock returns and characteristics. We start by describing our data construction for individual stock returns and then describe how we construct risk factors.

4.1 Individual stock data

We use data for 62,320 stocks from 22 DMs and 24 EMs. Our data are monthly, denominated in U.S. dollars, in excess of the U.S. one-month T-Bill rate, and cover the period January 1984 to February 2018.

We conduct a widespread comparative analysis of different global stock databases. Fama and French (2012, 2017) use data from Bloomberg, which they complement with Datastream, to obtain stock returns and accounting variables for 23 DMs. To obtain stock returns and accounting variables for the same 23 DMs, Asness et al. (2013) use S&P Compustat Global (xpressFeed). In contrast, Hou et al. (2011), Karolyi and Wu (2018), Lee (2011), and Moore and Sercu (2013) use data from Datastream, on which they apply different filters to handle data errors. In Appendix 3, we detail our construction methodology and comparative analysis of data coming from Compustat and Datastream and provide an exhaustive list of filters and data corrections. For reasons listed in Appendix 3, we use data from Compustat in this paper.

We retrieve all securities classified as common or ordinary shares, but keep only stocks listed on a country major stock exchange. We define a major stock exchange as the one with the highest number of equities listed. However, we include more than one stock exchange in some countries: Brazil (Rio de Janeiro and Bovespa), Canada (Toronto and TSX Venture), China (Shanghai and Shenzhen), Paris (Paris and NYSE Euronext), Germany (Deutsche Boerse and Xetra), India (BSE and National Stock Exchange), Japan (Tokyo and Osaka), Russia (Moscow and MICEX), South Korea (Korea and KOSDAQ), Switzerland (Swiss Exchange and Zurich), United Arab Emirates (Abu Dhabi and Dubai), and the U.S. (NYSE, NYSE Arca, AMEX, and NASDAQ).

We keep only countries with at least a 10-year continuous period. We combine the 22 DMs into three regions: (1) North America (Canada and United States); (2) Developed Europe (Austria, Belgium, Denmark, Finland, France, Germany, Ireland, Italy, the Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, and the United Kingdom); and (3) Asia Pacific (Australia, Hong Kong, Japan, New Zealand, and Singapore). We combine the 24 EMs into three regions: (1) Latin America (Argentina, Brazil, Chile, Mexico, and Peru), (2) Middle-East and Africa (Israel, Jordan, Morocco, Oman, Saudi Arabia, South Africa, Turkey, and United Arab Emirates), and (3) Emerging Asia (China, India, Indonesia, Malaysia, Pakistan, Philippines, South Korea, Sri Lanka, Taiwan, Thailand).[28]

[28] While Poland does not fall into one of these regions, we use European factors when estimating its regional models.

Table 1 provides summary statistics for our data. We provide summary statistics for countries in Panel A, the minima, averages, and maxima across countries in Panel B, and summary statistics for regions in Panel C. Except for the North American region, the return data for DMs usually start in the late 1980s. The EM return data start in the mid-1990s. The number of stocks varies over time and across countries. There are 39,792 stocks in DMs and 22,528 in EMs. The minimum (maximum) number of stocks in a country is 97 (11,595) for Morocco (U.S.), the minimum (maximum) number of stocks in a region is 1,091 (16,806) in Latin (North) America.

4.2 Risk factors

We consider four sets of risk factors. We first use the excess return on the value-weighted market portfolio of all stocks as in the CAPM model. Next, we use the four-factor model that adds momentum factor to the Fama-French three-factor model that includes market, size, and value (Carhart, 1997). Then, we consider the five-factor model of Fama and French (2015) that augments the Fama-French three-factor model with profitability and investment factors. Based on Hou et al. (2015), our final set of factors include the market, size, profitability, and investment factors.[29] Feng et al. (2017) also show that profitability and investment factors have significant explanatory power for expected returns of U.S. stocks.

We use the market capitalization of a stock to measure size, and the monthly updated book-to-price ratio constructed as in Asness and Frazzini (2013) to measure value. See Barillas and Shanken (2018) for recent evidence on the importance of using the monthly updated value factor.

Each month and for each country, we use all stocks with a valid market capitalization at the end of the previous month as test assets and to construct a value-weighted market factor. We use the subset of these stocks with a valid book-to-price ratio to construct the size and value factors. All stocks in a country are assigned to two size groups using the median market capitalization for U.S. stocks and the 80^{th} market capitalization percentile for non-U.S. stocks. Within each size group, we separate stocks into three value-weighted portfolios based on the 30^{th} and 70^{th} percentiles of the book-to-price ratios. We follow Asness et al. (2013) and use conditional sorts to ensure a balanced number of stocks in each portfolio. We require

[29]Our model differs from theirs as we use a double sort on size and cash profitability and they use a triple sort on size, ROE, and investment to build the profitability factor. The low number of stocks in some countries makes it difficult to do a triple sort. We rely on double sorts for all factors to keep the same construction methodology across countries.

at least 20 available stocks to compute a factor return.

The size factor is the average of the return of the three small-size portfolios minus the average return on the three large-size portfolios. The value factor is the average return of the small and large portfolios with high book-to-price ratios minus the average return of the small and large portfolios with low book-to-price ratios. The momentum, profitability, and investment factors are constructed similarly as the value factor replacing book-to-price ratios respectively by each characteristic in the second ranking. To measure momentum, we use the past 12-month cumulative return on the stock and skip the most recent month return (see, for example, Jegadeesh and Titman, 1993). We use cash profitability, which is gross profitability unaffected by accruals, divided by book value of equity to measure profitability (Ball et al., 2016; Fama and French, 2018).[30] We use the relative change in total asset values to measure investment (see Fama and French, 2017; Hou et al., 2015).

We form one set of factors in each country and compute aggregate factors for each region and at the world level by value-weighting country-specific factors using lagged country total market capitalizations denominated in USD.

Table 1 reports the annualized average returns in columns (iii) to (viii) and volatilities in columns (ix) to (xiv) of the market excess return, size, value, momentum, profitability, and investment factors for each country, as well as at the world level and by region. Figure 1 shows the annualized average factor returns against the factor volatilities for DMs and EMs.[31]

The annualized mean returns averaged across countries of these six factors respectively are 9.49%, 2.33%, 4.95%, 10.03%, 2.82%, and 2.27%. The average market excess return is positive for all markets. Factor average returns are positive for 71.74% of countries for size, 84.78% for value, 91.30% for momentum, 78.26% for profitability, and 78.26% for investment (unreported proportions). But there are substantial cross-country differences in the magnitude of these historical average returns. Annualized factor volatilities range from 5.6% to more than 47%. Higher factor average return is associated with higher risk; the

[30] Ball et al. (2016) and Fama and French (2018) show that a cash profitability factor outperforms a gross-profitability factor in explaining the cross-section of U.S. stock returns. Hou et al. (2011) show the importance of the closely-related cash-flow-to-price characteristic in pricing international stocks.

[31] We report in Figure 1 of the Online Appendix the annualized average factor returns against the factor volatilities for each region and factor.

correlation between average returns and volatilities across all countries and factors is 0.39 (unreported).

Panel C of Table 1 presents the average returns and volatilities for regional factors. All regional factor average returns are positive, except for profitability in Latin America. Across regions, the average market excess return is the highest in North America and lowest in Asia Pacific. Asia Pacific has the highest value and lowest momentum historical average returns. Thus, momentum returns in Asia remain weaker than those around the world as previously found in Griffin et al. (2003). Size average returns is the largest in North America and smallest in Developed Europe. North America has the highest average returns for both the profitability and investment factors. The correlation between regional factor average returns and volatilities is positive (0.39).

Figure 1 shows the average returns and volatilities of country factors aggregated across DMs and across EMs during the period October 1996 to February 2018. All average factor returns are positive. The EM market factor has higher volatility than the DM market factor, a fact well documented in the literature. However, value and momentum factors in EMs deliver lower volatility than their respective DM factors. The risk/return profiles of size, profitability, and investment are similar across DMs and EMs. But investment in EMs has lower average returns (less than 2% compared to $4 - 5\%$).[32] Notwithstanding the difference in sample period and cross-section of countries, the magnitude of our historical average returns for size, value, momentum, profitability, and investment is comparable to past studies. For example, the stronger investment effect in developed compared to emerging markets is also found in Titman et al. (2013). They report an average annual investment return of 2% and 4% for, respectively, EMs and DMs.

Finally, Figure 2 presents the risk/return profiles of DM and EM factors during world equity bear and bull markets. We report on the horizontal axis the Sharpe ratio using all returns and on the vertical axis the difference in Sharpe ratios between bear and bull markets. We define a bear market as any 12-month period during which the cumulative return on the world market factor is below -20%.[33] The points above the zero horizontal line correspond to a higher Sharpe ratio during bear markets than during bull markets and we call these factors defensive. Defensive factors include size, value, profitability, and to a lesser extent

[32] These observations are similar when looking at the longest available period for each set of factors.

[33] Based on this definition, the 1985-2018 period covers these bear markets: September 1989 to January 1991, February 2000 to January 2002, March 2002 to March 2003, and September 2007 to June 2009. We use the longest period available for each factor to have more precise estimates of their bear- and bull-market Sharpe ratios.

investment. In contrast, market and momentum factors in both DMs and EMs are aggressive factors; they deliver lower Sharpe ratios during bear markets.[34] Therefore, the historical size, value, momentum, and profitability factors have similar magnitude and performance across DMs and EMs, but also during bear and bull markets.

In the next section, we use different combinations of these factors to explain the cross-sectional differences in average returns across a large set of individual stock returns.

5 Empirical results

This section contains our main empirical results. We start by investigating which models capture the factor structure in stock returns. Then, we show the asset pricing performance of each model. Finally, we discuss whether and how factor risk premia vary over time and how they differ across regions.

5.1 Do factors capture equity and currency risk?

Underlying the validity of our international APT and the consistency of the risk premia estimator is the assumption that residuals are weakly cross-sectionally correlated. Residuals could be too highly correlated if there is a missing equity factor, a missing currency factor as discussed in Section 2.1, or a misspecified dynamics for the risk factor loadings.[35] In this section, we empirically evaluate whether the weak cross-correlation assumption is verified in the data. We adopt two approaches. First, we use the GOS2 diagnostic criterion, described in Section 3.3, to determine whether there remains a factor structure in model residuals. Second, we examine the correlation matrices of individual stock returns and model residuals to determine whether there remains high correlations between blocks of stocks potentially induced by currency conversion.

In Table 2, we report the proportion of negative diagnostic criteria across different models. A negative value indicates that the candidate factor model has successively captured all the factor structure in USD-

[34]This classification between aggressive and defensive factors is robust to using the common sample period starting in October 1996.

[35]As discussed in GOS2, even if the omitted factors are not priced, the risk premia estimates will not converge to the risk premia of the priced factors, and biases on betas and risk premia will not compensate each other.

denominated returns. A positive value would otherwise indicate that we are missing at least one factor that drives stock returns, whether it be a missing equity or currency factor. In Panel A, we report results for the CAPM, four-factor, five-factor, q-factor models using each country respective lagged dividend yield as a common instrument $Z_{c,t-1}$. For each model, we report the proportion across all countries, across all DMs, and across all EMs.

In column (i), we consider models with factors aggregated at the world level. We find that the factor structure in stock returns is captured by the world market factor in only 8.70% of countries.[36] Using world factors in multifactor models leads only to small increases in the proportions for DMs and EMs. Clearly, world factors do not capture the factor structure in stock returns.

Next, we report in column (ii) the proportions of countries when using factors aggregated at the regional level. For each country, we use its respective set of regional factors. The regional CAPM successively captures the factor structure for 40.91% of DMs, but only for 16.67% of EMs. Moving to multifactor models increases the proportion for DMs, but not as much for EMs. Regional factors fare better than world factors, but fail to capture the factor structure in many countries' stock returns.

In columns (iii) and (iv), we augment the world and regional models with the excess country-specific market factor. We construct the excess country market factor by computing the return of the country market factor in excess of the world market factor, $f_m^{Country} - f_m^{World}$, and of the regional market factor, $f_m^{Country} - f_m^{Region}$.[37] We denote this factor as the *excess* country market factor and these models as mixed world and mixed regional models. We find that mixed regional models capture the factor structure in all DMs and most EMs. Mixed world models perform slightly worse. Our results based on large unbalanced panels of

[36]Those inferred proportions are not subject to a multiple testing problem and do not require a Bonferroni correction or a false discovery approach (see, for example Barras et al., 2010; Bajgrowicz and Scaillet, 2012). GOS2 explain that we can view their model selection procedure as a conservative testing procedure with size zero by construction.

[37] Stehle (1977), Bekaert, Hodrick, and Zhang (2009) and Karolyi and Wu (2018) also propose mixed models with global and local factors, where the local factor is orthogonalized on the global factor. We use a simple return difference to avoid any look-ahead bias in the construction of a given factor possibly induced by the projection coefficient estimated from the full sample. This construction choice also eases the interpretation of the total country market risk premia as the sum of the world (regional) market and the excess country market risk premia. Section 6 of the Online Appendix shows the correlations across country factors in excess of the world and regional market factors. We find low cross-correlations further supporting the fact that excess country market factors are key in capturing the factor structure in many countries.

individual stock returns show that excess country market factors are required in addition to world factors or even regional factors to capture their factor structure.[38] We therefore focus on these mixed models in the next sections.

Finally, we examine in Panel B the extent to which our results come from using the country lagged dividend yield as an instrument. We report the proportions of countries based on models with no common instruments $Z_{c,t-1}$. We find that the proportions are slightly lower for the CAPM models and equal for many multifactor models indicating that the scaled factors are not key in achieving weak cross-sectional dependence in the errors. We nonetheless estimate models in the next section with the lagged dividend yields to investigate the time series dynamics of factor risk premia.

As an additional check, we graphically report the correlation matrices of individual stock returns and residuals from the mixed regional four-factor model. We compute the correlation matrix using all individual stocks that are kept in the model estimations. Of these stocks, we keep all pairs for which we have more than two years of overlapping monthly returns. In Figure 3, we report the average block correlations between countries. The blocks on the diagonal are the average correlations of all stocks within a country. The off-diagonal blocks are the average correlation between each stock in a country and each stock in another country. We order countries by region on both axes.

The highest country correlation averages are close to 0.4 and are found in Brazil, Saudi Arabia, and China. Cross-country average correlations are rarely higher than 0.25. Most notably, we can observe in the upper left corner the European block in which all countries exhibit high correlation between each other.

Return correlations in Figure 3 sharply contrast with correlations of their mixed regional four-factor model residuals in Figure 4. We find that, once we control for regional market, size, value, momentum, and excess country market factors, then all of the within- or cross-country average correlations are less than 0.10. These low correlations confirm that we capture all the factor structure, including currency factors, present in U.S. dollar-denominated returns. This also indicates that the risk exposure dynamics are correctly specified in that they achieve weak cross-sectional dependence in the errors. The small remaining correlations within a diagonal block might reflect the impact of currency conversion for that country, and the ones remaining in off-diagonal blocks the impact of currency-specific shocks correlated across currencies as discussed in

[38]The excess country market factors may partially capture currency risk.

Section 2.1. This exemplifies the need to allow for an approximate factor structure in an international setting. Therefore, our factor model specification is close to a block diagonal structure for the error covariance matrix with blocks corresponding to countries. Such a sparse matrix is compatible with the notion of weak cross-sectional dependence (see GOS Appendix F for a proof).

5.2 Asset pricing performance of the factor models

Guided by our diagnostic from the previous section, we estimate in this section mixed world models with world factors augmented with an excess country market factor and mixed regional models with regional factors augmented with an excess country market factor.[39]

For each country, we compute the test statistic and its p-value for the test of the asset pricing restrictions, $a_{i,c,t} = b_{i,c,t}\nu_{c,t}$, and for the case of tradable factors, $a_{i,c,t} = 0$. The latter corresponds to the traditional test that alphas from time-series regressions are jointly equal to zero (see Gibbons et al., 1989), but here with an inference suitable for large unbalanced panels.

Table 3 presents asset pricing test results for the four asset pricing models. We report in the first line the proportion of countries for which the model is not rejected. A model is not rejected when the set of factors is correctly specified and the no-arbitrage asset pricing restriction is not rejected, that is, the diagnostic criterion ζ in Equation (9) is negative and the p-value for the asset pricing restrictions are above the significance level. We use a 5% significance level with a Bonferroni correction (i.e., $5\%/46$). We display mixed world models in the second and third columns and mixed regional models in the last two columns.

Panel A displays asset pricing test results for the mixed CAPM models. The mixed CAPM models perform well. The mixed world model is correctly specified and is not rejected for 71.74% of countries (see column (i)). The mixed regional models perform slightly better with 73.91% of models not rejected (see column (iii)).

The mixed world and regional market models perform better for DMs than for EMs. The non-rejection rates of the world and regional market models are 77.27% and 81.82%, respectively for DMs compared to

[39]Ideally, we would include common factors across all countries and test market integration by testing equality of risk premia. This amounts to adding all excess country market factors. Unfortunately, the curse of dimensionality precludes the estimation and testing for market integration in such models. There are $p(p+1)/2 + pq + K(p+q)$ regressors which, for example, gives 34 regressors with $p = 2$, $q = 3$, and $K = 5$ as in our application of the mixed four-factor model.

66.67% and 66.67% for EMs. Strikingly, the mixed world model performs well in most Latin American countries but fails in many Emerging Asian countries. The mixed world and regional models are rejected for markets with a large number of securities like China, Japan, the U.K., and the U.S.[40]

For the world market models, we cannot reject the hypothesis that alphas are jointly equal to zero for all countries for which the asset pricing restrictions are not rejected. Therefore, the proportions of countries are identical for both tests. For regional market models, rejections from the asset pricing restriction test and from the test of alphas equal to zero generally agree.

Panels B-D report on the mixed four-, five-, and q-factor models. Based on the proportion of non-rejection across all countries, the mixed regional models perform better than the mixed world models except for the zero-alpha tests in the q-factor model. All three mixed regional models perform similarly, with non-rejection ranging from 74% to 91% of countries. Multifactor models almost always perform better for DMs than for EMs. The rejection rates for the mixed multifactor models tend to be higher in the Emerging Asia region.[41]

The test for the asset pricing restrictions, $a_{i,c,t} = b_{i,c,t}\nu_{c,t}$, has a lower number of degrees-of-freedom than the test for the alphas equal to zero.[42] The former test is therefore more demanding than the latter one. Accordingly, in Table 3 where we report the percentage of non-rejections, we observe for some multifactor models a higher number of non-rejections for the hypothesis $a_{i,c,t} = 0$ than for the asset pricing restrictions $a_{i,c,t} = b_{i,c,t}\nu_{c,t}$.

5.3 The significance and dynamics of factor risk premia

In this section, we test whether the factor risk premia are significant and whether they vary over time. We also explore the time series dynamics of the factor risk premia across DMs and EMs.

[40] Tables 1 and 5 of the Online Appendix provide the diagnostic criteria and asset pricing test results for each country for, respectively, the mixed world and regional CAPM models.

[41] Tables 2-4 of the Online Appendix provide results for each country for mixed world multifactor models and Tables 6-8 report on mixed regional multifactor models. We also report p-values for the asset pricing restriction test with fixed n for countries with less than 100 stocks. Results are overall robust in countries with smaller cross-sections of stocks.

[42] There are $K \times p$ parameters to estimate in the ν vector which equals 4 for the mixed regional CAPM, 10 for the mixed four-factor model, and 12 for the mixed five-factor model.

5.3.1 The significance of factor risk premia

Using the distribution for the parameters of the risk premium dynamics, Λ_c, we test the null hypothesis that factor k risk premium is zero, $\mathcal{H}_0 : \Lambda_{k,0} = \Lambda_{k,DY_{t-1}} = \Lambda_{k,DY_{c,t-1}} = 0$, and the null hypothesis that it is constant, $\mathcal{H}_0 : \Lambda_{k,DY_{t-1}} = \Lambda_{k,DY_{c,t-1}} = 0$.[43]

Tables 9-12 of the Online Appendix report the estimated coefficients and their significance for the constant $\Lambda_{k,0}$, the lagged world dividend yield $\Lambda_{k,DY_{t-1}}$, and the country dividend yield $\Lambda_{k,DY_{t-1}}$ for the excess country market factor in the mixed world CAPM, four-, five-, and the q-factor models. Tables 13-16 show the estimates for the regional models. Across models, the effect of world and regional dividend yields on the excess country market factor premium is insignificant for most countries. Hence the time variation in the excess country market premia is essentially captured by the country dividend yield.

Across all mixed world (regional) models, the excess country market is significantly priced at the 5% level in 76% (78%) of estimated models (untabulated results). We use a Bonferroni correction for multiple testing: the world (regional) excess country factor risk premium is estimated in four models and 46 countries for a total of 184 different tests. We reject the null hypothesis of no time variation in 64% (72%) of estimations.

As discussed in Section 2.2, if the country market in excess of the world market earns a significant risk premium then the country is segmented from the world market. Likewise, if we reject the null that the country market in excess of the regional market earns a zero risk premium then the country is segmented from the region. Hence our results inferred from individual stocks provide evidence of world and regional segmentation both in DMs and EMs. Past studies based on market indices and international equilibrium models show that country market risk is not priced in DMs (see, for example, De Santis and Gerard, 1997; Carrieri et al., 2013) but is priced in EMs (see, for example, Errunza and Losq, 1985; Bekaert and Harvey, 1995; Carrieri et al., 2007, 2013). Our new results show that country market risk is priced in both DMs and EMs.

Using a Bonferroni correction for multiple testing, we test the null hypothesis that the risk premium for each of the other factors is zero (untabulated results). Across all models in which a factor is included, we reject the null that the premium is insignificant in 73%, 81%, 79%, 89%, 77%, and 90%, of the estimated

[43] See Appendix 1 for the distribution of Λ_c.

models respectively for the world market, size, value, momentum, profitability, and investment factors. The corresponding proportions for the regional factors are 74%, 77%, 74%, 86%, 73%, and 73%.

We find weaker evidence of time-variations in the risk premia. Across all models, we reject the null that the premium is constant in 58%, 59%, 66%, 74%, 64%, and 76% of the estimated models for the world factors and 63%, 67%, 68%, 75%, 64%, and 64% for the regional factors.

We next examine how large are the premia for the different risk factors and how much they vary over time. For the sake of space, we focus our discussion on the four-factor and q-factor models.

5.3.2 Time-varying risk premia in the four-factor model

In Figure 5, we present the value-weighted averages of risk premia over time across DMs (left column) and across EMs (right column) for the mixed world four-factor model. We report the market risk premium averages in the upper graph and the size, value, and momentum risk premium averages in the bottom graph. We value-weight using each country lagged total market capitalization in USD. In the upper graph, we show the world market risk premium using a dark blue area. The light yellow area denotes the risk premium for the excess country market factor $f_m^{Country} - f_m^{World}$. Hence, the sum of the two areas, shown with a red line, measures the risk premium for the total country market factor $f_m^{World} + f_m^{Country} - f_m^{World} = f_m^{Country}$. Figure 6 repeats the analysis using the mixed regional four-factor model.

We use recession dates from the NBER for the U.S. and from the Economic Cycle Research Institute for non-U.S. countries. We build a recession indicator for each region which is equal to one when at least half of the countries in the region are in a recession.[44] We report in each figure gray areas to denote the recession periods using all regional recession indicators in the region (North America, Developed Europe, and Asia Pacific for DMs and Latin America, Middle East and Africa, and Emerging Asia for EMs).

The excess country market factor risk premium for DMs in the upper left graph of Figure 5 is small in absolute terms and relative to the world market factor risk premium. Therefore, the excess country market factor is required to capture the factor structure in DM stock returns and is priced in most DMs, but it carries on average a small risk premium. In sharp contrast, the excess country market factor risk premium for EMs

[44] Hirata et al. (2013) document the emergence of regional business cycles and show higher degree of business cycle synchronization across countries in a region.

in the upper right graph is large relative to the world market factor.

The bottom graphs in Figure 5 show the time-varying risk premia for world size, value, and momentum factors. Across DMs, we find positive premia for all factors except momentum which turn negative during and after the global financial crisis. Value and market premia spike during the global financial crisis. Periods with increasing value premium also show decreasing momentum premium. This pattern is consistent with the negative correlation between value and momentum (see Asness et al., 2013). Across EMs, the sign of the factor risk premia varies over time and momentum premia are highly volatile.

In regional models reported in Figure 6, the excess country market factor is trivial for DMs and is small for EMs relative to the regional market factor. Risk premia for the regional size, value and momentum factors in Figure 6 show similar patterns to the ones estimated from the world four-factor model, but the premium for regional momentum is less volatile.[45]

How large are the average conditional risk premia and how do they vary across countries? To address this question, we derive in Appendix 2 the distribution of the average risk premium, $\hat{\bar{\Lambda}}_{c,k} = \hat{\Lambda}_{c,k}\bar{Z}_c$ where $\bar{Z}_c$ is the instrument time-series averages. In Figure 7 and 8, we report the average conditional risk premia and their 95% confidence intervals for the mixed world four-factor model. In Figure 7, we use blue dots for the world market factor and red Xs for the excess country market factor. We report on other factors (size, value, and momentum) in Figure 8.[46]

We observe a lot of variations in the sign and magnitude of the market factor average conditional premia across countries. Most world market premia in DMs vary between 5% and 15%. The excess country market factor premia are insignificant in countries which have a large weight in the world market portfolio like Japan, the U.S., and U.K. The average excess country market premium is large in many EMs. For some countries, a large world market premium is offset by a large excess country market premium of the opposite sign. For example, the significantly negative average excess country market premium in Chile of about -10% is more than offset by a significantly positive average world market premium of 25% resulting in a positive total country market premium of about 15%. This is also the case for Austria, Denmark, Ireland, and Spain.

[45] We also report in the Online Appendix the value-weighted averages and cross-sectional dispersions of risk premia over time for each region.

[46] We report on the mixed regional models in the Online Appendix to save space.

The world size average premia reported in the top graph of Figure 8 is positive in most DMs, but we see more dispersions across EMs. World value average premia sign and size vary across countries. It is positive and significant in 14 markets but insignificant in many Developed Europe markets. World momentum average premium is insignificant not only in U.S. but also in some other DMs and EMs. Its sign and size vary a lot across countries.

5.3.3 Time-varying risk premia in the q-factor model

We report the value-weighted average risk premia for the mixed world q-factor model in Figure 9 and for the mixed regional model in Figure 10. We obtain the same conclusion as for the four-factor model. The excess country market risk premium is small in absolute terms and relative to regional market factor for DMs. Excess country market premium is larger in EMs, especially compared to the world market risk premium.

We obtain less volatile risk premia for profitability and investment than for market factors. The world nonmarket factor risk premia are almost always positive, though we obtain a small world profitability risk premium for DMs and EMs. Investment premium turns negative during the global financial crisis both in DMs and EMs.

Regional nonmarket factor premia are positive for DMs. Profitability and investment show little variation over time and earn respectively, 3% and 4% on average. The average conditional risk premia for profitability and investment are close to the time-series averages of the excess return of these factors aggregated at the world level (see Table 1).

In Figures 11 and 12, we plot the average conditional risk premia and their 95% confidence intervals from the mixed world q-factor model. We report on world and excess country market factors in Figure 11 and on other factors (size, profitability and investment) in Figure 12. World size average premium is positive and significant in a large proportion of countries. The magnitude of the average world size premium varies not only among EMs but also among Developed Europe countries. Overall, we find that the profitability and investment premia in the mixed q-factor model show less variations over time and across countries compared to the value and momentum premia estimated in the mixed four-factor model.

6 Conclusion

We estimate time-varying equity risk premia from large international individual stock returns. Our international database include $62,320$ stocks from 46 countries offering the largest cross-sectional dispersion in average returns that any asset pricing model should seek to explain.

Based on a diagnostic criterion for approximate factor structure, we find that the excess country market factor is required in addition to world or regional factors to capture the factor structure in equity returns for both DMs and EMs.

We test the time-varying specifications of four models: the CAPM, the four-factor, the five-factor, and the q-factor each augmented with the excess country market factor. Mixed CAPM models with regional and country market factors are not rejected for 72% of the countries. The mixed multifactor models are not rejected in 74% to 91% of countries. Both market and non-market factors earn significant risk premia in DMs and EMs. However, there is heterogeneity in the magnitude of these risk premia across and within DMs and EMs.

Whereas the excess country market factor is important to describe the covariance structure in international stock returns and is significantly priced, its risk premium is economically small for DMs. In contrast, the excess country market premium is still large for EMs despite the increase in market integration over time. Hence, country allocations continue to be an important consideration for active managers of global equity portfolios.

References

Adler, M., and B. Dumas. 1983. International portfolio choice and corporation finance: A synthesis. *Journal of Finance* 38:925–984.

Ahn, S. C., and A. R. Horenstein. 2013. Eigenvalue ratio test for the number of factors. *Econometrica* 81:1203–1227.

Andrews, D. W. K. 2005. Cross-section regression with common shocks. *Econometrica* 73:1551–1585.

Ang, A. 2014. *Asset management: a systematic approach to factor investing*. New York, NY: Oxford University Press.

Ang, A., J. Liu, and K. Schwarz. 2017a. Using individual stocks or portfolios in tests of factor models. *Working Paper, Wharton* .

Ang, A., A. Madhavan, and A. Sobczyk. 2017b. Estimating time-varying factor exposures. *Financial Analyst Journal* 73:41–54.

Asness, C. S., and A. Frazzini. 2013. The devil in HML's details. *Journal of Portfolio Management* 39:49–68.

Asness, C. S., A. Frazzini, and L. H. Pedersen. 2017. Quality minus junk. *Working paper, AQR Capital Management* .

Asness, C. S., T. J. Moskowitz, and L. H. Pedersen. 2013. Value and momentum everywhere. *Journal of Finance* 68:929–986.

Avramov, D., and T. Chordia. 2006. Asset pricing models and financial market anomalies. *The Review of Financial Studies* 19:1000–1040.

Bai, J. 2003. Inferential theory for factor models of large dimensions. *Econometrica* 71:135–171.

Bai, J. 2009. Panel data models with interactive fixed effects. *Econometrica* 77:1229–1279.

Bai, J., and S. Ng. 2002. Determining the number of factors in approximate factor models. *Econometrica* 70:191–221.

Bai, J., and S. Ng. 2006. Confidence intervals for diffusion index forecasts and inference for factor-augmented regressions. *Econometrica* 74:1133–1150.

Bajgrowicz, P., and O. Scaillet. 2012. Technical trading revisited: False discoveries, persistence tests, and transaction costs. *Journal of Financial Economics* 106:473–491.

Ball, R., J. Gerakos, J. T. Linnainmaa, and V. Nikolaev. 2016. Accruals, cash flows, and operating profitability in the cross section of stock returns. *Journal of Financial Economics* 121:28–45.

Barillas, F., and J. Shanken. 2018. Comparing asset pricing models. *Journal of Finance* 73:715–754.

Barras, L., O. Scaillet, and R. Wermers. 2010. False discoveries in mutual fund performance: measuring luck in estimated alphas. *Journal of Finance* 65:179–216.

Bekaert, G., and C. R. Harvey. 1995. Time-varying world market integration. *Jounal of Finance* 50:403–444.

Bekaert, G., C. R. Harvey, C. Lundblad, and S. Siegel. 2011. What segments equity markets? *Review of Financial Studies* 42:3841–3890.

Bekaert, G., R. Hodrick, and X. Zhang. 2009. International stock return comovements. *Jounal of Finance* 64:2591–2626.

Bekaert, G., and M. Urias. 1996. Diversification, integration, and emerging market closed-end funds,. *Jounal of Finance* 51:835–870.

Belsley, D., E. Kuh, and R. Welsch. 2004. *Regression diagnostics - Identifying influential data and sources of collinearity*. John Wiley & Sons, New York.

Bickel, P. J., and E. Levina. 2008. Covariance regularization by thresholding. *The Annals of Statistics* 36:2577–2604.

Black, F., M. Jensen, and M. Scholes. 1972. *The Capital Asset Pricing Model: Some empirical findings.* In Jensen, M.C. (Ed.), Studies in the Theory of Capital Markets. Praeger, New York.

Carhart, M. 1997. On persistence of mutual fund performance. *Journal of Finance* 52:57–82.

Carrieri, F., I. Chaieb, and V. Errunza. 2013. Do implicit barriers matter for globalization. *Review of*

Financial Studies 26:1694–1739.

Carrieri, F., V. Errunza, and K. Hogan. 2007. Characterizing world market integration through time. *Journal of Financial Quantitative Analysis* 42:915–940.

Cattaneo, M. D., R. K. Crump, M. H. Farrell, and E. Schaumburg. 2017. Characteristic-Sorted Portfolios: Estimation and Inference. *Working paper, University of Michigan* .

Chamberlain, G., and M. Rothschild. 1983. Arbitrage, factor structure, and mean-variance analysis on large asset markets. *Econometrica* 51:1281–1304.

Chan, K., A. Karolyi, and R. Stulz. 1992. Global financial markets and the risk premium on U.S. equity. *Journal of Financial Economics* 32:137–167.

Cho, C., C. Eun, and L. Senbet. 1986. International arbitrage pricing theory: An empirical investigation. *Jounal of Finance* 41:313–329.

Cochrane, J. H. 1996. A cross-sectional test of an investment-based asset pricing model. *Journal of Political Economy* 104:572–621.

Connor, G., M. Hagmann, and O. Linton. 2012. Efficient semiparametric estimation of the Fama-French model and extensions. *Econometrica* 80:713–754.

Connor, G., and R. A. Korajczyk. 1987. Estimating pervasive economic factors with missing observations. *Working Paper No. 34, Department of Finance, Northwestern University* .

Cooper, I., A. Mitrache, and R. Priestley. 2017. A global macroeconomic risk model for value, momentum, and other asset classes. *Working paper, BI Norwegian Business School* .

Cooper, I., and R. Priestley. 2011. Real investment and risk dynamics. *Journal of Financial Economics* 101:182–205.

De Roon, F., T. Nijman, and B. Werker. 2001. Testing for Mean-Variance Spanning with Short Sales Constraints and Transaction Costs: The Case of Emerging Markets. *Jounal of Finance* 56:721–742.

De Santis, G., and B. Gerard. 1997. International Asset Pricing and Portfolio Diversification with Time-Varying Risk. *Journal of Finance* 52:1881–1912.

Dumas, B., K. K. Lewis, and E. Osambela. 2017. Differences of opinion and international equity markets. *Review of Financial Studies* 30:750–800.

Dumas, B., and B. Solnik. 1995. The world price of foreign exchange risk. *Jounal of Finance* 50:445–479.

Eiling, E., and B. Gerard. 2015. Emerging Equity Market Comovements: Trends and Macroeconomic Fundamentals. *Review of Finance* 19:15431585.

Errunza, V., and E. Losq. 1985. International asset pricing under mild segmentation: Theory and test. *Journal of Finance* 40:105–124.

Errunza, V., and D. Miller. 2000. Market Segmentation and the Cost of Capital in International Equity Markets. *Journal of Financial and Quantitative Analysis* 35:577–600.

Eun, C., S. Lai, F. de Roon, and Z. Zhang. 2010. International diversification with factor funds. *Management Science* 56:1500–1518.

Eun, C. S., and J. Lee. 2010. Mean-variance convergence around the world. *Journal of Banking and Finance* 34:856–870.

Fama, E. F., and K. R. French. 1997. Industry costs of equity. *Journal of Financial Economics* 43:153–193.

Fama, E. F., and K. R. French. 1998. Value versus growth: The international evidence. *Journal of Finance* 53:1975–1999.

Fama, E. F., and K. R. French. 2012. Size, value, and momentum in international stock returns. *Journal of Financial Economics* 105:457–472.

Fama, E. F., and K. R. French. 2015. A five-factor asset pricing model. *Journal of Financial Economics* 116:1–22.

Fama, E. F., and K. R. French. 2017. International tests of a five-factor asset pricing model. *Journal of Financial Economics* 123:441–463.

Fama, E. F., and K. R. French. 2018. Choosing factors. *Journal of Financial Economics* 128:234–252.

Fama, E. F., and J. D. MacBeth. 1973. Risk, return, and equilibrium: empirical tests. *Journal of Political*

Economy 81:607–36.

Feng, G., S. Giglio, and D. Xiu. 2017. Taming the factor zoo. *Working paper, Yale School of Management* .

Ferson, W. E., and C. R. Harvey. 1991. The variation of economic risk premiums. *Journal of Political Economy* 99:385–415.

Ferson, W. E., and C. R. Harvey. 1993. The risk and predictability of international equity returns. *Review of Financial Studies* 6:527–566.

Forni, M., M. Hallin, M. Lippi, and L. Reichlin. 2000. The generalized dynamic factor model: Identification and estimation. *The Review of Economics and Statistics* 82:540–54.

Forni, M., M. Hallin, M. Lippi, and L. Reichlin. 2004. The generalized dynamic factor model: Consistency and rates. *Journal of Econometrics* 119:231–55.

Forni, M., M. Hallin, M. Lippi, and L. Reichlin. 2005. The generalized dynamic factor model: One-sided estimation and forecasting. *Journal of the American Statistical Society* 100:830–40.

Freyberger, J., A. Neuhierl, and M. Weber. 2017. Dissecting Characteristics Nonparametrically. *Working paper, University of Chicago* .

Gagliardini, P., E. Ossola, and O. Scaillet. 2016. Time-varying risk premium in large cross-sectional equity datasets. *Econometrica* 84:985–1046.

Gagliardini, P., E. Ossola, and O. Scaillet. 2017. A diagnostic criterion for approximate factor structure. *Working Paper, Swiss Finance Institute* .

Gibbons, R., S. A. Ross, and J. Shanken. 1989. A test of the efficiency of a given portfolio. *Econometrica* 57:1121–1152.

Giglio, S., and D. Xiu. 2017. Inference on risk premia in the presence of omitted factors. *Working paper, University of Chicago* .

Greene, W. 2008. *Econometric Analysis, 6th ed.* Prentice Hall.

Griffin, J. 2002. Are the Fama and French factors global or country specific? *Review of Financial Studies*

15:783–803.

Griffin, J., X. Ji, and S. Martin. 2003. Momentum investing and business cycle risk: evidence form pole to pole. *Jounal of Finance* 58:2515–2547.

Griffin, J., P. J. Kelly, and F. Nardari. 2010. Do market efficiency measures yield correct inferences? A comparison of developed and emerging markets. *Review of Financial Studies* 23:3225–3277.

Gultekin, M., N. Gultekin, and A. Penati. 1989. Capital controls and international capital market segmentation: The evidence from the Japanese and American stock markets. *Jounal of Finance* 44:849–869.

Hansen, L. P., and S. F. Richard. 1987. The role of conditioning information in deducing testable restrictions implied by dynamic asset pricing models. *Econometrica* 55:587–613.

Harvey, C. R. 1991. The world price of covariance risk. *Jounal of Finance* 46:111–157.

Heston, S. L., K. G. Rouwenhorst, and R. E. Wessels. 1995. The structure of international stock returns and the integration of capital markets. *Jounal of Empirical Finance* 2:173–197.

Hirata, H., A. M. Kose, and C. Otrok. 2013. Regionalization vs. Globalization. In Y.-W. Cheung and F. Westermann (eds.), *Global Interdependence, Decoupling, and Recoupling*, pp. 87–130. MIT Press.

Hou, K., A. Karolyi, and B.-C. Kho. 2011. What factors drive global stock returns? *Review of Financial Studies* 24:2527–2574.

Hou, K., C. Xue, and L. Zhang. 2015. Digesting anomalies: an investment approach. *Review of Financial Studies* 28:650–705.

Hsiao, C. 2003. *Analysis of Panel Data*. Econometric Society Monographs, 2nd edition, Cambridge University Press.

Ikeda, S. 1991. Arbitrage asset pricing under exchange risk. *Journal of Finance* 46:447–455.

Ince, O. S., and R. B. Porter. 2006. Individual equity return data from Thompson Datastream: handle with care! *Journal of Financial Research* 29:463–479.

Jagannathan, R., and Z. Wang. 1996. The conditional CAPM and the cross-section of expected returns.

Journal of Finance 51:3–53.

Jegadeesh, N., and S. Titman. 1993. Returns to buying winners and selling losers: Implications for stock market efficiency. *Journal of Finance* 48:65–91.

Karolyi, A. 2015. *Cracking the emerging markets enigma*. New York, NY: Oxford University Press.

Karolyi, A., and R. Stulz. 2003. *Are assets priced locally or globally?* Handbook of the Economics of Finance, eds G. Constantinides, M. Harris and R. Stulz, Elsevier.

Karolyi, A., and Y. Wu. 2018. A new partial-segmentation approach to modeling international stock returns. *Journal of Financial and Quantitative Analysis* 53:507–546.

Korajczyk, R., and C. Viallet. 1989. An empirical investigation of international asset pricing. *Review of Financial Studies* 2:553–585.

Kozak, S., S. Nagel, and S. Santosh. 2017. Shrinking the cross section. *Working paper, University of Chicago* .

Kozak, S., S. Nagel, and S. Santosh. 2018. Interpreting factor models. *Forthcoming in The Journal of Finance* .

Lakonishok, J., A. Shleifer, and R. W. Vishny. 1994. Contrarian investment extrapolation and risk. *Journal of Finance* 49:15411578.

Lee, K.-H. 2011. The world price of liquidity risk. *Journal of Financial Economics* 99:136–161.

Lewellen, J., S. Nagel, and J. Shanken. 2010. A skeptical appraisal of asset-pricing tests. *Journal of Financial Economics* 96:175–194.

Lewis, K. K. 2011. Global asset pricing. *Annual Review of Financial Economics* 3:435–466.

Liew, J. K.-S., and M. Vassalou. 2000. Can book-to-market, size, and momentum be risk factors that predict economic growth? *Journal of Financial Economics* 57:221245.

Ludvigson, S., and S. Ng. 2007. The empirical risk-return relation: A factor analysis approach. *Journal of Financial Economics* 83:171–222.

Ludvigson, S., and S. Ng. 2009. Macro factors in bond risk premia. *The Review of Financial Studies* 22:5027–5067.

Moore, L. D., and P. Sercu. 2013. The smallest firm effect: An international study. *Journal of International Money and Finance* 32:129–155.

Onatski, A. 2010. Determining the number of factors from empirical distribution of eigenvalues. *Review of Economics and Statistics* 92:1004–1016.

Pesaran, M. H. 2006. Estimation and inference in large heterogeneous panels with a multifactor error structure. *Econometrica* 74:967–1012.

Pukthuanthong, K., and R. Roll. 2009. Global market integration: an alternative measure and its application. *Journal of Financial Economics* 94:214–232.

Ross, S. A. 1976. The arbitrage theory of capital asset pricing. *Journal of Economic Theory* 13:341–360.

Rouwenhorst, G. 1999. Local return factors and turnover in emerging stock markets. *Jounal of Finance* 54:1439–1464.

Sentana, E. 2002. Did the EMS reduce the cost of capital? *The Economic Journal* 112:786–809.

Sercu, P. 1980. A generalization of the international asset pricing model. *Revue de lAssociation Francaise de Finance* 1:91–135.

Shanken, J. 1982. The arbitrage pricing theory: Is it testable? *Journal of Finance* 37:1129–1140.

Shanken, J. 1990. Intertemporal asset pricing: An empirical investigation. *Journal of Econometrics* 45:99–120.

Shumway, T. 1997. The delisting bias in CRSP data. *Journal of Finance* 52:327–340.

Solnik, B. 1974a. An equilibrium model of the international capital market. *Journal of Economic Theory* 8:500–524.

Solnik, B. 1974b. The international pricing of risk: An empirical investigation of the world capital market structure. *Journal of Finance* 29:48–54.

Solnik, B. 1983. International arbitrage pricing theory. *Journal of Finance* 38:449–457.

Stehle, R. 1977. An empirical test of the alternative hypotheses of national and international pricing of risky assets. *Journal of Finance* 32:493–502.

Stock, J. H., and M. W. Watson. 2002a. Forecasting using principal components from a large number of predictors. *Journal of the American Statistical Society* 97:1167–1179.

Stock, J. H., and M. W. Watson. 2002b. Macroeconomic forecasting using diffusion indexes. *Journal of Business and Economic Statistics* 20:147–62.

Stulz, R. 1981. A model of international asset pricing. *Journal of Financial Economics* 9:383–406.

Titman, S., K. J. Wei, and F. Xie. 2013. Market development and the asset growth effect: international evidence. *Journal of Financial and Quantitative Analysis* 48:1405–1432.

Uppal, R., and P. Zaffaroni. 2016. Portfolio choice with model misspecification: a foundation for alpha and beta portfolios. *Working paper, EDHEC* .

Watanabe, A., Y. Xu, T. Yao, and T. Yu. 2013. The asset growth effect: Insights from international equity markets. *Journal of Financial Economics* 108:529–563.

Figures and Tables

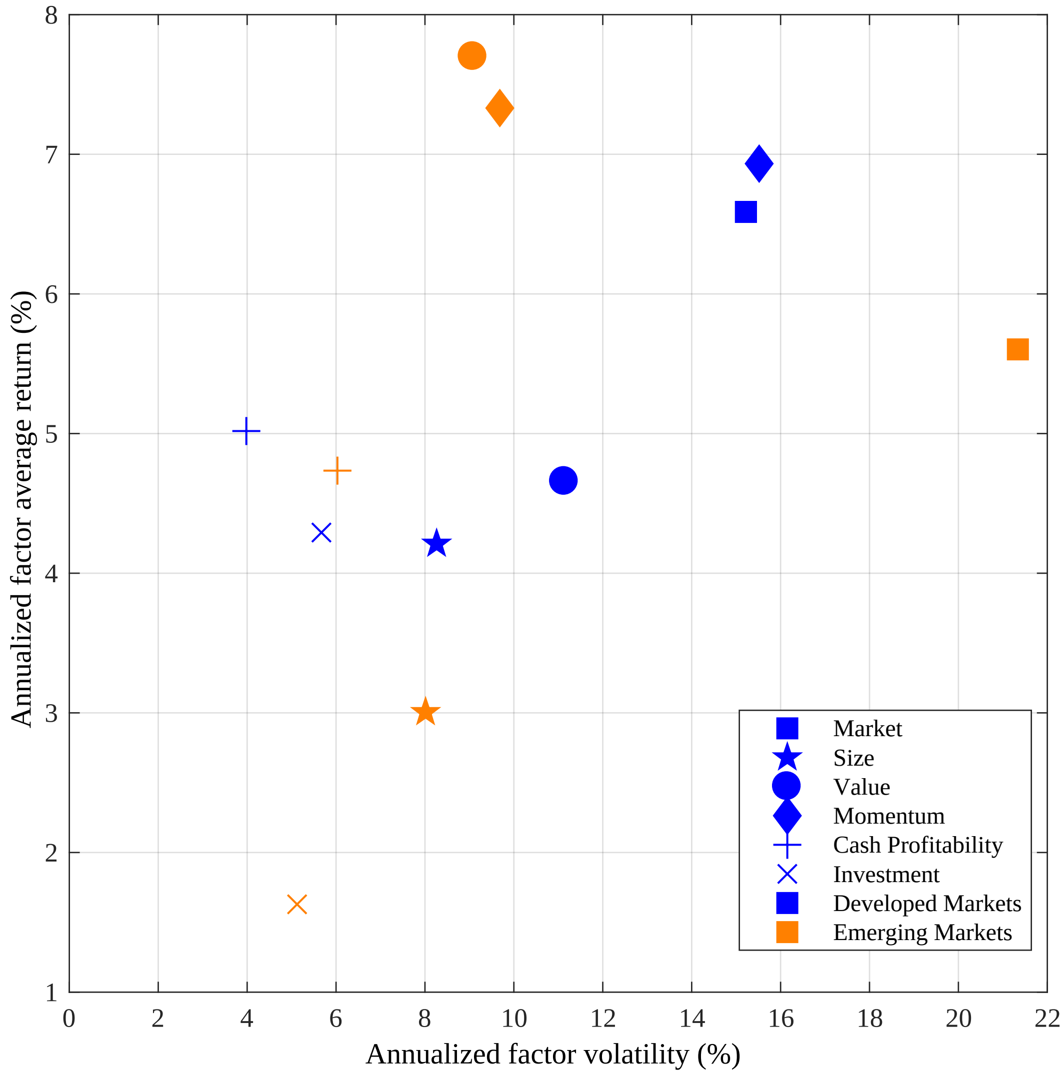

Figure 1 Developed versus emerging market factor risk and return
We report DM and EM factor average returns as a function of their volatility. We construct a market, size, value, momentum, profitability, and investment factor for each of our 46 countries. We build DM factors using all 22 developed countries and EM factors using all 24 emerging countries. For each factor, we use each country lagged total market capitalization in USD to compute value-weighted returns. All returns are monthly, in USD, start in October 1996 when all regions are available, and end in February 2018.

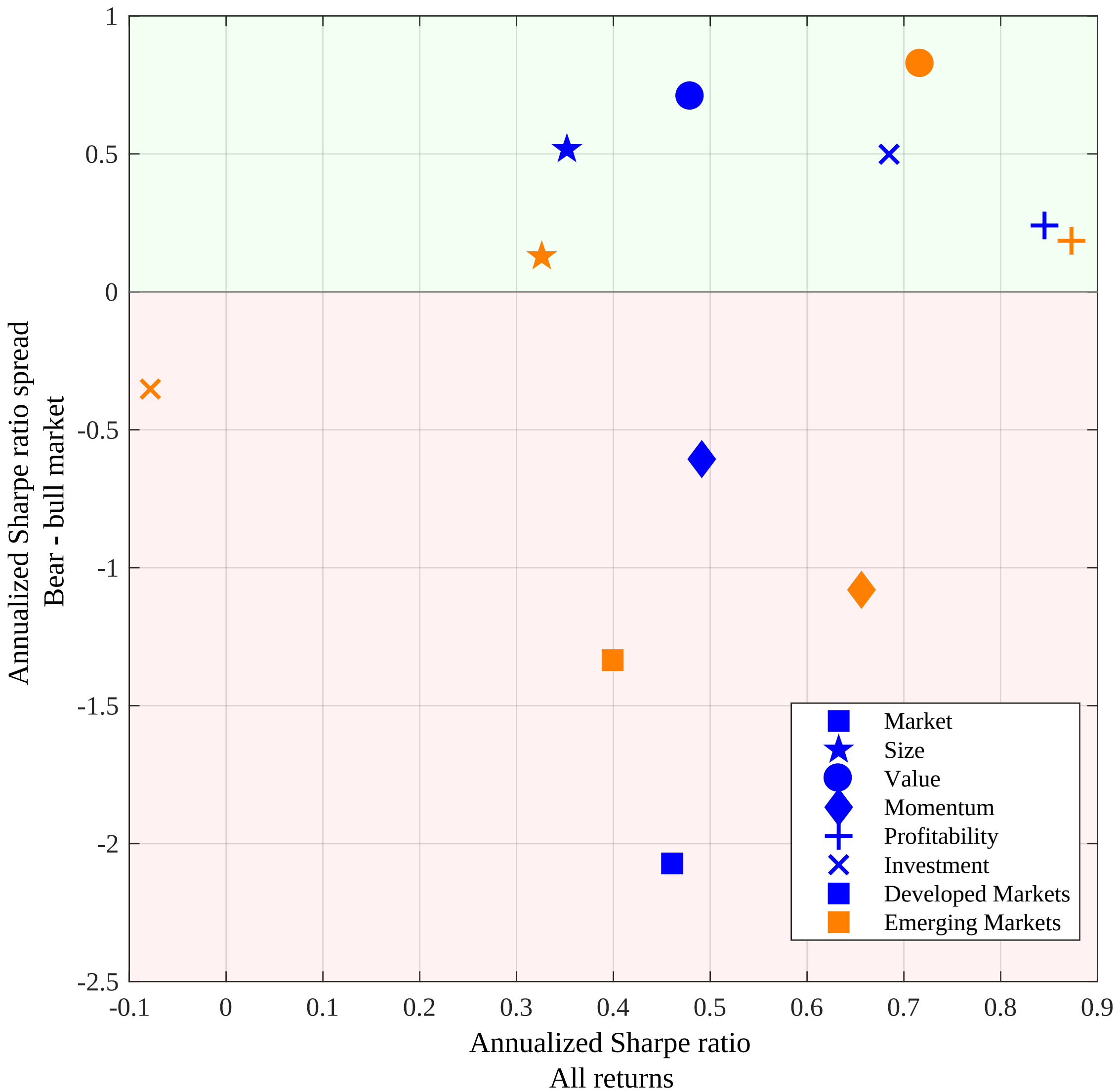

Figure 2 Factor Sharpe ratio spread between bear and bull markets
We report on the vertical axis the difference between risk factor Sharpe ratios computed during bear and bull world market and on the horizontal axis the Sharpe ratios using all returns. We define a bear market as any 12-month period during which the cumulative return on the world market factor is below -20%. Bear markets cover the periods September 1989 to January 1991, February 2000 to January 2002, March 2002 to March 2003, and September 2007 to June 2009. We construct a market, size, value, momentum, profitability, and investment factor for each of our 46 countries. We build DM factors using all 22 developed countries and EM factors using all 24 emerging countries. For each factor, we use each country's lagged total market capitalization in USD to compute value-weighted returns. All returns are monthly, in USD, and end in February 2018. The start dates differ across regions depending on data availability.

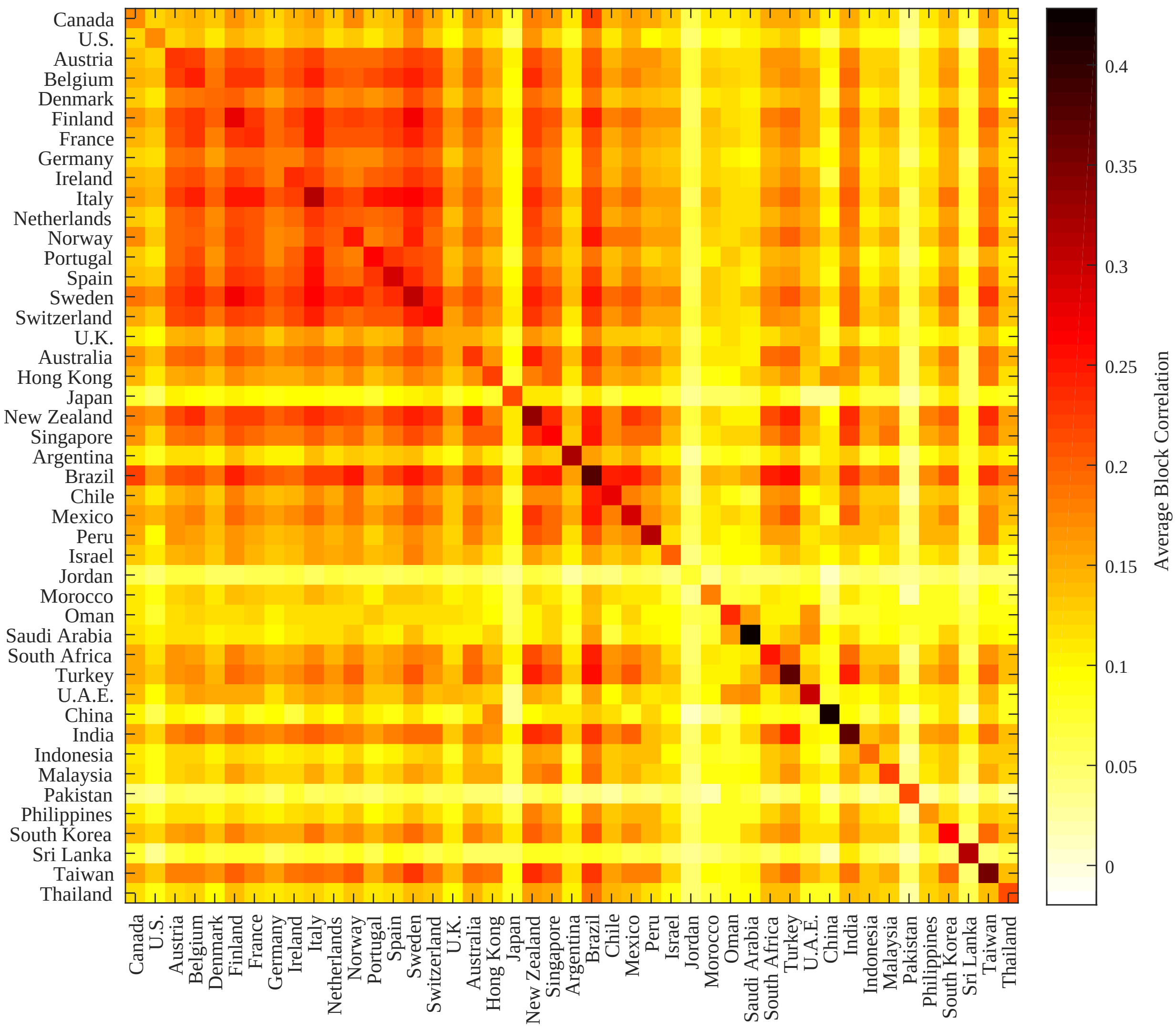

Figure 3 Average return correlation across countries
We compute the return correlation between each pair of stocks kept in the estimation of the mixed regional four-factor model and then compute country average correlations. The blocks on the diagonal are the average correlations between all stocks in a country. The off-diagonal blocks are the average correlations between each stock in a country and each stock in another country. We order countries by region (North America, Developed Europe, Asia Pacific, Latin America, Middle East and Africa, and Emerging Asia) and then alphabetically. We only keep pairs of stocks for which we have more than 24 months to compute their correlation. All returns are monthly and in USD.

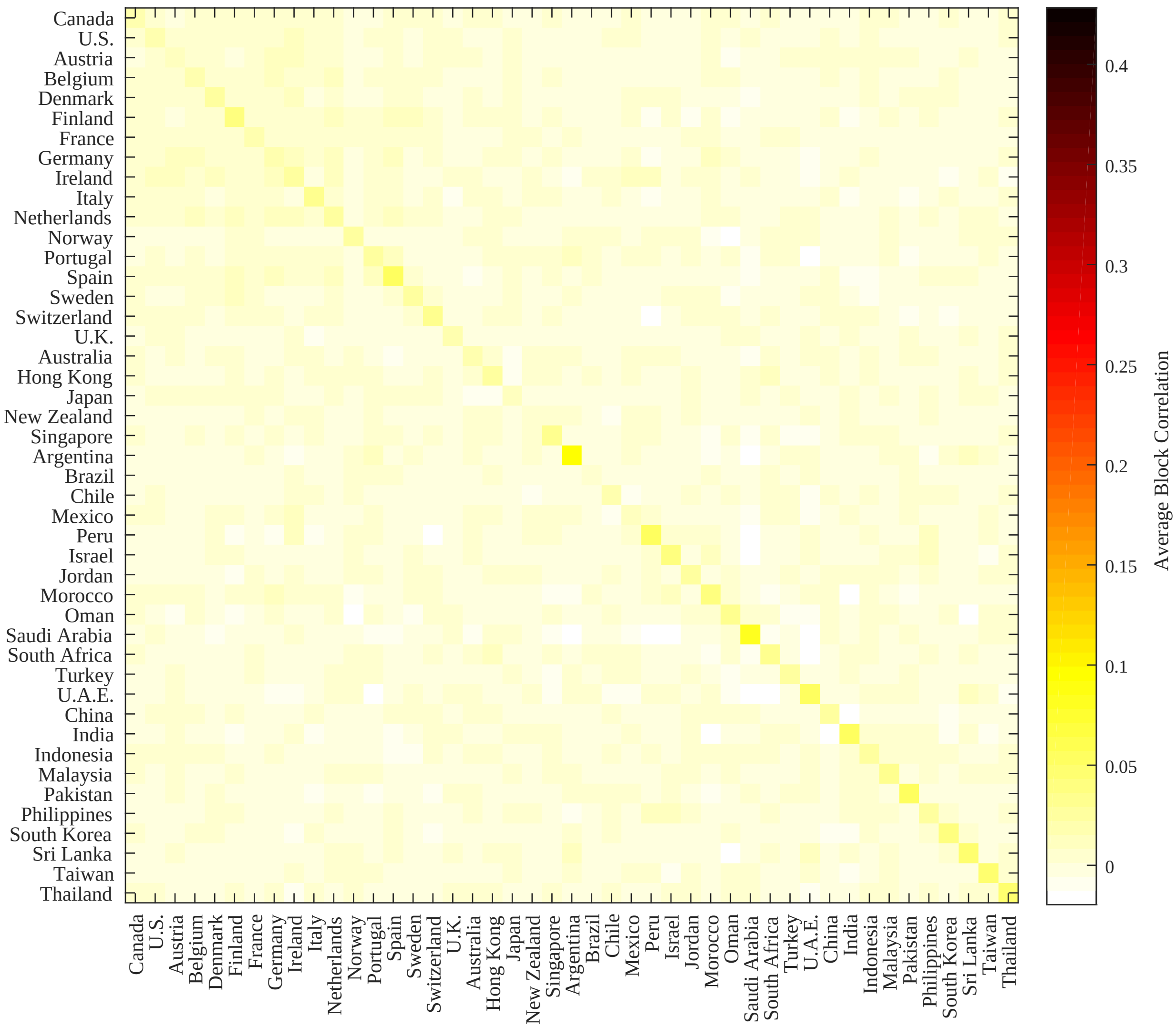

Figure 4 Average four-factor residual correlation across countries

We compute the residual correlation between each pair of stocks kept in the estimation and then compute country average correlations. We use residuals from the mixed regional four-factor model with regional market, size, value, and momentum factors, and an excess country market factor for each country. The blocks on the diagonal are the average correlations between all stocks in a country. The off-diagonal blocks are the average correlations between each stock in a country and each stock in another country. We order countries by region (North America, Developed Europe, Asia Pacific, Latin America, Middle East and Africa, and Emerging Asia) and then alphabetically. We only keep pairs of stocks for which we have more than 24 months to compute their correlation. All returns are monthly and in USD.

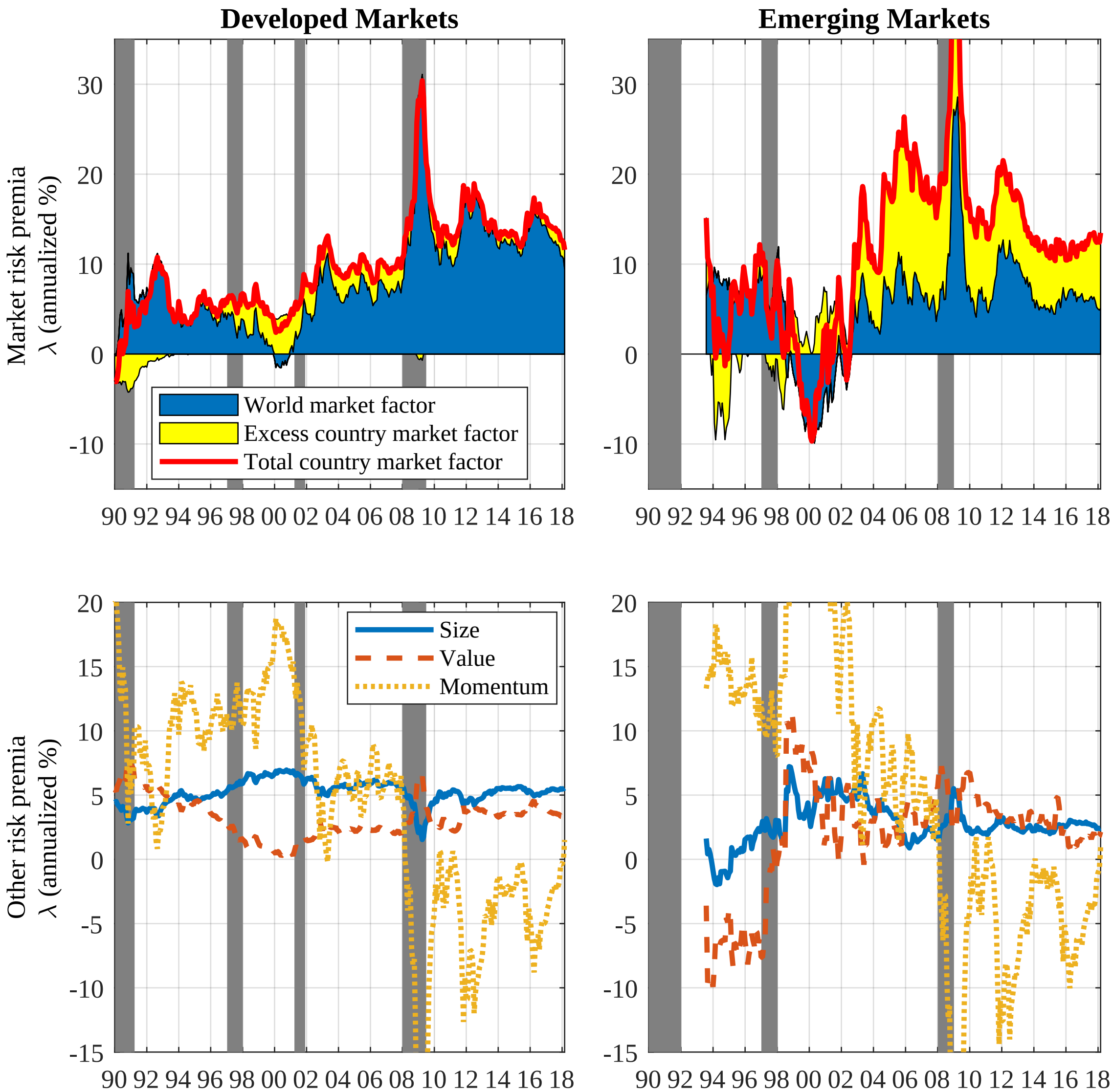

Figure 5 Time-varying world four-factor risk premia - Developed versus emerging Markets
We report each month the value-weighted average of factor risk premia λ across all DMs in the left column and all EMs in the right column. We use the mixed world four-factor models with world market, size, value, and momentum factors, and excess country market factor. We report in the upper graph the time-varying risk premia for the market factors. The dark blue area reports the world market factor risk premia and we superimpose a light yellow area to report the excess country market risk premia. The red line denotes the sum of the two premia. We report in the bottom graph the time-varying risk premia for the other factors. We compute value-weighted averages using each country lagged total market capitalization in USD for each country.

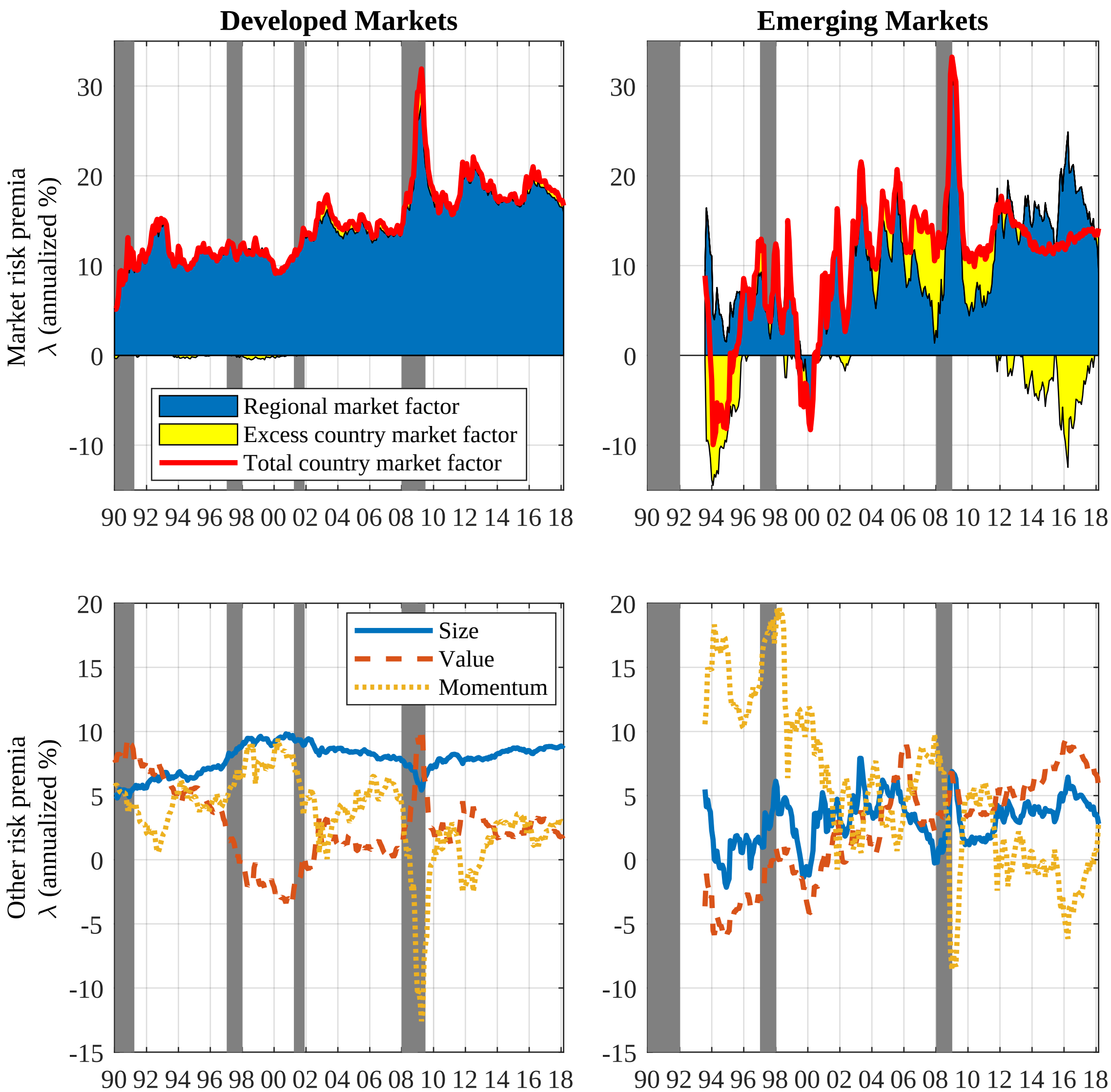

Figure 6 Time-varying regional four-factor risk premia - Developed versus emerging Markets
We report each month the value-weighted average of factor risk premia λ across all DMs in the left column and all EMs in the right column. We use the mixed regional four-factor models with regional market, size, value, and momentum factors, and excess country market factor. We report in the upper graph the time-varying risk premia for the market factors. The dark blue area reports the regional market factor risk premia and we superimpose a light yellow area to report the excess country market risk premia. The red line denotes the sum of the two premia. We report in the bottom graph the time-varying risk premia for the other factors. We compute value-weighted averages using each country lagged total market capitalization in USD for each country.

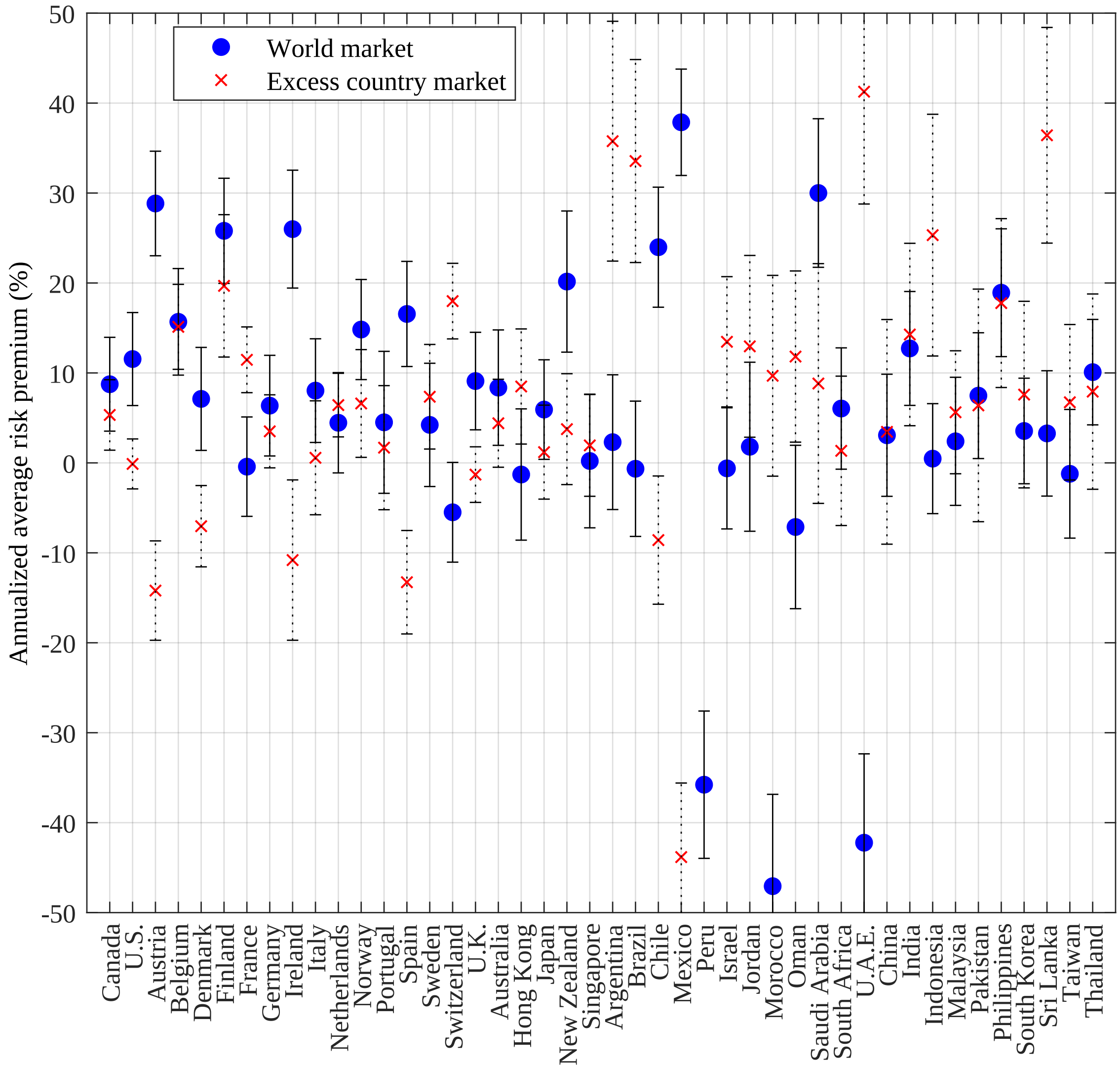

Figure 7 World and country market average risk premia - Mixed world four-factor model
We report for each country the time-series average world market risk premia using blue dots and the excess country market risk premia using red Xs, along with a 95% confidence interval for both. We use the mixed world four-factor model with world market, size, value, and momentum factors, and excess country market factor. We order countries by region (North America, Developed Europe, Asia Pacific, Latin America, Middle East and Africa, and Emerging Asia) and then alphabetically.

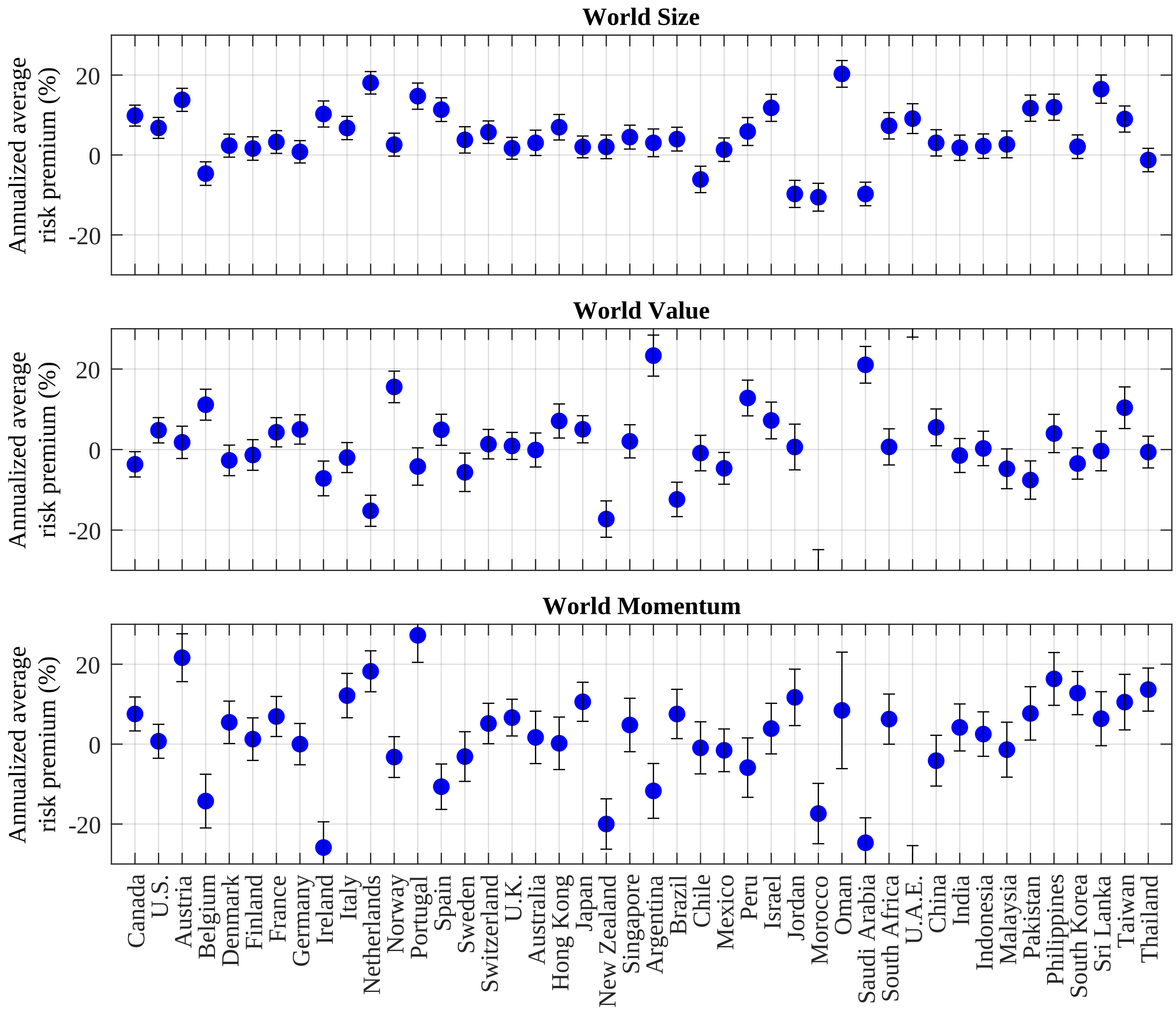

Figure 8 Other world factor average risk premia - Mixed world four-factor model
We report for each country the time-series average world factor risk premia using blue dots along with a 95% confidence interval. We report on factors other than market factors. We use the mixed world four-factor model with world market, size, value, and momentum factors, and excess country market factor. We order countries by region (North America, Developed Europe, Asia Pacific, Latin America, Middle East and Africa, and Emerging Asia) and then alphabetically.

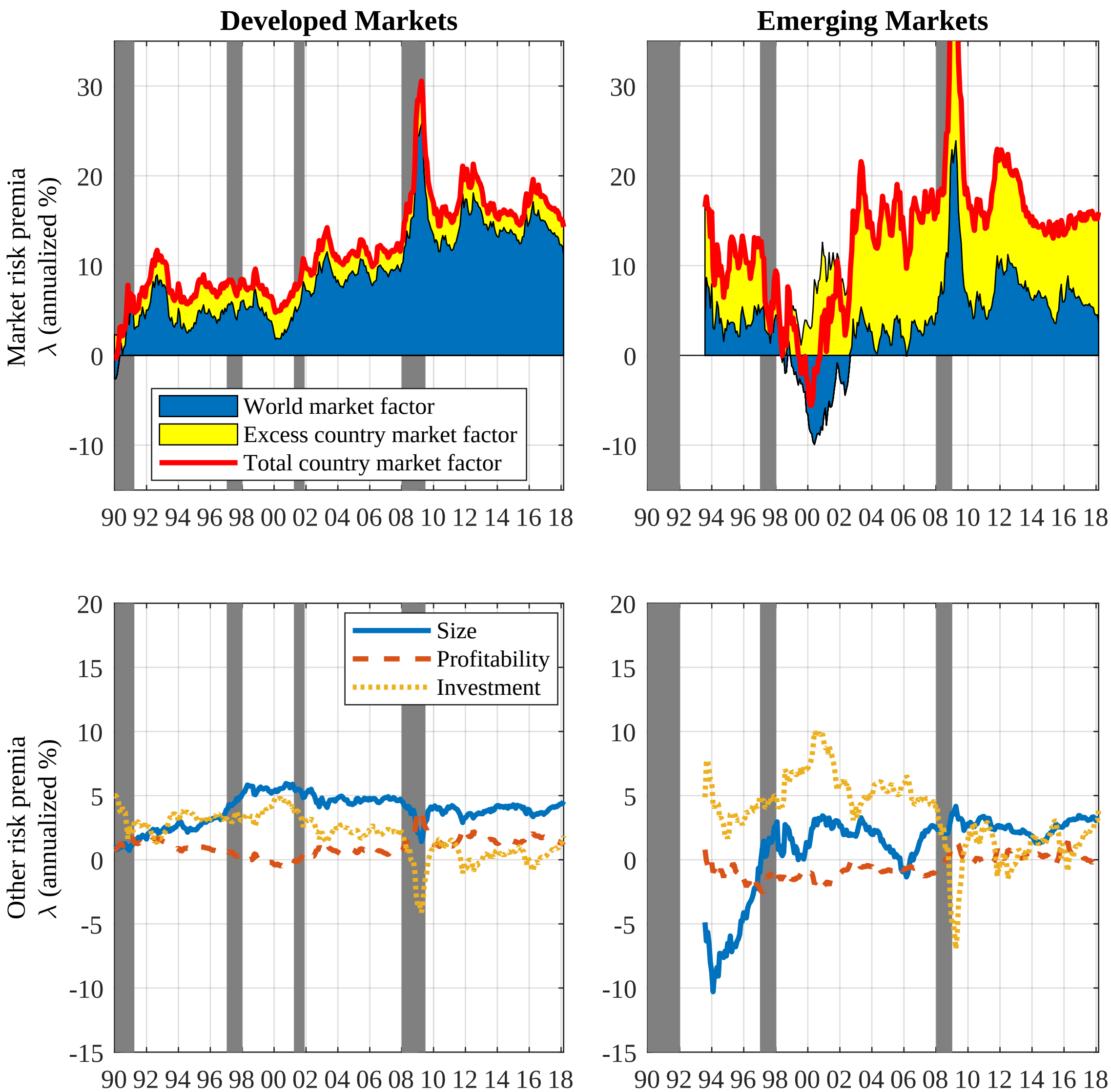

Figure 9 Time-varying world q-factor risk premia - Developed versus emerging Markets
We report each month the value-weighted average of factor risk premia λ across all DMs in the left column and all EMs in the right column. We use the mixed world q-model with world market, size, profitability, and investment factors, and excess country market factor. We report in the upper graph the time-varying risk premia for the market factors. The dark blue area reports the world market factor risk premia and we superimpose a light yellow area to report the excess country market risk premia. The red line denotes the sum of the two premia. We report in the bottom graph the time-varying risk premia for the other factors. We compute value-weighted averages using each country lagged total market capitalization in USD for each country.

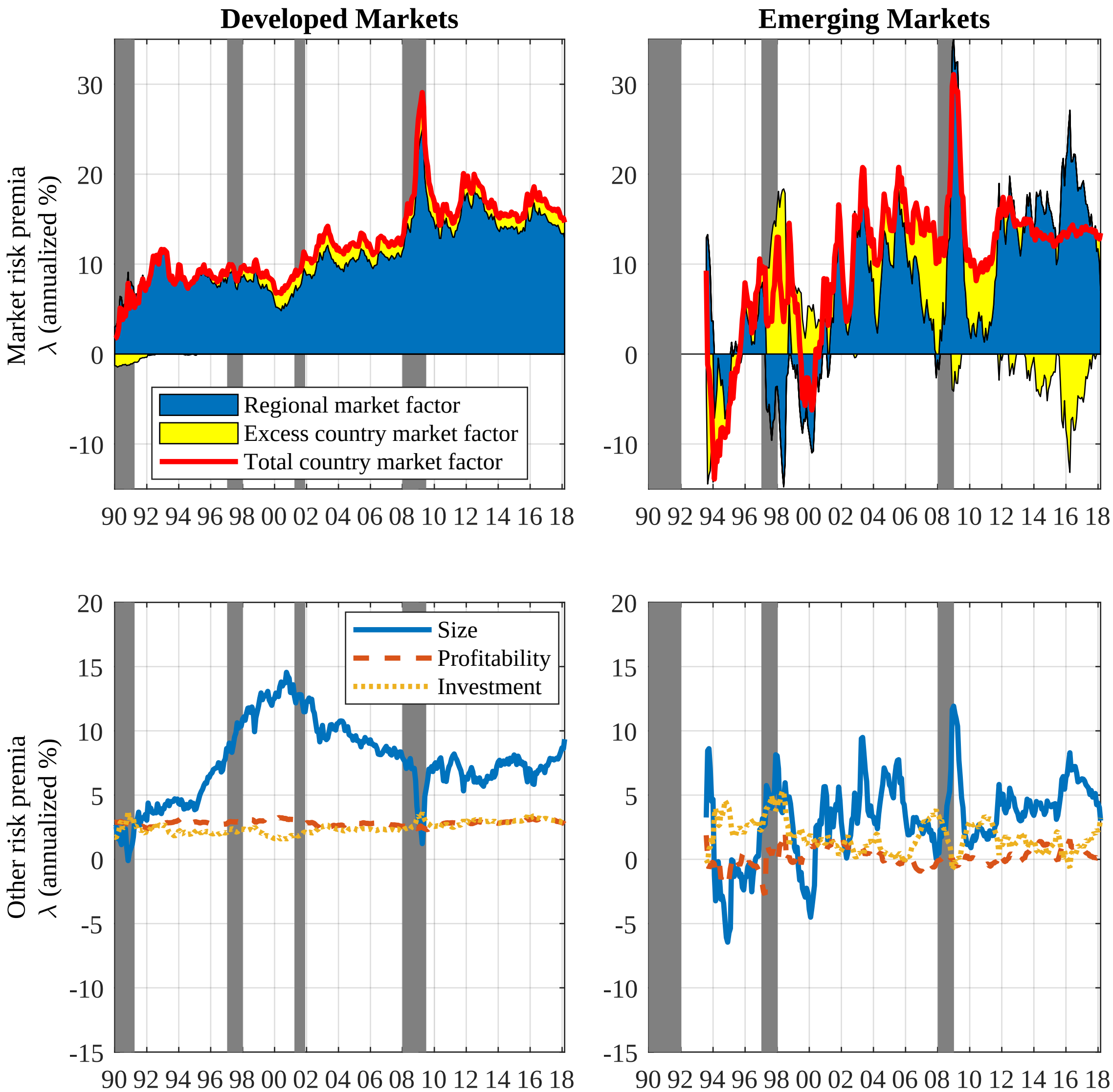

Figure 10 Time-varying regional q-factor risk premia - Developed versus emerging Markets
We report each month the value-weighted average of factor risk premia λ across all DMs in the left column and all EMs in the right column. We use the mixed regional q-model with regional market, size, profitability, and investment factors, and excess country market factor. We report in the upper graph the time-varying risk premia for the market factors. The dark blue area reports the regional market factor risk premia and we superimpose a light yellow area to report the excess country market risk premia. The red line denotes the sum of the two premia. We report in the bottom graph the time-varying risk premia for the other factors. We compute value-weighted averages using each country lagged total market capitalization in USD for each country.

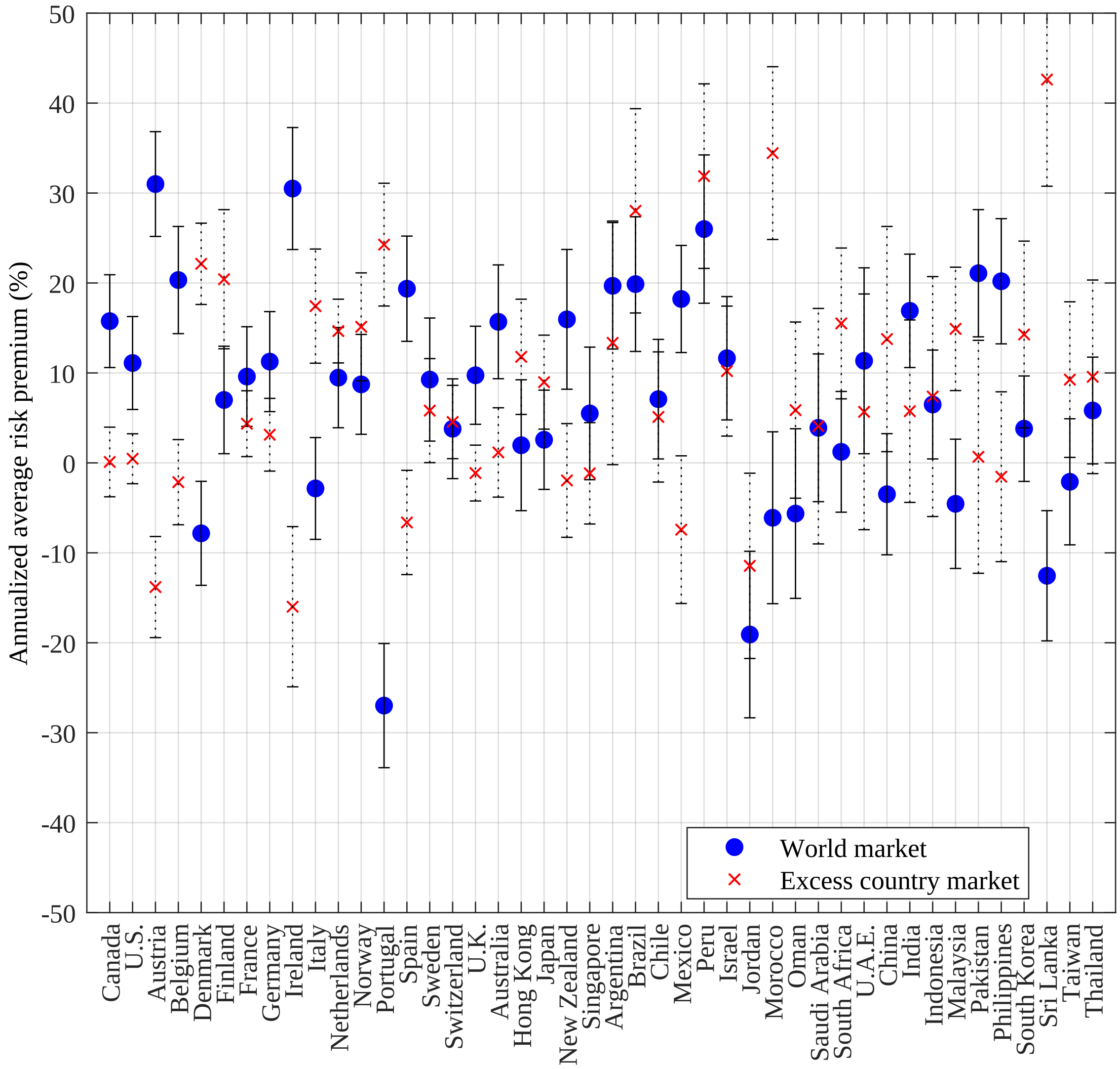

Figure 11 World and country market average risk premia - Mixed world q-factor model
We report for each country the time-series average world market risk premia using blue dots and the excess country market risk premia using red Xs, along with a 95% confidence interval for both. We use the mixed world q-factor model with world market, size, profitability, and investment factors, and excess country market factor. We order countries by region (North America, Developed Europe, Asia Pacific, Latin America, Middle East and Africa, and Emerging Asia) and then alphabetically.

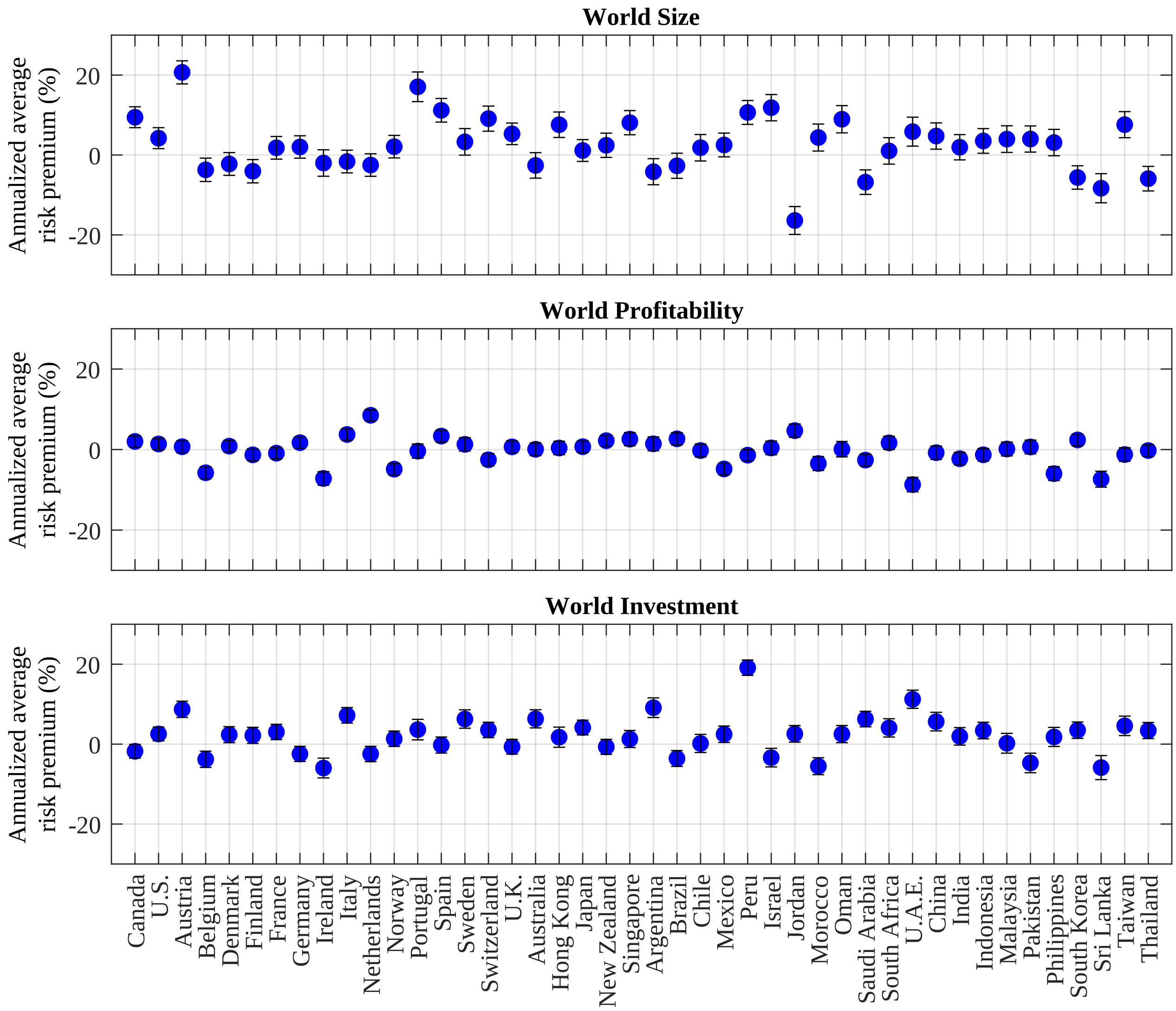

Figure 12 Other world factor average risk premia - Mixed world q-factor model
We report for each country the time-series average world factor risk premia using blue dots along with a 95% confidence interval. We report on factors other than market factors. We use the mixed world q-factor model with world market, size, profitability, and investment factors, and excess country market factor. We order countries by region (North America, Developed Europe, Asia Pacific, Latin America, Middle East and Africa, and Emerging Asia) and then alphabetically.

Table 1 Summary statistics across countries and regions

Country/ Region	Start date (i)	Number of stocks (ii)	Annualized average return (%) Market (iii)	Size (iv)	Value (v)	Mom. (vi)	Prof. (vii)	Inv. (viii)	Annualized volatility (%) Market (ix)	Size (x)	Value (xi)	Mom. (xii)	Prof. (xiii)	Inv. (xiv)
Panel A: Summary statistics for 46 countries														
Argentina	Jul 98	156	8.97	8.62	3.10	10.54	0.67	6.47	33.81	21.66	23.49	27.08	25.17	21.79
Australia	Dec 95	3,237	9.26	−0.88	0.81	20.90	8.43	6.44	20.87	12.67	12.04	15.18	16.60	10.13
Austria	Feb 93	179	7.41	3.39	7.81	6.24	−1.44	6.44	21.29	13.71	17.56	19.41	14.57	14.59
Belgium	Jul 93	297	8.85	2.92	2.98	11.92	−2.56	0.81	18.47	9.49	14.33	18.02	13.11	11.09
Brazil	Feb 02	288	18.26	0.27	6.16	17.61	1.65	0.66	32.27	17.16	18.72	21.46	19.78	16.67
Canada	Jan 85	5,211	7.40	−1.40	−0.95	17.38	5.11	7.13	18.30	11.70	15.88	18.77	12.85	10.44
Chile	Nov 96	264	9.23	4.78	1.51	8.50	−1.47	−0.85	21.33	11.23	11.16	14.71	13.50	12.30
China	Jul 97	3,588	10.20	9.77	12.73	−1.24	0.22	2.21	30.11	22.57	20.53	16.18	9.81	12.38
Denmark	Sep 92	364	10.97	−4.76	−0.94	13.83	1.93	3.82	18.03	12.05	15.85	16.08	12.99	12.67
Finland	Apr 93	257	13.96	−1.63	−0.67	12.83	−3.49	−2.84	28.00	14.14	21.56	20.93	16.72	15.98
France	Jul 91	1,769	7.76	2.46	2.71	10.33	5.17	3.77	18.45	10.39	13.36	16.65	7.64	8.03
Germany	May 91	1,274	6.99	−3.10	3.87	13.34	3.92	4.41	19.52	10.64	14.08	19.15	10.74	11.18
Hong Kong	Oct 00	1,663	10.97	−1.34	6.23	17.53	12.51	3.29	23.67	14.50	14.14	17.98	11.01	10.46
India	Oct 95	4,453	12.55	4.97	−3.88	16.04	12.28	3.48	30.75	21.63	25.06	25.06	19.32	17.03
Indonesia	Jun 94	675	9.51	10.39	12.10	7.38	6.42	1.94	38.84	27.13	33.68	28.32	22.34	20.73
Ireland	Mar 96	112	7.07	3.39	4.85	12.40	6.14	2.92	28.20	23.76	36.44	37.73	21.22	29.25
Israel	Jul 97	712	8.37	5.85	2.31	13.76	6.20	2.29	21.60	13.73	15.06	18.06	14.68	12.85
Italy	Dec 91	616	5.48	2.27	0.50	8.04	5.72	−2.53	23.41	11.01	16.42	18.76	11.95	11.66
Japan	Jan 89	4,746	0.32	3.88	9.51	1.93	1.52	1.34	20.33	11.55	11.57	15.78	6.88	7.20
Jordan	Jul 02	255	8.57	−3.39	2.48	9.83	7.23	0.77	18.57	11.84	18.30	17.18	18.30	18.35
Malaysia	Nov 99	1,317	7.87	−2.00	4.56	9.57	2.81	−2.87	17.96	10.72	9.58	13.21	8.64	7.59
Mexico	Aug 93	226	9.22	0.79	3.61	12.13	−0.20	4.36	27.34	12.59	15.70	19.74	16.52	16.07
Morocco	Aug 06	97	8.24	2.37	−1.90	13.05	3.00	8.09	15.24	12.88	15.62	15.58	14.12	11.90
Netherlands	Jul 91	355	9.02	−0.87	5.21	5.36	0.06	1.53	18.44	10.55	14.06	19.43	10.21	10.36
New Zealand	Sep 03	245	12.18	0.02	1.39	15.53	−4.06	−4.71	19.36	8.28	10.69	10.99	12.58	9.21
Norway	Aug 91	463	9.96	0.20	2.93	12.22	1.44	6.25	24.11	12.55	17.55	21.26	15.38	15.19
Oman	Jan 05	114	5.62	5.46	9.39	3.83	0.83	5.68	17.82	12.90	17.74	17.36	18.31	16.16
Pakistan	Aug 98	557	18.80	8.38	10.17	11.16	0.46	0.32	29.24	17.53	20.61	17.24	20.37	15.50
Peru	Aug 02	157	20.76	14.05	9.08	9.89	−7.97	8.50	24.32	19.15	30.12	32.88	39.09	22.64
Philippines	Jul 98	312	11.55	7.35	19.24	−0.79	2.14	−1.50	25.94	16.62	25.03	31.54	22.68	24.40
Poland	Jul 99	1,077	9.38	−0.98	5.34	16.15	6.53	1.14	29.83	15.62	16.03	17.45	14.12	13.70
Portugal	Feb 98	139	2.82	1.51	−9.07	15.76	−0.15	3.18	22.42	13.83	21.72	21.49	18.60	16.93
Saudi Arabia	Jul 04	194	8.59	2.74	9.18	−3.83	1.91	−3.73	27.17	20.60	17.59	17.44	21.78	12.98
Singapore	Jul 01	914	10.86	−2.61	10.66	11.21	3.47	1.24	21.05	10.66	11.13	17.53	9.76	9.49
South Africa	Apr 97	996	9.71	0.68	4.31	17.36	8.18	5.15	27.11	12.04	17.14	16.93	20.95	11.82
South Korea	Jul 93	2,901	7.70	−1.26	12.48	9.15	7.06	3.88	33.57	19.54	20.25	22.96	14.13	11.15
Spain	Jul 93	375	8.85	1.54	1.79	6.80	0.47	−3.48	22.42	12.01	14.59	18.61	11.37	10.73
Sri Lanka	Oct 98	333	13.34	7.65	10.10	10.63	1.35	1.26	25.75	15.21	16.44	14.32	20.74	13.04

Country/ Region	Start date (i)	Number of stocks (ii)	Annualized average return (%) Market (iii)	Size (iv)	Value (v)	Mom. (vi)	Prof. (vii)	Inv. (viii)	Annualized volatility (%) Market (ix)	Size (x)	Value (xi)	Mom. (xii)	Prof. (xiii)	Inv. (xiv)
... continued														
Sweden	Jul 97	1,045	9.57	0.11	2.07	13.38	2.48	5.57	24.51	11.77	16.52	21.60	13.25	13.27
Switzerland	Jul 91	427	8.98	1.43	−0.74	9.33	4.66	5.07	16.48	10.83	12.95	16.33	11.14	10.67
Taiwan	Jul 98	2,239	5.78	0.84	5.78	3.90	−0.68	−2.10	25.64	16.54	19.19	17.60	11.62	12.72
Thailand	Jul 93	989	8.12	2.98	9.32	5.84	10.70	2.85	32.73	21.72	25.24	29.50	15.19	16.58
Turkey	Jul 98	514	14.63	5.04	12.73	−0.33	−6.41	−3.09	47.03	17.59	17.77	18.62	18.09	12.94
U.A.E.	Apr 07	114	7.94	−1.05	12.90	7.41	3.26	3.49	24.06	14.71	19.44	21.91	23.61	18.89
U.K.	Oct 88	4,509	5.95	1.06	0.34	12.07	6.25	1.24	16.43	10.88	10.30	15.48	8.45	7.51
U.S.	Jan 85	11,595	9.07	5.18	3.62	5.32	6.04	5.20	14.94	11.51	12.74	16.32	5.60	7.42
Panel B: Summary statistics across 46 countries														
Minimum across countries	Jan 85	97	0.32	−4.76	−9.07	−3.83	−7.97	−4.71	14.94	8.28	9.58	10.99	5.60	7.20
Average across countries	Jul 96	1,354	9.49	2.33	4.95	10.03	2.82	2.27	24.28	14.60	17.72	19.69	15.55	13.77
Maximum across countries	Apr 07	11,595	20.76	14.05	19.24	20.90	12.51	8.50	47.03	27.13	36.44	37.73	39.09	29.25
Panel C: Summary statistics for regions														
World	Jan 85	62,320	7.39	3.36	4.64	6.69	5.34	3.53	15.23	7.69	9.23	12.53	4.19	5.09
Developed Markets	Jan 85	39,792	7.35	3.31	4.38	6.54	5.33	3.65	15.15	8.11	9.82	13.31	4.31	5.32
Emerging Markets	Jul 93	22,528	5.76	2.71	7.02	7.65	4.59	1.81	21.08	7.83	8.82	9.38	7.08	5.48
North America	Jan 85	16,806	8.91	4.71	3.34	6.11	6.01	5.29	14.96	11.15	12.47	16.07	5.44	7.24
Developed Europe	Oct 88	12,181	6.87	0.26	1.45	10.74	4.34	2.27	17.02	7.40	9.36	13.56	5.64	6.04
Asia Pacific	Jan 89	10,805	1.34	2.49	8.53	4.83	2.93	1.67	19.75	10.21	10.15	14.14	6.56	6.56
Latin America	Aug 93	1,091	8.69	3.81	3.74	11.59	−0.90	4.02	23.64	9.53	10.83	13.75	10.02	12.04
Middle East and Africa	Apr 97	2,482	6.71	3.37	6.84	11.57	4.80	2.21	20.04	10.24	13.37	12.36	16.87	9.13
Emerging Asia	Jul 93	17,364	5.70	1.87	7.15	6.92	5.84	1.99	23.22	10.03	10.96	11.29	8.59	6.75

We report the start date, total number of stocks, and annualized average returns and volatilities for risk factors across countries and regions. The end date is February 2018. We construct a market, size, value, momentum, profitability, and investment long-short factor for each country, and form regional factors by value-weighting country factors using the lagged total market capitalization in U.S. dollars. Panel A reports on 46 countries. We present in Panel B the minima, averages, and maxima across the 46 countries. In Panel C, we report the summary statistics for each region. All returns are monthly and are in U.S. dollars. The market factor is in excess of the U.S. one-month T-bill rate.

Table 2 Which models capture the factor structure in equity markets?

Model	Country	World (i)	Regional (ii)	World + Country Market (iii)	Regional + Country Market (iv)
Panel A: $Z_{c,t-1} = (1, DY_{c,t-1})'$					
CAPM	All Countries	8.70	28.26	71.74	76.09
	Developed Markets	9.09	40.91	77.27	86.36
	Emerging Markets	8.33	16.67	66.67	66.67
Four-factor model	All Countries	15.22	39.13	86.96	95.65
	Developed Markets	22.73	63.64	95.45	100.00
	Emerging Markets	8.33	16.67	79.17	91.67
Five-factor model	All Countries	28.26	54.35	89.13	95.65
	Developed Markets	40.91	77.27	95.45	100.00
	Emerging Markets	16.67	33.33	83.33	91.67
q-model	All Countries	26.09	50.00	86.96	95.65
	Developed Markets	31.82	72.73	95.45	100.00
	Emerging Markets	20.83	29.17	79.17	91.67
Panel B: $Z_{c,t-1} = 1$					
CAPM	All Countries	8.70	28.26	65.22	71.74
	Developed Markets	9.09	40.91	63.64	77.27
	Emerging Markets	8.33	16.67	66.67	66.67
Four-factor model	All Countries	15.22	39.13	84.78	95.65
	Developed Markets	22.73	63.64	95.45	100.00
	Emerging Markets	8.33	16.67	75.00	91.67
Five-factor model	All Countries	26.09	50.00	89.13	95.65
	Developed Markets	36.36	72.73	95.45	100.00
	Emerging Markets	16.67	29.17	83.33	91.67
q-model	All Countries	23.91	47.83	86.96	95.65
	Developed Markets	31.82	72.73	95.45	100.00
	Emerging Markets	16.67	25.00	79.17	91.67

We report the proportion in % of countries for which the GOS2 diagnostic criterion is negative. The diagnostic criterion, ζ in Equation (9), checks for a remaining factor structure in the time-series of residuals. A positive value for the diagnostic criterion indicates that there remains at least one factor in the residuals obtained from the first step time-series regressions. A negative value says that the factors used in the asset pricing model capture the factor structure in stock returns. Panel A reports on models with common instruments $Z_{c,t-1} = (1, DY_{c,t-1})'$, where $DY_{c,t-1}$ is the country dividend yield, and Panel B contains proportions for unconditional models with $Z_{c,t-1} = 1$. In each Panel, we present the proportions for the CAPM model with market factors, the four-factor model with market, size, value, and momentum factors, the five-factor model with market, size, value, profitability, and investment factors, and the q-model with market, size, profitability, and investment factors. For each model, we report the proportion of negative criteria across all 46 countries, across 22 developed markets, and across 24 emerging markets. We use a model-specific set of stock-specific instruments. We use the cross-sectional ranks of each factor-specific characteristic. For the CAPM models, we use the cross-sectional ranks of the size and value characteristics. Columns (i) and (ii) report, respectively, on models with factors aggregated at the world level and aggregated at the regional level. Columns (iii) and (iv) display world and regional models augmented with the country market factor (in excess of the world and regional factor, respectively).

Table 3 Can the factor models price single stocks?

Region	*Mixed World Model* $H_0 : a_{c,t}(\gamma) = b_{c,t}(\gamma)'\nu_{c,t}$ (i)	$H_0 : a_{c,t}(\gamma) = 0$ (ii)	*Mixed Regional Model* $H_0 : a_{c,t}(\gamma) = b_{c,t}(\gamma)'\nu_{c,t}$ (iii)	$H_0 : a_{c,t}(\gamma) = 0$ (iv)
Panel A: CAPM				
All countries	71.74%	71.74%	73.91%	71.74%
Developed Markets	77.27%	77.27%	81.82%	77.27%
Emerging Markets	66.67%	66.67%	66.67%	66.67%
North America	50.00%	50.00%	50.00%	50.00%
Developed Europe	80.00%	80.00%	93.33%	86.67%
Asia Pacific	80.00%	80.00%	60.00%	60.00%
Latin America	100.00%	100.00%	100.00%	100.00%
Middle East and Africa	71.43%	71.43%	71.43%	71.43%
Emerging Asia	40.00%	40.00%	40.00%	40.00%
Panel B: Four-factor model				
All countries	67.39%	78.26%	86.96%	91.30%
Developed Markets	77.27%	86.36%	95.45%	90.91%
Emerging Markets	58.33%	70.83%	79.17%	91.67%
North America	100.00%	100.00%	50.00%	50.00%
Developed Europe	80.00%	86.67%	100.00%	93.33%
Asia Pacific	60.00%	80.00%	100.00%	100.00%
Latin America	80.00%	100.00%	100.00%	100.00%
Middle East and Africa	57.14%	85.71%	85.71%	100.00%
Emerging Asia	40.00%	40.00%	60.00%	80.00%
Panel C: Five-factor model				
All countries	52.17%	71.74%	86.96%	86.96%
Developed Markets	63.64%	72.73%	95.45%	86.36%
Emerging Markets	41.67%	70.83%	79.17%	87.50%
North America	0.00%	0.00%	50.00%	50.00%
Developed Europe	80.00%	93.33%	100.00%	93.33%
Asia Pacific	40.00%	40.00%	100.00%	80.00%
Latin America	80.00%	100.00%	100.00%	100.00%
Middle East and Africa	57.14%	85.71%	85.71%	100.00%
Emerging Asia	20.00%	40.00%	60.00%	70.00%

Region	*Mixed World Model* $H_0 : a_{c,t}(\gamma) = b_{c,t}(\gamma)'\nu_{c,t}$ (i)	$H_0 : a_{c,t}(\gamma) = 0$ (ii)	*Mixed Regional Model* $H_0 : a_{c,t}(\gamma) = b_{c,t}(\gamma)'\nu_{c,t}$ (iii)	$H_0 : a_{c,t}(\gamma) = 0$ (iv)
... continued				
Panel D: q-factor model				
All countries	84.78%	86.96%	86.96%	73.91%
Developed Markets	95.45%	95.45%	95.45%	95.45%
Emerging Markets	75.00%	79.17%	79.17%	54.17%
North America	100.00%	100.00%	50.00%	50.00%
Developed Europe	100.00%	100.00%	100.00%	100.00%
Asia Pacific	80.00%	80.00%	100.00%	100.00%
Latin America	100.00%	100.00%	100.00%	60.00%
Middle East and Africa	71.43%	85.71%	85.71%	42.86%
Emerging Asia	60.00%	60.00%	60.00%	60.00%

We report for different models and different regions the proportion of countries for which the model is not rejected. A model is not rejected when the diagnostic criterion is negative and the p-value for the asset pricing restrictions is above the significance level. We use a significance level of 5% using a Bonferroni correction (i.e., $5\%/46$). In columns (i) and (iii), we report on the test for the asset pricing restrictions, $a_{c,t}(\gamma) = b_{c,t}(\gamma)'\nu_{c,t}$. In columns (ii) and (iv), we report on the test for the asset pricing restrictions with traded factors, $a_{c,t}(\gamma) = 0$. We report on mixed world models in columns (i) and (ii) and on mixed regional models in columns (iii) and (iv). Panel A contains results for the mixed CAPM models with a world (regional) market factor and a country excess market factor. Panel B contains results for the mixed four-factor models with world (regional) market, size, value, and momentum factors and a country excess market factor. Panel C contains results for the mixed five-factor models with world (regional) market, size, value, profitability, and investment factors and a country excess market factor. Panel D contains results for the mixed q-factor models with world (regional) market, size, profitability, and investment factors and a country excess market factor.

Appendix 1 Estimation methodology in the time-varying case

We detail in this section the estimation methodology in the time-varying case with common instruments $Z_{c,t}$ for stocks of country c and stock-specific instruments $Z_{i,c,t}$.

1. **Time-series regressions:** Our specification choices for factor exposures and factor risk premia (Equations (5), (6) and (7) combined with the asset pricing restrictions, $a_{c,t}(\gamma) = b_{c,t}(\gamma)\nu_{c,t}$, imply that a stock intercept is,

$$a_{i,c,t} = Z'_{c,t-1}B'_{i,c}\left(\Lambda_c - F_c\right)Z_{c,t-1} + Z'_{i,c,t-1}C'_{i,c}\left(\Lambda_c - F_c\right)Z_{c,t-1},$$

   using the simplifying notations $a_{c,t}(\gamma_i) = a_{i,c,t}$ and $b_{c,t}(\gamma_i) = b_{i,c,t}$. To handle the time-varying case, we define the $d_1 = p(p+1)/2 + pq$ vector of predetermined variables

$$x_{i,c,t,1} = \left(\text{vech}(X_t)', Z'_{c,t-1} \otimes Z'_{i,c,t-1}\right)', \tag{14}$$

   and the $d_2 = K(p+q)$ vector of factors scaled by $Z_{c,t-1}$ (scaled factors) and by $Z_{i,c,t-1}$

$$x_{i,c,t,2} = \left(f'_t \otimes Z'_{c,t-1}, f'_t \otimes Z'_{i,c,t-1}\right)', \tag{15}$$

   using the simplifying notation $Z_{c,t}(\gamma_i) = Z_{i,c,t}$, and where the matrix X_t has typical diagonal elements $X_{k,k,t} = Z^2_{t-1,k}$ and off-diagonal elements $X_{k,l,t} = 2Z_{t-1,k}Z_{t-1,l}$. Then, we can use the compact notation with the $d = d_1 + d_2$ vector $x_{i,c,t} = \left(x'_{i,c,t,1}, x'_{i,c,t,2}\right)'$,

$$r_{i,c,t} = \beta'_{i,c}x_{i,c,t} + \varepsilon_{i,c,t}, \tag{16}$$

   with $\beta_{i,c} = \left(\beta'_{i,c,1}, \beta'_{i,c,2}\right)'$.

2. **Cross-sectional regressions:** We can estimate the second-step cross-sectional regressions as

$$\beta_{i,c,1} = \beta_{i,c,3}\nu_c, \tag{17}$$

where

$$
\begin{aligned}
\nu_c &= \text{vec}\left(\Lambda_c' - F_c'\right), \\
\text{vec}(\beta_{i,c,3}) &= J_a \beta_{i,c,2}, \\
J_a &= \begin{pmatrix} J_1 & 0 \\ 0 & J_2 \end{pmatrix}, \\
J_1 &= W_{p(p+1)/2,pK}\left(I_K \otimes [(I_p \otimes N_p)(W_p \otimes I_p)(I_p \otimes \text{vec}(I_p))]\right), \\
J_2 &= W_{pq,pK}\left(I_K \otimes [(I_p \otimes W_{p,q})(W_{p,q} \otimes I_p)(I_q \otimes \text{vec}(I_p))]\right), \\
N_p &= \frac{1}{2} D_p^+ \left(W_{p,p} + I_{p^2}\right),
\end{aligned}
$$

where $W_{p,q}$ is the commutation matrix such that $\text{vec}(A') = W_{p,q}\text{vec}(A)$ for a p-by-q matrix A, I_p is the identity matrix of size p, and D_p^+ is the $p(p+1)/2$-by-p^2 matrix such that $\text{vech}(A) = D_p^+\text{vec}(A)$.

3. **Estimation of the risk premium bias:** The bias term for the estimate $\hat{\nu}_c$ of the risk premia is estimated as

$$
\hat{B}_{\nu_c} = \hat{Q}_{\beta_3}^{-1} J_b \frac{1}{n_c} \sum_{i=1} \tau_{i,c} \text{vec}\left(E_2' \hat{Q}_{x,i,c}^{-1} \hat{S}_{ii,c} \hat{Q}_{x,i,c}^{-1} C_{\hat{\nu}_c} \hat{w}_{i,c}\right), \tag{18}
$$

with

$$
\begin{aligned}
\hat{Q}_{\beta_3} &= \frac{1}{n_c} \sum_i \hat{\beta}_{i,c,3}' \hat{w}_{i,c} \hat{\beta}_{i,c,3}, \\
J_b &= (\text{vec}(I_{d_1})' \otimes I_{Kp})(I_{d_1} \otimes J_a), \\
C_{\hat{\nu}_c} &= (E_1' - (I_{d_1} \otimes \hat{\nu}_c') J_a E_2')', \\
E_1 &= (I_{d_1}, \mathbf{0}_{d_1,d_2})', \\
E_2 &= (\mathbf{0}_{d_2,d_1}, I_{d_2})'.
\end{aligned}
$$

where $\mathbf{0}_{d_1,d_2}$ is a d_1-by-d_2 matrix of zeros.

4. **Estimation of the risk premium covariance matrix:** The covariance matrix for the risk premia estimate $\hat{\nu}_c$ is estimated as

$$
\hat{\Sigma}_{\nu_c} = \left(\text{vec}(C_{\hat{\nu}_c}')' \otimes \hat{Q}_{\beta_3}^{-1}\right) \hat{S}_{\upsilon 3} \left(\text{vec}(C_{\hat{\nu}_c}')' \otimes \hat{Q}_{\beta_3}^{-1}\right)
$$

where

$$\hat{S}_{\upsilon_3} = \frac{1}{n_c}\sum_{i,j}\frac{\tau_i\tau_j}{\tau_{i,j}}\left(\hat{S}_{Q,ij}\otimes\upsilon_{3,i}\upsilon'_{3,j}\right),$$

$$\tau_{i,j,c} = \frac{T_c}{T_{ij,c}},$$

$$T_{ij,c} = \sum_t I_{i,c,t}I_{j,c,t},$$

$$\hat{S}_{Q,ij} = \hat{Q}^{-1}_{x,i,c}\hat{S}_{ij,c}\hat{Q}_{x,i,c},$$

$$\hat{S}_{ij,c} = \frac{1}{T_{ij,c}}\sum_t I_{i,c,t}I_{j,c,t}\hat{\varepsilon}_{i,c,t}\hat{\varepsilon}_{i,c,t}x_{i,c,t}x'_{j,c,t},$$

$$\upsilon_{3,i} = \text{vec}(\beta_{i,c,3}w_i).$$

The estimation of this covariance matrix is complicated since $\hat{S}_{\upsilon_3}$ involves a sum on i and j but is standardized only by n_c (and not n_c^2. Hence, the usual sample estimator is not consistent. We employ a hard thresholding technique to set the smallest elements of $\hat{S}_{\upsilon_3}$ to zero and therefore obtain a consistent estimator. We use the threshold proposed in Bickel and Levina (2008) extended by GOS to a random coefficient setting,

$$\tilde{S}_{ij,c} = \hat{S}_{ij,c}\mathbf{1}_{||\hat{S}_{ij,c}||\leq\kappa_{n_c,T_c}},$$

where $||\hat{S}_{ij,c}||$ is the Frobenius norm, $\kappa_{n_c,T_c} = M\sqrt{\frac{log(n_c)}{T_c}}$ is a data-dependent threshold, and M is a positive number set by cross-validation (see GOS for details).

5. **Estimation of the bias term and covariance matrix for asset pricing tests:**

   The test for asset pricing restrictions is based on the weighted sum of squared residuals $\hat{Q}_e = \frac{1}{n_c}\sum_i \hat{e}'_{i,c}\hat{w}_{i,c}\hat{e}_{i,c}$, where $\hat{e}_{i,c} = \hat{\beta}_{i,c,1} - \hat{\beta}'_{i,c,3}\hat{\nu}_c$. The distribution of the re-centered sum of squared residuals is

$$\tilde{\Sigma}_e^{-1/2}T_c\sqrt{n_c}\left(\hat{Q}_e - \frac{d_1}{T_c}\right) \sim N(0,1).$$

   where

$$\tilde{\Sigma}_e = \frac{2}{n_c}\sum_{i,j}\frac{\tau^2_{i,c}\tau^2_{j,c}}{\tau^2_{i,j,c}}Tr\left[\left(C'_{\hat{\nu}_c}\hat{Q}^{-1}_{x,i}\tilde{S}_{ij}\hat{Q}^{-1}_{x,j}C_{\hat{\nu}_c}\right)\hat{w}_{j,c}\left(C'_{\hat{\nu}_c}\hat{Q}^{-1}_{x,j}\tilde{S}_{ji}\hat{Q}^{-1}_{x,i}C_{\hat{\nu}_c}\right)\hat{w}_{i,c}\right].$$

6. **Distribution of the risk premium dynamic parameters Λ_c**

The parameters for the dynamics of the risk premia, Λ_c, follow a normal distribution

$$\sqrt{T_c} vec[\hat{\Lambda}_c' - \Lambda_c'] \sim N\left(0, \Sigma_{\Lambda_c}\right)$$

where

$$\begin{aligned} \Sigma_{\Lambda_c} &= \left(\mathbb{I}_K \otimes Q_z^{-1}\right) \Sigma_u \left(\mathbb{I}_K \otimes Q_z^{-1}\right), \\ \Sigma_u &= E\left[u_t u_t' \otimes Z_{c,t-1} Z_{c,t-1}'\right], \\ u_t &= f_{c,t} - F_c Z_{c,t-1}, \\ Q_z &= E\left[Z_{c,t-1} Z_{c,t-1}'\right]. \end{aligned}$$

Appendix 2 Distribution of the average conditional risk premium

In this section, we derive the distribution of the average conditional risk premium, $\hat{\bar{\Lambda}}_{c,k} = \hat{\Lambda}_{c,k} \bar{Z}_c$ where $\bar{Z}_c$ is the average of $Z_{c,t}$, $\bar{Z}_c = \frac{1}{T} \sum_{t=2}^{T} Z_{c,t-1}$. The distribution of $\bar{Z}_c$ is

$$\sqrt{T}(\bar{Z}_c - E[Z_{c,t}]) \sim N\left(0, Q_z - E[Z_{c,t}]E[Z_{c,t}]'\right)$$

where $Q_z = E[Z_{c,t} Z_{c,t}']$. Using Proposition 4 in GOS, we also have

$$\sqrt{T}\left(\hat{\Lambda}_{c,k} - \Lambda_{c,k}\right) \sim N\left(0, Q_z^{-1} \Sigma_{u_k} Q_z^{-1}\right)$$

where $\Sigma_u = E[u_{k,t}^2 Z_{c,t-1} Z_{c,t-1}']$, $u_{k,t} = f_{k,t} - F_{c,k} Z_{c,t-1}$, and $\Lambda_{c,k}$ and $F_{c,k}$ are the k^{th} row of matrix Λ_c and F_c respectively.

Using the Delta method, we obtain the distribution of the average risk premium of factor k, $\hat{\bar{\Lambda}}_{c,k} = \hat{\Lambda}_{c,k} \bar{Z}_c$, as

$$\sqrt{T}\left(\hat{\bar{\Lambda}}_{c,k} - E[\bar{\Lambda}_{c,k}]\right) \sim N\left(0, \Sigma_{\bar{\Lambda}_{c,k}}\right)$$

where

$$\Sigma_{\bar{\Lambda}_{c,k}} = \bar{Z}_c' Q_z^{-1} \Sigma_{u_k} Q_z^{-1} \bar{Z}_c + \hat{\Lambda}_{c,k}\left(Q_z - E[Z_{c,t}]E[Z_{c,t}]'\right)\hat{\Lambda}_{c,k}'. \tag{19}$$

In our empirical implementation, $Z_{c,t-1} = (1, DY_{t-1}, DY_{c,t-1})'$. In this case, the variance in Equation (19) simplifies to

$$\Sigma_{\bar{\Lambda}_{c,k}} = \Sigma_{\Lambda_{c,k,0}} + \hat{\Lambda}_{c,k,DY_{t-1}}^2 + \hat{\Lambda}_{c,k,DY_{c,t-1}}^2 + 2\hat{\Lambda}_{c,k,DY_{t-1}} \hat{\Lambda}_{c,k,DY_{c,t-1}} Q_{z,DY_{t-1},DY_{c,t-1}}$$

where $\Sigma_{\Lambda_{c,k,0}}$ is the first element of $Q_z^{-1}\Sigma_{u_k}Q_z^{-1}$ and $Q_{z,DY_{t-1},DY_{c,t-1}}$ is the covariance between DY_{t-1} and $DY_{c,t-1}$.

Appendix 3 Equity data construction

A.3.1 Methodology

Our objective is to build a database of common stocks traded on major stock exchanges. We examine the pros and cons of using Datastream versus Compustat Global/xpressfeed. Given the longer time series found on CRSP for US stocks, we focus on non-US countries.

Our main conclusions are as follows. Datastream has longer time series for some but not all stocks. However, it contains many errors. Compustat has less data errors, the history of SEDOLs and ISINs, and the type of daily quote which to our knowledge is not available on Datastream (only the current identifiers are available).

The following steps describe how we construct the data for each country. By visual inspection of value- and equal-weighted indexes, we investigate each discrepancy. In some cases, we can confirm a mistake in Datastream (Compustat) by using data from Compustat (Datastream). For example, a spike in the total return index on Datastream is identified and removed by looking at the total return index on Compustat. In other cases, we can not conclude which of the two databases has an error and further check on Bloomberg and/or MSCI.

Given the advantages listed above, we use data from Compustat/xpressfeed in this paper. We describe the filters and error corrections we use for each of the two databases in the following steps. Therefore, this guide can be used for research based on Datastream or Compustat data.

1. **Stock Universe:**

    - Datastream: We retrieve all securities which are classified as equity (*instrument_type = 'Equity'*).

    - Compustat: We retrieve all securities which are classified as common or ordinary shares (*tpci = '0'*).

2. **Major Stock Exchanges:** We keep only stocks listed on a country major stock exchange. We define the major stock exchange as the one with the highest number of listed stocks. In most cases, the choice is obvious. However, we include more than one stock exchanges in a few countries. We provide a Table in the Online Appendix that lists all the stock exchanges for each country and their correspondance between Datastream and Compustat.

3. **Refining the common stock universe:** Securities are misclassified in both databases. We apply the following additional filter on the security name:

   - Datastream: We apply the name and industry filters as in Griffin et al. (2010). We add "BDR" to the list of keywords to remove Brazilean Depositary Receipts. We also use additional keyword filters used by Lee (2011): "AFV" in Belgium due to their preferential tax treatment, "INC.FD." in Canada because they are income trusts, and "RSP" in Italy due to their nonvoting provisions.
   - Compustat: We remove non-common stocks based on the presence of the same keywords in their issue description (*dsci*).

4. **Preliminary cleaning of times series:**

   - Compustat: We use only days for which a price (*prccd*) is available with a price code status (*prcstd*) either equal to 3 (high, low and close prices) or 10 (prices as reported). We also include price code status 4 (bid, ask, average/last volume close) for Canadian issues because Compustat historically delivered prices as the average of the bid/ask pricing for U.S. and Canadian issues.
   - Datastream: We use only days for which the unadjusted price (*UP*) is available. Datastream does not provide any indication as to the type of quote it provides. In many cases, total return indexes (*RI*) continue after the price stops quoting. Datastream repeats the last price after a stock stops. For each stock, we verify each day if the rest of the time series is the same price and remove the rest of the time series in such case. This procedure does not capture cases in which a stock stops quoting for a few months and then starts again. In this case, we get a series of zero returns.

     At this stage, indexes built from Datastream have longer time series for many countries compared to Compustat indexes. This is especially the case for some developed countries whose indexes start in the early 1970s whereas all non-North American data on Compustat starts in the early 1980s. However, many unexplained spikes in Datastream time series come from days for which only the price is available. We can match several of these cases to Compustat data and confirm that they correspond to a price standard (*prcstd*) equal to 5 (no price is available, the last price is carried forward). Unfortunately, we cannot match these cases with Compustat data in the pre-1980s period. Therefore, we keep only quotes for which either the volume, low, or high is available as a sign of real market activity. This filter solves many of the initial discrepancies between the two data providers.

5. **Controlling for spikes that are reversed:**

   - Datastream: Following Ince and Porter (2006), we control for extreme daily returns that are reversed the following day. If the total return over two consecutive days is below 50% and any of the two daily total return is above 100%, we remove both daily observations.
   - Compustat: None.

6. **Computing monthly returns:** We build monthly returns by using the last available total return index value during the previous month and the last available value in the current month.

   - Datastream: We use the total return index (*RI*). We convert the local total return index to U.S. dollars and keep nine decimals such that monthly returns are not impacted by rounding (using the function *DPL#(X(RI) U$,9)*).
   - Compustat: We build total return indexes using prices (*prccd*), adjustment factors (*ajexdi*), quotation units (*qunit*), exchange rates (*exratd*), and total return factors (*trfd*). We follow Shumway (1997) and apply a -30% delisting return when delisting is performance related (using the delisting reason *dlrsni*).

7. **Computing market capitalizations:** We build monthly lagged market capitalizations by using the last available market capitalization during the previous month.

   - Datastream: We use the market value (*MV*) converted to U.S. dollars.

- Compustat: We build market capitalization by multiplying the number of shares by prices (*prccd*). For non-North American stocks, we use the current number of shares outstanding (*cshoc*). For North-American stocks, we use the last report number of shares outstanding (*cshoi*).

8. **Manual data corrections:** We investigate and identify in Table 4 for Compustat and in a table available upon request for Datastream errors not captured by the filters above.

   In unreported figures available upon request, we plot for each country the returns of the value-weighted and equal-weighted market portfolios as well as the number of stocks over time using both databases.

A.3.2 Corrections for Compustat

gvkey/iid	Error
202192/01W, 203051/01W, 207206/01W, 208514/01W	In January 1992 in Argentina, there are four stocks for which the transition from the old currency code ARA to ARS creates 10,000+ returns. We remove them for this month.
203579/01W, 205247/01W	Before January 1992 in Argentina, these two stocks' USD market capitalization are off by a factor 10. We multiply the market capitalization by 0.1.
029178/01W	This Argentinean stock's market cap is too large and erratic, and there are some holes. Its data on Datastream starts on January 1992. We start in October 1990 after the last hole when the market capitalization is not erratic.
208536/01W	The adjustment factor *ajexdi* does not adjust for the 0.0513-to-1 stock split on May 20^{th}, 2015. We remove the stock for this month.
030581/01W	Before February 1992, this stock in Brazil has extreme market values.
All stocks in Brazil	In January 1989, the 1-to-1,000 change from the Cruzado to the Cruzado novo is not reflected in Compustat's exchange rate table (nor is the one in 1986). We divide returns by 1,000.
206477/01W	There is an error in the adjustment factor (*ajexdi*) from 01/09/2007 to 20/3/2007, it should be 1 instead of 10, verified on Bloomberg.
208194/02W 203187/01W 229956/02W 208200/01W 203462/01W 203682/01W 208603/01W 208366/01W 209409/01W	Spike for these Chinese stocks in March and June 1993. Spike for 203187/01W in June 1993 is confirmed with Bloomberg (but return of 700% happens in July). Datastream show missing infrequent returns for these months. We check all large returns on June 1993 with Bloomberg and we can confirm all but one. We multiply the return in March by 10 and divide by 10 in June.
213573/01W	In February 2002 in Estonia, we replace the 25^{th} return with the 21^{st}, Datastream ends on the 21^{st}. We set R = 0.0111301630700127 / 0.0645498918825071 - 1.
103255/01W, 210759/01W, 240641/01W	There are errors caused by the change of currency to the Euro for these three European stocks. We remove them for January 1999.
All stocks in Iceland	For Iceland, the currency plummets on Oct 8^{th}, 2008 and doubles on February 2^{nd}, 2009. We cannot find this plunge on Bloomberg nor on Yahoo. We use Datastream exchange rates, namely, FX rate 0.009452, 0.008440, 0.006994, 0.008246, 0.008773, and 0.008778 for the month of September 2008 through February 2009.
200503/01W	Spike in price creates a return of 15. This Peruvian stock is not on Datastream and it starts in 1996 on Bloomberg. We remove it for December 1992.

All Peruvian stocks	In January 1992, the 1,000,000-to-1 change described below (from Wikipedia) is not reflected on CSXF. "Because of the bad state of economy and hyperinflation in the late 1980s the government was forced to abandon the inti and introduce the sol as the country's new currency. The currency was put into use on July 1, 1991 (by Law No. 25,295) to replace the inti at a rate of 1 sol to 1,000,000 intis. Coins denominated in the new unit were introduced on October 1, 1991 and the first banknotes on November 13, 1991. Hitherto, the sol has retained a low inflation rate of 1.5%, the lowest inflation rate ever in both Latin and South America. Since the new currency was put into effect, it has managed to maintain a stable exchange rate between 2.2 and 3.66 per United States dollar." We divide returns by 1,000,000.
201673/01W	In July 1998, this New Zealand stock has the same price as on Datastream, but its adjustment factor (*ajexdi*) and total return factor (*trfd*) create a huge difference compared to Datastream. We remove it for this month.
206463/03W	Moscow City Telephone Network Co has random 1000x spikes in the price time series, it would take too many corrections to solve the problem. We remove the complete time series.
284439/01W	In January 2005, there is an error in the adjustment factor (*ajexdi*) when the currency changed. Other stocks' prices (*prccd*) and *ajexdi* adjust. This stock *prccd* adjusts, but not its *ajexdi*. We remove it for this month.
217719/01W	In February and March 1994, there is an error for this Colombian stock (verified with Datastream) and remove it for those two months.
185208/01C	This Canadian stock is delisted on January 1^{st} 2017, there is a spike in the price on December 30^{th}, 2016, and the time series ends on December 2^{nd}, 2016, on Bloomberg. We remove it for December 2016. CSXF is also missing the total return adjustment for the 100-to-1 conversion on November 1^{st}, 2013, which creates a 100+ % return. We remove it for November 2013.
202022/01W	This Chilean stock has erratic and infrequent quotes before January 2004. There are price spikes on days with unavailable volumes, but classified as "prices as reported" (*prcstd*=10). There are no quotes on these days on Bloomberg. We remove infrequent returns before January 2004.
149822/01C	The number of shares outstanding (*cshoc*) is off by a factor 100 for the last two days of June 2004. We then correct the number of shares.

Table 4 We report in this table the manual data corrections to data on Compustat/xpressfeed.