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The Cross-Section of Risk and Return.

Kent Daniel[†‡], Lira Mota[†], Simon Rottke[§†], and Tano Santos[†‡]

- Abstract -

In the finance literature, a common practice is to create *factor-portfolios* by sorting on characteristics (such as book-to-market, past return or profitability) associated with average returns. The goal of this exercise is to create a parsimonious set of factor-portfolios that explain the cross-section of average returns, in the sense that the returns of these factor-portfolios span the mean-variance efficient portfolio. We argue that this is unlikely to be the case, as factor portfolios constructed in this way fail to incorporate information about the covariance structure of returns. By using a high statistical power methodology to forecast future covariances, we are able to construct a set of portfolios which captures the characteristic premia, but hedges out much of the factor risk. We apply our methodology to hedge out unpriced risk in the Fama and French (2015) five-factors. We find that the squared Sharpe ratio of the optimal combination of the resulting hedged-factor portfolios is 2.29, compared with 1.31 for the unhedged portfolios, and is highly statistically significant.

[†]Columbia Business School, [‡]NBER, and [§]University of Münster. We thank Lars Lochstoer, Ravi Jagannathan, Paul Tetlock, and Brian Weller and the participants of seminars at Columbia and Kellogg/Northwestern, and conferences at Fordham, and HKUST for helpful comments and suggestions.

1 Introduction

A common practice in the academic finance literature has been to create *factor-portfolios* by sorting on characteristics positively associated with expected returns. The resulting set of zero-investment factor-portfolios, which go long a portfolio of high-characteristic firms and short a portfolio of low-characteristic firms, then serve as a model for returns in that asset space. Prominent examples of this are the three- and five-factor models of Fama and French (1993, 2015), but there are numerous others, developed both to explain the equity market anomalies, and also the cross-section of returns in other asset classes.[1]

Consistent with this, Fama and French (2015, FF) argue that a standard dividend-discount model implies that a combination of individual-firm metrics based on valuation, profitability and investment should forecast these firms' average returns. Based on this they develop a five factor model—consisting of the Mkt-Rf, SMB, HML, RMW, and CMA factor-portfolios—and argue that this model does a fairly good job explaining the cross-section of average returns for a variety of test portfolios, based on a set of time-series regressions like:

$$\begin{aligned} R_{p,t}-R_{f,t} &= \alpha_p + \beta_m \cdot (R_{m,t}-R_{f,t}) + \beta_{HML} \cdot HML_t + \beta_{SMB} \cdot SMB_t \\ &\quad +\beta_{CMA} \cdot CMA_t + \beta_{RMW} \cdot RMW_t + \epsilon_{p,t} \end{aligned}$$

where a set of portfolios is chosen for which the excess returns, $R_{p,t}-R_{f,t}$, exhibit a considerable average spread.[2]

Standard projection theory shows that the αs from such regressions will all be zero

[1] Examples are: UMD (Carhart, 1997); LIQ (Pastor and Stambaugh, 2003); BAB (Frazzini and Pedersen, 2014); QMJ (Asness, Frazzini, and Pedersen, 2013); and RX and HML-FX (Lustig, Roussanov, and Verdelhan, 2011). We concentrate on the factors of Fama and French (2015). However, the critique we develop in Section 2 applies to any factors constructed using this method.

[2] The Fama and French (2015) test portfolios SMB, HML, RMW, and CMA are formed by sorting on various combinations of firm size, valuation ratios, profitability and investment respectively.

for all assets if and only if the mean-variance efficient (MVE) portfolio is in the span of the factor portfolios, or equivalently if the maximum Sharpe ratio in the economy is the maximum Sharpe-ratio achievable with the factor portfolios alone. For the case of the five factor-portfolios examined by Fama and French (2015), the *ex-post* optimal combination of these five-factors has an annualized Sharpe ratio of 1.14 over 1963/07 - 2017/06 time period. Despite several critiques of this methodology it remains popular in the finance literature.[3]

The objective of this paper is to refine our understanding of the relationship between firm characteristics and the risk and average returns of individual firms. Our argument is that, if characteristics are a good proxy for expected returns, then forming factor portfolios by sorting on characteristics will generally *not* explain the cross-section of returns in the way proposed in the papers in this literature.

The argument is straightforward, and based on the early insights of Markowitz (1952) and Roll (1977): suppose a set of characteristics are positively associated with average returns, and a corresponding set of long-short factor-portfolios are constructed by buying high-characteristic stocks and shorting low-characteristic stocks. This set of portfolios will explain the returns of portfolios sorted on the same characteristics, but are unlikely to span the MVE portfolio of all assets, because they do not take into account the asset covariance structure. The intuition underlying this comes from a stylized example: assume there is a single characteristic which is a perfect proxy for expected returns, i.e., $\mathbf{c} = \kappa\boldsymbol{\mu}$, where $\mathbf{c}$ is the characteristic vector, $\boldsymbol{\mu}$ is a vector of expected returns and κ is a constant of proportionality. A portfolio formed with weights proportional to firm-characteristics, i.e., with $\mathbf{w}^c \propto \mathbf{c} = \kappa\boldsymbol{\mu}$, will be MVE only if $\mathbf{w}^c \propto \mathbf{w}^* = \boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}$. In Section 2, we develop stylized model where we develop

[3]Daniel and Titman (1997) critique the original Fama and French (1993) technique. Our critique here is closely related to that paper. Also related to our discussion here are Lewellen, Nagel, and Shanken (2010) and Daniel and Titman (2012) who argue that the space of test assets used in numerous recent asset pricing tests is too low-dimensional to provide adequate statistical-power against reasonable alternative hypotheses. Our focus in this paper is also expanding the dimensionality of the asset return space, but we do so with a different set of techniques.

this argument formally.

When will $\mathbf{w}^c$ be proportional to $\mathbf{w}^*$? That is, when will the characteristic sorted portfolio be MVE? As we show in Section 2, this will be the case only in a few selected settings. For example, it will always be true in a single factor world framework in which the law of one price holds. However, it will not generally hold in settings where the number of factors exceeds the number of characteristics. Specifically, we show that any cross-sectional correlation between firm-characteristics and firm exposures to unpriced factors will result in the factor-portfolio being inefficient.

Of course our theoretical argument does not address the *magnitude* of the inefficiency of the characteristic-based factor portfolios. Intuitively, our theoretical argument is that forming factor-portfolios on the basis of characteristics alone leads to these portfolios being exposed to unpriced factor risk, risk which is hedged-out in the MVE portfolio. In Sections 3 and 4, respectively, we address the questions of how large the loadings on unpriced factors are likely to be, and how much improvement in the efficiency of the factor-portfolios can be obtained by hedging out the unpriced factor risk.

As we discuss in Section 3, extant evidence on the value effect suggests that the industry component of many characteristic measures, such as book-to-price, are not helpful in forecasting average returns. This suggests that any exposure of HML to industry factors is unpriced. Therefore, if this exposure were hedged out, it would result in a factor-portfolio with lower risk, but the same expected return, i.e., with a higher Sharpe ratio. Our analysis in Section 3 shows that the HML exposure on industry factors varies dramatically over time, but that at selected times the exposure can be very high. We highlight two episodes in particular in which the correlation between HML and industry factors exceeds 95%: in late-2000/early-2001 as the prices of high-technology firms earned large negative returns and became highly volatile, and 2008-2009 during the financial-crisis, a parallel episode for financial firms. In both of these episodes the past return performance of the industry led to the vast majority of the firms in the industry becoming either growth or value firms—that is, there was a high cross-sectional correlation between valuation ratios and industry membership—leading to HML becoming highly correlated with that industry factor.

However the evidence that the FF factor-portfolios sometimes load heavily on presumably unpriced industry factors, while suggestive, does not establish that these portfolios are inefficient. Therefore in Section 4, we address the question of what fraction of the risk of the FF factor-portfolios is unpriced and can therefore be hedged out, and how much improvement in Sharpe-ratio results from doing so. The method that we use for constructing our hedge portfolio builds on that developed in Daniel and Titman (1997). However, through the use of higher frequency data, industry adjustment and differential windows for calculating volatilities and correlations we are able to construct hedge portfolios that have both a higher spread in factor loadings and lower idiosyncratic risk. That is, they are more efficient hedge portfolios. Using this technique, we construct hedge portfolios for the five factor portfolios of Fama and French (2015). We are conservative in the way that we construct these portfolios; consistent with the methodology employed by Fama and French, we form these portfolios once per-year, in July, and hold the composition of the portfolios fixed for 12 months. The portfolios are value-weighted buy-and-hold portfolios. Except for the size (SMB) hedge portfolio, these all earn economically and statistically significant five-factor alphas. Using the combined Market-, HML-, RMW- and CMA-hedge portfolios, we construct a combination portfolio that has zero exposure to any of the five FF factors, and yet earns an annualized Sharpe-ratio of 0.99, close to that of the 1.14 Sharpe-ratio of the *ex-post* optimal combination of the five FF factor-portfolios. Thus, by hedging out the unpriced factor risk in the FF portfolios, we increase the squared-Sharpe ratio of this optimal combination from from 1.31 to 2.29.

This result is important for several reasons. First it increases the hurdle for standard asset pricing models, in that pricing kernel variance that is required to explain the returns of our hedged factor portfolios is double what is required to explain the returns of the Fama and French (2015) five factor-portfolios.

Second, while the characteristics approach to measure managed portfolio performance (see, e.g., Daniel, Grinblatt, Titman, and Wermers, 1997) has gained some popularity, the regression based approach initially employed by Jensen (1968) (and later by Fama and French (2010) and numerous others) remains the more popular. A good reason for this is that the characteristics approach can only be used to estimate the alpha of a portfolio when the holdings of the managed portfolio are known, and frequently sampled. In contrast, the Jensen-style regression approach can be used in the absence of holdings data, as long as a time series of portfolio returns are available.

However, as pointed out originally by Roll (1977), to use the regression approach, the multi-factor benchmark used in the regression test must be efficient, or the conclusions of the regression test will be invalid. What we show in this paper is that, with the historical return data, efficiency of the proposed factor-portfolios can be rejected. However, the hedged versions of the factor-portfolios, that we construct here and which incorporate the information both from the characteristics and from the historical covariance structure, are efficient with respect to both of these information sources. Thus, alphas equivalent to what would be obtained with the DGTW characteristics-approach can be generated with the regression approach, if the hedged factor portfolios are used, without the need for portfolio holdings data.

The layout of the remainder of the paper is as follows: In Section 2 we lay out the underlying econometric theory that motivates our analysis. Section 3 provides a descriptive analysis of the industry loadings of the Fama and French factors. In Section 4 we perform the construction of the hedge portfolios, and empirically test the effectiveness of this hedging. Section 6 concludes.

2 Theory

The usual procedure to construct factors involves two steps. The starting point is the identification of a particular characteristic $c_{i,t}$, where $i \in \mathcal{I}$ is the index denoting a particular stock and $\mathcal{I}$ is the set of stocks, that correlates with average returns in the cross section. The stocks are then sorted according to this characteristic. The second step involves building a portfolio that is long stocks, say, with high values of the characteristic and short stocks with low value of the characteristics. The claim is that the return of a factor so constructed is the projection on the space of returns $\mathcal{R}$ of a factor f_t which drives the investors' marginal rate of substitution and that as a result is a source of premia. The projection should result in a mean variance efficient portfolio. We argue instead that the usual procedure to construct proxies for these true factors should not be expected to produce mean-variance efficient portfolios. As a result the Sharpe ratios associated with those factors produce too low a bound for the volatility of the stochastic discount factor, which diminishes the power of asset pricing tests. This observation is not related to whether characteristics or covariances are the drivers of average returns. To put it simply, factors based on characteristics' sorts are likely to pick up sources of common variation that are not compensated with premia. To illustrate this point we start with a simple example, then we generalize it in a formal model.

2.1 Example

Consider a standard asset pricing model in a world where the Law of One Price holds:

$$R_{i,t} = \beta_i \left(f_t + \lambda_{t-1} \right) + \beta_i^u f_t^u + \varepsilon_{i,t}, \tag{1}$$

with $\mathbb{E}_{t-1}f_t = \mathbb{E}_{t-1}f_t^u = \mathbb{E}_{t-1}\varepsilon_{i,t} = 0$ for all $i \in \mathcal{I}$, $\mathsf{var}(f_t) = \sigma_f^2$, $\mathsf{var}(\varepsilon_{i,t}) = \sigma_\varepsilon^2$, $\mathsf{var}(f_t^u) = \sigma_{f^u}^2$, $\mathsf{cov}(f_t, f_t^u) = 0$, and $\mathsf{cov}(\varepsilon_{i,t}, \varepsilon_{j,t}) = 0$ for all $i \neq j$, $i, j \in \mathcal{I}$.

In model (1), $\lambda_{f^u} = 0$ and thus f_t^u is an unpriced factor and β_i^u is the corresponding loading. As long as $\beta_i^u \neq 0$ there are common sources of variation that do not result in cross sectional dispersion in average returns. Many argue that the "return covariance structure essentially dictates that the first few PC factors must explain the cross-section of expected returns. Otherwise near-arbitrage opportunities would exist" (Kozak, Nagel and Santosh, 2015). But this does not need to be the case. It is natural to look for sources of premia amongst principal components but theory does not dictate that these two things are the same.

Researchers typically use the characteristic $c_{i,t}$ to sort stocks and construct the proxy for factor f_t, which we denote by $f_t^{(1)}$. Even when the characteristic lines up perfectly with average returns there is no guarantee that $f_t^{(1)}$ will be mean variance-efficient. To see this point consider a simple example: a stock market with only six stocks all with equal market capitalization. The six stocks have characteristics and covariances with the unpriced factor, f_t^u, as in Figure 1. Notice that assets 1 and 2 have identical loadings and characteristics, the same holds for assets 5 and 6 . Suppose further that it is a one period problem, so we can drop the $t-1$ subscript. Furthermore, suppose that the risk premium associated with factor f_t is $\lambda = 1$.

Suppose that we do not observe f_t, but there exists an observable characteristic $c_{i,t-1}$ that lines up perfectly with expected returns:

$$\mathbb{E}[r_{i,t}] = \kappa \cdot c_{t-1} \tag{2}$$

In order for equations (1) and (2) to hold at the same time, it must be the case that:

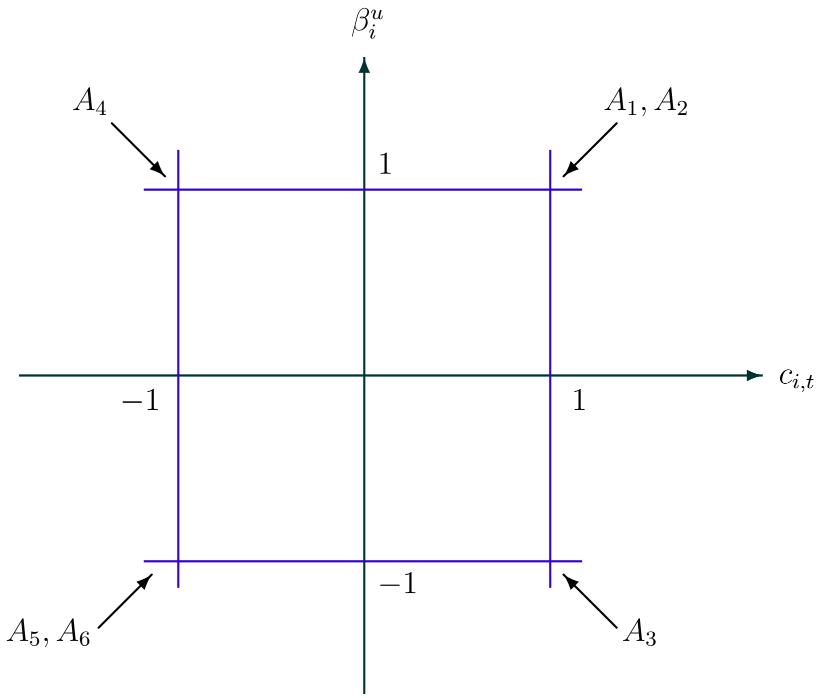

Figure 1: *Six assets in the space of loadings and characteristics*

$$\beta_{i,t-1} = \kappa c^4 \tag{3}$$

Substituting (3), we can rewrite model 1 as:

$$R_{i,t} = \kappa c \left(f_t + 1\right) + \beta_i^u f_t^u + \varepsilon_{i,t}, \tag{4}$$

[4]An example may be helpful here. Consider the Gordon growth model in which

$$\frac{E_i}{P_i} = r_i - \delta_i,$$

where E_i are the earnings of stock i, P_i the price, δ_i the rate of growth of earnings, $r_i = r + \beta_i \lambda_f$, the discount rate, and r is the risk free rate. Define a characteristic

$$c_i \equiv \frac{E_i}{P_i} + \delta_i - r \qquad \Rightarrow \qquad c_i = \beta_i \lambda_f.$$

The usual procedure to construct a proxy for factor f_t involves forming a portfolio that is long stocks with high characteristics and short stocks with low characteristics,[5] that is:

$$f_t^{(1)} = \frac{1}{3} \times \left[\sum_{j=1}^{3} R_{j,t} - \sum_{j=4}^{6} R_{j,t} \right] = 2\kappa(f_t + 1) + \frac{2}{3} f_t + \frac{1}{3} \left[\sum_{j=1}^{3} \varepsilon_{j,t} - \sum_{j=4}^{6} \varepsilon_{j,t} \right].$$

Factor portfolio $f_t^{(1)}$ does indeed capture the common source of variation in expected returns, since it loads on f_t . Though, it also loads on the unpriced source of variation f_t^u. The factor portfolio $f_t^{(1)}$ loads on the factor f_t with $\beta_{f^{(1)}} = 2\kappa$, loads on f_t^u with $\beta^u_{f^{(1)}} = \frac{2}{3}$ and its characteristic is $c_{f^{(1)}} = 2$. As a result, from equation (4), the expected excess return of portfolio $f_t^{(1)}$ is $\mathbb{E} f_t^{(1)} = 2\kappa$ and the variance is

$$\mathsf{var}\left(f_t^{(1)}\right) = 4\kappa^2\sigma_f^2 + \frac{4}{9}\sigma_f^2 + \frac{2}{3}\sigma_\varepsilon^2.$$

The Sharpe ratio is thus

$$\mathsf{SR}_{f^{(1)}} = \frac{2\lambda}{\sqrt{4\kappa^2\sigma_f^2 + \frac{4}{9}\sigma_f^2 + \frac{2}{3}\sigma_\varepsilon^2}} \tag{5}$$

Factor portfolio $f_t^{(1)}$ is not mean variance efficient and thus cannot be the projection of the stochastic discount factor on the space of returns. To see this consider forming the following portfolio

$$h_t = \frac{1}{2}\left[R_{3,t} + R_{6,t}\right] - \frac{1}{2}\left[R_{1,t} + R_{4,t}\right] = -2f_t + \frac{1}{2}\left[\varepsilon_{3,t} + \varepsilon_{6,t}\right] - \frac{1}{2}\left[\varepsilon_{1,t} + \varepsilon_{4,t}\right].$$

[5] Because in this simple example all stocks have equal weight there is no difference between equal and value weighted. The usual Fama-French construction uses value weighted portfolios.

This portfolio thus goes long stocks with low loadings and short stocks with high loadings on f_t^u. Each leg of this portfolio is characteristics balanced. Thus $c_h = 0$ and $\mathbb{E}h_t = 0$. The loading of this portfolio h_t on f_t^u is $\beta_h^u = -2$. We can use h_t to improve on $f_t^{(1)}$. Indeed consider next the following portfolio

$$f_t^{(2)} = f_t^{(1)} - \gamma h_t. \tag{6}$$

It is straightforward to show that setting $\gamma = -\frac{1}{3}$ results in a portfolio $f_t^{(2)}$ that has a zero loading on factor f_t^u, $\beta_{f^{(2)}}^u = 0$. Moreover

$$\mathbb{E}f_t^{(2)} = 2\kappa \qquad \text{and} \qquad \mathsf{var}\left(f_t^{(2)}\right) = 4\kappa^2\sigma_f^2 + \frac{7}{9}\sigma_\varepsilon^2$$

and thus the Sharpe ratio

$$\mathsf{SR}_{f^{(2)}} = \frac{2\lambda}{\sqrt{4\kappa^2\sigma_f^2 + \frac{7}{9}\sigma_\varepsilon^2}}. \tag{7}$$

Comparing the Sharpe ratios of $f_t^{(1)}$ and $f_t^{(2)}$ in equations (5) and (7), respectively, one can immediately see that if σ_ε^2 is low compared to $\sigma_{f^u}^2$ then $\mathsf{SR}_{f^{(1)}} << \mathsf{SR}_{f^{(2)}}$. Notice that given that the number of assets is finite the investor cannot achieve an infinite Sharpe ratio.[6]

We have chosen γ to eliminate the exposure of $f^{(2)}$ to the unpriced factor f_t^u. In general though we will choose the parameter γ in order to minimize the variance of the resulting factor, $f_t^{(2)}$:

[6] In the limit as the number of assets grows $\mathsf{SR}_{f^{(2)}} \to \infty$.

$$\min_{\gamma} \mathsf{var}\left(f_t^{(2)}\right) \qquad \Rightarrow \qquad \widehat{\gamma} = \rho_{1,h} \frac{\sigma\left(f_t^{(1)}\right)}{\sigma\left(h_t\right)}, \tag{8}$$

where $\sigma\left(f_t^{(1)}\right)$ is the standard deviation of returns of the original factor-portfolio $f_t^{(1)}$ and $\sigma\left(h_t\right)$ is the standard deviation of the characteristics balanced hedge portfolio. Setting $\gamma = \widehat{\gamma}$ guarantees that, under the null of model 1, the Sharpe ratio of $f_t^{(2)}$ is maximized. In general then, if model 1 holds, it can be shown that the improvement in the Sharpe ratio of the original factor-portfolio $f_t^{(1)}$ is

$$\frac{\mathsf{SR}^{(2)}}{\mathsf{SR}^{(1)}} = \frac{1}{\sqrt{1-\rho_{1,h}^2}}. \tag{9}$$

The question of whether one can improve on the Sharpe ratio associated with exposure to factor f_t is thus an empirical one. In Section 4 we construct characteristics balanced portfolios for each of the Fama and French (2015) five factors and show that "subtracting them" as in (6) from each of the factors considerably improves the Sharpe ratio of each of them.

The procedure proposed in this paper has the considerable advantage of being able to improve upon standard factors without the need of identifying specifically what those unpriced sources of common variation are. Finding these sources of unpriced common variation involves searching among likely candidates. This is what we do in Section 4. For instance, HML_t is typically thought to be a proxy for an unknown distress factor. Its construction involves going long value and short growth, where value and growth are defined to be stocks with high and low book-to-market, respectively. We show though that HML_t loads heavily on industry factors at particular points in time, which is not surprising. For instance HML_t loads heavily on financials during the Great Recession and on "tech stocks" before and after the bursting of the Nasdaq bubble. Importantly those industries *were* indeed distressed and thus the "over-representation" of these particular stocks in the value portfolio may be warranted. Still, we show that removing these industry factors improves considerably the Sharpe ratios: There may be indeed an industry component to distress but not all of it carries a premium.

We now formalize further these ideas. Specifically, the next section shows the relation that exists between the cross sectional correlation of the characteristic used to effect the sort and the loadings of unpriced factors, on the one hand, and the improvement in the Sharpe ratio of the portfolio that is proposed as a projection of the priced factor on the space of returns, on the other.

2.2 General Case

2.2.1 Factor representations

Consider a single-period setting, with N risky assets and a risk-free asset whose returns are generated according to a K factor structure:

$$\mathbf{R}_t = \boldsymbol{\beta}_{t-1} \left(\mathbf{f}_t + \boldsymbol{\lambda}_{t-1} \right) + \boldsymbol{\varepsilon}_t \tag{10}$$

where $\mathbf{R}_t$ is $N \times 1$ vector of the period t realized excess returns of the N assets; $\mathbf{f}_t$ is a $K \times 1$ vector of the period t unanticipated factor returns, with $\mathbb{E}_{t-1}[\mathbf{f}_t] = \mathbf{0}$, and $\boldsymbol{\lambda}_t$ is the $K \times 1$ vector of premia associated with these factors. $\boldsymbol{\beta}_{t-1}$ is the $N \times K$ matrix of factor loadings, and $\boldsymbol{\varepsilon}_t$ is the $N \times 1$ vector of (uncorrelated) residuals. We assume that $N \gg K$, and that N is sufficiently large so that well diversified portfolios can be constructed with any factor loadings.[7]

As it is well known, there is a degree of ambiguity in the choice of the factors. Specif-

[7] We note that, in a finite economy, the breakdown of risk into systematic and idiosyncratic is problematic. See Grinblatt and Titman (1983), Bray (1994) and others.

ically, any set of the factors that span the K-dimensional space of non-diversifiable risk can be chosen, and the factors can be arbitrarily scaled. Therefore, without loss of generality, we rotate and scale the factors so that:[8]

$$\boldsymbol{\lambda}_{t-1} = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad \text{and} \quad \boldsymbol{\Omega}_{t-1} = \mathbb{E}_{t-1}[\mathbf{f}_t \mathbf{f}_t'] = \begin{bmatrix} \sigma_1^2 & 0 & \cdots & 0 \\ 0 & \sigma_2^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma_K^2 \end{bmatrix} \tag{11}$$

We further define:

$$\boldsymbol{\mu}_{t-1} = \mathbb{E}_{t-1}[\mathbf{R}_t] \quad \boldsymbol{\Sigma}^{\varepsilon}_{t-1} = \mathbb{E}_{t-1}[\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t{}'] \quad \text{and} \quad \boldsymbol{\Sigma}_{t-1} = \mathbb{E}_{t-1}[\mathbf{R}_t \mathbf{R}_t'] = \boldsymbol{\beta}_{t-1} \boldsymbol{\Omega}_{t-1} \boldsymbol{\beta}_{t-1}' + \boldsymbol{\Sigma}^{\varepsilon}_{t-1}$$

where $\boldsymbol{\mu}_{t-1}$ and $\boldsymbol{\sigma}^2_{\varepsilon}$ are $N \times 1$ vectors. Given we have chosen the K factors to summarize the asset covariance structure, $\boldsymbol{\Sigma}^{\varepsilon}_{t-1} = \mathbb{E}_{t-1}[\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t']$ is a diagonal matrix, (i.e., with the residual variances on the diagonal, and zeros elsewhere).

2.2.2 Characteristic-Based Return Factors

Over that last several decades, academic studies have documented that certain characteristics (market capitalization, price-to-book values ratios, past returns, etc.) are related to expected returns. In response to this evidence, Fama and French (1993; 2015), Carhart (1997), Pastor and Stambaugh (2003), Frazzini and Pedersen (2014) and numerous other researchers have introduced "return factors" based on characteristics. The literature has then tested whether these characteristic-weighted factors can explain the cross-section of returns, in the sense that some linear combination of the factor portfolios is mean-variance-efficient.

[8] The rotation is such that the first factor captures all of the premium. The scaling of the first factor is such that its expected return is 1. The other factors form an orthogonal basis for the space of non-diversifiable risk, but the scaling for all but the first factor is arbitrary

Assume that we can identify a vector of characteristics that perfectly captures expected returns, that is such that: $\mathbf{c}_{t-1} = \kappa \boldsymbol{\mu}_{t-1}$ (see (3) above). Moreover $\mathbf{c}_{t-1}$ is an $N \times 1$ vector, that is, a single characteristic summarizes expected returns. Following the usual procedure we assume that the factor-portfolio is formed based on our single vector of characteristics $\mathbf{c}_{t-1}$ or to put it differently that the weights of the portfolio are assumed to be proportional to the characteristic. We normalize this portfolio so as to guarantee that it has a unit expected return:[9] [10]

$$\mathbf{w}_{c,t-1} = \kappa \left(\frac{\mathbf{c}_{t-1}}{\mathbf{c}'_{t-1}\mathbf{c}_{t-1}} \right) = \frac{\boldsymbol{\mu}_{t-1}}{\boldsymbol{\mu}'_{t-1}\boldsymbol{\mu}_{t-1}} \tag{12}$$

Note that, given this normalization, $\mathbf{w}'_{c,t-1}\boldsymbol{\mu}_{t-1} = 1$, as desired.

2.2.3 Relation between the characteristic-weighted and MVE portfolio

Assuming no arbitrage in the economy, there exists a stochastic discount factor that prices all assets, and a corresponding mean-variance-efficient portfolio. In our setting the weights of the MVE portfolio are:

$$\mathbf{w}_{\mathrm{MVE,t-1}} = \left(\boldsymbol{\mu}'_{t-1}\boldsymbol{\Sigma}^{-1}_{t-1}\boldsymbol{\mu}_{t-1} \right)^{-1} \boldsymbol{\Sigma}^{-1}_{t-1}\boldsymbol{\mu}_{t-1}, \tag{13}$$

which have been scaled so as to give the portfolio a unit expected return. The variance of the portfolio is $\sigma^2_{\mathrm{MVE,t-1}} = \left(\boldsymbol{\mu}'_{t-1}\boldsymbol{\Sigma}^{-1}_{t-1}\boldsymbol{\mu}_{t-1} \right)^{-1}$, so the Sharpe-ratio of the portfolio is $SR_{\mathrm{MVE}} = \sqrt{\boldsymbol{\mu}'_{t-1}\boldsymbol{\Sigma}^{-1}_{t-1}\boldsymbol{\mu}_{t-1}}$.

Given our scaling of returns, the βs of the risky asset w.r.t the MVE portfolio are

[9] The typical normalization in building factor portfolios is that they are "\$1-long, \$1 short" zero investment portfolios. However since we are dealing with excess returns, this normalization is arbitrary and has no effect on the ability of the factor-portfolios to explain the cross-section of average returns.

[10] To make the notation as simple as possible we drop the time subscript.

equal to the assets' expected excess returns:[11]

$$\boldsymbol{\beta}_{\text{MVE,t}-1} = \frac{\mathsf{cov}_{t-1}\left(\mathbf{R}_t, R_{\text{MVE,t}}\right)}{\mathsf{var}_{t-1}\left(R_{MVE,t}\right)} = \frac{\boldsymbol{\Sigma}_{t-1}\mathbf{w}_{\text{MVE},t-1}}{\mathbf{w}_{\text{MVE,t}-1}\boldsymbol{\Sigma}_{t-1}\mathbf{w}_{\text{MVE,t}-1}} = \boldsymbol{\mu}_{t-1}$$

We can then project each asset's return onto the MVE portfolio:

$$\mathbf{R}_t = \boldsymbol{\beta}_{\text{MVE,t}-1} R_{\text{MVE,t}} + \mathbf{u}_t = \boldsymbol{\mu}_{t-1} R_{\text{MVE,t}} + \mathbf{u}_t \tag{14}$$

$\mathbf{u}$ is the component of each asset's return that is uncorrelated with the return on the MVE portfolio, which is therefore unpriced risk.

Given the structure of the economy laid out in equations (10) and (11),

$$R_{\text{MVE,t}} = f_{1,t} + 1$$

where f_1 denotes the first element of $\mathbf{f}$ (and the only priced factor). This means that, referencing equation (14),

$$\boldsymbol{\beta}_{\text{MVE,t}-1} = \boldsymbol{\mu}_{t-1} = \boldsymbol{\beta}_{1,t-1} = \kappa^{-1}\mathbf{c}_{t-1} \tag{15}$$

Finally, this means that we can write the residual from the regression in equation (14) as:

$$\mathbf{u}_t = \boldsymbol{\beta}^u_{t-1}\mathbf{f}^u_t + \boldsymbol{\varepsilon}_t \tag{16}$$

where $\boldsymbol{\beta}^u_{t-1}$ is the $N \times (K-1)$ matrix which is $\boldsymbol{\beta}_{t-1}$ with the first column deleted (i.e., the loadings of the N assets on the $K-1$ unpriced factors), and $\mathbf{f}^u_t$ is the $(K-1) \times 1$ vector consisting of the 2nd through Kth elements of $\mathbf{f}_t$ (i.e., the Unpriced factors).

[11]For the third equality, just substitute $\mathbf{w}_{MVE,t-1}$ from equation (13) into the second.

We will use this projection to study the efficiency of the characteristic-weighted portfolio. Since both the characteristic-weighted and MVE portfolio have unit expected returns, the increase in variance in moving from the MVE portfolio to the characteristic portfolio can tell us how inefficient the characteristic-weighted portfolio is. From equations (12) and (14), we have:

$$\begin{aligned} R_{c,t} &= \mathbf{w}'_{t-1,c}\mathbf{R}_t = R_{MVE,t} + (\boldsymbol{\mu}'_{t-1}\boldsymbol{\mu}_{t-1})^{-1}\boldsymbol{\mu}'_{t-1}\mathbf{u}_t \quad \Rightarrow \\ R_{c,t} - R_{MVE,t} &= \left(\boldsymbol{\mu}'_{t-1}\boldsymbol{\mu}_{t-1}\right)^{-1}\boldsymbol{\mu}'_{t-1}\left[\boldsymbol{\beta}^u_{t-1}\mathbf{f}^u_t + \boldsymbol{\varepsilon}_t\right] \end{aligned}$$

Thus given (16)

$$\mathsf{var}_{t-1}(R_{c,t} - R_{\text{MVE,t}}) = \sum_{k=2}^{K} [\underbrace{(\mathbf{c}'_{t-1}\mathbf{c}_{t-1})^{-1}(\mathbf{c}'_{t-1}\beta^u_{k,t-1})}_{\equiv \gamma_{k,c}}]^2 \sigma^2_{k,t-1} \tag{17}$$

What is the interpretation of (17)? $\gamma_{k,c}$ is the coefficient from a cross-sectional regression of the kth (unpriced) factor loading on the characteristic.[12] Even though the K factors are uncorrelated, the *loadings on the factors in the cross-section* are potentially correlated with each other, and this regression coefficient could potentially be large for some factors. Indeed, the necessary and sufficient conditions for the characteristic-sorted portfolio to price all assets are that

$$\gamma_{k,c} = 0 \quad \forall \quad k \in \{2, \ldots, K\}.$$

This condition is unlikely hold even approximately. For example, as we show later, in the middle of the financial crisis, many firms in the financial sector were high expected return (high μ). However, these firms also had a high loading on the finance

[12]Note that we get the same expression, up to a multiplicative constant, if we instead regress the unpriced factor loadings on the the priced factor loadings, or on the expected returns, given the equivalence in equation (15).

industry factor ($\sigma^2_{k,t-1}$ was high). Because $\boldsymbol{\mu}_{t-1}$ (the expected return based on the characteristics) and $\boldsymbol{\beta}^u_{k,t-1}$ (the loading on the unpriced finance industry factor) were highly correlated, the characteristics-sorted portfolio has high industry factor risk, meaning that it has a lower Sharpe-ratio than the MVE portfolio. Because $\sigma^2_{k,t-1}$ was quite high in this period, the extra variance of the characteristic-sorted portfolio was arguably also large. In Section 4, we show how this extra variance can be diagnosed and taken into account.

2.2.4 An optimized characteristic-based portfolio

It follows from the previous discussion that the optimized characteristic-based portfolio is

$$\mathbf{w}^*_{c,t-1} = \kappa \left(\frac{\boldsymbol{\Sigma}^{-1}_{t-1}\mathbf{c}_{t-1}}{\mathbf{c}'_{t-1}\boldsymbol{\Sigma}^{-1}_{t-1}\mathbf{c}_{t-1}} \right) = \frac{\boldsymbol{\Sigma}^{-1}_{t-1}\boldsymbol{\mu}_{t-1}}{\boldsymbol{\mu}'_{t-1}\boldsymbol{\Sigma}^{-1}_{t-1}\boldsymbol{\mu}_{t-1}} \tag{18}$$

Clearly the challenge is the actual construction of such a portfolio. For instance, there are well known issues associated with estimating $\boldsymbol{\Sigma}_{t-1}$ and using it to do portfolio formation. In the next subsection, we develop an alternative approach for testing portfolio optimality.

Assuming the characteristics model is correct, and one observes the characteristics, it is straightforward to test the optimality of the characteristics-sorted portfolio. All that is needed is some (ex-ante) instrument to forecast the component of the covariances which is orthogonal to the characteristics. If the characteristic sorted portfolio is optimal (i.e., MVE) then characteristics must line up with betas with the characteristics sorted-portfolio *perfectly*. If they don't (and the characteristics model holds) then the portfolio can't be optimal.

Moreover, one can improve on the optimality of the portfolio by following the procedure advocated in this paper, by, first, identifying assets with high (low) alphas relative to the characteristic-sorted portfolio (again based on the characteristic model) and, second, building a portfolio with the highest possible expected alpha relative to the characteristic sorted portfolio, under the characteristic hypothesis. If this portfolio has a positive alpha then the optimality of the characteristics-sorted portfolio is established. This is the empirical approach we take in this paper.

In sum, our point is that if a particular characteristic is used to construct a factor-portfolio then whenever there is a correlation between the characteristic and the loadings on unpriced sources of variation the factor-portfolio will fail to be main variance efficient. Thus the factor cannot be a proxy for the true, underlying, stochastic discount factor. In this section and the next we show that the point is not just of theoretical interest but that its quantitative importance is substantial. We do so in two different ways. In the next section we focus in one particular factor, Fama and French (1993) HML factor and show that it loads heavily on particular industries at particular times. This source of variation is unpriced and thus one can improve on this factor by removing its dependence of industry factors. In Section 4 we use the more general procedure developed in this section to improve upon the standard Fama and French (2015) five factors.

3 Sources of common variation: Industry Factors

Asness, Porter, and Stevens (2000), Cohen and Polk (1995) and others[13] have shown that if book-to-price ratios are decomposed into an industry-component and a within-industry component, then only the within-industry component— that is, the difference between a firm's book-to-price ratio and the book-to-price ratio of the industry portfolio—forecasts future returns. This suggests that any exposure of HML to in-

[13]See also Lewellen (1999) and Cohen, Polk, and Vuolteenaho (2003).

dustry factors is likely unpriced, and therefore that if this exposure were hedged out, it would result in a factor-portfolio with lower risk, but the same expected return, that is, with a higher Sharpe ratio. But, does HML load on industry factors?

Figure 2 plots the R^2 from 126-day rolling regressions of daily HML returns on the twelve daily Fama and French (1997) value-weighted industry excess returns. The time period is January 1981 to December 2015.[14] The plot shows that, while there are short periods where the realized R^2 dips below 50%, there are also several periods where it exceeds 90%. The R^2 fluctuates considerably but the average is well above 70%. The upper Panel of Figure 3 plots, for the same set of daily, 126-day rolling regressions, the regression coefficients for each of the 12 industries. As it is apparent these coefficients display considerable variation: sometimes the HML portfolio loads more heavily on some industries than on others.

To provide a little clarity let's focus on two particular industries: 'Business Equipment', which comprises many of the high technology firms, and 'Money' which includes banks and other financial firms. The two industries are selected because HML had the lowest and highest exposure, respectively, to them in the post-1995 period. Start with 'Business Equipment' and focus in the late 1990s and 2000. As one can see the regression coefficient of HML on this particular industry started falling in the mid to late 90s, as the "high-tech" sector started posting impressive returns. These firms were, in addition, heavy on intangible capital which was not reflected in book. As their book-to-market shrank these companies were classified into the growth portfolio: The L in HML became a short on high tech companies, which became to dominate the 'Business Equipment' industry. Simultaneously the volatility of returns in this industry started increasing consistently around 1997, reaching a peak in early 2001, as illustrated in Figure 4, which plots the rolling-126 day volatil-

[14] The daily HML returns, the daily industry returns and the risk-free data are taken from Ken French's data library at `http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/Data_Library`. This web site also provides a breakdown of the standard industrial classification (SIC) codes that are included in each.

ity of returns.[15] The annualized volatility of 'Business Equipment' returns hovered below 20% for almost two decades but then shot up in the mid 90s to well above 60%, at the peak of the Nasdaq cycle. The increase in the absolute value of the regression coefficient and the high volatility of returns result on the high R^2 of the regression of HML on industry factors.

The behavior of the 'Money' industry during and after the Great Recession of 2008 is an even more striking example of the large industry effect on HML. The regression coefficient associated with 'Money' increased dramatically between 2007 and early 2009 as stock prices for firms in this segment collapsed and quickly became classified as value. [16] As shown in Figure 4 the volatility of returns also increased dramatically. As a result of these two effects 'Money' explained a substantial amount of the variation of HML returns during those years. Indeed Figure 5 plots the R^2 or a regression of the return on HML on the 'Money' industry excess returns. Between late 2008 and late 2010 the R^2 was well above 60%. Why was it so high? As of December 2007, the top 4 firms by market capitalization in the "Money" industry were Bank of America, AIG, Citigroup and J.P. Morgan. Three of these four were in the large value portfolio portfolio (Big/High-BM to use the standard terminology). Interestingly, the one that wasn't was AIG – it was in the middle portfolio. While the market capitalization of these firms falls dramatically through 2008, they remain large and, particularly as the volatility of the returns on the 'Money' industry increases, these firms and others like them drive the returns both of the HML portfolio and the 'Money' industry portfolio.

However, there are firms in the 'Money' industry that do not have high book-to-market ratios, even in the depths of the financial crisis. For example, in 2008 US

[15] Note that for this plot, like the other "rolling" plots in this section, the x-axis label indicates the date on which the 126-day interval ends.

[16] As shown in Huizinga and Laeven (2009) banks during the crisis used accounting discretion to avoid writing down the value of distressed assets. As a result the value of bank equity was overstated. The market knew better and as a result the book-to-market of bank stocks shot up during the crisis.

Bancorp (USB) and American Express (AXP) were both "L" (low book-to-market) firms. Yet both USB and AXP have large positive loadings on HML at this point in time (see Table 1). The reason is that both USB and AXP covary strongly with the returns on the 'Money' industry, as does HML at this point in time. We use this variation within the 'Money' Industry to construct hedge portfolios for each of the Fama and French (2015) factors, as illustrated in the the example previous section. In particular we construct a characteristics balanced portfolio, f^{cb}. The short side of the characteristics balanced portfolio features firms with high loadings on HML and low and high book-to-market, such as American Express and Citi, respectively. In the example in Figure 1 American Express would be asset A_4 and Citi would be A_1. The long side of the characteristics balanced portfolio is comprised of stocks with low loadings on HML. Then we combine a long position in the HML portfolio with an appropriately sized position on the characteristics balanced portfolio to hedge the exposure of the HML portfolio to the 'Money' industry, as in expression (6). This procedure thus succeeds in creating a more efficient "hedged" HML portfolio, on that has the same expected return, but lower return variance and therefore a higher Sharpe-ratio, than the original Fama and French (1993) HML portfolio.

4 Hedge Portfolios

4.1 Construction

The empirical goal is to construct the best possible hedge portfolios, as introduced in model (1). To achieve this, if $f_t^{(1)}$ is a well diversified portfolio, we only need to maximize the hedge portfolio loading on the unpriced source of common variation, f_t^u. However, empirically, this is not observable. What we can observe ex-ante, though, is the loading on a candidate factor portfolio that captures any type of common variation, e.g., $f_t^{(1)}$, i.e., this captures common variation through f_t^u as well as through f_t. To disentangle the two from each other, we use a procedure first introduced by Daniel and Titman (1997). The idea is to use the ex-ante loading of each stock i on the candidate factor portfolio $f_t^{(1)}$ and construct portfolios that maximize the loading on $f_t^{(1)}$. At the same time, these portfolios are constructed in such a way that they have zero exposure to characteristics, and consequently zero expected return. Effectively, this shuts out the component of the loading that is linked to the potentially priced factor and thereby only hedges out unpriced risk.

Our focus is on the five factor Fama and French (2015) model and we follow these authors in the construction of their factor portfolios. In the following, we will explain the procedure based on the example of HML. We first rank NYSE firms by their, in this case, book-to-market (BM) ratios at the end of December of a given year and their market capitalization (ME) at the end of June of the following year. Break points are selected at the 33.3% and 66.7% marks for both the book-to-market and market capitalization sorts. Then in July of a given year all NYSE/Amex and Nasdaq stocks are placed into one of the nine resulting bins. There is an important difference though in the way the sorting procedure is implemented relative to Fama and French (1992, 1993 and 2015) or Daniel and Titman (1997) and it is that our characteristics sorted portfolios are industry adjusted. That is, whether a stock has, for example, a high or low book-to-market ratio depends on whether it is above or below the corresponding value-weighted industry average. [17] Our industries are the 49 industries of Fama and French (1997).

Next, each of the stocks in one of these nine bins is sorted into one of three additional bins formed based on the stocks' expected future loading on the HML factor portfolio. The firms remain in those portfolios between July and June of next year. Sorting on the characteristic and the expected loading itself identifies to what extent the variation in returns is driven by the characteristic or the loading. This last sort results

[17] The reason we use industry adjusted characteristics is because they have been shown to be better proxies of expected returns (see Cohen et al. (2003)).

in portfolios of stocks with similar characteristics (BEME and ME) but different loadings on HML.

Finally, we construct our hedge portfolio for the HML factor portfolio, as in the example in section 2, by going long an equal-weight combination of all low-loading portfolios and short an equal-weight combination of all high-loading portfolios. Thereby, the long and short sides of this portfolio have zero exposure to the characteristic and we maximize the spread in expected loading on the unpriced sources of common variation.

The hedge portfolios for RMW and CMA are constructed in exactly the same way. For SMB, we follow Fama and French (2015) and construct three different hedge portfolios: One where the first sorts are based on BEME and ME, and then within these 3x3 bins, we conditionally sort on the loading on SMB. The second and third versions use OP and INV instead of BEME in the first sort. Then, an equal weighted portfolio of the three different SMB hedge portfolios is used as the hedge portfolio for SMB. We do exactly the same for the hedge portfolio for the market.

Clearly a key ingredient of the last step of the sorting procedure is the estimation of the expected loading on the corresponding factor. Our purpose is to obtain estimates of the future loadings in the five factor model of Fama and French (2015):

$$\begin{aligned} R_{i,t} - R_{F,t} = a_{i,t-1} + \beta_{Mkt-RF,i,t-1}(R_{Mkt,t} - R_{F,t}) + \beta_{SMB,i,t-1} R_{SMB_t} \\ + \beta_{HML,i,t-1} HML_t + \beta_{RMW,i,t-1} RMW_t + \beta_{CMA,i,t-1} CMA_t + e_{i,t} \end{aligned} \tag{19}$$

We instrument future expected loadings with preformation loadings of each stock with the candidate factor portfolios. The resulting estimation method is intuitive and is close to the method proposed by Frazzini and Pedersen (2014). These authors build on the observation that correlations are more persistent than variances (see, among others, de Santis and Gerard (1997)) and propose estimating covariances and variances separately and then combine these estimates to produce the preformation loadings. Specifically, covariances are estimated using a five-year window with overlapping log-return observations aggregated over three trading days, to account for non-synchronicity of trading. Variances of factor portfolios and stocks are estimated on daily log-returns over a one-year horizon. In addition, we introduce an additional intercept in the pre-formation regressions for returns in the six months preceding portfolio formation, i.e., from January to June of the rank-year (see Figure 1 in Daniel and Titman (1997) for an illustration). We refer to this estimation methodology as the 'high power' methodology. Intuitively, if our forecasts of future loadings are very noisy, then sorting on the basis of forecast-loading will produce no variation in the actual *ex-post* loadings of the sorted portfolios. In contrast, if the forecasts are accurate, then our hedge portfolio—which goes long the low-forecast-loading portfolio-and short the high-forecast-loading portfolio—will indeed be strongly correlated with the corresponding FF portfolio. Also, since this portfolio is "characteristic-neutral," meaning the long and short-sides of the portfolio have equal characteristics and, if the characteristic model is correct, will have zero expected excess return. Such a portfolio would be an optimal hedge portfolio, in that it maximizes the correlation with the FF portfolio subject to the constraint that it is characteristic neutral. Also, such a portfolio would have the highest possible likelihood of rejecting the FF model, under the hypothesis that the characteristic model is correct.

This estimation method contrasts with the traditional approach of simply using as instruments for future factor loadings the result of regressing stock excess returns on factor portfolios over a moving fixed-sized window based on, e.g., 36 or 60 monthly observations, skipping the most recent 6 months, i.e., those that already fall in the rank-year (see, e.g., Daniel and Titman (1997) or Davis, Fama, and French (2000)).[18] We refer to this method, which is effectively the one used by Daniel and Titman (1997), as the 'low power' method and use it to construct an alternative set of hedge portfolios. In addition, this set of hedge portfolios is not industry adjusted.

[18]Notice that in contrast, the high power method avoids discarding the most recent data.

In sum, the high and low power sets of hedge portfolios differ in two dimensions: the estimation method for the expected loading and whether the characteristics are industry adjusted or not. In what follows, we examine to what extent these portfolios differ and whether we succeed in maximizing our ability to hedge out unpriced sources of common variation.

4.2 Average returns and characteristics of test portfolios

Table 2 presents average monthly excess returns for the portfolios that we combine to form our hedge portfolios. Each panel presents a set of sorts with respect to size and to one characteristic—either value (Panel A), profitability (Panel B) or investment (Panel C)—and the corresponding loading.

For example, to form the 27 portfolios in Panel A, we first perform independent sorts of all firms in our universe into three portfolios based on book-to-market (BEME) and based on size (ME). We then sort each of these nine portfolios into three subportfolios, each with an equal number of firms, based on the *ex-ante* forecast loading on HML for each firm. In the upper subpanels, the loading sorts are done using the low-power methodology; and in the lower panels use the high-power methodology.

For each of the 27 portfolios in each subpanel, we report average monthly excess returns. The column labeled "Avg." gives the average across the 9 portfolios for a given characteristic.

First, note that the average returns in the "Avg." column are consistent with empirical regularities well known in the literature: The average return of value portfolios are higher than those of growth, historically profitable firms beat unprofitable, and historically low investment firms beat high investment firms. We will present the *ex-post* loadings in a moment (in Table 4), but we will see that there are large differences between the *ex-post* betas of the low-forecast-loading ("1") and high-forecast-loading

("3") portfolios in for every size-characteristic portfolio, particularly when these sorts are done using the high-power-methodology. For the value, profitability, and investment sorts, the *ex-post* differences in loading of the "3" and "1" portfolios are 0.93, 0.76, and 1.09 respectively. Given these large differences in loadings for the high-power sorts, it is remarkable that the difference in the average monthly returns for the high- and low-loading portfolios are portfolios are 6, 14, and 2 bp/month for the value, profitability and investment-loading sorts, respectively.[19] This is consistent with the Daniel and Titman (1997) conjecture that average returns are a function of characteristics, and are unrelated to the FF factor loadings after controlling for the characteristics.

Moreover, these small observed return differences may be related to the fact that, in sorting on factor loadings, we are picking up variation in characteristics within each of the nine size-characteristic-sorted portfolios. For example, among the firms in the small-cap, high book-to-price portfolios in Panel A, there is considerable variation in book-to-market ratios. In sorting into sub-portfolios on the basis of forecast HML-factor loading, we are undoubtedly picking up variation in the characteristic of the individual firms, since characteristics and factor-loadings are highly correlated (i.e, value firms typically have high HML factor loadings).

We explore this possibility in Table 3 where we show the average of the relevant characteristic for each of the portfolios. Consistent with our hypothesis, there is generally a relation between factor loadings and characteristics within each of the nine portfolios.

[19] For comparison, the average excess returns of the HML, RMW, and CMA portfolios over the same period are 34, 28, and 23 bp/month, respectively.

4.3 Postformation loadings

We estimate the post-formation loadings by running a time series regression of the monthly excess returns for each of the portfolios on the Fama and French (2015) five factors (see equation (19)). To compare whether our high power methodology results in larger dispersion of the post-formation loadings when compared to the low power methodology, Figure 6 shows the postformation loadings for each of the 27 portfolios. Panels A and B correspond to the low and high power methodology, respectively.

Consider for example the top panels in Figure 6, which focus on the loadings on HML for each of the two estimation methodologies. There are three groups of estimates, each corresponding to a particular book-to-market bin. Each of the groups has in turn three lines, corresponding each to a particular size grouping. Finally each of those lines have three points corresponding to a particular estimate of the post-formation loading. The plot thus reports book-to-market on the y-axis for each of the 27 portfolios and the postformation loading on the x-axis. The actual point estimates for the loadings on HML, together with the corresponding t-statistics, are reported in Table 4 Panel A. To illustrate the point further focus on the loadings on HML for the large value portfolios (portfolio (3,3)). The low power methodology generates post-formation loadings on HML, β_{HML}, for each of the three portfolios of .41, .72 and 1.07, respectively. The high power methodology instead generates post-formation HML loadings of .06, .46 and .91, respectively. The last column of the panel reports the post-formation loading on HML of the portfolio that goes long the low loading portfolio and short the high loading portfolio amongst the large value firms, portfolio. The loading is $-.67$ for the low power methodology with a $t-$statistic of -9.24. For the high power methodology the same post-formation loading is $-.85$ with a $t-$statistic of -12.30.

Notice that, reassuringly, both methodologies generate a positive correlation between pre- and post-formation loadings for each of the book-to-market and size groupings.

This positive correlation between pre- and post extents to the case of CMA. But in the case of the loadings on RMW, the low power methodology does not produce a consistent positive association between pre and post-formation loadings, whereas the high power methodology does. Indeed turn to Table 4 Panel B, which reports the post-formation loadings [20] on the profitability factor, RMW, and focus on the portfolios (3,1), that is small firms with high operating profitability. The low power methodology generates loadings of .34, .38 and .32, a non monotone relation. Instead the post-formation loadings for the same set of portfolios as estimated by the high-power methodology are $-.04$, .34 and .36.

As it is readily apparent from Figure 6, the high power methodology generates substantially more cross sectional dispersion in post-formation loadings than the low power methodology, which is key to generating hedge portfolios that are maximally correlated with the candidate factor. Each of the panels of Table 4 reports the difference in the post-formation loadings between the low and and high pre-formation loading sorted portfolio for each of the characteristic-size bin. Consistently, this difference is much larger with the high power methodology than the low. In sum then our high power methodology forecasts future loadings much better than the one used by Daniel and Titman (1997) or Davis et al. (2000) and as a result they translate into better hedge portfolios as well as asset pricing tests with more power.

5 Empirical Results

In this section we describe the two main empirical results of this paper. First we show how the use of the high power methodology advanced in this paper to forecast loadings increases the power of standard asset pricing tests. We illustrate how standard methodologies used to estimate the loadings lead to a failure to reject asset

[20]The alphas of these regressions are also reported in Table 4. We will turn to the asset pricing implications in Section 5.1.

pricing models and thus impose too low a bound on the volatility of the stochastic discount factor. We do so by constructing characteristics balanced portfolios and showing that the ability of standard asset pricing models to properly account for their average returns depends critically on whether one uses the low or high power methodology.

Our second contribution is to show how to improve the Sharpe ratios of factor-portfolios by combining them optimally with the characteristics balanced portfolios. We argue that these hedged factor portfolios have a better chance of spanning the mean variance frontier than the standard factor models proposed in the literature.

5.1 Pricing the characteristics balanced hedge portfolios

We turn now to the characteristics balanced hedge portfolios. We construct them as follows. For each of the five factors in the Fama and French (2015) model we form a portfolio that goes long the portfolios with low loading on the corresponding factor, averaging across the corresponding characteristic and size, and short the high loading portfolios. For instance consider the line labeled HML in Table 5. There, we take a long position in the low loading portfolios, weighting the corresponding nine book-to-market size sorted portfolios equally, and a short position in the nine high loading portfolios in the same manner.

We then run a single time series regression of the returns of these hedge portfolios $h_{k,t}$ on the five Fama and French (2015) factor portfolios. Table 5 reports the alphas and loadings as well as the corresponding $t-$statistics. Panel A focuses on the set of hedge portfolios where preformation loadings are estimated with the low power methodology and Panel B focuses on the high power one. We first assess the hedge portfolios' ability to hedge out unpriced risk by looking at their post-formation loading on the corresponding factor. As expected, each hedge portfolio exhibits a strong negatively significant loading on their corresponding factor. For example, the hedge portfolio for HML has a loading on HML of -0.5 with a $t-$statistic of -17.05, for the low power methodology. All of these numbers are larger in magnitude for the high power methodology - in the case of HML, the loading is -0.94 now, with a $t-$statistic of -36.15. To check whether these are unpriced, as was intended by constructing the portfolios to be characteristic-neutral, we turn to the average realized excess-return of the test-portfolios. It is statistically indistinguishable from zero for all hedge portfolios.

This directly translates into pricing implications, as indicated by the alphas. Whereas, when using the low-power methodology, the five Fama and French factors price all hedge portfolios correctly, the model fails to price four out of five of the high-power long-short hedge portfolios. [21] The last line of each of the panels constructs equal weighted combinations of these portfolios. The alphas for all of them are strongly statistically significant in the high power test whereas this is not the case for the low power methodology.

5.2 Ex-ante determination of the optimal hedge-ratio

Now that the hedge portfolios' effectiveness to hedge out unpriced risk is established, the next step is to construct improved or hedged factors, i.e.,

$$f_{k,t}^{(2)} = f_{k,t}^{(1)} - \hat{\gamma}'_{\boldsymbol{k,t-1}} \boldsymbol{h_t}$$

where $k \in \{HML, RMW, CMA, SMB, MktRF\}$.

The optimal hedge ratio $\hat{\boldsymbol{\gamma}}_{\boldsymbol{k,t-1}}$ is determined ex-ante, in the spirit of equation (8).

[21] The only one for which the Fama and French model cannot be rejected is the "SMB" portfolio. The fact that Fama and French model succeeds in pricing h_{SMB} is consistent with the notion that there is little to price there, as we know that the size premium is relatively weak.

We employ the same loading forecast techniques as described before to forecast $\hat{\boldsymbol{\gamma}}_{\boldsymbol{k,t-1}}$, i.e., we first calculate five years of constant weight and constant allocation pre-formation returns of $f_{k,t}^{(1)}$ and $\boldsymbol{h_t}$. We then calculate correlations over the whole five years of 3-day overlapping return observations and variances by utilizing only the most recent 12 months of daily observations. Note that this is done in a multi-variate framework, i.e., we consider the covariance of each candidate factor portfolio with all five hedge portfolios, to account for the correlation structure among the hedge portfolios. Consequently, both $\hat{\boldsymbol{\gamma}}_{\boldsymbol{k,t-1}}$ and $\boldsymbol{h_t}$ are length-K vectors, where K=5 in the case of the Fama and French model examined here. Note furthermore, that the factor portfolios $f^(2)_{k,t}$ are (approximately) orthogonal to the hedged portfolios $h_{k,t}$. The reason why they are only approximately orthogonal is because the $\hat{\boldsymbol{\gamma}}_{\boldsymbol{k,t-1}}$ is estimated ex-ante, i.e., up to $t-1$.

5.3 Hedged Fama and French Factor Portfolios

Table 6 reports the main result of the paper. For each of the five Fama and French (2015) factors we report the annualized average returns in percentages, the annualized volatility of returns and the Sharpe ratio. The next column reports the same magnitudes for the hedge portfolios constructed in subsection 5.1 orthogonalized with respect to the five Fama and French factors. Consequently, the mean here can be interpreted as the alpha of the hedge portfolio, the volatility is the volatility of the residuals and SR can alternatively be interpreted as the information ratio of the hedge portfolio with respect to the five Fama and French factors. The last column reports the same three quantities for the improved factor portfolios, $f_t^{(2)}$. These portfolios are constructed exactly as in expression (6).[22]

When we move from $f_{k,t}^{(1)}$ to $f_{k,t}^{(2)}$, we see that the mean return of all factors decreases,

[22]We calculate the Sharpe ratio of the factor-portfolio $f_t^{(2)}$ using the usual procedure rather than using expression (9), which only holds under the null of model (1).

but the volatility also decreases considerably more. This leads to an increase in the Sharpe ratio for each of the individual Fama and French factor portfolios. For example, the squared Sharpe ratio of the improved version of HML is 0.32, where the original HML factor's squared Sharpe ratio is 0.17.

While the result that we improve on each factor portfolio individually is promising, the ultimate goal of the exercise was to move the candidate factor representation of the stochastic discount factor closer to being mean-variance efficient. Hence, in the bottom panel of Table 6, we compute the in-sample optimal combination of both the original Fama and French factors (column $f_{k,t}^{(1)}$) and the improved versions ($f_{k,t}^{(2)}$). The maximum achievable squared Sharpe ratio with the original Fama and French factors in the sample period covered in this paper (1963/07 - 2017/06) turns out to be 1.31. The squared Sharpe ratio of the optimal combination of the improved versions of these five factors is 2.29.

Notice that each individual improved factor portfolio $f_{k,t}^{(2)}$ is perfectly tradable, as all information used to construct them is known to an investor ex-ante. Only the weights of optimal combinations of the five (traditional as well as improved) factor portfolios, as reported in the bottom panel of Table 6, are calculated in-sample. Additionally, we want to emphasize that the way we construct our portfolios is very conservative, in that we only rebalance once every year. By increasing the frequency of rebalancing it is most likely possible to improve the Sharpe ratios even further.

6 Conclusions

A set of factor portfolios can only explain the cross-section of average returns if the mean-variance efficient portfolio is in the span of these factor-portfolios. There are numerous sources of information from which to construct such a set of factors. In the cross-sectional asset pricing literature, the most widely utilized source of information used to form factor-portfolios have been observable firm characteristics such as the ones we examine here: firm size, book-to-market ratio, and accounting-based measures of profitability and investment. Portfolios formed going long high-characteristic firms and short low-characteristic firms ignore the forecastable part of the covariance structure, and thus cannot explain the returns of portfolios formed using the characteristics and past-returns. Factor-portfolios formed in this way are therefore inefficient with respect to this information set.

In the empirical part of this paper, we have examined one particular model in this literature: the five-factor model of Fama and French (2015). Our empirical findings show that the factor-portfolios that underlie this model contain large unpriced components, which we show are at least correlated with unpriced factors such as industry risk. When we add information from the historical covariance structure of returns we can vastly improve the efficiency of these factor portfolios, generating a portfolio that is orthogonal to the original five factors and has a Sharpe-ratio of $2.29 - 1.31 = 0.98$. It is important to note that we are extremely conservative in the way in which we construct these hedged-portfolios: following Fama and French (1993), we form portfolios annually, and value-weight these portfolios. By hedging out the ex-ante identifiable, unpriced risk in the five-factors, we increase the annualized squared-Sharpe ratio achievable with these factors.

Hedged factors like those we construct here raise the bar for standard asset pricing tests. By the logic of Hansen and Jagannathan (1991), a pricing kernel variance of at least 2.29 (annualized) is required to explain the returns of the hedged-factor-portfolios. Also, because the hedged factor portfolios are far less correlated with industry factors, etc., they are also far less likely to be correlated with variables that might serve as plausible proxies for marginal utility.

In addition, the hedged factor portfolios we generate can serve as an efficient set of benchmark portfolios for doing performance measurement using Jensen (1968) style time-series regressions. Such an approach will deliver the same conclusions as the characteristics approach (Daniel, Grinblatt, Titman, and Wermers, 1997), while maintaining the convenience of the factor regression approach.

Figures

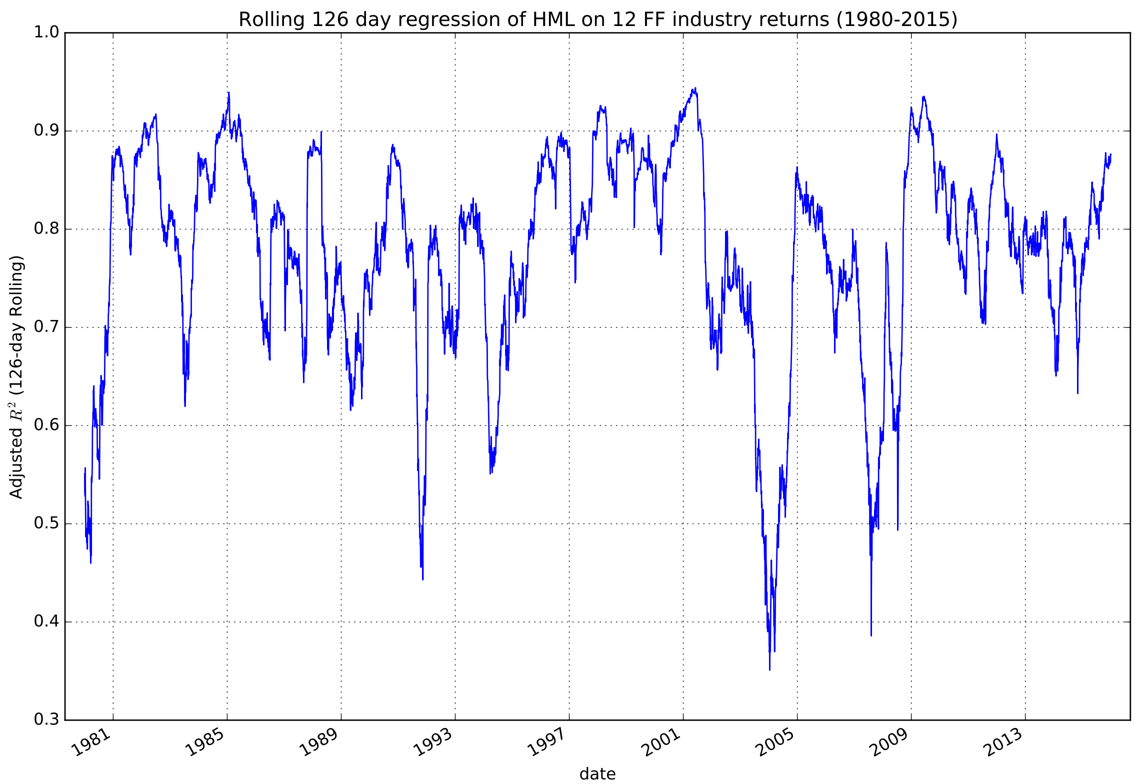

Figure 2: Rolling regression R^2s – HML returns on industry returns This figure plots the adjusted R^2 from 126-day rolling regressions of daily HML returns on the twelve daily Fama and French (1997) industry excess returns. The time period is January 1981-December 2015.

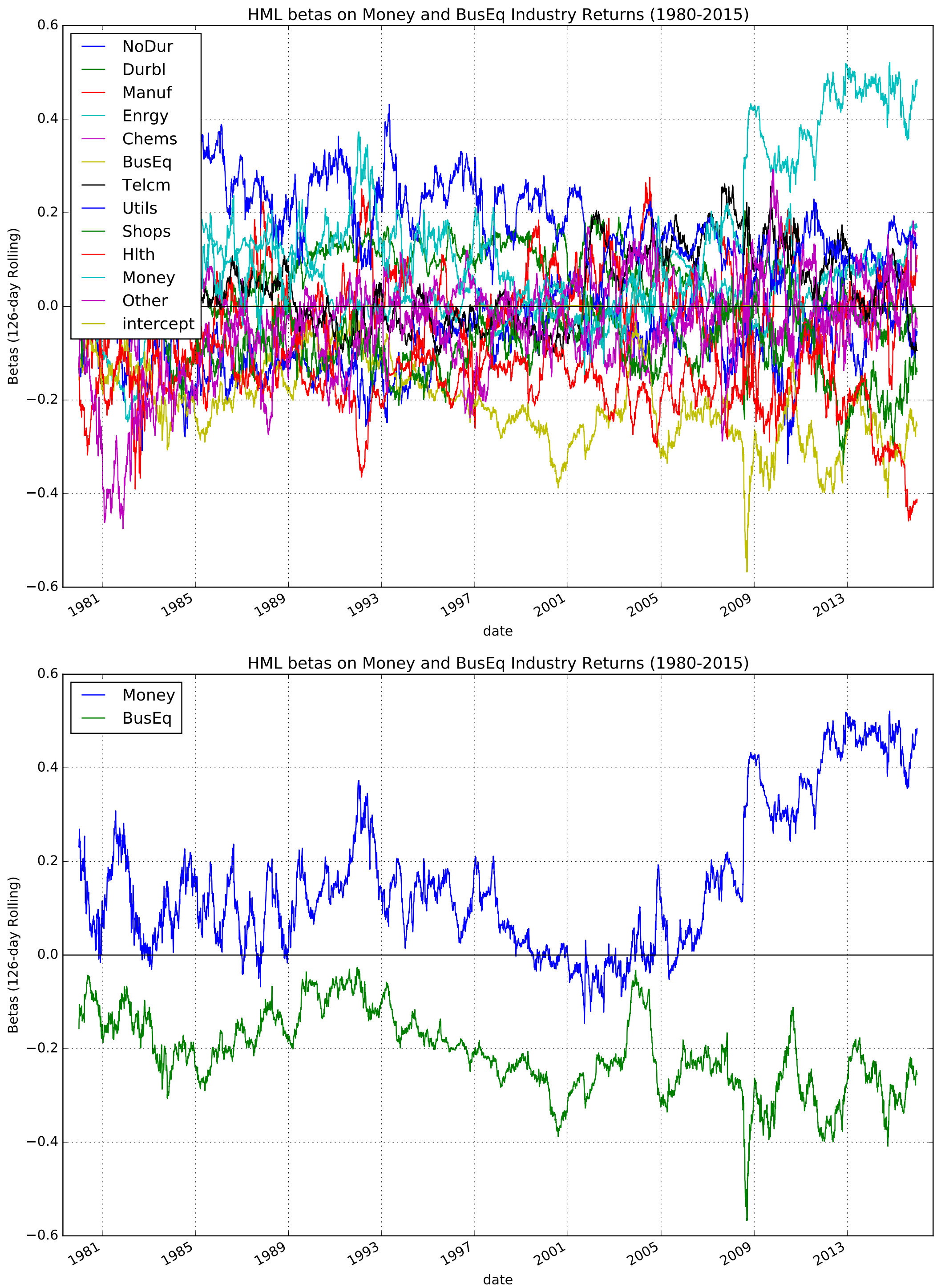

Figure 3: HML loadings on industry factors. The upper panel of this figure plots the betas from rolling 126-day regressions of the daily returns to the HML-factor portfolio on the twelve daily Fama and French (1997) industry excess returns over the January 1981-December 2015 time period. The lower panel plots only the betas for the Money and Business Equipment industry portfolios, and excludes the other 10 industry factors.

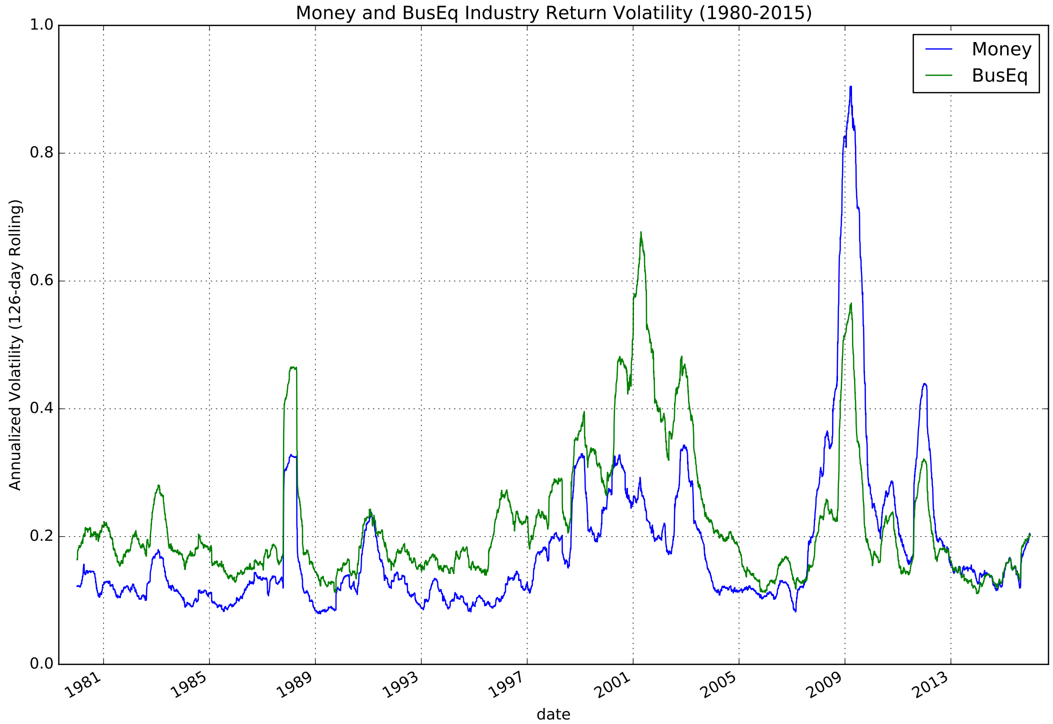

Figure 4: Volatility of the money and business equipment factors. This figure plots 126-day volatility of the daily returns to the Money and the Business Equipment factors over the January 1981-December 2015 time period.

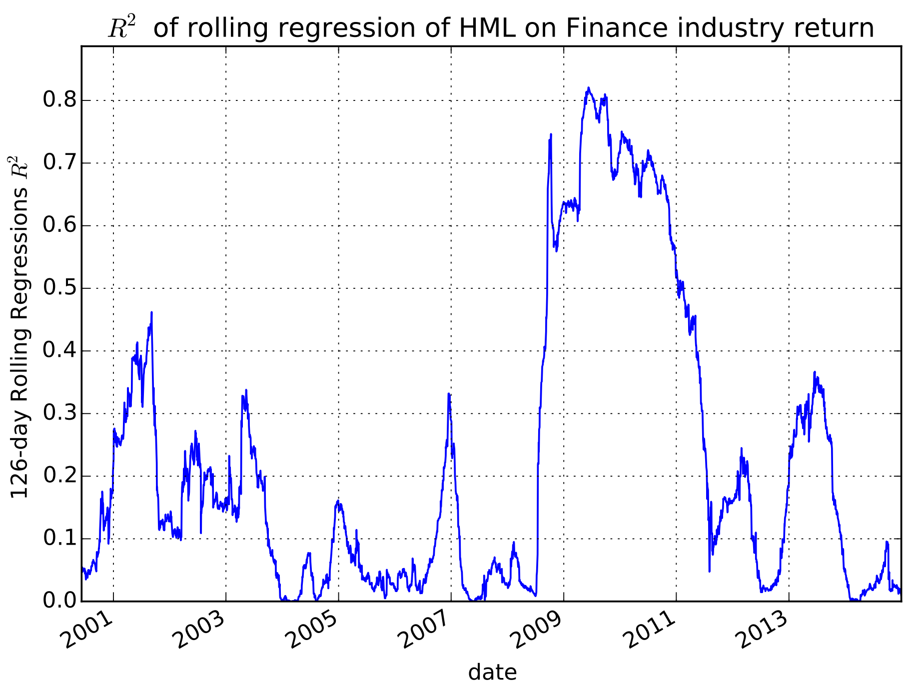

Figure 5: Rolling regression R^2s – HML returns on *Money* industry returns. This figure plots the adjusted R^2 from 126-day rolling regressions of daily HML returns on the daily *Money* industry returns from the 12 Fama and French (1997) industry returns. The time period is January 1981-December 2015.

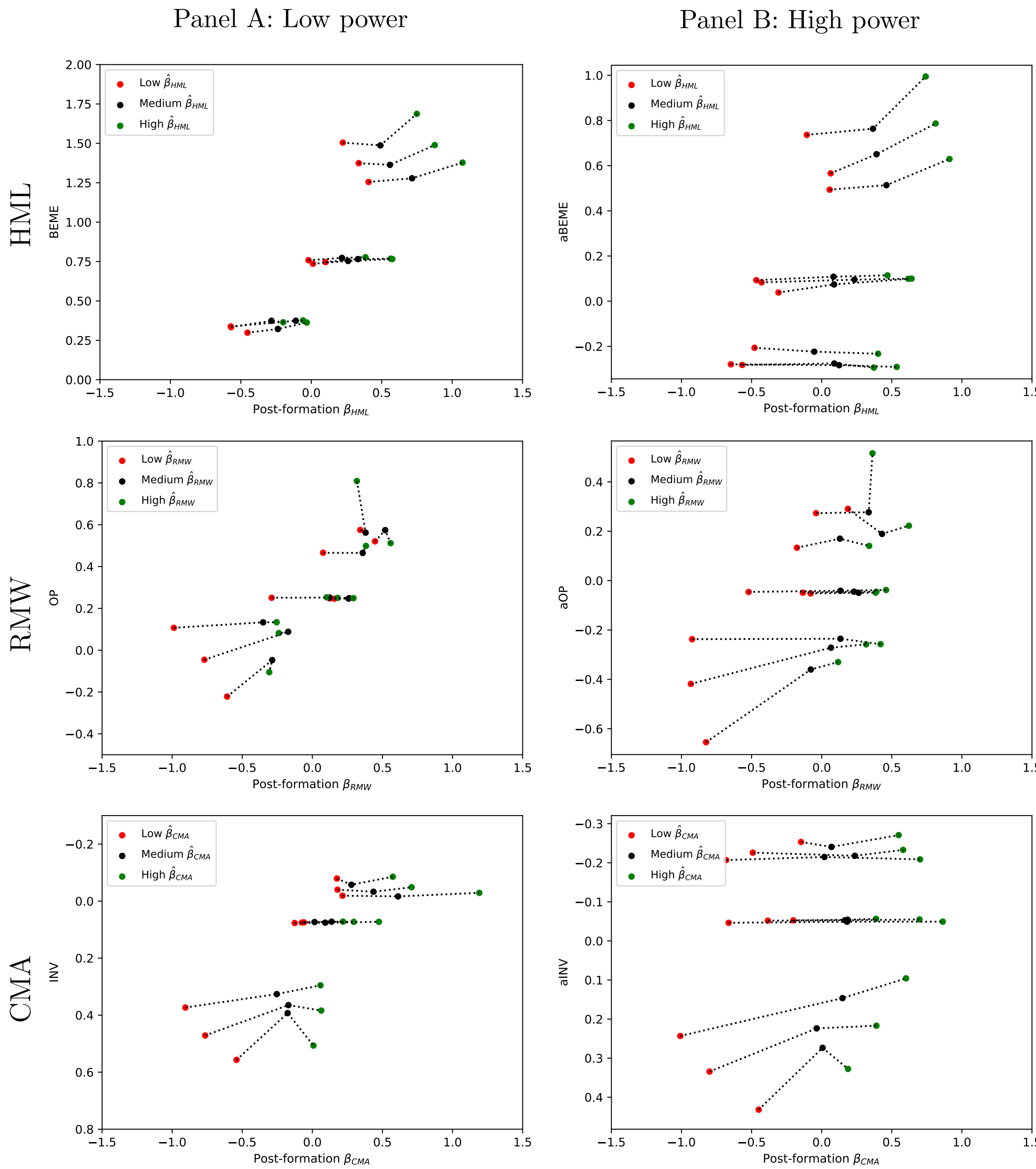

Figure 6: Ex-post loading vs. characteristic. This figure shows the time series average of post-formation factor-loading on the x-axis and the time series average of the respective characteristic on the y-axis of each of the 27 portfolios formed on size, characteristic and factor-loading. Panels A uses the low power methodology and B uses the high power methodology. The first row uses sorts on book-to-market and HML-loading, the second one operating profitability and RMW-loading and the last one investment and CMA-loading.

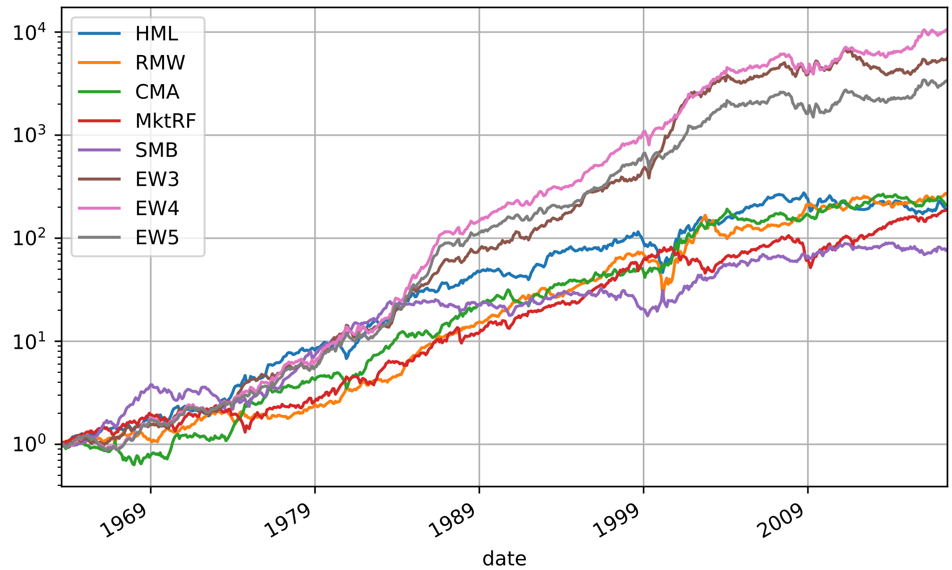

Figure 7: Portfolio Cumulative Returns. This figure plots the cumulative returns of of the five FF(2015) portfolios, and the residual portfolio. The residual portfolio is the equal-weighted combination of the HML, RMW, and CMA hedge portfolios, orthogonalized to the five-factors. Each portfolio assumes an investment of \$1 at close on the last trading day of June 1963, and earns a return of $(1+r_{LS,t}+r_{f,t})$ in each month t, where $r_{LS,t}$ is the long-short portfolio return, and $r_{f,t}$ is the one month risk free rate.

Tables

Table 1: Low book-to-market stocks in the Money industry as of June 2008. The first column reports the largest fifteen stocks in the Money industry in the low book-to-market bin by market capitalization. The second column reports the book-to-market and the third reports the HML loading portfolio to which the stock belongs as of June 30th, 2008.

Firm	BE/ME	β_{HML}-portfolio
US Bancorp	0.39	3
American Express	0.19	3
United Health	0.27	2
Aflac	0.36	2
State Street	0.36	2
Charles Schwab	0.13	1
Franklin Resources	0.27	3
Aetna	0.36	1
American Tower	0.18	3
Northern Trust	0.30	3
Price T. Rowe	0.17	2
Progressive	0.38	3
Crown Castle	0.29	3
TD Ameritrade	0.21	2
Cigna	0.34	1

Table 2: Average monthly excess returns for the test portfolios.
The sample period is 1963/07 - 2017/06. Stocks are sorted into 3 portfolios based on the respective characteristic - book-to-market (BEME), operating profitability (OP) or investment (INV) and independently into 3 size (ME) groups. These are depicted row-wise and indicated in the first two columns. Last, portfolios are sorted into 3 further porfolios based on the loadings forecast, conditional on the first two sorts. These portfolios are displayed column-wise. The last column shows average returns of all 9 respective characteristic portfolios. The last row shows averages of all 9 respective loadings portfolios. In the top panels we use the low power and in the bottom panels we use the high power methodology.

Panel A: HML

	Char-Portfolio		$\hat{\beta}_{HML}$-Portfolio			
	BEME	ME	1	2	3	Avg.
Low power	1	1	0.47	0.58	0.53	0.54
		2	0.48	0.63	0.65	
		3	0.57	0.38	0.56	
	2	1	0.87	0.86	0.93	0.73
		2	0.8	0.71	0.77	
		3	0.55	0.44	0.61	
	3	1	1.06	1.01	1	0.87
		2	0.94	0.9	0.98	
		3	0.73	0.6	0.66	
	Avg.		0.72	0.68	0.74	
High power	1	1	0.37	0.57	0.68	0.53
		2	0.49	0.62	0.72	
		3	0.41	0.5	0.41	
	2	1	0.78	0.84	0.91	0.72
		2	0.64	0.8	0.8	
		3	0.55	0.6	0.55	
	3	1	1.05	0.98	1.02	0.88
		2	0.98	0.83	1.07	
		3	0.81	0.64	0.57	
	Avg.		0.68	0.71	0.75	

Panel B: RMW

	Char-Portfolio		$\hat{\beta}_{RMW}$-Portfolio			
	OP	ME	1	2	3	Avg.
Low power	1	1	0.63	0.8	0.67	0.57
		2	0.64	0.62	0.66	
		3	0.21	0.42	0.47	
	2	1	0.87	0.86	0.82	0.69
		2	0.76	0.71	0.78	
		3	0.49	0.37	0.59	
	3	1	0.92	1.01	0.96	0.78
		2	0.74	0.77	0.94	
		3	0.59	0.45	0.62	
	Avg.		0.65	0.67	0.72	
High power	1	1	0.57	0.83	0.82	0.65
		2	0.61	0.67	0.83	
		3	0.28	0.55	0.65	
	2	1	0.86	0.95	0.88	0.7
		2	0.69	0.7	0.84	
		3	0.4	0.48	0.54	
	3	1	0.91	0.92	0.99	0.74
		2	0.78	0.74	0.85	
		3	0.51	0.47	0.49	
	Avg.		0.62	0.7	0.76	

Panel C: CMA

	Char-Portfolio		$\hat{\beta}_{CMA}$-Portfolio			
	INV	ME	1	2	3	Avg.
Low power	1	1	0.89	0.97	0.93	0.79
		2	0.76	0.85	0.8	
		3	0.55	0.71	0.66	
	2	1	0.97	0.88	0.97	0.76
		2	0.83	0.79	0.88	
		3	0.52	0.42	0.6	
	3	1	0.57	0.68	0.58	0.58
		2	0.5	0.68	0.77	
		3	0.46	0.41	0.59	
	Avg.		0.67	0.71	0.75	
High power	1	1	0.83	0.93	0.95	0.76
		2	0.82	0.92	0.73	
		3	0.55	0.52	0.55	
	2	1	0.95	0.97	0.93	0.75
		2	0.84	0.85	0.75	
		3	0.42	0.44	0.58	
	3	1	0.57	0.73	0.63	0.62
		2	0.6	0.71	0.68	
		3	0.53	0.52	0.62	
	Avg.		0.68	0.73	0.71	

Table 3: Average monthly characteristics for the test portfolios.

Stocks are sorted into 3 portfolios based on the respective characteristic - book-to-market (BEME), operating profitability (OP) or investment (INV) and independently into 3 size (ME) groups. These are depicted row-wise and indicated in the first two columns. Last, portfolios are sorted into 3 further porfolios based on the loadings forecast, conditional on the first two sorts. These portfolios are displayed column-wise. At each yearly formation date, the average respective characteristic (BEME, OP, or INV) for each portfolio is calculated, using value weighting. At each point, the characteristic is divided by the NYSE median at that point in time. The time series from 1963 - 2016 is then averaged to get the numbers that are presented in the table below. Note that the characteristics reported in the high power panels are industry-adjusted, i.e., for each firm we first subtract the value-weighted average characteristic of its correspinding industry. The last column shows average characteristics of all 9 respective characteristic portfolios. The last row shows averages of all 9 respective loadings portfolios. In the top panels we use the low power and in the bottom panels we use the high power methodology.

Panel A: HML

Low power

Char-Portfolio		$\hat{\beta}_{HML}$-Portfolio			
BEME	ME	1	2	3	Avg.
1	1	0.43	0.49	0.48	0.46
	2	0.44	0.49	0.49	
	3	0.39	0.42	0.47	
2	1	1	1.02	1.02	1
	2	0.98	1.01	1.01	
	3	0.97	0.99	1.01	
3	1	1.99	1.97	2.25	1.89
	2	1.82	1.82	1.99	
	3	1.68	1.7	1.84	
Avg.		1.08	1.1	1.17	

High power

Char-Portfolio		$\hat{\beta}_{HML}$-Portfolio			
BEME	ME	1	2	3	Avg.
1	1	-4.68	-4.68	-4.84	-4.42
	2	-4.62	-4.76	-4.72	
	3	-3.58	-3.9	-3.98	
2	1	1.01	1.23	1.26	0.98
	2	0.87	1.05	1.07	
	3	0.31	0.86	1.18	
3	1	10.52	10.81	13.59	9.59
	2	8.14	9.25	11.15	
	3	7.02	7.4	8.46	
Avg.		1.67	1.92	2.57	

Panel B: RMW

Low power

Char-Portfolio		$\hat{\beta}_{RMW}$-Portfolio			
OP	ME	1	2	3	Avg.
1	1	-0.95	-0.21	-0.45	0.04
	2	-0.22	0.34	0.32	
	3	0.43	0.53	0.54	
2	1	1	1	1.01	1.01
	2	1	1.01	1.01	
	3	1.01	1.02	1.02	
3	1	2.36	2.3	3.36	2.27
	2	2.13	2.33	2.09	
	3	1.9	1.9	2.03	
Avg.		0.96	1.13	1.21	

High power

Char-Portfolio		$\hat{\beta}_{RMW}$-Portfolio			
OP	ME	1	2	3	Avg.
1	1	16.8	8.89	8.17	8
	2	9.49	6.3	5.85	
	3	5.52	5.31	5.71	
2	1	1.14	1.08	1.04	1
	2	1.09	0.97	0.96	
	3	1.03	0.89	0.8	
3	1	-6.88	-6.71	-10.21	-5.79
	2	-6.92	-4.73	-5.54	
	3	-3.6	-4.07	-3.44	
Avg.		1.96	0.88	0.37	

Panel C: CMA

Low power

Char-Portfolio		$\hat{\beta}_{CMA}$-Portfolio			
INV	ME	1	2	3	Avg.
1	1	-2.36	-1.89	-2.49	-1.66
	2	-1.58	-1.46	-1.78	
	3	-1.08	-0.89	-1.42	
2	1	1.06	1.03	1.02	1.05
	2	1.06	1.05	0.98	
	3	1.09	1.12	1.02	
3	1	10.27	7.21	9.89	7.71
	2	8.73	6.7	6.97	
	3	7.81	5.91	5.88	
Avg.		2.78	2.09	2.23	

High power

Char-Portfolio		$\hat{\beta}_{CMA}$-Portfolio			
INV	ME	1	2	3	Avg.
1	1	5.12	4.91	5.53	4.59
	2	4.48	4.34	4.71	
	3	4.02	4.15	4.07	
2	1	1	1.05	1.11	1
	2	1	1.02	1.07	
	3	0.85	0.93	0.94	
3	1	-8.73	-5.74	-6.78	-5.13
	2	-6.66	-4.6	-4.32	
	3	-4.41	-3	-1.97	
Avg.		-0.37	0.34	0.49	

Table 4: Sorting-factor exposures and five-factor alphas.

The last column shows the return of long low-loading short high-loading hedge-portfolios. The last row shows averages of all 9 loadings portfolios. In the top panels we use the low power and in the bottom panels we use the high power methodology. Alphas and ex-post loadings on the relevant factor are obtained from a regression of monthly excess returns of the test-portfolios on the 5 Fama and French factors from 1963/07 - 2017/06.

Panel A: HML

Low power

Char-Portfolio		pre-formation $\hat{\beta}_{\text{HML}}$-sorted portfolios							
BE/ME	ME	1	2	3	1-3	1	2	3	1-3
		α				$t(\alpha)$			
1	1	0.01	-0.06	-0.22	0.23	0.12	-0.93	-2.95	2.04
	2	0.03	-0.08	-0.07	0.10	0.38	-1.25	-0.96	0.99
	3	0.10	-0.09	0.12	-0.03	1.46	-1.64	1.87	-0.24
2	1	0.05	-0.00	0.06	-0.01	0.82	-0.02	1.05	-0.07
	2	-0.01	-0.13	-0.09	0.08	-0.20	-1.98	-1.25	0.83
	3	-0.06	-0.19	-0.06	-0.00	-0.73	-2.41	-0.64	-0.03
3	1	0.18	0.10	-0.05	0.23	2.74	2.02	-0.83	2.55
	2	0.06	0.03	0.01	0.06	0.80	0.47	0.06	0.44
	3	-0.02	-0.18	-0.04	0.02	-0.25	-2.20	-0.40	0.12
Avg. Portfolio		0.04	-0.07	-0.04	0.07	0.97	-2.17	-0.95	1.19
		post-formation β_{HML}				$t(\beta_{HML})$			
1	1	-0.57	-0.28	-0.20	-0.37	-13.45	-9.24	-5.72	-6.99
	2	-0.57	-0.11	-0.06	-0.51	-15.65	-3.78	-1.81	-11.30
	3	-0.45	-0.24	-0.03	-0.42	-14.43	-9.20	-1.02	-8.12
2	1	-0.02	0.22	0.38	-0.40	-0.64	8.78	14.42	-10.09
	2	0.10	0.33	0.56	-0.46	3.19	10.85	16.43	-10.54
	3	0.01	0.26	0.58	-0.56	0.31	7.15	13.82	-9.52
3	1	0.22	0.49	0.75	-0.53	7.36	21.27	25.23	-12.42
	2	0.34	0.56	0.88	-0.54	9.51	17.37	21.16	-9.19
	3	0.41	0.72	1.07	-0.67	9.60	19.04	22.78	-9.24
Avg. Portfolio		-0.06	0.22	0.44	-0.50	-3.30	15.21	23.75	-16.86

High power

Char-Portfolio		pre-formation $\hat{\beta}_{\text{HML}}$-sorted portfolios							
BE/ME	ME	1	2	3	1-3	1	2	3	1-3
		α				$t(\alpha)$			
1	1	-0.10	-0.18	-0.21	0.11	-1.00	-2.65	-2.99	0.91
	2	0.08	-0.17	-0.15	0.22	0.95	-2.43	-2.01	2.02
	3	0.02	-0.09	-0.14	0.16	0.36	-1.57	-1.97	1.47
2	1	0.20	0.05	0.03	0.17	2.65	0.87	0.54	1.75
	2	0.07	-0.01	-0.01	0.08	0.94	-0.23	-0.17	0.85
	3	0.15	0.03	-0.07	0.23	2.25	0.41	-0.96	1.98
3	1	0.26	0.07	-0.10	0.35	3.56	1.41	-1.59	3.73
	2	0.15	-0.08	0.04	0.11	1.78	-1.10	0.46	0.91
	3	0.15	-0.19	-0.19	0.34	1.58	-2.19	-1.80	2.26
Avg. Portfolio		0.11	-0.06	-0.09	0.20	2.96	-1.82	-2.38	3.48
		post-formation β_{HML}				$t(\beta_{HML})$			
1	1	-0.57	0.09	0.37	-0.94	-12.23	2.79	11.58	-17.08
	2	-0.65	0.12	0.54	-1.18	-16.91	3.88	15.78	-22.86
	3	-0.48	-0.05	0.40	-0.88	-16.58	-1.95	12.46	-17.52
2	1	-0.47	0.08	0.47	-0.94	-13.30	3.14	16.92	-21.01
	2	-0.43	0.23	0.64	-1.07	-11.87	7.72	20.04	-23.11
	3	-0.31	0.09	0.62	-0.93	-9.62	2.74	17.65	-17.42
3	1	-0.11	0.37	0.74	-0.85	-3.17	14.90	25.99	-19.19
	2	0.06	0.39	0.81	-0.75	1.59	11.12	19.93	-12.85
	3	0.06	0.46	0.91	-0.85	1.31	11.31	18.57	-12.30
Avg. Portfolio		-0.32	0.20	0.61	-0.93	-18.58	12.06	35.68	-35.37

Panel B: RMW

Low power

Char-Portfolio		pre-formation $\hat{\beta}_{\text{HML}}$-sorted portfolios							
OP	ME	1	2	3	1-3	1	2	3	1-3
		α				$t(\alpha)$			
1	1	-0.02	0.07	-0.07	0.05	-0.27	1.22	-1.11	0.58
	2	0.22	-0.02	0.00	0.22	2.30	-0.31	0.03	1.87
	3	0.04	-0.02	-0.05	0.09	0.45	-0.34	-0.61	0.70
2	1	0.01	-0.01	-0.09	0.10	0.18	-0.12	-1.50	1.22
	2	0.02	-0.07	0.01	0.01	0.31	-1.05	0.14	0.14
	3	0.12	-0.24	-0.00	0.12	1.41	-3.51	-0.02	0.98
3	1	-0.03	0.06	0.01	-0.05	-0.42	1.05	0.22	-0.46
	2	-0.14	-0.13	-0.00	-0.14	-2.03	-2.06	-0.04	-1.46
	3	0.16	-0.07	0.07	0.08	2.34	-1.30	1.06	0.73
Avg. Portfolio		0.04	-0.05	-0.01	0.05	1.13	-1.72	-0.38	1.00
		post-formation β_{RMW}				$t(\beta_{RMW})$			
1	1	-0.61	-0.29	-0.31	-0.30	-15.43	-10.27	-9.11	-6.62
	2	-0.77	-0.17	-0.24	-0.53	-15.42	-4.90	-6.42	-8.68
	3	-0.99	-0.35	-0.25	-0.73	-21.73	-9.42	-6.13	-11.10
2	1	0.16	0.26	0.29	-0.14	4.56	8.75	9.53	-3.18
	2	0.12	0.26	0.18	-0.06	3.44	7.97	5.28	-1.23
	3	-0.29	0.12	0.10	-0.39	-6.70	3.47	2.65	-6.24
3	1	0.34	0.38	0.32	0.02	8.86	12.12	8.85	0.47
	2	0.45	0.52	0.56	-0.11	12.10	15.83	14.29	-2.21
	3	0.08	0.36	0.38	-0.31	2.20	12.05	10.68	-5.20
Avg. Portfolio		-0.17	0.12	0.11	-0.28	-8.80	8.31	6.63	-10.03

High power

Char-Portfolio		pre-formation $\hat{\beta}_{\text{HML}}$-sorted portfolios							
OP	ME	1	2	3	1-3	1	2	3	1-3
		α				$t(\alpha)$			
1	1	0.02	0.02	-0.15	0.17	0.25	0.30	-2.55	1.71
	2	0.31	-0.09	-0.08	0.39	3.16	-1.29	-1.07	3.19
	3	0.10	-0.03	-0.04	0.14	1.18	-0.40	-0.42	1.08
2	1	0.07	0.12	-0.10	0.17	0.95	1.86	-1.46	1.76
	2	0.05	-0.05	-0.06	0.11	0.66	-0.79	-0.87	1.15
	3	0.06	-0.05	-0.14	0.20	0.91	-0.81	-2.15	1.91
3	1	0.12	0.01	0.01	0.11	1.49	0.17	0.13	0.98
	2	0.03	-0.08	-0.18	0.21	0.35	-1.21	-2.26	1.92
	3	0.13	0.01	-0.03	0.16	1.83	0.18	-0.49	1.44
Avg. Portfolio		0.10	-0.02	-0.08	0.18	2.57	-0.49	-2.59	3.30
		post-formation β_{RMW}				$t(\beta_{RMW})$			
1	1	-0.82	-0.08	0.12	-0.94	-18.54	-2.76	3.90	-18.39
	2	-0.93	0.07	0.32	-1.25	-18.29	1.82	8.21	-19.70
	3	-0.92	0.13	0.42	-1.34	-20.22	3.14	9.04	-19.79
2	1	-0.08	0.26	0.38	-0.46	-2.11	7.85	11.03	-9.44
	2	-0.13	0.23	0.39	-0.52	-3.49	7.13	11.36	-10.89
	3	-0.52	0.13	0.46	-0.98	-14.03	4.16	13.94	-17.93
3	1	-0.04	0.34	0.36	-0.40	-0.93	9.59	9.17	-6.62
	2	0.19	0.43	0.62	-0.44	4.66	12.65	15.11	-7.80
	3	-0.18	0.13	0.34	-0.52	-4.70	4.32	10.35	-8.72
Avg. Portfolio		-0.38	0.18	0.38	-0.76	-18.90	10.90	22.42	-26.22

Panel C: CMA

Low power

Char-Portfolio		pre-formation $\hat{\beta}_{\text{HML}}$-sorted portfolios							
INV	ME	1	2	3	1-3	1	2	3	1-3
		α				$t(\alpha)$			
1	1	0.04	0.09	0.11	-0.07	0.63	1.87	1.51	-0.73
	2	-0.04	0.03	-0.08	0.04	-0.54	0.45	-1.01	0.37
	3	-0.07	0.01	-0.12	0.05	-0.84	0.08	-1.52	0.40
2	1	0.11	0.03	0.17	-0.06	1.70	0.43	2.50	-0.65
	2	0.07	0.02	0.11	-0.04	1.07	0.29	1.68	-0.46
	3	0.03	-0.10	-0.04	0.07	0.43	-1.67	-0.59	0.65
3	1	-0.18	-0.15	-0.13	-0.05	-2.54	-2.78	-2.08	-0.57
	2	-0.13	-0.07	0.12	-0.24	-1.49	-1.07	1.63	-2.17
	3	0.28	-0.10	0.08	0.21	3.21	-1.49	1.12	1.54
Avg. Portfolio		0.01	-0.03	0.02	-0.01	0.35	-0.93	0.70	-0.18
		post-formation β_{CMA}				$t(\beta_{CMA})$			
1	1	0.18	0.28	0.57	-0.40	3.31	7.30	10.46	-5.89
	2	0.18	0.44	0.71	-0.53	3.19	8.94	11.91	-6.62
	3	0.22	0.61	1.19	-0.98	3.54	11.67	20.60	-10.45
2	1	-0.06	0.14	0.30	-0.36	-1.24	3.02	5.84	-5.18
	2	-0.07	0.02	0.22	-0.29	-1.47	0.31	4.34	-4.41
	3	-0.13	0.09	0.47	-0.60	-2.27	1.97	9.66	-7.37
3	1	-0.54	-0.18	0.01	-0.55	-9.88	-4.18	0.17	-7.74
	2	-0.76	-0.17	0.06	-0.83	-12.03	-3.68	1.19	-9.87
	3	-0.91	-0.25	0.06	-0.96	-13.66	-4.84	1.14	-9.58
Avg. Portfolio		-0.21	0.11	0.40	-0.61	-7.35	4.74	15.32	-13.59

High power

Char-Portfolio		pre-formation $\hat{\beta}_{\text{HML}}$-sorted portfolios							
INV	ME	1	2	3	1-3	1	2	3	1-3
		α				$t(\alpha)$			
1	1	0.04	0.13	0.14	-0.10	0.64	2.46	1.72	-0.92
	2	0.20	0.08	-0.10	0.31	2.63	1.10	-1.24	2.79
	3	0.31	-0.02	-0.09	0.40	2.99	-0.25	-1.20	2.87
2	1	0.09	0.12	0.03	0.06	1.35	1.74	0.46	0.57
	2	0.15	0.03	-0.11	0.26	2.02	0.48	-1.41	2.47
	3	0.12	-0.12	-0.12	0.25	1.60	-1.74	-1.92	2.15
3	1	-0.24	-0.09	-0.14	-0.09	-3.06	-1.59	-2.05	-0.85
	2	-0.04	-0.08	-0.05	0.01	-0.52	-1.01	-0.64	0.05
	3	0.36	-0.10	-0.13	0.49	4.34	-1.46	-1.65	3.69
Avg. Portfolio		0.11	-0.01	-0.06	0.17	2.73	-0.16	-1.71	2.67
		post-formation β_{CMA}				$t(\beta_{CMA})$			
1	1	-0.15	0.07	0.55	-0.70	-2.86	1.70	8.77	-8.65
	2	-0.49	0.24	0.58	-1.07	-8.36	4.50	9.38	-12.93
	3	-0.68	0.02	0.70	-1.38	-8.79	0.33	12.05	-13.17
2	1	-0.20	0.18	0.39	-0.59	-4.06	3.61	7.21	-7.89
	2	-0.38	0.17	0.70	-1.08	-7.00	3.57	11.96	-13.82
	3	-0.66	0.18	0.86	-1.53	-11.36	3.49	18.13	-17.77
3	1	-0.45	0.01	0.19	-0.64	-7.72	0.15	3.54	-7.79
	2	-0.80	-0.04	0.39	-1.19	-12.62	-0.62	6.75	-13.66
	3	-1.01	0.15	0.60	-1.61	-15.94	2.93	10.24	-16.01
Avg. Portfolio		-0.54	0.11	0.55	-1.09	-17.48	4.11	19.64	-22.03

Table 5: Results of time-series regressions on characteristics-balanced hedge-portfolios.

Stocks are first sorted based on size and one of book-to-market, profitability or investment into 3x3 portfolios. Conditional on those sorts, they are subsequently sorted into 3 portfolios based on the respective loading, i.e., on HML, RMW or CMA. For Mkt-RF and SMB we use the prior sort on size and book-to-market. The "hedge-portfolio" then goes long the low loading and short the high loading portfolios. On the bottom, we form combination-portfolios that put equal weight on three (HML, RMW, CMA), four (HML, RMW, CMA, Mkt-RF) or five (HML, RMW, CMA, Mkt-RF, SMB) hedge-portfolios portfolios. Monthly returns of these portfolios are then regressed on the 5 Fama and French factors in the sample period from 1963/07 - 2017/06. In Panel A we use the low power and in Panel B we use the high power methodology.

Panel A: Low power

Hedge-Portfolio	Avg.	α	β_{Mkt-RF}	β_{SMB}	β_{HML}	β_{RMW}	β_{CMA}	R^2
h_{MktRF}	-0.07	0.09	-0.27	-0.17	0.07	0.08	-0.07	0.5
	(-0.8)	(1.38)	(-17.35)	(-7.77)	(2.44)	(2.36)	(-1.38)	
h_{SMB}	-0.1	-0.02	-0.12	-0.34	0.06	0.05	0.13	0.56
	(-1.34)	(-0.28)	(-9.36)	(-18.39)	(2.44)	(1.93)	(3.29)	
h_{HML}	-0.02	0.08	-0.02	-0.02	-0.5	-0.04	0.42	0.35
	(-0.23)	(1.33)	(-1.61)	(-0.74)	(-17.05)	(-1.16)	(8.94)	
h_{RMW}	-0.07	0.05	0.05	-0.06	-0.14	-0.28	-0.01	0.29
	(-1.19)	(0.95)	(3.6)	(-3.22)	(-5.76)	(-10.02)	(-0.29)	
h_{CMA}	-0.08	-0.01	-0.01	-0.01	0.27	-0.04	-0.61	0.26
	(-1.26)	(-0.24)	(-0.78)	(-0.37)	(9.89)	(-1.15)	(-13.8)	
$(h_{HML} + h_{RMW} + h_{CMA})/3$	-0.06	0.04	0	-0.03	-0.12	-0.12	-0.07	0.37
	(-1.69)	(1.43)	(0.55)	(-2.86)	(-9.44)	(-8.04)	(-3.2)	
$(h_{HML} + h_{RMW} + h_{CMA} + h_{MktRF})/4$	-0.06	0.05	-0.06	-0.06	-0.07	-0.07	-0.07	0.25
	(-1.9)	(1.81)	(-9.14)	(-6.38)	(-5.63)	(-4.64)	(-3.12)	
$(h_{HML} + h_{RMW} + h_{CMA} + h_{MktRF} + h_{SMB})/5$	-0.06	0.04	-0.06	-0.1	-0.06	-0.06	-0.04	0.29
	(-2.07)	(1.45)	(-9.14)	(-10.12)	(-4.77)	(-3.99)	(-1.82)	

Panel B: High power

Hedge-Portfolio	Avg.	α	β_{Mkt-RF}	β_{SMB}	β_{HML}	β_{RMW}	β_{CMA}	R^2
h_{MktRF}	-0.07	0.22	-0.42	-0.38	-0.04	0.13	0.05	0.65
	(-0.57)	(2.81)	(-21.97)	(-14.31)	(-1.09)	(3.1)	(0.81)	
h_{SMB}	-0.18	-0.02	-0.18	-0.56	0.03	0.15	0.13	0.73
	(-1.72)	(-0.41)	(-12.49)	(-28.34)	(1.15)	(5.0)	(3.12)	
h_{HML}	-0.07	0.2	-0.01	0.08	-0.94	-0.24	0.45	0.77
	(-0.66)	(3.57)	(-0.96)	(4.34)	(-36.15)	(-8.35)	(10.57)	
h_{RMW}	-0.14	0.18	0.04	-0.06	-0.27	-0.76	-0.05	0.67
	(-1.5)	(3.25)	(2.67)	(-3.32)	(-10.55)	(-26.26)	(-1.1)	
h_{CMA}	-0.03	0.17	-0.02	-0.03	0.3	-0.11	-1.09	0.49
	(-0.29)	(2.67)	(-1.26)	(-1.51)	(10.09)	(-3.11)	(-22.29)	
$(h_{HML} + h_{RMW} + h_{CMA})/3$	-0.08	0.18	0	-0	-0.3	-0.37	-0.23	0.78
	(-1.21)	(5.79)	(0.13)	(-0.4)	(-20.48)	(-22.22)	(-9.56)	
$(h_{HML} + h_{RMW} + h_{CMA} + h_{MktRF})/4$	-0.08	0.19	-0.1	-0.1	-0.24	-0.25	-0.16	0.65
	(-1.58)	(6.34)	(-13.9)	(-9.43)	(-16.81)	(-15.5)	(-7.0)	
$(h_{HML} + h_{RMW} + h_{CMA} + h_{MktRF} + h_{SMB})/5$	-0.09	0.16	-0.1	-0.15	-0.21	-0.21	-0.13	0.61
	(-2.0)	(5.36)	(-13.44)	(-14.6)	(-14.96)	(-13.22)	(-5.67)	

Table 6: Sharpe Ratio improvement.
For each of the five Fama and French (2015) factor portfolios, its corresponding hedge portfolio portfolio (orthogonalized with respect to the five original factors), and the resulting improved factor portfolio, we report the annualized average return in percentages, annualized volatility of returns and the corresponding squared Sharpe ratio. The bottom panel reports the statistics for the in-sample optimal combination of the original and the improved five factors. The sample period is 1963/07 - 2017/06.

	$f_k^{(1)}$	$h_k^{(1)} - \boldsymbol{\beta^T f^{(1)}}$	$f_k^{(2)}$
HML			
Mean	4.09	2.39	2.62
Vol	9.93	4.65	4.60
SR^2	0.17	0.26	0.32
RMW			
Mean	3.40	2.15	2.31
Vol	7.48	4.59	4.45
SR^2	0.21	0.22	0.27
CMA			
Mean	2.77	2.06	2.27
Vol	6.50	5.38	3.83
SR^2	0.18	0.15	0.35
SMB			
Mean	3.14	-0.28	2.50
Vol	10.38	4.74	6.56
SR^2	0.09	0.00	0.15
MktRF			
Mean	6.22	2.60	5.79
Vol	15.07	6.43	10.13
SR^2	0.17	0.16	0.33
In-sample optimal combination			
Mean	3.52		2.74
Vol	3.08		1.81
SR^2	1.31		2.29

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