Measuring Skewness Premia*

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Abstract

We provide a new methodology to empirically investigate the respective roles of systematic and idiosyncratic skewness in explaining expected stock returns. Using a large number of predictors, we forecast the *cross-sectional ranks* of systematic and idiosyncratic skewness which are easier to predict than their actual values. Compared to other measures of *ex ante* systematic skewness, our forecasts create a significant spread in *ex post* systematic skewness. A predicted systematic skewness risk factor carries a significant risk premium that ranges from 7% to 12% per year and is robust to the inclusion of downside beta, size, value, momentum, profitability, and investment factors. In contrast to systematic skewness, the role of idiosyncratic skewness in pricing stocks is less robust. Finally, we document how the determinants of systematic and idiosyncratic skewness differ.

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1 Introduction

Stocks with positively skewed returns are attractive to most investors because they occasionally pay large returns. Stocks with negatively skewed returns are less attractive because they sometimes drastically fall in value. However, not all kinds of return skewness are equal. Stocks with higher *systematic* return skewness are appealing because they offer defensive returns during bad times; these stocks provide downside protection. On the other hand, stocks with positive *idiosyncratic* skewness are sought for their potential for high returns regardless of broad market movements; these stocks provide a lottery payoff.

In studying the theoretical link between skewness and asset prices, previous research has proposed models in which only systematic skewness carries a risk premium (see Rubinstein, 1973; Kraus and Litzenberger, 1976; Harvey and Siddique, 2000) and models in which total skewness—both systematic and idiosyncratic—is important in pricing securities (see Brunnermeier, Gollier, and Parker, 2007; Barberis and Huang, 2008; Mitton and Vorkink, 2007).[1] These models mainly differ by their assumptions about investors' preferences.

In this paper, we empirically investigate the respective importance of systematic and idiosyncratic skewness in explaining differences in expected returns across stocks. Answering this question requires to measure each type of skewness. While measuring return skewness is a challenging task, distinguishing between types of skewnesses is harder still. Two broad methodologies exist. First, one can use past return skewness to predict its future value. But given the low time-series persistence of skewness measures, this methodology is bound to generate poor results. In their textbook treatment of this literature, Bali, Engle, and Murray (2016) (p.330) state that "these variables are likely to be very noisy, perhaps to the point of being ineffective at measuring the characteristics of the stock that they are designed to capture".

A second methodology is to measure skewness from option prices, but this approach also suffers from limitations. The risk-neutral skewness obtained from options needs to be translated to physical skewness at the cost of some assumption on how the two are related. Similarly, extracting the systematic part of option-implied skewness requires further assumptions.[2] In addition, given the availability of option data, empirical tests are restricted

[1] For systematic skewness, see also Simaan (1993), Dittmar (2002), Dahlquist, Farago, and Tedongap (2017), and Chabi-Yo, Leisen, and Renault (2014).

[2] Following Bakshi, Kapadia, and Madan (2003), Conrad, Dittmar, and Ghysels (2013) assume a one-factor structure under the risk-neutral measure to recover risk-neutral systematic skewness from option-implied moments of individual stocks and of the market portfolio. Because there are no options traded on the market portfolio implied by theoretical models, they use S&P 500 Index options as a proxy. Schneider, Wagner, and Zechner (2017) rely on a structural model of levered firms to empirically relate option-implied skewness to systematic skewness.

to a shorter period and a smaller cross-section than when using only returns.[3]

This paper develops a novel methodology for predicting differences in systematic and idiosyncratic skewness across stocks. We use our methodology to build a predicted systematic skewness factor and idiosyncratic skewness sorted portfolios as well as to describe which variables best predict each type of skewness. The predicted systematic skewness factor captures future systematic skewness risk better than other measures, is distinct from leading equity risk factors, and carries a significant risk premium. In contrast, we find weaker evidence that predicted idiosyncratic skewness is priced in U.S. stocks.

Our results are important because previous research has found that idiosyncratic skewness has a more robust pricing importance than systematic skewness. When comparing systematic to idiosyncratic skewness, different empirical methodologies are often used to measure each type of risk. Instead, we use the same empirical methodology to predict each measure, show that we successfully forecast their respective future realized values, and find that systematic skewness, in contrast to idiosyncratic skewness, has a robust and economically large premium.

We make three main contributions. First, we develop a novel methodology to forecast differences in systematic skewness between stocks. Buying a diversified portfolio of stocks with low measures of systematic skewness and short-selling a diversified portfolio of stocks with high values should isolate systematic skewness risk. Unfortunately, sorting stocks on past systematic skewness measures to create a low-minus-high factor produces insignificant realized, i.e., *ex post* systematic skewness. These realized systematic skewness measures are even significantly positive in some cases, the opposite sign of what these factors are designed to produce.[4]

Our main insight is that forming a low-minus-high systematic skewness factor only requires the cross-sectional ordering of stock systematic skewness, not their actual values. Therefore, we predict the former which is considerably easier than predicting the latter. We use large panel regressions to predict the future *cross-sectional ranks* of individual stock systematic skewness using the cross-sectional ranks of a large number of past risk measures and firm characteristics. We form each month a portfolio that buys stocks with low predicted systematic skewness cross-sectional ranks and short-sells stocks with high predicted systematic skewness cross-sectional ranks. We find that this predicted systematic skewness (PSS) factor generates from July 1963 to December 2017 a significantly positive average

[3]Data for U.S. equity option on Optionmetrics starts in 1996. Conrad et al. (2013) report an average of 92 stocks in their bottom and top portfolios that contain stocks with the lowest 30% and the highest 30% option-implied skewness, respectively. Schneider et al. (2017) use an average of 1,800 U.S. stocks from January 1996 to August 2014 whereas there are on average 5,361 stocks during the same period.

[4]We explore three different systematic skewness measures which we estimate using either monthly or daily returns.

excess return of 5.37% per year with a Sharpe ratio of 0.38. The corresponding values for the market portfolio are 6.37% and 0.42, respectively. Most importantly, the PSS factor has a significantly negative realized systematic skewness.

Despite the large number of predictors used, we obtain stable regression coefficients over time which increases our confidence in our results. This result is surprising given that using multiple predictors in a predictive model—the *kitchen sink* approach (see Goyal and Welch, 2008)—generally performs badly when predicting expected returns. The key feature of our approach is that we use cross-sectional ranks of predictors and the dependent variable. For example, we do not ask whether a large market capitalization predicts a high systematic skewness. Rather, we estimate whether being among the largest firms is related to having the highest systematic skewness across stocks.

Our second contribution is to use the PSS factor in formal asset pricing tests. In the main exercise, we use 25 size and book-to-market ratio sorted portfolios and 25 size and momentum sorted portfolios of U.S. stocks as test assets. We test different models: the CAPM with the market factor, the Fama-French-Carhart four-factor model with market, size, value, and momentum factors (Fama and French, 1993; Carhart, 1997), and the five-factor model with market, size, value, profitability, and investment factors (Fama and French, 2015). We also test another model designed to capture downside risk, the downside-beta CAPM of Ang, Chen, and Xing (2006) and Lettau, Maggiori, and Weber (2014). To assess the value of adding the PSS factor to each model, we run time-series regressions of test portfolio excess returns on the factors and run a cross-sectional regression of their average returns on their factor exposures to estimate risk premia. Remarkably, the PSS risk premium is positive and significant in all models, ranging from 0.59% to 0.98% per month.

The t-ratios of the PSS factor range from 2.33 to 3.06, which is impressive for several reasons. First, given the growing number of risk factors that have been tested over the years, many of them not based on an economic model, Harvey, Liu, and Zhu (2016) advocate using a higher standard than the traditional t-ratio of 2 when assessing the value of a new risk factor. In this paper, however, we do not propose a new risk factor, but rather a new methodology to measure a traditional risk factor that has been tested as far back as in Kraus and Litzenberger (1976). Its risk premium is supported by an economic model in which investors require a compensation for bearing lower systematic skewness.[5] Second, we obtain t-ratios using the methodology of Kan, Robotti, and Shanken (2013) which accounts for model misspecification. Finally, we find that the PSS risk premium is significant even when factors designed to capture the cross-sectional variation in average returns of the test

[5]See Rubinstein (1973), Kraus and Litzenberger (1976), Harvey and Siddique (2000), and Chabi-Yo et al. (2014).

assets are included in our tests (e.g., size and value factors with size and value sorted test portfolios).

As a robustness check, we also estimate the PSS risk premium using the methodology of Giglio and Xiu (2017) which accounts for omitted risk factors and measurement error in the risk factor of interest. Using either the size, book-to-market ratio, and momentum portfolios or the 202 test portfolios used in Giglio and Xiu (2017), we find that the risk premium for PSS is always significantly positive.

Our third contribution is to examine the extent to which idiosyncratic skewness is priced in U.S. stocks. We use our panel regression model to predict the cross-sectional ranks of future idiosyncratic skewness. Stocks with low predicted idiosyncratic skewness ranks have lower realized idiosyncratic skewness than stocks with high predicted ranks, and the spread in realized idiosyncratic skewness is larger than when using other idiosyncratic skewness predictors.

Then, we form equal-weighted and value-weighted portfolios of stocks sorted by their predicted idiosyncratic skewness rank and a long-short portfolio that buys low idiosyncratic skewness stocks and short-sells high idiosyncratic skewness stocks. We run time-series regressions of their returns on different factor models that include the PSS factor. Our first result is that PSS has a strong explanatory power for idiosyncratic skewness sorted portfolios: Low idiosyncratic skewness stocks are negatively exposed to the PSS factor. However, the risk-adjusted performance of the low-minus-high idiosyncratic skewness portfolio is significant in some of the specifications, in particular for value-weighted portfolios. Hence, the PSS, while important, is not sufficient to fully explain the idiosyncratic skewness effect. When controlling for other risk factors such as momentum and profitability, the idiosyncratic skewness risk-adjusted performances are never significant. Our results are robust to predicting total instead of idiosyncratic skewness and to using an alternative quantile-based measure of either idiosyncratic or total skewness.

Our methodology identifies what are the determinants of systematic and idiosyncratic skewness. Except for higher momentum, higher price impact, and lower beta which predict both lower systematic and idiosyncratic skewness, predictors of systematic skewness are different from those of idiosyncratic skewness. For example, we confirm previous findings that skewness is negatively related to firm size.[6] Hence, it is a poor candidate to explain the size effect: small firms have higher average returns than large firms, but also higher skewness which should be accompanied by lower average returns. However, large firms also have higher systematic skewness compared to small firms.[7] Consequently, we show that once we control

[6]See, among others, Chen, Hong, and Stein (2001), Boyer, Mitton, and Vorkink (2010), and Conrad et al. (2013).

[7]Bali et al. (2016) also find that coskewness (skewness) is positively (negatively) correlated with firm

for the PSS factor, the size effect disappears and the size factor is no longer needed in asset pricing tests.

Another example of the difference in skewness predictors is idiosyncratic volatility. A higher cross-sectional rank of idiosyncratic volatility predicts a lower systematic skewness rank, but also a higher idiosyncratic skewness rank. Boyer et al. (2010) (hereafter BMV) show that past idiosyncratic volatility is a stronger predictor of idiosyncratic skewness than past idiosyncratic skewness. In contrast, we find that the lagged cross-sectional rank of idiosyncratic skewness is a stronger predictor of its future value than lagged idiosyncratic volatility rank, with an average coefficient twice as large.

Our work is related to different strands of literature. There is a large literature on the link between return skewness and asset prices, see Bali et al. (2016) (Ch. 14) for a review. Models with well diversified expected utility maximizing investors imply a compensation for negative systematic coskewness, which measures the contribution of an asset to the skewness of the market portfolio (see Rubinstein, 1973; Kraus and Litzenberger, 1976; Harvey and Siddique, 2000; Chabi-Yo et al., 2014). See also Dittmar (2002) and Smith (2007) for additional empirical evidence. We provide a new methodology to measure the coskewness risk premium.

Models with other types of investor preferences, in contrast, imply that total skewness of an individual stock is priced, (see Brunnermeier et al., 2007; Barberis and Huang, 2008). Mitton and Vorkink (2007) also study preference for skewness in retail investor portfolios. Empirical evidence of the negative relation between total stock skewness and returns implied by these models are provided in BMV and using a variable related to idiosyncratic skewness in Bali, Cakici, and Whitelaw (2011). Our results provide empirical evidence to compare both types of models using return skewnesses estimated using the same methodology.

Our work is also related to papers that use past return skewness measures and firm characteristics to predict skewness. Chen et al. (2001) (CHS hereafter) and BMV run cross-sectional regressions to predict future total and idiosyncratic skewness, respectively, using past risk measures and firm characteristics.[8] We run cross-sectional regressions to predict the ordering of individual stock skewness, not their values. We also use panel regressions to predict the systematic part of skewness. Zhang (2005) groups stocks with similar characteristics and use all their recent returns to compute a stock's skewness. In our panel regression framework, stocks with similar characteristics will have similar forecasted skewness ranks. Freyberger, Neuhierl, and Weber (2017) use the adaptive group LASSO to determine which stock characteristic ranks have explanatory power for stocks returns. We use stock charac-

size. See also Albuquerque (2012) for a model to explain the difference between stock-level and market-level skewness.

[8]See Cosemans, Frehen, Schotman, and Bauer (2015) for a similar panel regression approach to predicting stock betas using firm characteristics.

teristic ranks to predict skewness measures instead of returns, and then build a risk factor suggested by an asset pricing model. Using intra-day returns from 1993 to 2013, Amaya, Christoffersen, Jacobs, and Vasquez (2015) show that realized skewness negatively predicts future returns. We relate future skewness to both past risk measures and firm characteristics and also use our methodology to predict systematic skewness.

Our work is also related to papers using option-implied information about third moments. Christoffersen, Fournier, Jacobs, and Karoui (2017) show that the conditional price of coskewness risk can be obtained from the index option-implied variance risk premium. Our approach uses standard cross-sectional regressions to estimate the risk premia of several risk factors and show that the price of coskewness risk is robustly significant. Conrad et al. (2013) show that future stock returns are negatively related to their option-implied skewness, but not to risk-neutral coskewness. Whereas they average daily option-implied skewness within a quarter, Rehman and Vilkov (2012) find a positive relation with future returns using the latest daily option-implied skewness. Schneider et al. (2017) use option-implied skewness to explain returns on low-risk anomalies.[9] Our methodology does not require option data and uses a longer sample period and larger cross-section of stocks (3,261 stocks on average from 1963 to 2017) to disentangle the role of systematic and idiosyncratic skewness. Whereas we focus on the cross-section of stock expected returns, Bali and Murray (2013) and Boyer and Vorkink (2014) find that stock-option portfolio and option returns, respectively, are negatively related to option-implied skewness.[10]

More broadly, our paper is related to the literature on the importance of return skewness for portfolio choice. Dahlquist et al. (2017) theoretically derive the optimal portfolio choice of a generalized disappointment aversion investor in the presence of return skewness and show that it can explain patterns in empirically observed asset allocation choices. Ghysels, Plazzi, and Valkanov (2016) empirically show that skewness is an important factor in allocating a portfolio across emerging market equity indexes. DeMiguel, Plyakha, Uppal, and Vilkov (2013) use option-implied skewness as a predictor of stock returns to improve the performance of mean-variance portfolios. Our results provide an estimate of the cost of hedging coskewness risk or alternatively the extra return an investor earns by bearing coskewness risk.

Section 2 presents a new systematic skewness factor, Section 3 runs asset pricing tests, Section 4 analyzes the performance of idiosyncratic skewness portfolios, and Section 5 concludes.

[9] Leland (1999) also shows that skewed strategies are misevaluated by the CAPM.

[10] See also Cremers and Weinbaum (2010) and Xing, Zhang, and Zhao (2010).

2 Measuring Systematic Skewness

In this section, we start by discussing the asset pricing implication of systematic skewness. We then show in Section 2.2 the inability of past measures of systematic skewness to capture future skewness risk which will motivate our methodology that we introduce in Section 2.3. Finally, we discuss the performance of our new systematic skewness factor in Section 2.4 and the predictors of systematic skewness in Section 2.5.

2.1 How Does Systematic Skewness Affect Risk Premia?

We consider the conditional Three-moment CAPM in which the expected return of stock i in month t, $r_{i,t}$, is given by

$$E_{t-1}\left[r_{i,t} - r_{f,t}\right] = \gamma_{M,t} Cov_{t-1}\left(r_{i,t}, r_{M,t}\right) + \gamma_{M^2,t} Cov_{t-1}\left(r_{i,t}, r_{M,t}^2\right) \quad (1)$$

where $r_{f,t}$ is the risk-free rate for period t, $r_{M,t}$ is the market portfolio return for period t, $\gamma_{M,t}$ and $\gamma_{M^2,t}$ are respectively the time t prices of covariance and coskewness risk, and $E_{t-1}[\cdot]$ and $Cov_{t-1}(\cdot)$ denote respectively the expected value and covariance conditional on information at time $t-1$.

In this model, investors have a preference for higher portfolio return skewness. The contribution of a stock to the market portfolio skewness is captured by the coskewness term $Cov_{t-1}\left(r_{i,t}, r_{M,t}^2\right)$. For example, adding a stock with a negative coskewness to the market portfolio makes the latter more negatively skewed. The impact on market portfolio skewness is smaller when adding a stock with a less negative coskewness. In contrast, adding a positive coskewness stock increases the market portfolio's skewness.

Investors require higher expected returns for holding stocks that decrease the market portfolio's skewness, and hence the price of coskewness risk, $\gamma_{M^2,t}$, is negative. The economic implication of the Three-Moment CAPM is that coskewness is the right measure of systematic skewness. We therefore use the term coskewness in the following sections to refer to the systematic part of skewness.

There are different ways of obtaining the Three-moment CAPM. Using a Taylor expansion for marginal utility, Rubinstein (1973) and Kraus and Litzenberger (1976) impose some restrictions on investor preferences to obtain Equation (1) in an unconditional setting. Harvey and Siddique (2000) obtain this model in a representative agent framework using either a quadratic stochastic discount factor or a Taylor expansion of marginal utility. Dittmar (2002) further considers cokurtosis and human capital in addition to coskewness.

Another way of obtaining a model related to Equation (1) is to impose an assumption on the distribution of return shocks. Simaan (1993) provides a model with spherical shocks and one common non-spherical shock to create systematic skewness. Dahlquist et al. (2017) use gaussian shocks and one common exponential shock to create systematic skewness. Both papers derive the asset pricing implications of systematic skewness in stock returns, the first in an expected utility framework and the second with generalized disappointment aversion preferences. Dahlquist et al. (2017) show that using generalized disappointment aversion instead of expected utility will lead to a larger importance for return skewness.

In the next section, we turn to the empirical task of building a risk factor to capture coskewness risk. In Section 4, we empirically examine whether idiosyncratic skewness is also priced.

2.2 Are Systematic Skewness Measures Persistent?

To test the Three-moment CAPM in Equation (1), we build a tradable factor to capture the source of risk that generates systematic skewness in asset returns. We form a well diversified portfolio of stocks with low coskewness and a well diversified portfolio of stocks with high coskewness. The return spread between the former and the latter captures the coskewness factor shock. We show in this section that coskewness estimated on past returns is a weak predictor of future coskewness, and that sorting stocks on past coskewness will fail to capture the systematic skewness risk premium.

We consider several measures of systematic skewness that have been used in past studies. First, we denote as $Cos_{i,t}$ the coskewness from Equation (1)

$$Cos_{i,t} = Cov_{t-1}\left(r_{i,t}, r_{M,t}^2\right). \tag{2}$$

Second, we use the regression coefficient $\beta_{M^2,i,t}$ in a regression of excess stock returns on a constant, the excess market return, and the squared excess market return. This measure is motivated by the beta representation of the Three-moment CAPM, $E[r_{i,t}-r_{f,t}] = \beta_{M,i,t}\mu_{M,t}+\beta_{M^2,i,t}\mu_{M^2,t}$, where $\mu_{M,t}$ and $\mu_{M^2,t}$ are the market and coskewness risk premia, respectively.

Third, we follow Harvey and Siddique (2000) and compute a measure of standardized coskewness, $\beta_{HS,i,t}$, for stock i defined as

$$\beta_{HS,i,t} = \frac{E_{t-1}\left[\epsilon_{i,t}\epsilon_{M,t}^2\right]}{\sqrt{E_{t-1}\left[\epsilon_{i,t}^2\right]}E_{t-1}\left[\epsilon_{M,t}^2\right]} \tag{3}$$

where $\epsilon_{i,t} = r_{i,t}-r_{f,t}-\alpha_i-\beta_{M,i}\left(r_{M,t}-r_{f,t}\right)$ is the residual from a regression of stock i's excess return on a constant and the market excess return and $\epsilon_{M,t} = r_{M,t}-r_{f,t}-\mu_M$ is the deviation of market excess returns from its mean. The three measures $Cos_{i,t}$, $\beta_{M^2,i,t}$, and $\beta_{HS,i,t}$ are related through the specification of a regression of r on r_M^2 or on both r_M and r_M^2. The advantages of using $\beta_{HS,i,t}$ are that it is zero for the market portfolio, unit free, and akin to a factor loading. Because the market portfolio has a benchmark value of zero, we use $\beta_{HS,i,t}$ to measure and compare the realized systematic skewness of different risk factors constructed below. However, our conclusions are robust to using any other coskewness measure.

For each month t and each coskewness measure, we create two sets of risk factors. First, we use monthly returns over the last 60 months, from $t-60$ to $t-1$, to estimate each of the three systematic skewness measures. Second, we also investigate the added value of higher frequency data by using daily returns over the last year, from the first day of month $t-12$ to the last day of month $t-1$, to compute daily versions of the three measures. To measure coskweness, Harvey and Siddique (2000) use the monthly version of $\beta_{HS,i,t}$ and Bali, Brown, Murray, and Tang (2017) use the daily version of $\beta_{M^2,i,t}$.

We use daily and monthly returns of all common stocks listed on NYSE, AMEX, Nasdaq and NYSE Arca from July 1963 to December 2017. Appendix A provides more details on our data construction. Each month, we compute the different coskewness measures for all available stocks. For each measure, we form two value-weighted portfolios: one containing the 30% of stocks with the lowest coskewness and one containing the top 30% stocks with the highest coskewness. The return on the factor is the return on the low-coskewness portfolio minus the return on the high-coskewness portfolio. The asset pricing relation in Equation (1) implies that such a long-short portfolio should have positive average returns.

We report their summary performance statistics in the first six rows of Table 1. We first report annualized average excess returns, volatilities, and Sharpe ratios. Across the coskewness factors, average excess returns range from -0.78% to 2.77%, but all are insignificant. The maximum Sharpe ratio across factors is 0.23, approximately half of the level for the market portfolio reported in the last line.

Most importantly, we report in the sixth column for each factor their realized, i.e., *ex post* standardized coskewness measure in Equation (3). Factors built from monthly coskewness measures have significant realized coskewness, but of the wrong sign, i.e., positive instead of negative. Therefore, past measures of coskewness do not create significantly negative spreads in *ex post* coskewness. Daily coskewness measures fare better; they have negative realized coskewness measures, but all are insignificant. These results are consistent with the low autocorrelations of coskewness measured at the stock level found in Bali et al. (2016) (see Table 14.7). In unreported results, we show that we can create a significant spread in future β_M by sorting stocks on past daily β_M (as in Bali et al., 2017, for example). Hence, the lack of persistence problem is specific to coskewness.

The significant positive realized coskewness for monthly measures is surprising. In unreported results, we compute the contingency table of past stock coskewness computed over months $t-60$ to $t-1$ and the month t values of $r_{i,t} \times r_{M,t}^2$. We find that stocks in the bottom (top) 30% of past coskewness are equally likely to be in the bottom (top) 30% of $r_{i,t} \times r_{M,t}^2$ values as they are to move to the top (bottom) 30% group, which is indicative of the low time-series persistence of coskewness. Therefore, we should expect the coskewness of this factor to be close to zero. Instead, we find significantly positive coskewness values in Table 1 because of the impact of the October 1987 crash. During that month, the coskewness factors built using monthly measures of past coskewness experience positive returns while the market factor plummets by more than 23%. Once we remove this monthly observation, the realized coskewness of these three factors are still positive, but not significant anymore.

Another perspective is presented in the last three columns. We follow the insight of Barillas and Shanken (2016) and report the coskewness factors' α from multi-factor time-series regressions. Barillas and Shanken (2016) show that to judge the added value of a factor in explaining the cross-section of stock average returns, it is sufficient to show that it has a significant α when regressed against a benchmark set of risk factors.

We consider different models: the CAPM with the market factor (MKT), the Carhart-Fama-French four-factor model with MKT, size (SMB), value (HML), and momentum (MOM) factors, and the Fama-French five-factor model with MKT, SMB, HML, profitability (RMW), and investment CMA factors.[11] In all but one case, the α of coskewness factors are insignificant suggesting that the coskewness factors do not add any explanatory power to existing factor models.

We can conclude from these results that past measures of coskewness are not persistent enough to generate a spread in future coskewness. In addition, the low-minus-high coskewness factors do not have positive risk-adjusted returns, either by themselves or when judged against other risk factors. Accordingly, we present in the next section a novel methodology to form the coskewness factor.

2.3 A Predictive Systematic Skewness Factor

In this section, we present a model for predicting the *cross-sectional rank* of conditional systematic skewness. Our methodology has three distinct features. First, we use a large number of risk measures and firm characteristics to predict future coskewness. Second, as the dependent variable we use the cross-sectional ranks of stock coskewness, not their values. Given that the compositions of the long and short portfolios in a systematic skewness factor

[11] All factor data are from Kenneth French's website.

are determined by how stocks are ordered, we only need to forecast the cross-sectional ranks of stock coskewness values. Finally, we use the cross-sectional ranks of predictors. We show that using the cross-sectional ranks of predictors results in stable predictive relations.

Each month t (say, January 2018), we run the following panel regression using all available stocks and historical data

$$F\left(Cos_{i,k-12\to k-1}\right) = \kappa + F\left(Y_{i,k-24\to k-13}\right)\theta + F\left(X_{i,k-13}\right)\phi + \varepsilon_{i,k-12\to k-1}, \tag{4}$$

where $k = 25, 26, \ldots, t$, $i = 1, ..., N_k$, and N_k is the number of stocks available at time k. In this equation, $Cos_{i,k-12\to k-1}$ is the coskewness Cos of stock i from Equation (2) computed using daily returns from month $k-12$ to month $k-1$ (e.g., January to December 2017), the K_Y variables $Y_{i,k-24\to k-13}$ are risk measures (volatility, β_M, coskewness, etc.) computed using stock i's daily returns from month $k-24$ to month $k-13$ (e.g., January to December 2016), the K_X-vector $X_{i,k-13}$ are characteristics (size, book-to-price ratio, momentum, etc.) of stock i observed at the end of month $k-13$, and $\varepsilon_{i,k-12\to k-1}$ are random shocks. We use a period of 12 months to measure risk measures like coskewness because it provides a reasonable trade-off between having enough returns while allowing for variations over time.

The function $F(x_{i,t}) = \frac{Rank(x_{i,t})}{N_t+1}$ computes the normalized rank of variable $x_{i,t}$ in the cross-section of x_t. The $Rank(x_{i,t})$ function gives the order (1,2,...,N_t) of variable $x_{i,t}$ in all x_t values sorted in ascending order. We divide by $N_t + 1$ to obtain a variable that falls between 0 and 1.

The K_Y-by-one vector of coefficients θ and K_X-by-one vector of coefficients ϕ measure the ability of past ranks of risk measures and characteristics, respectively, to predict the future rank of a stock coskewness in the cross-section. κ is a constant. We run the panel regression (4) using all monthly observations from month 25 to month t. Each month, we use all stocks in the cross-section for which the values Cos, Y, and X are available. By estimating regression (4), we model how past cross-sectional ranks of risk measures and characteristics predict future coskewness ranks.[12] For example, we identify whether being among the largest market capitalization firms is associated with having an above median coskewness over the next 12 months.

To form our coskewness factor we first compute the model predicted coskewness ranks for month t using our regression estimates, $\hat{\kappa}$, $\hat{\theta}$, and $\hat{\phi}$, as

$$F\left(\widehat{Cos_{i,t\to t+11}}\right) = \hat{\kappa} + F\left(Y_{i,t-12\to t-1}\right)\hat{\theta} + F\left(X_{i,t-1}\right)\hat{\phi}. \tag{5}$$

[12] Given that we estimate linear regressions, the predicted cross-sectional ranks may fall outside of the 0 to 1 interval. As we use predicted ranks only to classify stocks into high and low predicted coskewness portfolios, this does not affect our methodology.

Finally, we form our coskewness factor as the return spread between the value-weighted portfolio containing the bottom 30% of stocks with the lowest predicted coskewness cross-sectional ranks and the value-weighted portfolio containing the top 30% of stocks with the highest predicted coskewness cross-sectional ranks. We denote the resulting factor as the predicted systematic skewness factor or PSS for short.[13]

In our empirical implementation, we use β_M, idiosyncratic volatility, coskewness Cos, and idiosyncratic skewness as past risk measures Y. These measures capture the systematic and idiosyncratic second and third order moments and therefore describe the shape of the distribution of past returns.

As firm characteristics, we use variables identified in the literature as being related to either average returns or to future skewness values. We use market capitalization and book-to-price ratio (Fama and French, 1993), profitability and investment (Fama and French, 2015; Novy-Marx, 2013), net payout yield (Boudoukh, Michaely, Richardson, and Roberts, 2007), momentum (Jegadeesh and Titman, 1993; Carhart, 1997), intermediate horizon return (Novy-Marx, 2012), the lagged monthly return to capture return reversal (Jegadeesh, 1990), price impact (Amihud, 2002), turnover (CHS), and the maximum return measure of Bali et al. (2011) (the average of the five highest daily returns measure within a month).[14] The construction of all these measures is detailed in Appendix A.

Our methodology differs from BMV and CHS in that we use cross-sectional ranks of predictors on the right hand side of Equation (4) and we forecast the cross-sectional rank of the risk measure on the left-hand side. Our methodology further differs from BMV because we use all past observations instead of estimating cross-sectional regressions separately for each month. In addition to predicting only the information needed to form the coskewness factor, another advantage of our approach is that computing cross-section ranks ensures that we use stationary variables and that the regression coefficients are not impacted by outliers possibly caused by data errors. Given that all regressors are transformed into cross-sectional ranks, our method also allows to easily compare coefficient values across risk measures and firm characteristics.

[13]Inference about the panel regression coefficients is complicated by two aspects: the period used to compute the risk measures overlap and we estimate cross-sectional ranks with error. We avoid these complications because we rely only on the predicted values $F\left(\widehat{CS_{t\to t+11,i}}\right)$ to form a long-short portfolio and do not make any inference on the regression coefficients.

[14]In a robustness check, we also used industry dummies. Results are available from the author.

2.4 Performance of the Predicted Systematic Skewness Factor

We next present in the seventh row of Table 1 (see row label PSS) the summary performance statistics of the PSS factor. The factor has a positive and significant average excess return of 5.37% per year and a Sharpe ratio of 0.38, almost double the maximum Sharpe ratio of the other coskewness factors. Most importantly, it has a significantly negative realized coskewness of -0.27 compared to a value of 0 for the market portfolio. Although we only report the realized monthly standardized coskewness measure β_{HS}, we also compute the daily standardized coskewness, the daily and monthly realized coskewness Cos, and the daily and monthly realized β_{M^2}. In all cases, we find that the PSS factor creates negative realized coskewness measures that are the lowest across all factors.

When judged against factor models, we find an annualized α of 4.02% for the CAPM, -0.36% for the four-factor model, and 5.91% for the five-factor model, the first and last of which are significant. Hence, the PSS factor adds the least amount of information when the momentum factor is included in the regression, an issue we explore further in Section 3.2. Therefore, the PSS factor is successful in creating an *ex post* spread in coskewness, has a significant and positive average excess return, and adds value to other leading risk factors used in the literature.[15]

Figure 1 reports the cumulative log-return of the PSS and MKT factors along with gray bars for NBER recessions. The PSS factor crashes before the 1970 recession, at the onset of the Asian crisis in 1997, before the 2001 recession, and during the financial crisis in 2008. On the other hand, the PSS factor does not go down as the market plummets during the 1973-1975 and 1981-1982 recessions. This behaviour is in line with the low market β_M of 0.21 reported in the fifth column of Table 1. Figure 1 also shows that the positive performance of the PSS cannot be attributed to a specific period. Overall, we find that the PSS factor is distinct from the market factor MKT.

The better performance of our methodology can be understood as follows. Sorting stocks based on a lagged measure of coskewness, using either the past 60 monthly returns or one year of daily returns, is bound to have limited predictive ability for future coskewness. To distinguish how different stocks react to extreme events, one needs an estimation sample during which such events have occurred (or at least large values of r_M^2). When no such event has occurred during the estimation period, past coskewness has a limited ability to predict future coskewness.

In contrast, our panel approach considers all past observations and how stock coskewnesses were related to lagged risk measures and firm characteristics. It also captures on a

[15] In unreported results, we find that the significance of the αs are unchanged when we augment the four-factor and five-factor models with the illiquidity factor of Pastor and Stambaugh (2003).

timely basis the changing nature of individual stocks. Consider an example in which coskewness is positively related to firm size; large firms have higher coskewness than small firms. When a firm goes from being a small firm to a large firm, a rolling estimate of coskewness is not only dependent on the presence of extreme events in the lookback period, but it would also lag the true higher coskewness level of the newly large firm. In contrast, using the cross-sectional rank of its firm size would readily capture its new coskewness level.

To further motivate the importance of using cross-sectional ranks in our panel regression, the next line in Table 1 reports the results for a long-short factor built using the raw (i.e., unranked) coskewness values on the left-hand side of regression (4). The average return of 2.52 is lower and insignificant, the Sharpe ratio of 0.19 is half the 0.38 value for the PSS factor, the factor regression α is significant for the five-factor model, but smaller in magnitude, and the realized standardized coskewness measure, though significant, is less negative (-0.16 compared to -0.27). We also build a factor using the unranked values of both the predictors on the right-hand side and coskewness on the left-hand side (unreported results). This factor performs even worse: it has a realized coskewness close to 0 and all its factor regression αs are insignificant. In the next section, we also discuss that the panel regression coefficients θ and ϕ for these two factors are unstable whereas we find stable coefficients when using cross-sectional ranks of coskewness and the predictors. Therefore, we better capture *ex ante* the coskewness risk factor by predicting cross-sectional ranks using the cross-sectional ranks of predictors than by using unranked variables.

We also run other robustness checks. First, we estimate Equation (4) using 10-year rolling windows. The results are not as good suggesting that there are no significant time variations in how future coskewness is related to past risk measures and firm characteristics. Second, we consider a forecast combination method by estimating Equation (4) using only a constant and one predictor at a time and then computing the average forecast from all models (see, for example, Rapach, Strauss, and Zhou, 2010). Using either all past data or rolling estimation windows, the results are not as good indicating that our methodology benefits from cross-correlations between predictors to obtain better coskewness rank predictions.

2.5 What Predicts Systematic Skewness?

In this section, we describe the predictive power of each risk measure and firm characteristic. Figure 2 reports the panel regression coefficients $\hat{\theta}$ and $\hat{\phi}$ in Equation (4) estimated over time using expanding samples while Table 2 provides the time-series averages of coefficients as well as their 5^{th} and 95^{th} percentiles.

Working with cross-sectional ranks results in stable regression estimates, especially in the later part of the sample period when the estimation sample is longer.[16] Lagged coskewness, market capitalization, intermediate horizon return, and maximum return always positively predict coskewness. Idiosyncratic volatility, momentum, and lagged monthly return always negatively predict coskewness, though idiosyncratic volatility's coefficient is briefly positive at the start of the sample period.

CHS document that returns over the last six months negatively predicts daily total return skewness. Harvey and Siddique (2000) find a strong negative relation between return momentum (measured from month $t-12$ to $t-2$) and coskewness. Novy-Marx (2012) show that the momentum effect mainly comes from intermediate horizon returns (from month $t-12$ to $t-7$), not from recent returns. The coefficients for momentum and lagged monthly return (return for month $t-1$) robustly predict lower coskewness whereas intermediate horizon return predicts higher coskewness. Therefore, returns over the last six months have more importance in predicting lower coskewness, similar to CHS' results for total skewness.

We find that market capitalization is positively related to coskewness; large firms have higher coskewness than small firms. In constrast, Conrad et al. (2013) use option-implied moments over the 1996-2005 period to compute a risk-neutral estimate of coskewness and find that it is negatively related to firm size. CHS and BMV find that large firms have more negative daily total skewness and monthly idiosyncratic skewness, respectively. In Section 4, we use a predictive panel regression for idiosyncratic skewness to distinguish the determinants of systematic from those of idiosyncratic skewness. We find that large firms have lower idiosyncratic skewness, in line with their results. We further show that predictors for coskewness differ from those for predictive-idiosyncratic skewness, which demonstrate that the coskewness factor is a distinct source of risk.

The time-series coefficient averages in Table 2 shed light on the superior performance of the PSS factor in Table 1 compared to factors based only on past coskewness measures. The coefficient θ for lagged coskewness is always positive, indicating that coskewness is persistent. But its average value of 0.059 is small and past coskewness is only one of the different predictors. Hence, the PSS factor better captures future coskewness risk by using more information.

Having established that the PSS factor successively captures future realized coskewness risk, we proceed in the next section to formal asset pricing tests.

[16] In unreported results, we found that using unranked variables resulted in unstable regression coefficient values.

3 Systematic Skewness and Expected Stock Returns

We explore the pricing performance of the PSS factor in conjunction to leading factor models. We use two sets of test assets; 25 size and book-to-market ratio sorted portfolios and 25 size and momentum sorted U.S. stock portfolios. Following the advice of Lewellen, Nagel, and Shanken (2010), we report the GLS R^2 and include a constant to allow for a zero-beta rate different from the risk-free rate. Further, we use the methodology of Kan et al. (2013) to compute model misspecification robust t-ratios of risk premia estimates, prices of risk, and standard errors of the cross-sectional R^2.

For each of the $N = 25$ test portfolios, we run time-series regressions of their excess returns on a constant and factors returns. Then, we run a cross-sectional regression of the test portfolio average returns on their estimated time-series factor loadings to estimate the factor risk premia. We estimate different factor models to investigate the added pricing power of the PSS factor. We start in the next section with the CAPM.

3.1 Test of the CAPM

We start in this section with a comparison of the CAPM and the Three-moment CAPM in which the market excess return MKT is augmented with the PSS factor. Table 3 reports estimation results for the 25 size and book-to-price portfolios in Panel A and the 25 size and momentum portfolios in Panel B.

For all model, we report the model misspecification robust t-ratios in parentheses below the estimates of the constant and risk premia. We then report the R^2 and the p-value in parenthesis for the increase in R^2 for the Three-moment CAPM compared to the CAPM. All parameters are estimated by GLS, but we report both OLS and GLS R^2.

In both panels, we first report the pricing performance for the CAPM. The risk premium is negative, in line with previous results (see, for example, Kan et al., 2013). When we add the PSS factor, we obtain a positive risk premium of 0.64% (0.74%) per month with a model misspecification robust t-ratio of 2.62 (3.06) using the 25 size and book-to-price ratio (momentum) portfolios.

The increase in model fit is striking. The OLS R^2s increase from 0.08 to 0.51 for size and book-to-market portfolios and from 0.11 to 0.77 for size and momentum portfolios. The GLS R^2s increase from 0.11 to 0.29 and from 0.02 to 0.20, respectively. All R^2 increases are significant.

To judge the added value of a risk factor, Kan et al. (2013) stress that it is the t-ratio of its price of risk, not its risk premium, that should be compared to a critical value. Accordingly, we report in the last two rows for each model the price of risk estimates obtained by estimating the covariance-form of the asset pricing model instead of the beta-form. We find in Panel A that the PSS factor carries a price of risk of 5.55 with a t-ratio of 2.65. The corresponding values using the size and momentum portfolios are 5.69 with a t-ratio of 2.91.

We next compare the PSS pricing performance to the downside β_M measure of Ang et al. (2006) and Lettau et al. (2014). To capture downside risk, they measure a portfolio's sensitivity to market returns when it is below a threshold. We follow Lettau et al. (2014) and define the threshold as the average excess market return minus one standard deviation.[17] In the first step time-series regression, we regress returns on the PSS, a MKT^- factor which is equal to $r_{M,t}$ if $r_{M,t}$ is below the threshold and zero otherwise, and a MKT^+ factor which is equal to $r_{M,t}$ if $r_{M,t}$ is above the threshold and zero otherwise. To capture a downside risk premium, we should expect the risk premium for MKT^- to be higher than the premium for MKT^+.

We find for both sets of test portfolios that the PSS risk premium and prices of risk remain significant whereas coefficients for both MKT^- and MKT^+ are not significant. Overall, we conclude that the PSS factor carries a significant amount of information in pricing size, book-to-market, and momentum sorted portfolios.

3.2 Test of Multi-Factor Models

In this section, we examine whether the PSS factor adds any pricing information in multi-factor models. We report on the four-factor model with MKT, SMB, HML, and MOM in Table 4 and on the five-factor model with MKT, SMB, HML, RMW, and CMA in Table 5. These Tables have the same structure as Table 3.

In the first two rows of each panel in Table 4, we report on the estimates for the four-factor model. HML is only significant for size and book-to-market portfolios, MOM is only significant for size and momentum portfolios, whereas SMB is significant for both sets of portfolios. The OLS R^2 are respectively 0.76 and 0.85 and the GLS R^2 are 0.40 and 0.26.

When augmenting the four-factor model with PSS (see row *Four-factor + PSS*), we initially find that the prices of risk for PSS and SMB are not significant and that the R^2 is not significantly higher than for the four-factor model. This result can be understood as follows. SMB and PSS have a high correlation of 0.73 during the 1963-2017 period.[18] In unreported results, we find that the size factor *never* has a significant α when regressed against factor models augmented with PSS, which indicates that PSS subsumes SMB.

[17] Over the July 1963 to December 2017 sample period, 114 of the 816 market returns are below this threshold.

[18] The size factor in the Fama-French five-factor model examined below is constructed differently. That size factor has a correlation of 0.71 with PSS.

Therefore, we remove SMB from all factor models. The last four lines in each panel report on the models with MKT, PSS, HML, and MOM. Both coskewness risk premium and price of risk are positive and significant at the 5% level. We further find that the R^2 is not significantly different from the four-factor model R^2 (neither lower nor higher).[19] Hence, we replaced the SMB factor with the theoretically motivated PSS factor without impacting the model fit.

Finally, we examine the five-factor model in Table 5. For size and book-to-market ratio test portfolios in Panel A, the PSS factor price of risk is positive and significant, in the presence of SMB (middle rows) or not (bottom rows). We find that the increases in R^2 from the five-factor model to the models that include the PSS factor are significant at the 1% level. The only exception is the increase in OLS R^2 for the last model which is marginally significant with a p-value of 0.12.

For size and momentum portfolios in Panel B, we find that, similar to the four-factor model, the PSS and SMB prices of risk are not significant when they are both included. The last rows report on a model in which we remove the SMB factor. Both PSS's risk premium of 0.59% per month and price of risk of 7.27 are significant. The p-values for model comparison tests reported in parenthesis below the R^2 indicate that they are not significantly different. Hence, we replaced SMB with the theoretically motivated PSS without impacting the model fit.

Interestingly, we find that the modified five-factor model with MKT, PSS, HML, RMW, and CMA reported in the last rows of Panel A and B of Table 5 has better performance, as indicated by its R^2s, than the four-factor model reported in the first rows of Panel A and B of Table 4 in pricing both the size and book-to-market ratio and the size and momentum sorted portfolios.

Overall, our results show that measuring ex ante systematic skewness is important for explaining the cross-section of stock returns. The main conclusion is that the size factor is completely subsumed by the systematic skewness factor, and that the latter remains significant even in the presence of factors built to explain the cross-section of average returns of the chosen test portfolios (e.g., MOM for momentum sorted portfolios).

3.3 Robustness Check

As a robustness check, we use the methodology of Giglio and Xiu (2017) that accounts for omitted factors and measurement error when estimating a factor risk premium. Table 6 contains the estimated constant and risk premia for the MKT and PSS factors.[20] Their

[19] We use the normal test for non-nested models of Kan et al. (2013).

[20] We thank Stefano Giglio for making their estimation code available on his website.

methodology first obtains the first p principal components from test portfolio returns, computes the loadings by regressing portfolio returns on the principal components, and then obtains the principal components' risk premia from a cross-sectional regression of average test portfolio returns on their loadings. Then, the risk premia of the factors of interest are obtained by combining the principal components' risk premia and the time-series regression loadings of risk factors on principal components.

Their methodology relies on a large cross-section of test portfolios to extract a set of principal components. Therefore, we combine the 25 size and book-to-market ratio portfolios and the 25 size and momentum portfolios. The second column in Table 6 reports the number of principal components p used in the tests. We follow Giglio and Xiu (2017) and report results using 4, 5, and 6 principal components. The optimal number identified by their methodology is 5. The risk premium for PSS reported in the last column varies from 0.36% to 0.47% per month and is highly significant in all cases.

We also report model estimates using the set of 202 portfolios used in their paper which assembles the most well-known stock market anomalies. We find a significant risk premium for PSS ranging from 0.28% to 0.30%. While the premium based on such a large cross-section of portfolios is smaller, all estimates are significantly positive. The optimal number of principal components for this set of portfolios is 6.

Now that we have shown the importance of systematic skewness, we turn our attention to the relative importance of idiosyncratic skewness in the next section.

4 Idiosyncratic Skewness

In this section, we investigate whether idiosyncratic skewness is priced once we account for systematic risk. In Section 4.1, we first predict the cross-sectional rank of idiosyncratic skewness and show that we better capture differences in future realized idiosyncratic skewness across stocks. Then, we run factor analyses for predicted idiosyncratic skewness sorted portfolios in Section 4.2 and run some robustness checks in Section 4.3.

Models in Barberis and Huang (2008), Brunnermeier et al. (2007), and Mitton and Vorkink (2007) imply lower average returns for stocks with higher idiosyncratic skewness. In Barberis and Huang (2008), cumulative prospect theory investors' preference for positively skewed assets results in these assets earning lower average returns. In Brunnermeier et al. (2007), investors optimally tradeoff the *ex ante* utility derived from and the *ex post* pain caused by their subjective beliefs. As a result, good states with small probabilities—states in which positively skewed assets pay off—earn relatively lower expected returns. In Mitton and Vorkink (2007), investors with preference for skewness optimally choose to remain underdiversified to preserve their portfolio skewness. As a result, positively skewed assets earn lower expected returns. We empirically investigate in this section whether these effects are important in a large cross-section of stock returns.

4.1 What Predicts Idiosyncratic Skewness?

We run predictive panel regression as in Equation (4), but replacing coskewness ranks on the left hand side by the cross-sectional rank of idiosyncratic skewness. We measure stock i's idiosyncratic skewness by estimating the Three-moment CAPM using daily excess returns over 12-month periods and computing the skewness of residuals, see Appendix A for more details.

As shown in Bali et al. (2016), the idiosyncratic skewness of individual stocks' Fama and French (1993) three-factor model residuals are very similar to their total skewness suggesting that the exposures to the market, size, and value factors do not capture the systematic part of skewness. Hence, analyzing the effect of the idiosyncratic skewness of three-factor model residuals is tantamount to analyzing total skewness, which includes systematic skewness. In this section, we instead regress daily returns on the market portfolio returns and its squared values to remove systematic skewness from returns. As a robustness check, we repeat our analysis for total skewness instead of idiosyncratic skewness in Section 4.3.1.

We report in Figure 3 the time-series of regression coefficients and in Table 7 their time-series averages and 5^{th} and 95^{th} percentiles obtained by estimating panel regression (4) each month. Figure 3 can be directly compared to Figure 2 and Table 7 has the same structure as Table 2.

The rank of idiosyncratic skewness seems more persistent than for coskewness; its coefficient is always positive and on average 0.135 (compared to 0.059 for coskewness). Idiosyncratic volatility is also a strong predictor of idiosyncratic skewness as shown in BMV, although its coefficient is briefly negative at the start of the sample period. In contrast to BMV, however, we find that the idiosyncratic skewness rank is a much stronger predictor than idiosyncratic volatility rank, with an average coefficient twice as large.

There is a strong and negative relation between firm size and idiosyncratic skewness, in line with BMV, CHS, and Conrad et al. (2013). This is in sharp contrast with coskewness which is positively associated with firm size. Hence, large firms have more positive systematic skewness, but more negative idiosyncratic skewness. On the other hand, small firms have a higher likelihood of experiencing a large and positive returns, but they also tend to carry more systematic downside risk.

CHS document that skewness is strongly negatively related to returns over the last six months. We find that coskewness and idiosyncratic skewness are both strongly negatively related to momentum and the lagged monthly return (hence, the return over the last 12 months). The coefficient for intermediate horizon return (from month $t-12$ to $t-7$) is always positive for coskewness, suggesting that returns from month $t-6$ to $t-1$ are more important in predicting coskewness. We do not find the same result for idiosyncratic skewness; the coefficient for intermediate horizon return is close to zero.

Future idiosyncratic skewness ranks are also related to other risk measures and firm characteristics. High β_M and high book-to-price ratio always positively predict idiosyncratic skewness ranks whereas higher net payout yield and investment predict lower idiosyncratic skewness.

How well do we capture future realized idiosyncratic skewness? In Table 1 we analyzed the realized coskewness of different long-short portfolios and showed that our PSS factor generated the most negative realized coskewness. The economics of idiosyncratic skewness is different; it is a potentially priced characteristic, not a risk factor. Skewness-seeking investors modeled in Barberis and Huang (2008), Brunnermeier et al. (2007), and Mitton and Vorkink (2007) are not interested by the idiosyncratic skewness of a portfolio, but rather in the high return potential of a single stock. Therefore, we examine the equal-weighted average *stock-specific* idiosyncratic skewness. This measure conveys the idiosyncratic skewness one can expect by picking one stock among the low-idiosyncratic-skewness group, not the skewness of an equal-weighted diversified portfolio.

Figure 4 presents evidence on the ability of different predictors in forecasting future realized idiosyncratic skewness. Each month t, we use the predicted cross-sectional idiosyncratic skewness rank to sort stocks into a bottom 30% group and a top 30% group. The values reported with the thick blue line is the equal-weighted average of realized idiosyncratic skewness from month t to $t+11$ of individual stocks in the bottom group minus the average for stocks in the top group. We also report the average spread using two other idiosyncratic skewness predictors to sort stocks: lagged idiosyncratic volatility and idiosyncratic skewness.

All measures create a negative spread, indicating that the bottom 30% of stocks indeed have lower realized idiosyncratic skewness than the top 30%. The only exception is the November 1986 to August 1987 period during which the spread becomes slightly positive. But most importantly, the predicted idiosyncratic skewness measure creates the lowest (i.e., most negative) difference between the bottom stocks' idiosyncratic skewness and the top stocks. Therefore, our predictive panel regression for cross-sectional ranks is successful in predicting differences across stocks in both realized coskewness and idiosyncratic skewness.[21]

[21] The ordering in realized idiosyncratic skewness is robust to using value-weighted averages.

4.2 Factor Analysis for Idiosyncratic Skewness Sorted Portfolios

In this section, we examine the risk-adjusted performance of idiosyncratic skewness sorted portfolios. We run time-series regressions of idiosyncratic skewness sorted portfolios on the different factor models examined in Section 3. In each panel in Table 8 and 9, we run a time-series regression for the portfolio that holds each month the bottom 30% of stocks with the lowest predicted idiosyncratic skewness, the portfolio with the middle 40% of stocks, the portfolio with the top 30% of stocks, and finally the long-short portfolio that buys the bottom 30% portfolio and short-sells the top 30% portfolio. Table 8 reports on equal-weighted portfolios and Table 9 on value-weighted portfolios. We use the best model in each of Tables 3-5: Panel A reports on the Three-moment CAPM with MKT and PSS, Panel B on the model with MKT, PSS, HML, and MOM, and Panel C on the model with the MKT, PSS, HML, RMW, and CMA.

We find that across all three factor models and portfolio weighting, the low-minus-high portfolio has negative and significant loadings on the MKT and PSS factors that vary from -0.39 to -0.14 for MKT and from -0.90 to -0.44 for PSS. The loadings on PSS increase going from the low to the high idiosyncratic skewness portfolios. This is in line with the negative correlation reported in Bali et al. (2016) (see Table 14.5) between coskewness and idiosyncratic skewness. In both Tables, the low-minus-high portfolio has a significantly positive loading on MOM in Panel B and on RMW in Panel C. HML also has a significantly negative loading for value-weighted portfolios.

If idiosyncratic skewness is priced, then these time-series regressions should reveal a higher α for the low portfolio than for the high portfolio and a positive α for the low-minus-high portfolio.

Across factor models, the regression αs for the equal-weighted low-minus-high idiosyncratic skewness portfolio range from -0.25% to 0.16% per month and are all insignificant. The Three-moment CAPM in Panel A is not sufficient in all cases: Both the equal- and value-weighted Low portfolios and the value-weighted Low-High portfolios have significant αs. However, when we move to other factor models in Panels B and C, all αs become insignificant.

Overall, we show that we can forecast the relative ranking of each stock's future idiosyncratic skewness. The PSS is an important factor in explaining the return of idiosyncratic skewness sorted portfolios, but the Three-moment CAPM does not suffice in explaining the higher performance of equal-weighted portfolios with low idiosyncratic skewness and of value-weighted portfolios. But these αs are not robust to the inclusion of other risk factors.

4.3 Robustness Checks

In this section, we run robustness checks for the risk-adjusted performance of low idiosyncratic skewness stocks. We examine portfolios sorted by total instead of idiosyncratic skewness and sorted by a quantile-based measure of skewness.

4.3.1 Portfolios Sorted by Total Predicted Skewness

In the previous section, we used the skewness of daily residuals from the Three-moment CAPM to measure idiosyncratic skewness. To verify that our results are robust to this choice of risk adjustment, we run predictive panel regressions for total skewness. If the PSS factor captures systematic skewness risk, then the α of total skewness sorted portfolios, if any, can be attributed to idiosyncratic skewness. On the other hand, if only systematic skewness is priced, then controlling for systematic skewness should leave no pricing relation for total skewness.

We report in Section 1 of the Online Appendix a figure with time-varying panel regression coefficients, a table with coefficient averages and percentiles, and a figure for the stock-specific realized skewness average of low predicted skewness stocks minus the average for high predicted skewness stocks. These figures and table are directly comparable to Figures 3 and 4 and Table 7. Overall, we find that predictors of total skewness are very similar to the ones for idiosyncratic skewness and that the spread in realized total skewness is the lowest across predictors. These results show that our predictive panel regression model is also successful in predicting the cross-sectional ranks in total skewness.

Tables 10 and 11 have the same structure as Tables 8-9, but run time-series regressions for predicted skewness sorted equal-weighted and value-weighted portfolios, respectively.

In line with CHS who do not find much differences in their predictive model when using either total skewness or market adjusted-return skewness, our results are largely unchanged. The low-minus-high total skewness portfolio has a significantly negative loading on PSS. Its α ranges from -0.25% to 0.33% per month across models, which is smaller than the Fama-French three-factor α of slightly more than 1% for low-minus-high risk-neutral skewness portfolios reported in Conrad et al. (2013) (see their Table 4). All low-minus-high portfolio αs are insignificant, except for the Three-Moment CAPM with value-weighted portfolios. Therefore, value-weighted skewness portfolios are strongly exposed to MKT and PSS, but these two factor are not sufficient to explain the higher performance of stocks with low idiosyncratic skewness. When controlling for other risk factors, the risk-adjusted performance of low skewness stocks is not distinguishable from zero.

4.3.2 Predicting Quantile-based Total and Idiosyncratic Skewness

In this section, we use a different measure for idiosyncratic skewness and total skewness. Given that the sample skewness estimator is sensitive to large values, we follow Ghysels et al. (2016) and use a quantile-based measure of skewness,

$$QSK(x_t) = \frac{(q_{0.95}(x_t) - q_{0.50}(x_t)) - (q_{0.50}(x_t) - q_{0.05}(x_t))}{q_{0.95}(x_t) - q_{0.05}(x_t)} \tag{6}$$

where x_t are either daily time-series residuals from the Three-Moment CAPM to capture idiosyncratic skewness or daily returns to capture total skewness, and $q_{0.05}(x_t)$, $q_{0.50}(x_t)$, and $q_{0.95}(x_t)$ are respectively the 5^{th}, 50^{th}, and 95^{th} empirical percentiles of x_t. QSK measures the standardized difference between the distance between a top percentile and the median and the distance between the median and a bottom percentile. QSK is zero for a symmetric distribution and negative (positive) for a negatively (positively) skewed distribution. The advantage of the quantile-based skewness measure QSK is that it is robust to the presence of outliers.

We run predictive panel regressions (4) to predict future quantile-based idiosyncratic and total skewness. We then run time-series regressions for predicted QSK rank sorted portfolios. Tables 2-5 of the Online Appendix reports on equal- and value-weighted portfolios sorted by either quantile-based idiosyncratic skewness or quantile-based total skewness.

The low-minus-high skewness portfolios' loadings on the PSS factor range from -0.87 to -0.39 and are highly significant in all cases. As before, loadings on MOM and RMW are positive and significant everywhere.

The αs for the Three-moment CAPM are positive and significant, ranging from 0.37% to 0.58% per month. Similar to the results using predicted idiosyncratic or total skewness, PSS does not capture all of the outperformance. In contrast, none of the low-minus-high portfolio αs are significantly positive in Panels B and C of Tables 2 to 5 of the Online Appendix. Therefore, we find stronger evidence of low skewness portfolios outperforming high skewness portfolios when predicting a quantile-based measure of skewness, but this outperformance is not robust to the inclusion other risk factors.

5 Conclusion

We provide a novel empirical methodology to predict future differences in systematic and idiosyncratic skewness across stocks. We form a new systematic skewness risk factor and find that it has a robust and economically sizeable risk premium. Finally, we find that idiosyncratic skewness sorted portfolios have a significantly negative loading on idiosyncratic skewness. While the idiosyncratic skewness α is not fully explained by its exposure to the systematic skewness factor, it is not robust to the inclusion of other risk factors such as momentum and profitability.

Our results are important for understanding the relative impact of systematic and idiosyncratic skewness on asset prices. We have relied on models that link risk measures to expected returns and showed which variables best predict these risk measures. A natural extension of our research is to come up with micro-fondations for the link between the identified firm characteristics and future risk measures. We leave this aspect for future research.

Appendix A Data Construction

In this Appendix, we detail our data construction. We use market data from CRSP which we merge with accounting data from Compustat. We use daily and monthly delisting-adjusted returns for all common stocks with a share code 10 or 11, and the one-month U.S. T-bill rate as the risk-free rate.

For each stock i, we construct the following risk measures. For all measures, we use daily data for days t_d in a 12-month period.

- β_M: We estimate a stock $\beta_{M,i,t\to t+11}$ by running a regression of daily excess returns on a constant and the excess returns on the value-weighted CRSP market portfolio:

$$r_{i,t_d} - r_{f,t_d} = \alpha_{i,t\to t+11} + \beta_{M,i,t\to t+11}\left(r_{M,t_d} - r_{f,t_d}\right) + \epsilon_{i,t_d}. \tag{A.7}$$

- **Idiosyncratic volatility**: Volatility of the CAPM regression residuals ϵ_{i,t_d} in Equation (A.7).
- **Coskewness** Cos: We measure coskewness as the covariance between r_{i,t_d} and r^2_{M,t_d}.
- β_{M^2}: We estimate a stock $\beta_{M^2,i,t\to t+11}$ by running a regression of daily excess returns on a constant, the excess returns on the value-weighted CRSP market portfolio and its square:

$$r_{i,t_d} - r_{f,t_d} = \alpha_{i,t\to t+11} + \beta_{M,i,t\to t+11}\left(r_{M,t_d} - r_{f,t_d}\right) + \beta_{M^2,i,t\to t+11}\left(r_{M,t_d} - r_{f,t_d}\right)^2 + \upsilon_{i,t_d}. \tag{A.8}$$

- β_{HS}: We compute the average of $\epsilon_{i,t_d}\times\epsilon^2_{M,t_d}$ where $\epsilon_{M,t_d} = r_{M,t_d} - \frac{1}{T_{d,t\to t+11}}\sum_{t_d=1}^{T_{d,t\to t+11}} r_{M,t_d}$ and $T_{d,t\to t+11}$ is the number of daily returns from month t to month $t+11$. We divide by the square root of the average of ϵ^2_{i,t_d} times the average of ϵ^2_{M,t_d}.
- **Skewness**: Mean of the cubed standardized daily returns r_{i,t_d}.
- **Quantile-based skewness**: We measure robust skewness as in Equation (6) with daily returns r_{i,t_d}.
- **Idiosyncratic skewness**: Mean of the cubed standardized regression residuals υ_{i,t_d} in Equation (A.8).
- **Quantile-based idiosyncratic skewness**: We measure robust idiosyncratic skewness as in Equation (6) with daily residuals υ_{i,t_d}.

We construct the following firm characteristics. We impose a six-month lag on all accounting data to ensure data was available at each point in time.

- **Market Capitalization**: Number of shares outstanding multiplied by the stock price.

- **Book-to-price ratio**: We measure book-to-price ratio as in Asness and Frazzini (2013). For book value of equity, we use in order of availability stockholder's equity, the sum of common equity and preferred stocks, or total assets minus the sum of total liabilities, minority interest, and preferred stocks. We divide by common shares outstanding or, if it is not available, the sum of shares outstanding for all company issues with an earnings participation flag. We divide the book value per share by the most recent stock price. We set to missing if either the book equity is negative or the stock price is missing.
- **Net payout yield**: We measure net payout as in Boudoukh et al. (2007). We compute the sum of common stock dividends and the purchase of common and preferred stocks minus the sale of common and preferred stocks. We divide total payout by the most recent market value of equity.
- **Profitability-to-asset ratio**: We divide gross profitability by total assets.
- **Investment**: We measure total asset growth on an annual basis.
- **Momentum**: Total return from month $t - 12$ to month $t - 2$.
- **Intermediate horizon return**: Total return from month $t - 12$ to month $t - 7$.
- **Lagged monthly return**: Total return for month $t - 1$.
- **Price impact**: The absolute daily returns divided by daily dollar volume averaged over all days in a month for which we have at least five observations.
- **Turnover**: The sum of dollar volume during a month divided by the market capitalization at the end of the previous month.
- **Maximum return**: The average of the highest five daily returns in a given month for months for which there are at least 15 daily returns.

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Table 1 Summary statistics for factors built from different coskewness measures

Factor	Annualized average excess return	Annualized volatility	Sharpe ratio	β_M	Standardized systematic skewness β_{HS}	Annualized CAPM α	Annualized Four-factor α	Annualized Five-factor α
Monthly Cos	2.77	12.24	0.23	0.27**	0.32**	1.03	2.64	3.22*
Monthly β_{M^2}	1.80	9.45	0.19	0.04	0.12**	1.51	0.76	1.22
Monthly β_{HS}	1.89	8.55	0.22	0.00	0.14**	1.86	0.61	0.52
Daily Cos	1.46	12.13	0.12	0.16**	−0.01	0.41	−2.97	−0.76
Daily β_{M^2}	0.00	10.70	0.09	0.08	−0.02	0.52	−2.59	−1.36
Daily β_{HS}	−0.78	9.21	−0.08	−0.01	−0.04	−0.73	2.01	1.05
PSS	5.37**	14.19	0.38	0.21**	−0.27**	4.02*	−0.36	5.91**
PSS (unranked)	2.52	13.40	0.19	0.14	−0.16**	1.61	−0.95	4.55**
Market	6.37**	15.20	0.42		–			

We report summary statistics of monthly returns of different coskewness factors and the value-weighted market portfolio from July 1963 to December 2017. We report the annualized average return (in %), volatility (in %), and Sharpe ratio. Next, we report the market β_M with the value-weighted market portfolio and the systematic skewness β_{HS} defined in Equation (3). Finally, we report the regression α (annualized in %) using different factor models. Each month and for each coskewness measure, we compute the return of a factor that is long a value-weighted portfolio containing the stocks with the lowest 30% values and short a value-weighted portfolio containing the stocks with the top 30% values. Cos is the covariance between stock returns and squared market returns. β_{M^2} is the regression coefficient on squared market returns in a regression of excess stock returns on a constant, the market excess return, and its square. β_{HS} is the standardized coskewness measure of Harvey and Siddique (2000) defined in Equation (3). We compute a monthly version of each of these three measures using monthly returns over the past 60 months and a daily version using daily returns over the past 12 months. We run each month the panel regression (4) to predict the cross-sectional rank of future coskewness using cross-sectional ranks of predictors on the right-hand side. We form the predicted systematic skewness factors PSS by forming a value-weighted portfolio with the stocks with the bottom 30% predicted coskewness and shorting a value-weighted portfolio with the stocks with the top 30% predicted coskewness. The PSS (unranked) uses a predictive panel regression without computing the cross-sectional ranks of future coskewness on the left hand side. The market return is the value-weighted portfolio of all stocks on NYSE, AMEX, and NASDAQ. For each factor and the market portfolio, we simulate 10,000 samples from a bivariate normal distribution with the same means and covariance matrix and compute the standardized coskewness to obtain the significance level. All other significance levels are obtained using a Newey-West estimator with $T^{0.25} \approx 6$ lags. * and ** denote significance at the 5% and 1% level, respectively.

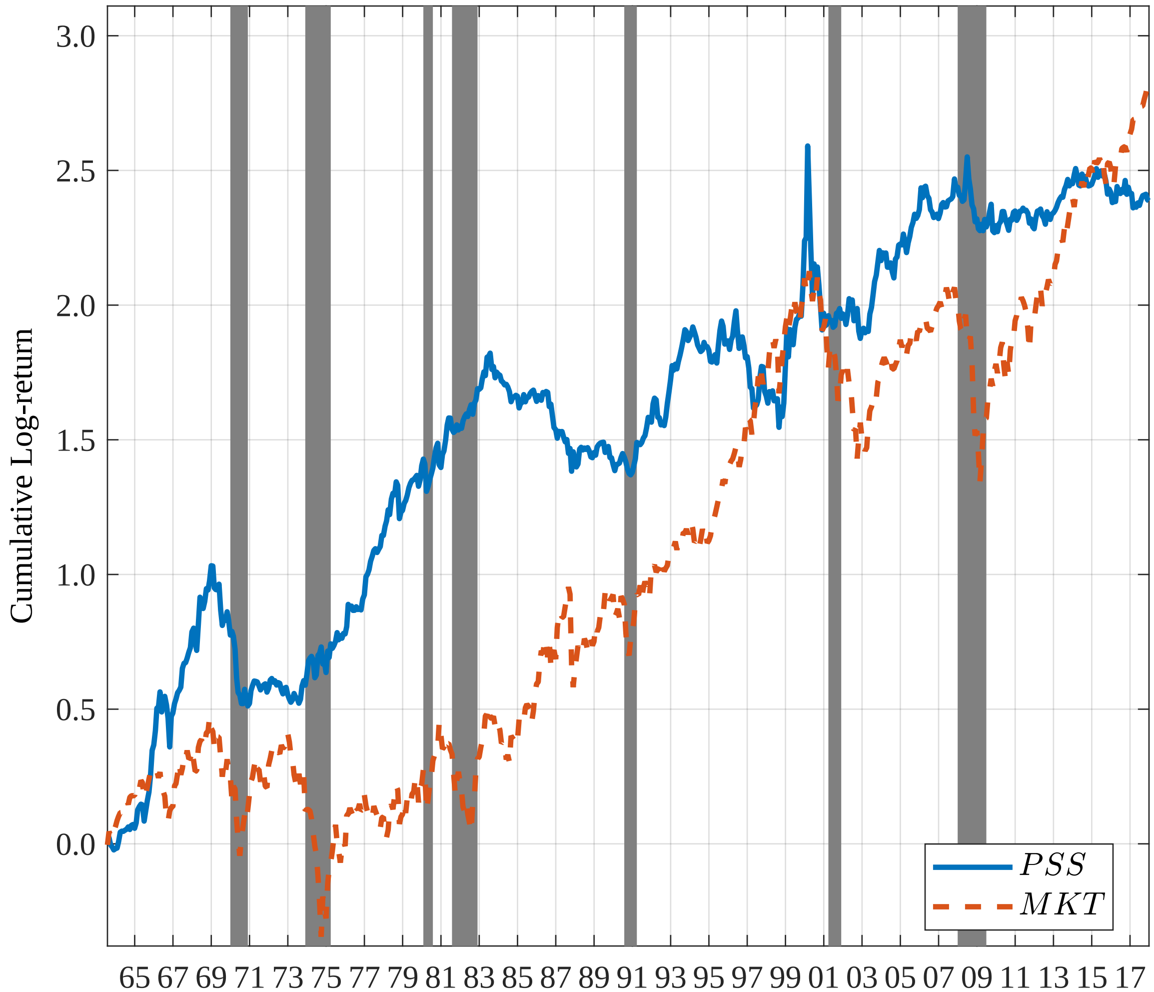

Figure 1 Cumulative log-return for the PSS and MKT factors
We report the cumulative log-return of 1$ invested in the predicted systematic skewness factor PSS and the excess return on the market portfolio MKT from July 1963 to December 2017. Gray areas denote NBER recessions. Both factors are self-financed.

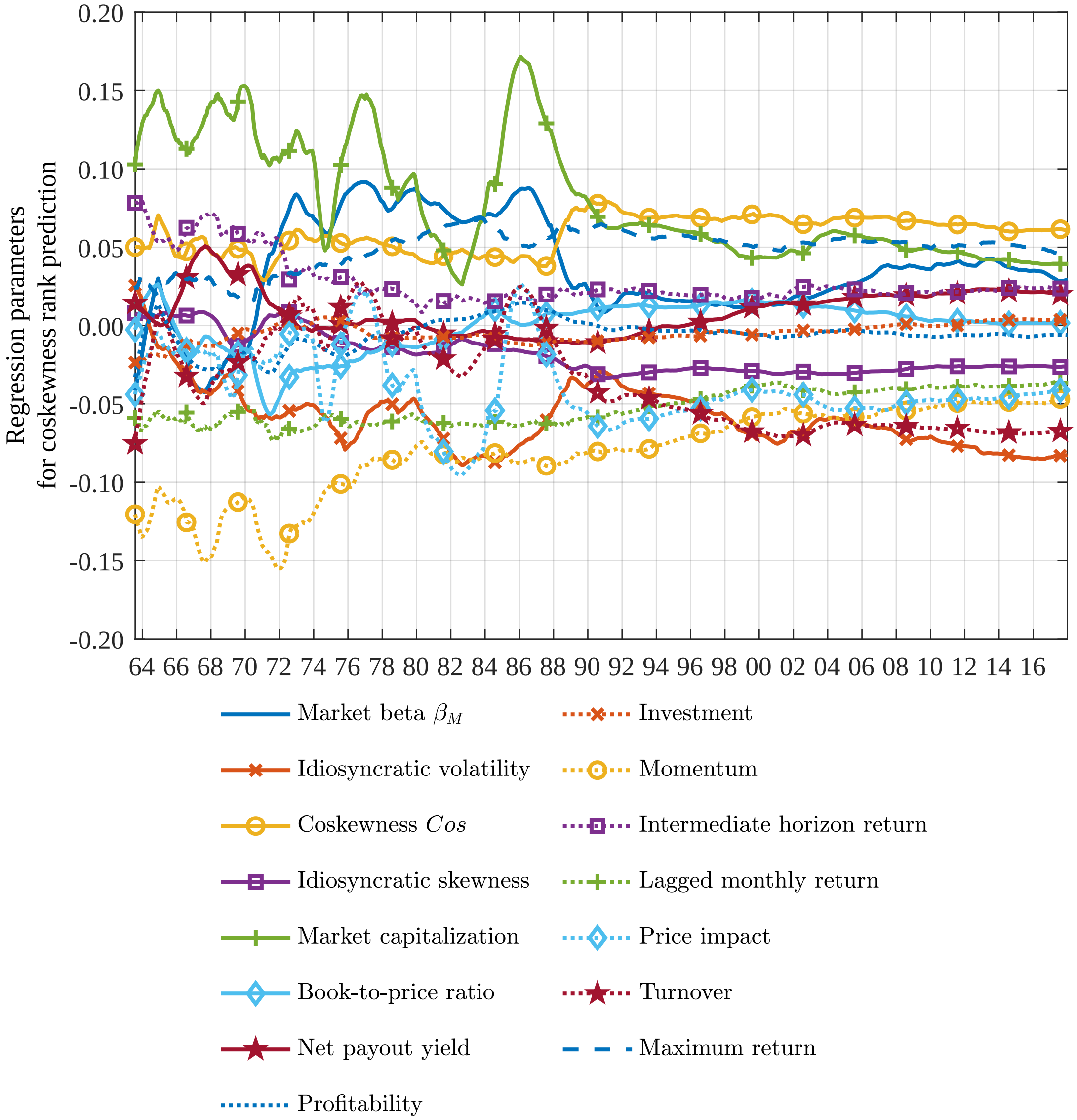

Figure 2 Coefficients of predictive panel regressions for coskewness ranks
We report the panel regression coefficients $\hat{\theta}$ and $\hat{\phi}$ in Equation (4) from July 1963 to December 2017. Each month, we run a panel regression that predicts the next 12-month realized daily coskewness using past risk measures and stock characteristics. We use the cross-sectional rank of coskewness as the dependent variable and the cross-sectional ranks of past risk measures and characteristics as predictors. We use all past observations to estimate at each point in time the panel regression. The construction of all variables is detailed in Appendix A.

Table 2 Summary statistics for panel regression coefficients to predict the coskewness ranks

	Average	5^{th} percentile	95^{th} percentile
Market beta β_M	0.036	−0.022	0.086
Idiosyncratic volatility	−0.059	−0.085	−0.028
Coskewness Cos	0.059	0.041	0.074
Idiosyncratic skewness	−0.019	−0.031	0.007
Market capitalization	0.079	0.039	0.147
Book-to-price ratio	−0.000	−0.029	0.015
Net payout yield	0.009	−0.010	0.036
Profitability	−0.005	−0.026	0.010
Investment	−0.005	−0.015	0.004
Momentum	−0.081	−0.138	−0.049
Intermediate horizon return	0.028	0.016	0.062
Lagged monthly return	−0.052	−0.066	−0.038
Price impact	−0.039	−0.073	0.009
Turnover	−0.036	−0.069	0.018
Maximum return	0.049	0.023	0.064

We report summary statistics of regression coefficients $\hat{\theta}$ and $\hat{\phi}$ in Equation (4) from July 1963 to December 2017. We compute the time-series average and 5^{th} and 95^{th} percentiles. Each month, we run a panel regression that predicts the cross-sectional rank of the daily coskewness computed over the next year using past risk measures and stock characteristics. We use the cross-sectional rank of coskewness as the dependent variable and the cross-sectional ranks of past risk measures and characteristics as predictors. We use all past observations to estimate at each point in time the panel regression. The construction of all variables is detailed in Appendix A.

Table 3 Is the PSS factor significant?

Model		*Constant*	MKT	PSS	MKT^{-}	MKT^{+}	*OLS* R^2	*GLS* R^2
Panel A: 25 Size and Book-to-price Ratio Portfolios								
CAPM	Risk premium	1.36	−0.80				0.08	0.11
		(5.27)	(−2.62)					
	Price of risk	0.01	−4.16					
		(5.27)	(−2.65)					
Three-moment CAPM	Risk premium	1.71	−1.15	0.64			0.51	0.29
		(6.08)	(−3.64)	(2.62)			(0.04)	(0.01)
	Price of risk	0.02	−7.13	5.55				
		(6.08)	(−3.77)	(2.65)				
PSS, β_M^-, β_M^+	Risk premium	1.68		0.62	−0.79	−0.34	0.52	0.30
		(6.00)		(2.47)	(−1.41)	(−0.56)	(0.66)	(0.76)
	Price of risk	0.02		5.78	−12.35	−2.69		
		(6.00)		(2.70)	(−1.29)	(−0.33)		
Panel B: 25 Size and Momentum Portfolios								
CAPM	Risk premium	0.99	−0.36				0.11	0.02
		(4.14)	(−1.26)					
	Price of risk	0.01	−1.85					
		(4.14)	(−1.27)					
Three-moment CAPM	Risk premium	1.41	−0.77	0.74			0.77	0.20
		(4.45)	(−2.29)	(3.06)			(0.00)	(0.00)
	Price of risk	0.01	−5.21	5.69				
		(4.45)	(−2.55)	(2.91)				
PSS, β_M^-, β_M^+	Risk premium	1.61		0.82	−0.93	−0.06	0.77	0.22
		(2.87)		(2.64)	(−0.72)	(−0.08)	(1.00)	(0.65)
	Price of risk	0.02		7.13	−15.52	0.67		
		(2.87)		(2.02)	(−0.75)	(0.06)		

We report asset pricing tests for the CAPM and the Three-moment CAPM in which the market excess return (MKT) is augmented with the predicted systematic skewness factor (PSS). We also report on a model where the MKT factor is separated into low returns (MKT^{-}) and high returns (MKT^{+}). We define low returns as those below the average market excess return minus one standard deviation. As test assets, we use 25 size and book-to-price ratio sorted U.S. equity portfolios in Panel A and 25 size and momentum sorted U.S. equity portfolios in Panel B. For each test portfolio, we run a time-series regression of its excess returns on a constant and factor returns. Then, we run a cross-sectional regression of average test portfolio excess returns on a constant and betas obtained from the time-series regressions. For each model, we estimate the beta form and report the risk premia in % per month for each factor in the first two rows. We then estimate the model in its covariance form and report the prices of risk in the last two rows for each model. We report the model misspecification robust t-ratios from Kan et al. (2013) below risk premia and prices of risk. Below the R^2s for the Three-moment CAPM, we report the p-value for the one-sided test that the model has a significantly higher R^2 than the CAPM. Below the R^2s for the last model, we report the p-value for the two-sided test for non-nested models that the model has a significantly different R^2 than the Three-moment CAPM (see Kan et al., 2013). The sample period is July 1963 to December 2017.

Table 4 Is the PSS factor significant in the four-factor model?

Model		*Constant*	MKT	PSS	SMB	HML	MOM	$OLS\ R^2$	$GLS\ R^2$
Panel A: 25 Size and Book-to-price Ratio Portfolios									
Four-factor	Risk premium	1.07	−0.50		0.22	0.35	1.57	0.76	0.40
		(2.50)	(−1.12)		(1.72)	(2.64)	(1.47)		
	Price of risk	0.01	−0.96		4.05	7.52	9.74		
		(2.50)	(−0.25)		(2.01)	(2.09)	(1.38)		
Four-factor + PSS	Risk premium	1.18	−0.61	0.99	0.22	0.35	1.60	0.76	0.42
		(2.67)	(−1.34)	(2.03)	(1.73)	(2.65)	(1.50)	(0.92)	(0.34)
	Price of risk	0.01	−1.88	4.61	−0.14	7.07	8.26		
		(2.67)	(−0.49)	(0.96)	(−0.03)	(1.98)	(1.17)		
MKT, PSS, HML, UMD	Risk premium	1.18	−0.60	0.98		0.35	1.59	0.76	0.42
		(2.69)	(−1.34)	(2.39)		(2.65)	(1.51)	(0.93)	(0.65)
	Price of risk	0.01	−1.87	4.48		7.07	8.29		
		(2.69)	(−0.49)	(1.95)		(1.98)	(1.18)		
Panel B: 25 Size and Momentum Portfolios									
Four-factor	Risk premium	1.06	−0.44		0.33	0.02	0.69	0.85	0.26
		(1.75)	(−0.76)		(2.25)	(0.04)	(3.80)		
	Price of risk	0.01	−2.59		4.87	1.20	3.71		
		(1.75)	(−0.56)		(2.11)	(0.15)	(1.66)		
Four-factor + PSS	Risk premium	1.02	−0.39	0.37	0.36	−0.07	0.69	0.86	0.27
		(1.65)	(−0.68)	(0.82)	(2.25)	(−0.12)	(3.72)	(0.78)	(0.68)
	Price of risk	0.01	−2.41	−2.76	7.55	0.26	4.46		
		(1.65)	(−0.51)	(−0.41)	(1.07)	(0.03)	(1.83)		
MKT, PSS, HML, UMD	Risk premium	0.98	−0.36	0.66		0.20	0.70	0.86	0.24
		(1.64)	(−0.64)	(2.61)		(0.50)	(3.89)	(0.99)	(0.51)
	Price of risk	0.01	−1.72	3.79		3.63	2.99		
		(1.64)	(−0.39)	(1.77)		(0.54)	(1.33)		

We report asset pricing tests for the four-factor model with market (MKT), size (SMB), value (HML), and momentum (MOM) factors to which we add the predicted systematic skewness factor (PSS). As test assets, we use 25 size and book-to-price ratio sorted U.S. equity portfolios in Panel A and 25 size and momentum sorted U.S. equity portfolios in Panel B. For each test portfolio, we run a time-series regression of its excess returns on a constant and factor returns. Then, we run a cross-sectional regression of average test portfolio excess returns on a constant and betas obtained from the time-series regressions. For each model, we estimate the beta form and report the risk premia in % per month for each factor in the first two rows. We then estimate the model in its covariance form and report the prices of risk in the last two rows for each model. We report the model misspecification robust t-ratios from Kan et al. (2013) below risk premia and prices of risk. Below the R^2s for the four-factor + PSS model, we report the p-value for the one-sided test that the model has a significantly higher R^2 than the four-factor model. Below the R^2s for the last model, we report the p-value for the two-sided test for non-nested models that the model has a significantly different R^2 than the four-factor model (see Kan et al., 2013). The sample period is July 1963 to December 2017.

Table 5 Is the PSS factor significant in the five-factor model?

Model		*Constant*	*MKT*	*PSS*	*SMB*	*HML*	*RMW*	*CMA*	*OLS* R^2	*GLS* R^2
Panel A: 25 Size and Book-to-price Ratio Portfolios										
Five-factor	Risk premium	1.37	−0.83		0.26	0.34	0.23	0.19	0.78	0.32
		(4.18)	(−2.31)		(2.04)	(2.57)	(1.03)	(0.79)		
	Price of risk	0.01	−4.48		6.05	3.20	5.24	−1.07		
		(4.18)	(−1.66)		(3.34)	(0.56)	(1.10)	(−0.09)		
Five-factor + PSS	Risk premium	1.17	−0.61	1.44	0.27	0.35	0.32	0.26	0.88	0.56
		(3.50)	(−1.60)	(3.09)	(2.11)	(2.60)	(1.84)	(1.25)	(0.00)	(0.00)
	Price of risk	0.01	−1.14	19.94	−10.14	2.21	18.30	10.37		
		(3.50)	(−0.35)	(2.87)	(−1.70)	(0.41)	(2.48)	(0.79)		
MKT, PSS, HML, RMW, CMA	Risk premium	1.32	−0.77	0.80		0.34	0.33	0.20	0.84	0.48
		(4.35)	(−2.29)	(3.01)		(2.57)	(1.86)	(1.00)	(0.12)	(0.00)
	Price of risk	0.01	−3.39	9.57		3.06	13.10	3.39		
		(4.35)	(−1.34)	(4.39)		(0.62)	(2.72)	(0.33)		
Panel B: 25 Size and Momentum Portfolios										
Five-factor	Risk premium	1.12	−0.49		0.44	−0.63	0.25	0.52	0.89	0.42
		(2.04)	(−0.90)		(3.27)	(−1.66)	(0.69)	(1.27)		
	Price of risk	0.01	−0.01		9.17	−29.77	13.23	43.82		
		(2.04)	(0.00)		(2.70)	(−2.48)	(1.39)	(2.21)		
Five-factor + PSS	Risk premium	0.99	−0.37	0.44	0.41	−0.35	0.16	0.63	0.90	0.44
		(1.85)	(−0.69)	(1.14)	(3.02)	(−0.71)	(0.41)	(1.45)	(0.28)	(0.44)
	Price of risk	0.01	1.45	4.45	4.30	−24.87	13.09	44.00		
		(1.85)	(0.32)	(0.76)	(0.65)	(−2.14)	(1.32)	(2.26)		
MKT, PSS, HML, RMW, CMA	Risk premium	0.89	−0.26	0.59		−0.10	0.02	0.75	0.90	0.43
		(1.72)	(−0.52)	(2.33)		(−0.34)	(0.06)	(2.06)	(0.75)	(0.82)
	Price of risk	0.01	2.73	7.27		−21.22	11.00	44.92		
		(1.72)	(0.68)	(2.40)		(−1.95)	(1.04)	(2.37)		

We report asset pricing tests for the five-factor model with market (MKT), size (SMB), value (HML), profitability (RMW), and investment (CMA) factors to which we add the predicted systematic skewness factor (PSS). As test assets, we use 25 size and book-to-price ratio sorted U.S. equity portfolios in Panel A and 25 size and momentum sorted U.S. equity portfolios in Panel B. For each test portfolio, we run a time-series regression of its excess returns on a constant and factor returns. Then, we run a cross-sectional regression of average test portfolio excess returns on a constant and betas obtained from the time-series regressions. For each model, we estimate the beta form and report the risk premia in % per month for each factor in the first two rows. We then estimate the model in its covariance form and report the prices of risk in the last two rows for each model. We report the model misspecification robust t-ratios from Kan et al. (2013) below risk premia and prices of risk. Below the R^2s for the five-factor + PSS model, we report the p-value for the one-sided test that the model has a significantly higher R^2 than the five-factor model. Below the R^2s for the last model, we report the p-value for the two-sided test for non-nested models that the model has a significantly different R^2 than the five-factor model (see Kan et al., 2013). The sample period is July 1963 to December 2017.

Table 6 Robustness check - asset pricing tests with omitted factors

Test Portfolios	p	*Constant*	MKT	PSS
50 Size, book-to-price ratio, and momentum Portfolios	4	0.18	0.38	0.36
		(0.80)	(1.40)	(2.37)
	5	0.36	0.20	0.36
		(1.33)	(0.64)	(2.37)
	6	1.06	−0.46	0.47
		(2.34)	(−0.99)	(2.65)
202 Portfolios from Giglio and Xiu (2017)	4	0.21	0.35	0.28
		(2.26)	(1.73)	(1.89)
	5	0.26	0.29	0.30
		(2.52)	(1.38)	(2.00)
	6	0.21	0.34	0.30
		(1.69)	(1.56)	(1.99)

We report estimated constants and risk premia for the Three-moment CAPM in which the market excess return (MKT) is augmented with the predicted systematic skewness factor (PSS). We use the estimation methodology of Giglio and Xiu (2017) which is robust to omitted factors and measurement error. As test assets, we use 25 size and book-to-price ratio sorted and 25 size and momentum sorted U.S. equity portfolios in the top rows. We use the 202 U.S. equity portfolios from Giglio and Xiu (2017) in the bottom rows (25 portfolios sorted by size and book-to-market ratio, 17 industry portfolios, 25 portfolios sorted by operating profitability and investment, 25 portfolios sorted by size and variance, 35 portfolios sorted by size and net issuance, 25 portfolios sorted by size and accruals, 25 portfolios sorted by size and momentum, and 25 portfolios sorted by size and beta). The second column reports on the number of principal components used to span the space of asset returns. The last three columns report on the constant, the risk premium for MKT and the risk premium for PSS, all reported in % per month. t-ratios are below risk premia in parentheses. The sample period is July 1963 to December 2017.

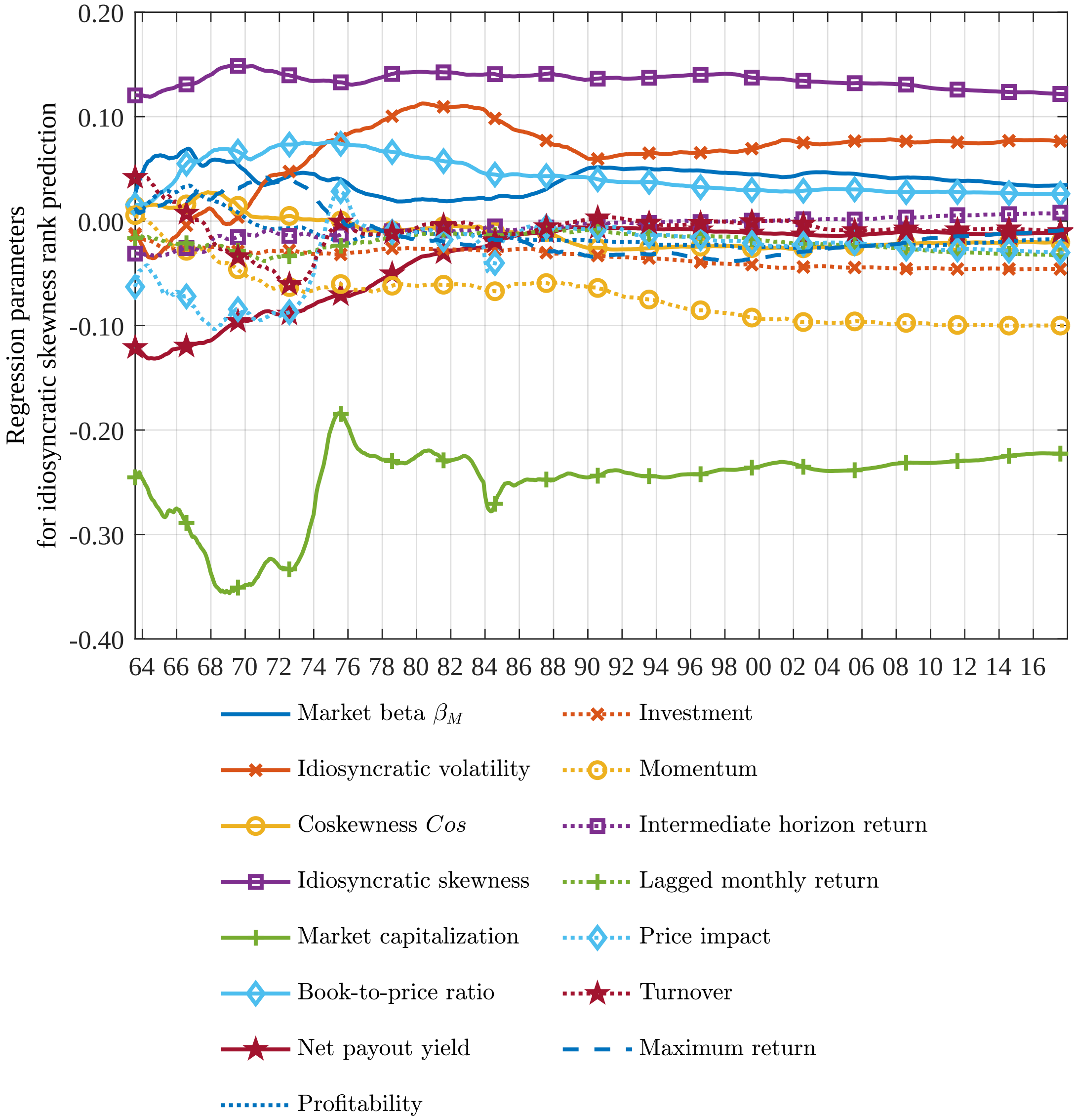

Figure 3 Coefficients of predictive panel regressions for idiosyncratic skewness ranks

We report the panel regression coefficients $\hat{\theta}$ and $\hat{\phi}$ in Equation (4) from July 1963 to December 2017. Each month, we run a panel regression that predicts the next 12-month realized daily idiosyncratic skewness using past risk measures and stock characteristics. We use the cross-sectional rank of idiosyncratic skewness as the dependent variable and the cross-sectional ranks of past risk measures and characteristics as predictors. We use all past observations to estimate at each point in time the panel regression. The construction of all variables is detailed in Appendix A.

Table 7 Summary statistics for panel regression coefficients to predict idiosyncratic skewness ranks

	Average	5^{th} percentile	95^{th} percentile
Market beta β_M	0.040	0.020	0.059
Idiosyncratic volatility	0.066	−0.009	0.109
Coskewness Cos	−0.012	−0.026	0.017
Idiosyncratic skewness	0.135	0.122	0.144
Market capitalization	−0.249	−0.340	−0.221
Book-to-price ratio	0.043	0.026	0.074
Net payout yield	−0.036	−0.121	−0.007
Profitability	−0.014	−0.025	0.024
Investment	−0.035	−0.046	−0.025
Momentum	−0.073	−0.100	−0.026
Intermediate horizon return	−0.005	−0.029	0.007
Lagged monthly return	−0.021	−0.033	−0.010
Price impact	−0.029	−0.091	−0.005
Turnover	−0.008	−0.044	0.012
Maximum return	−0.012	−0.036	0.037

We report summary statistics of regression coefficients $\hat{\theta}$ and $\hat{\phi}$ in Equation (4) from July 1963 to December 2017. We compute the time-series average and 5^{th} and 95^{th} percentiles. Each month, we run a panel regression that predicts the cross-sectional rank of the daily idiosyncratic skewness computed over the next year using past risk measures and stock characteristics. We use the cross-sectional rank of idiosyncratic skewness as the dependent variable and the cross-sectional ranks of past risk measures and characteristics as predictors. We use all past observations to estimate at each point in time the panel regression. The construction of all variables is detailed in Appendix A.

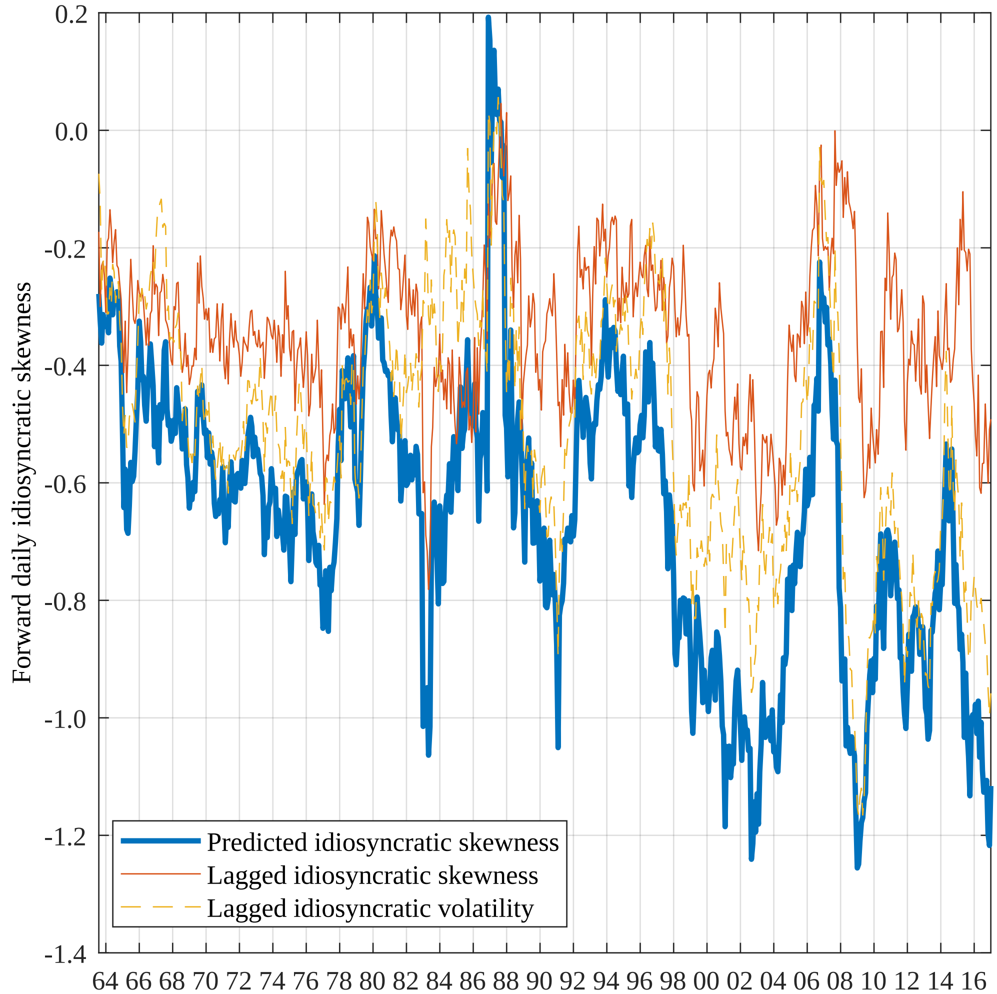

Figure 4 Equal-weighted average of stock-specific realized idiosyncratic skewness
We report equal-weighted average of stock-specific realized idiosyncratic skewness. Each month, we rank stocks based on a predictor of future idiosyncratic skewness. As predictors, we use daily return idiosyncratic skewness computed over the last year, daily return idiosyncratic volatility computed over the last year, and the panel regression predicted idiosyncratic skewness cross-sectional ranks. We then compute each stock's daily return idiosyncratic skewness over the next year. For each predictor and each month, we report the equal-weighted average idiosyncratic skewness of the bottom 30% stocks minus the equal-weighted average idiosyncratic skewness of the top 30% stocks.

Table 8 Factor analysis of equal-weighted portfolios sorted by predicted idiosyncratic skewness

Portfolio	α(%)	β_{MKT}	β_{PSS}	β_{HML}	β_{MOM}	β_{RMW}	β_{CMA}	Adj. R^2
Panel A: MKT, PSS								
Low	0.15	0.91	−0.04					0.92
	(2.43)	(45.06)	(1.09)					
Medium	0.21	1.04	0.21					0.86
	(2.17)	(36.15)	(3.59)					
High	−0.01	1.19	0.54					0.70
	(0.04)	(24.83)	(6.88)					
Low-High	0.16	−0.28	−0.59					0.33
	(1.11)	(5.21)	(9.46)					
Panel B: MKT, PSS, HML, MOM								
Low	0.05	0.94	−0.04	0.18	0.04			0.93
	(0.99)	(73.09)	(1.51)	(3.97)	(1.57)			
Medium	0.22	1.03	0.31	0.25	−0.21			0.91
	(2.78)	(48.68)	(8.40)	(6.03)	(8.34)			
High	0.29	1.09	0.78	0.12	−0.59			0.81
	(1.78)	(23.60)	(13.61)	(1.47)	(7.53)			
Low-High	−0.25	−0.14	−0.82	0.06	0.63			0.55
	(1.50)	(3.13)	(15.70)	(0.59)	(6.78)			
Panel C: MKT, PSS, HML, RMW, CMA								
Low	−0.06	0.97	0.03	0.09		0.22	0.19	0.94
	(1.31)	(78.13)	(1.10)	(3.32)		(6.25)	(4.09)	
Medium	0.04	1.10	0.24	0.27		0.02	0.10	0.89
	(0.46)	(52.87)	(5.84)	(5.64)		(0.40)	(1.51)	
High	0.05	1.19	0.46	0.29		−0.44	−0.05	0.73
	(0.23)	(24.66)	(4.65)	(2.90)		(2.98)	(0.25)	
Low-High	−0.11	−0.22	−0.44	−0.20		0.67	0.25	0.39
	(0.46)	(4.40)	(4.28)	(1.89)		(3.96)	(0.00)	

We run time-series regressions of portfolio excess returns on different factor models. Each month, we run a panel regression that predicts the next 12-month realized daily idiosyncratic skewness using past risk measures and stock characteristics. We use the cross-sectional rank of idiosyncratic skewness as the dependent variable and the cross-sectional ranks of past risk measures and characteristics as predictors. We form equal-weighted portfolios: one with the bottom 30% stocks with the lowest predicted idiosyncratic skewness ranks (Low), one with the middle 40% stocks (Medium), one with the top 30% stocks with the highest predicted idiosyncratic skewness ranks (High), and a low-minus-high portfolio. We use the Three-moment CAPM in which the market excess return (MKT) is augmented with the predicted systematic skewness factor (PSS) in Panel A, the modified four-factor model with MKT, PSS, value (HML), and momentum (MOM) factors in Panel B, and the modified five-factor model with MKT, PSS, profitability (RMW), and investment (CMA) factors in Panel C. For each regression, we report the monthly α in %, the factor exposures, and adjusted R^2. We report in parentheses the t-ratios using a Newey-West estimator with $T^{0.25} \approx 6$ lags. The sample period is July 1963 to December 2017.

Table 9 Factor analysis of value-weighted portfolios sorted by predicted idiosyncratic skewness

Portfolio	α(%)	β_{MKT}	β_{PSS}	β_{HML}	β_{MOM}	β_{RMW}	β_{CMA}	Adj. R^2
Panel A: MKT, PSS								
Low	0.08	0.91	−0.14					0.97
	(2.78)	(78.43)	(12.32)					
Medium	0.04	1.13	0.04					0.86
	(0.43)	(30.63)	(0.75)					
High	−0.21	1.31	0.47					0.75
	(1.44)	(24.22)	(6.11)					
Low-High	0.28	−0.39	−0.62					0.40
	(1.85)	(6.36)	(7.63)					
Panel B: MKT, PSS, HML, MOM								
Low	0.00	0.94	−0.17	0.04	0.09			0.98
	(0.04)	(148.90)	(16.97)	(2.36)	(8.80)			
Medium	0.07	1.11	0.17	0.29	−0.28			0.93
	(1.08)	(52.78)	(6.13)	(7.96)	(12.14)			
High	0.05	1.22	0.72	0.24	−0.61			0.87
	(0.36)	(31.05)	(14.01)	(4.13)	(9.65)			
Low-High	−0.05	−0.28	−0.90	−0.20	0.70			0.69
	(0.33)	(6.70)	(15.95)	(2.79)	(10.16)			
Panel C: MKT, PSS, HML, RMW, CMA								
Low	−0.02	0.94	−0.10	−0.03		0.15	0.12	0.98
	(0.50)	(118.45)	(8.47)	(1.77)		(6.66)	(3.97)	
Medium	−0.06	1.16	0.03	0.39		−0.16	−0.06	0.90
	(0.71)	(44.58)	(0.92)	(6.03)		(3.12)	(0.81)	
High	−0.18	1.32	0.40	0.45		−0.47	−0.12	0.78
	(0.93)	(27.57)	(4.60)	(4.66)		(3.67)	(0.68)	
Low-High	0.17	−0.38	−0.50	−0.49		0.62	0.24	0.49
	(0.76)	(7.14)	(5.21)	(4.38)		(4.21)	(1.18)	

We run time-series regressions of portfolio excess returns on different factor models. Each month, we run a panel regression that predicts the next 12-month realized daily idiosyncratic skewness using past risk measures and stock characteristics. We use the cross-sectional rank of idiosyncratic skewness as the dependent variable and the cross-sectional ranks of past risk measures and characteristics as predictors. We form value-weighted portfolios: one with the bottom 30% stocks with the lowest predicted idiosyncratic skewness ranks (Low), one with the middle 40% stocks (Medium), one with the top 30% stocks with the highest predicted idiosyncratic skewness ranks (High), and a low-minus-high portfolio. We use the Three-moment CAPM in which the market excess return (MKT) is augmented with the predicted systematic skewness factor (PSS) in Panel A, the modified four-factor model with MKT, PSS, value (HML), and momentum (MOM) factors in Panel B, and the modified five-factor model with MKT, PSS, profitability (RMW), and investment (CMA) factors in Panel C. For each regression, we report the monthly α in %, the factor exposures, and adjusted R^2. We report in parentheses the t-ratios using a Newey-West estimator with $T^{0.25} \approx 6$ lags. The sample period is July 1963 to December 2017.

Table 10 Factor analysis of equal-weighted portfolios sorted by predicted skewness

Portfolio	α(%)	β_{MKT}	β_{PSS}	β_{HML}	β_{MOM}	β_{RMW}	β_{CMA}	Adj. R^2
Panel A: MKT, PSS								
Low	0.16	0.91	−0.04					0.92
	(2.63)	(42.80)	(0.94)					
Medium	0.22	1.04	0.21					0.86
	(2.15)	(35.77)	(3.80)					
High	−0.02	1.19	0.53					0.69
	(0.12)	(24.44)	(6.59)					
Low-High	0.18	−0.28	−0.57					0.31
	(1.25)	(4.96)	(9.03)					
Panel B: MKT, PSS, HML, MOM								
Low	0.04	0.95	−0.04	0.20	0.05			0.93
	(0.83)	(73.57)	(1.52)	(4.41)	(2.37)			
Medium	0.23	1.04	0.31	0.25	−0.21			0.91
	(2.83)	(48.49)	(8.74)	(5.91)	(8.67)			
High	0.29	1.08	0.76	0.11	−0.60			0.80
	(1.76)	(23.04)	(13.21)	(1.37)	(7.70)			
Low-High	−0.25	−0.13	−0.81	0.09	0.66			0.55
	(1.54)	(2.79)	(15.45)	(0.81)	(7.16)			
Panel C: MKT, PSS, HML, RMW, CMA								
Low	−0.06	0.97	0.04	0.11		0.24	0.19	0.94
	(1.34)	(78.43)	(1.51)	(3.86)		(6.70)	(3.97)	
Medium	0.05	1.10	0.25	0.26		0.02	0.11	0.88
	(0.50)	(51.79)	(5.97)	(5.40)		(0.37)	(1.53)	
High	0.05	1.19	0.45	0.29		−0.45	−0.05	0.72
	(0.20)	(24.13)	(4.44)	(2.80)		(3.05)	(0.25)	
Low-High	−0.10	−0.22	−0.41	−0.18		0.69	0.24	0.38
	(0.43)	(4.15)	(3.98)	(1.67)		(4.08)	(0.97)	

We run time-series regressions of portfolio excess returns on different factor models. Each month, we run a panel regression that predicts the next 12-month realized daily skewness using past risk measures and stock characteristics. We use the cross-sectional rank of skewness as the dependent variable and the cross-sectional ranks of past risk measures and characteristics as predictors. We form equal-weighted portfolios: one with the bottom 30% stocks with the lowest predicted skewness ranks (Low), one with the middle 40% stocks (Medium), one with the top 30% stocks with the highest predicted skewness ranks (High), and a low-minus-high portfolio. We use the Three-moment CAPM in which the market excess return (MKT) is augmented with the predicted systematic skewness factor (PSS) in Panel A, the modified four-factor model with MKT, PSS, value (HML), and momentum (MOM) factors in Panel B, and the modified five-factor model with MKT, PSS, profitability (RMW), and investment (CMA) factors in Panel C. For each regression, we report the monthly α in %, the factor exposures, and adjusted R^2. We report in parentheses the t-ratios using a Newey-West estimator with $T^{0.25} \approx 6$ lags. The sample period is July 1963 to December 2017.

Table 11 Factor analysis of value-weighted portfolios sorted by predicted skewness

Portfolio	α(%)	β_{MKT}	β_{PSS}	β_{HML}	β_{MOM}	β_{RMW}	β_{CMA}	Adj. R^2
Panel A: MKT, PSS								
Low	0.08	0.91	−0.14					0.97
	(2.93)	(77.17)	(12.02)					
Medium	−0.00	1.14	0.07					0.86
	(0.02)	(29.28)	(1.41)					
High	−0.25	1.31	0.45					0.73
	(1.65)	(23.13)	(5.74)					
Low-High	0.33	−0.40	−0.59					0.37
	(2.05)	(6.14)	(7.29)					
Panel B: MKT, PSS, HML, MOM								
Low	−0.00	0.94	−0.17	0.05	0.09			0.98
	(0.02)	(149.20)	(16.87)	(2.92)	(9.07)			
Medium	0.07	1.11	0.21	0.26	−0.32			0.94
	(1.14)	(53.31)	(7.95)	(6.70)	(13.81)			
High	0.05	1.20	0.71	0.21	−0.65			0.86
	(0.36)	(30.42)	(13.63)	(3.60)	(9.89)			
Low-High	−0.05	−0.27	−0.88	−0.16	0.75			0.69
	(0.34)	(6.40)	(15.68)	(2.29)	(10.66)			
Panel C: MKT, PSS, HML, RMW, CMA								
Low	−0.02	0.94	−0.10	−0.02		0.15	0.12	0.98
	(0.51)	(117.89)	(8.51)	(1.32)		(6.91)	(4.03)	
Medium	−0.07	1.17	0.05	0.37		−0.20	−0.06	0.89
	(0.78)	(40.99)	(1.37)	(5.60)		(3.58)	(0.85)	
High	−0.19	1.32	0.36	0.43		−0.52	−0.13	0.77
	(0.94)	(26.57)	(4.12)	(4.40)		(4.05)	(0.72)	
Low-High	0.18	−0.38	−0.46	−0.46		0.67	0.25	0.46
	(0.77)	(6.85)	(4.78)	(4.11)		(4.57)	(1.21)	

We run time-series regressions of portfolio excess returns on different factor models. Each month, we run a panel regression that predicts the next 12-month realized daily skewness using past risk measures and stock characteristics. We use the cross-sectional rank of skewness as the dependent variable and the cross-sectional ranks of past risk measures and characteristics as predictors. We form value-weighted portfolios: one with the bottom 30% stocks with the lowest predicted skewness ranks (Low), one with the middle 40% stocks (Medium), one with the top 30% stocks with the highest predicted skewness ranks (High), and a low-minus-high portfolio. We use the Three-moment CAPM in which the market excess return (MKT) is augmented with the predicted systematic skewness factor (PSS) in Panel A, the modified four-factor model with MKT, PSS, value (HML), and momentum (MOM) factors in Panel B, and the modified five-factor model with MKT, PSS, profitability (RMW), and investment (CMA) factors in Panel C. For each regression, we report the monthly α in %, the factor exposures, and adjusted R^2. We report in parentheses the t-ratios using a Newey-West estimator with $T^{0.25} \approx 6$ lags. The sample period is July 1963 to December 2017.