Corporate Finance Responses to Exogenous Tax Changes: What Is the Null and Where Did It Come From?

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Abstract

Absent theoretical guidance, empiricists have been forced to rely upon comparative statics analysis derived from constant tax rate models in formulating testable implications of tradeoff theory in the context of natural policy experiments. We fill the theoretical void by solving in closed-form a dynamic tradeoff theoretic model in which corporate taxes follow a (two-state) Markov process with exogenous rate changes. We simulate ideal difference-in-differences estimations, finding that constant tax rate models offer poor guidance regarding testable implications of the underlying theory in experimental settings. While constant rate models predict large, symmetric and statistically significant responses to tax rate changes, our model with stochastic tax rates predicts small, asymmetric, and often statistically insignificant responses. Under plausible parameterizations with decade-long regime lives, the true underlying theory, that taxes matter, is incorrectly rejected in roughly half the simulated natural experiments. These results suggest the need for theorists and empiricists to move beyond comparative statics analysis for inference, instead relying upon dynamic models actually speaking to the policy experiments being exploited.

1 Introduction

Slemrod (1990) opens his volume analyzing the empirical evidence on the landmark Tax Reform Act of 1986 (TRA86) by posing a startling question: Do taxes matter? Slemrod (1992) offers an equally startling answer to this question, writing, "I do not mean to suggest that, for all aspects of behavior, all economists have lowered toward near zero their best guess of the tax elasticity... However, I do believe that this is an appropriate generalization about how views have changed in the past decade."

For public finance economists, Slemrod's question borders on the existential, for if taxes do not matter for decisions, a good deal of theoretical and empirical analysis is destined for the historical dustbin. The normative importance of correctly answering this question is also hard to overstate. After all, a negative answer implies tax-induced deadweight losses are a non-issue, implying policymakers can focus on redistribution. Indeed, having the final word in the Slemrod volume, Aaron (1990) states, "The kind of results reported here would, if replicated and sustained in future work, cast a doubt over the celebrated conclusion of optimal tax literature that welfare-maximizing tax rates leave little room for progressivity..."

In analyzing the optimal tax mix, public finance economists have devoted considerable attention to understanding the effect of corporate taxation on capital structure decisions. The sensitivity of leverage to tax rates is of first-order importance in policy debates. For example, in making the U.S. Treasury's case for integration of the individual and corporate tax systems, Hubbard (1993) contends, "tax-induced distortions in corporations' comparisons of nontax advantages and disadvantages of debt entail significant efficiency costs." Understanding the tax-sensitivity of leverage is also important for estimating revenue impacts. For example, Gruber and Rauh (2007) estimate the tax elasticity of corporate income is only -0.2, evidence that would appear to contradict Hubbard's notion that corporations aggressively change capital structures in response to tax incentives.

For corporate finance economists, understanding whether taxes affect leverage is also a first-order question. After all, trade-off theory stands as one of the field's leading theories. According to this theory, firms weigh tax advantages of debt against costs of increasing leverage, such as distress costs, in optimizing leverage. Clearly, the failure to reject the null of zero tax effect would constitute strong evidence against trade-off theory. To wit, in making his case against the trade-off theory, and in favor of his pecking-order, Myers (1984) writes, "I know of no study clearly demonstrating that a firm's tax status has predictable, material effects on its debt policy."

Gordon and MacKie-Mason (1990) offer what has by now become a consensus view regarding the empirical evidence on taxes and leverage, writing, "Past work has presumed that taxes play an important role in these decisions. If so, then the extensive changes in the tax law that were enacted in 1986 should have led to noticeable changes in these decisions... We find that the actual change in debt-to-value ratios has been substantially smaller than the models predict." The finding of a weak corporate leverage response to tax changes has been confirmed over much longer sample windows. For example, in their recent study of long-term changes in U.S. corporate capital structures, Graham, Leary, and Roberts (2015) state, "Corporate taxes underwent 30 revisions over the past century and increased from 10% to 52% between 1920 and 1950. Yet we find no significant time-series relation between taxes and the margin between debt usage and common equity, in large part because of a near decade-long delay in the response of leverage to tax changes."[1]

Reflecting the primacy today's empiricists place upon the avoidance of selection bias, many would argue that the chief barrier to our understanding of tax-elasticities is the fact that tax changes are endogenous decisions. Consistent with this view, Auerbach (1996) states,"Tax reforms have proven difficult to assess for a variety of reasons, all related to the nonexperimental nature of empirical economic analysis." Although we do not dispute the importance of avoiding selection bias, we begin this paper by pointing out a much more fundamental constraint on our ability to use empirical tests in order to reach valid conclusions here: *We actually have no idea what kind of regression coefficients to expect from trade-off theoretic firms.*

To see the point, suppose one were actually able to identify the type of ideal exogenous change in corporate taxation that today's best empiricists seek out, with, say, an ideal control group unaffected by the change. Before hitting the return key, some obvious questions arise. Do we actually have an informed prior regarding what coefficient magnitudes to expect? And does the interpretation of the evidence hinge upon the sign of the tax rate change? Finally, do we actually know the properties of the estimator here, such as the probability of Type II errors?

The problem is not that empiricists are not sufficiently careful, or that they have failed to pay sufficient attention to existing theoretical models. Rather, *the problem is the absence of models*

[1]In contrast, Graham (1999) does document a cross-sectional relationship between leverage and tax shield value.

that actually speak to the empirical tests. This has not been for lack of effort. For example, early models, such as that of Stiglitz (1973), failed to deliver interior optimal leverage ratios. Lacking interior optimal capital structures, computational general equilibrium (CGE) models, e.g. Ballard, et. al (1985), posited exogenous financing rules. In the absence of closed models, public finance economists such as Gordon and MacKie-Mason (1990) and Nadeau (1993) were forced into positing ad hoc costs of financial distress.

In an important contribution, Leland (1994) showed how to develop a logically closed model of capital structure for firms facing taxation and costs of distress using contingent-claims pricing methods. Equally important for empiricists seeking an interpretable model, Leland's framework is analytically tractable, in contrast to CGE models. A limitation of the Leland model, however, is that the firm chooses its debt level once and for all at date zero. The model of Goldstein, Ju and Leland (2001), following in the footsteps of Fischer, Heinkel and Zechner (1989), made an important advance, offering a tractable fully dynamic model of optimal leverage policy for firms facing taxation, bankruptcy costs, and proportional debt issuance costs. This model has become a staple in the capital structure literature, with Strebulaev (2007) and Danis, Rettl and Whited (2014) showing that the framework sheds light on a number of capital structure puzzles.

With such models in-hand, it is tempting to proceed to the data, using comparative statics analysis to formulate testable hypotheses regarding tax effects. In fact, analytical or numerical comparative statics analysis has been the workhorse for extracting empirical predictions. For example, Auerbach and Slemrod (1997) state that "we must specify why firms do not finance exclusively with debt... There are a variety of potential explantations in the literature, and they may give rise to different comparative static predictions regarding the impact of TRA86." Welch (2010, 2013) uses comparative statics as a basis for concluding that leverage ratios should respond positively to increases in the corporate tax rate according to trade-off theory. Leland (1994) reaches the same conclusion using his contingent-claims model. Goldstein, Ju and Leland (2001) present a similar numerical comparative static result in their dynamic model. With such comparative static results serving as a backdrop, the consensus interpretation of the empirical evidence looks uncontroversial: The theoretical canon predicts large symmetric leverage responses to tax rate changes, so apparently tax motives are a second-order effect, if they are an effect at all.

However, introspection suggests there is still a wide gap between the empirical tests for tax effects and the models, even those representing the state-of-the-art, e.g. Goldstein, Ju and Leland (2001). After all, the econometrician is exploiting stochastic fluctuations in tax rates, yet the tax rate in the model is a parameter. In other words, the econometrician and the corporations inside the model apparently occupy different worlds. The econometrician exploits real-world data with periodic changes in tax shield value. Yet the modeled firms expect tax shield value to remain constant forever, until the next tax rate innovation, after which they again expect the tax rate to remain constant forever.

The first contribution of our paper is to develop a theoretical model that remedies this internal inconsistency, narrowing the gap between model and data. Specifically, we extend the dynamic capital structure model of Goldstein, Ju and Leland (2001) to incorporate a stochastic corporate tax rate, taking the form of a two-state Markov chain with arbitrary expected regime durations. Despite the substantial increase in complexity, all components of firm value are derived in closed-form. Since there is no need for numerical solutions for value functions, the model represents a tractable and transparent framework for empiricists to contemplate various empirical thought-experiments, as we do below.

We use the simulated model as an ideal laboratory for conducting tests of empirical tests. To begin, we analyze the question of whether responses to tax rate changes should be expected to be symmetric in real-world data. The workhorse approach in the literature, numerical comparative statics, mechanically predicts symmetric responses. In contrast, our model shows that financial policy should be more responsive to tax rate increases than to decreases. Intuitively, if the tax rate increases (decreases), the optimal response is to decrease (increase) the interest coverage ratio, which imposes a negative (positive) externality on the tax collector. Thus, after a jump upward (downward) in the tax rate, simulated firms accelerate (delay) recapitalization transactions.

In simulated data, we find that this asymmetry manifests itself at both the extensive (percentage of firms recapitalizing) and intensive (interest coverage ratio) margins. For example, consider a 10 percentage point change in the effective tax rate (30% versus 40%) for firms facing decade-long expected tax regime durations. Applying textbook difference-in-differences (DD) estimation to 1000 simulated tax changes, with 1000 treated firms paired with 1000 ideal control firms, we find a high probability of Type II error, especially for tax rate decreases. In terms of refinancing events, roughly 40% (8%) of simulated DD estimations analyzing tax rate decreases (increases) find zero effect. In terms of interest coverage ratios roughly 35% (8%) of simulated DD estimations analyzing tax rate decreases (increases) find zero effect.

We next focus our attention on the likely probability of Type II error in real-world data which generally focuses on the behavior of corporate leverage ratios. Even with the fundamental tax overhaul of TRA86, Gordon and MacKie-Mason (1990) estimate a change in effective tax shield value of only 2.5 percentage points. We consider a larger change of 4 percentage points (36% versus 40%), focusing on a tax rate increase where statistical power is higher. In comparing leverage ratios across economies facing permanent tax rates, the average difference in leverage ratios would be 0.0157. In contrast, with decade-long expected regime lives, the average DD regression coefficient is only 0.0057. That is, the coefficient estimate obtained under a long, yet realistic, regime life of one decade is only 36.4% of the average causal effect associated with such tax rate differentials. Worse still, in about 45% of the simulated tax reforms studied, one fails to find a statistically significant coefficient at the 5% level of confidence. More generally, if the expected regime life falls just below one decade, the majority of simulated experiments fail to reject the null that taxes do not matter for leverage decisions– despite the fact that taxes are *the* cause of corporate leverage in the posited economy.

What explains these troubling findings? A number of mechanisms are at work. First, due to transactions costs, only a subset of firms react quickly to tax rate changes. Second, transient tax rates cause firms to be less aggressive conditional upon engaging in financial restructuring. Finally, although we impose common aggregate shocks on all firms, treated and control groups are not identical, as they necessarily differ in terms of idiosyncratic shocks. We do not claim to be the first to point out these three challenges to inference in the context of capital structure research. Rather, we merely claim to be the first to develop an operational model incorporating the underlying frictions, allowing us to assess their quantitative importance in the context of state-of-the-art empirical tests featuring ideal exogenous policy changes. Our findings show that rather than being rattled off in the boilerplate list of caveats, these concerns may be fatal, leading to false falsifications of a correct theory or erroneous conclusions regarding whether, and how much, taxes matter for capital structure decisions.

As mentioned above, the frictions at work in the model have been discussed in qualitative terms by empirical researchers who were, alas, at a loss in assessing their quantitative importance.

For example, Gordon and MacKie-Mason (1990) and Aaron (1990) mention adjustment costs as potential caveats–an idea explored by Auerbach (1985) in reduced-form regressions in real-world data. Similarly, in his closing remarks to the Slemrod volume, Aaron (1990) notes that tax rate transience may have dampened responsiveness, an idea that dates back to Abel (1982), for example, who considered investment decisions in a perfect foresight model.

Danis, Rettl and Whited (2014) provide empirical evidence in favor of lumpy leverage adjustment models, such as we present here, documenting that market leverage decreases for several quarters for those firms that subsequently engage in large deliberate leverage increases. The most direct support in favor of our model is offered by Heider and Ljungqvist (2015) who use difference-in-differences estimation to document a positive response of corporate leverage to increases in state income tax rates, especially for more profitable and higher rated firms, but find zero response to decreases in tax rates. The difference-in-differences methodology is also applied by Panier, Perez-Gonzalez and Villanueva (2015) who document an increase in equity usage in conjunction with a novel Belgium tax reform allowing deductibility of notional interest on equity capital. Our model suggests that the causal effect of taxation on capital structure must be strong indeed in order to consistently generate statistically significant coefficients.

Hennessy and Strebulaev (2016) consider a stylized model with instantaneous investment and quadratic adjustment costs. In their setting, one would always find statistically significant responses to policy variable changes, since there is no region of optimal inaction. Further, in their paper, the focus is on the severe challenges to inference arising when the policy variable can take on more than two states. As they show, in such a setting, it is possible to observe attenuation, overshooting, and sign reversals relative to the underlying comparative statics. In the present paper's binary setting, the problem is always one of economic and statistical attenuation. The present paper considers a simpler policy generating process while tackling a much more complex optimization and pricing problem. But the general message of the papers is the same in that they call for a tighter nexus between models and empirical tests, with greater attention devoted to the policy generating process.

The rest of the paper is organized as follows. Section 2 develops the model. Section 3 presents a standard numerical comparative statics exercise to set a benchmark. Section 4 describes the simulation procedure and analyzes asymmetric tax responses. Section 5 considers statistical power in detail. Section 6 offers concluding remarks. The appendix derives the price of key primitive option claims under regime-shifts.

2 Model

This section develops a model of capital structure decisions in an economy in which corporate income tax rates are stochastic. We build on the standard contingent-claims framework á la Leland (1994) where firms face a corporate income tax, with bankruptcy being costly. Departing from Leland, firms can increase leverage dynamically, with each new bond flotation incurring proportional issuance costs. In these respects, our model follows a rich literature analyzing dynamic financing decisions, e.g. Fischer, Heinkel, and Zechner (1989), Goldstein, Ju, and Leland (2001), and Strebulaev (2007).

2.1 EBIT and tax rate dynamics

Time is continuous and the horizon is infinite. There is a risk-free asset with interest rate $r > 0$. The economy is populated by N firms, each of which has monopolistic access to a project that generates an instantaneous flow of Earnings Before Interest and Taxes (EBIT). Each firm n has an EBIT process evolving as a geometric Brownian motion under the pricing measure, with:

$$\frac{dX_t^n}{X_t^n} = \mu dt + \sigma dZ_t^n.$$

In the preceding equation $X_0^n = 1$, while Z_t^n is a standard Brownian motion defined on a filtered probability space $(\Omega, \mathcal{F}, \mathbb{Q}, (\mathcal{F}_t)_{t\geq 0})$. The parameters μ and σ represent the risk-neutral drift and volatility of EBIT, respectively. For notational simplicity, the superscript n is dropped when obvious.

There is a linear corporate income tax, with the tax rate evolving as a two-state Markov chain. For example, one can think of the model as approximating an economy with a two-party system, e.g. the U.S., where one party favors a relatively higher tax rate, and the other a relatively low tax rate. In reality, the tax advantage to debt varies with complex tax code provisions such as depreciation schedules, tax credits, loss limitations, and alternative minimum taxes. Since we abstract from such complications, one should think of the tax rate as capturing the effective debt tax shield.

There are two tax rate states, $i \in \{l, h\}$. The process $\nu_t \in \{l, h\}$ denotes the current tax state. In state i, the corporate income tax rate is τ_i. It is assumed $0 \leq \tau_l < \tau_h \leq 1$. The tax rate switches from τ_i to τ_j $(j \neq i)$ with instantaneous probability $\lambda_i dt$. The parameter λ_i therefore determines the expected time the tax rate will occupy regime i. Specifically, if the current tax regime is i, then the expected remaining occupation time in that regime is equal to $1/\lambda_i$. The model therefore converges to a constant tax rate model if one were to let each transition rate tend to zero.

In reality, empiricists exert considerable energy trying to identify orthogonal tax rate changes, with the concern being that causal inference will be contaminated if the government changes the tax rate in response to changes in economic conditions. Endogeneity will be ruled out by construction, as we consider that the tax rate process is independent of each EBIT process.

2.2 Unlevered firm value

To understand how stochastic tax rates affect valuation, consider first the valuation of an unlevered firm. The value of unlevered assets at time t, denoted A_t, is equal to the present value of expected future after-tax earnings. This value depends upon current EBIT value, as well as the current tax regime. We have:

$$A(x, i) = \mathbb{E}\left[\int_0^\infty e^{-rs}(1-\tau_i)X_s ds \middle| X_t = x, \nu_t = i\right].$$

This value function must satisfy the following system of ordinary differential equations (ODEs) which demand that at each instant the expected holding return is just equal to the opportunity cost r:

$$\begin{aligned} rA(x,l) &= \mu x A_x(x,l) + \frac{1}{2}\sigma^2 x^2 A_{xx}(x,l) + \lambda_l[A(x,h) - A(x,l)] + (1-\tau_l)x; \\ rA(x,h) &= \mu x A_x(x,h) + \frac{1}{2}\sigma^2 x^2 A_{xx}(x,h) + \lambda_h[A(x,l) - A(x,h)] + (1-\tau_h)x. \end{aligned} \tag{1}$$

The first two terms on the right hand side of both equations reflect expected capital gains due to the instantaneous evolution of EBIT, computed via Ito's lemma. The third term on the right hand side of both equations captures the discrete jump in unlevered asset value that will occur in the event of a tax rate transition. The final term in both equations captures the dividend flow. By solving the preceding system, one obtains an analytic expression of unlevered asset value, with value equal to a regime-contingent constant times x:

$$A(x,i) = \left[\frac{1-\tau_i}{r-\mu} + \frac{\lambda_i(\tau_i - \tau_{\bar{i}})}{(r-\mu)(r-\mu+\lambda_l+\lambda_h)}\right] x. \tag{2}$$

The first term in the preceding equation captures unlevered asset value under a constant tax rate. The second term captures the value adjustment attributable to potential tax rate transitions.

2.3 Dynamic capital structure policy

This subsection considers dynamic capital structure policy. Following standard EBIT-based capital structure models (e.g., Goldstein, Ju, and Leland 2001), EBIT is split between equityholders, debtholders, and the government. More specifically, suppose the firm issues debt promising lenders an instantaneous coupon payment c. Then after paying corporate taxes and lenders, equityholders receive the residual amount $(1-\tau_i)(X_t - c)$ as a dividend. It follows that debt coupons increase the net flow to equity and debt by $\tau_i c$, coming at the expense of the tax collector.

Consider now capital structure dynamics. Suppose the tax state at date zero is i. Then at this point in time equityholders will choose a state-contingent coupon c_i to be written into the first bond issued. In order to ensure that re-levering is done optimally, maximizing total firm value, the debt contract will also specify a pair of mandatory refinancing thresholds (γ_i^l, γ_i^h). Here the subscript indexes the tax state at the time the bond was issued, with the superscripts denoting the two potential tax states post-issuance. For example, if the tax regime at some future date, post-issuance, is equal to j, then the corporation is obligated to call the outstanding bond if EBIT is greater than or equal to γ_i^j. By writing the call thresholds into the bond at the time of issuance, value losses due to debt-equity agency conflicts regarding call timing are avoided, as in Goldstein, Ju and Leland (2001). The contractual call price is par value.[2]

Once it has called its outstanding bond, the firm is free to issue another bond. Refinancing is costly since new bond flotations force the firm to incur issuance costs equal to a fraction q of the proceeds raised. As is well known in the literature, in such a setting the firm will prefer to refinance only if EBIT is sufficiently high to justify incurring the issuance costs.

At all future restructuring dates, the so-called scaling property holds in that each element of the relevant tax-state-contingent debt contract will scale up in proportion to the increase in

[2] Instead of contracting on the call threshold, state-contingent call prices could be set to achieve the same outcome.

EBIT. To understand the mechanics here, for each possible date zero state $i \in \{l, h\}$, let the triple $\Omega_i \equiv (c_i, \gamma_i^l, \gamma_i^h)$ refer to the debt contract that would be optimally chosen by the firm at this point in time. Suppose then the date zero tax state was i, with the current tax state being j. And suppose further that the first upward restructuring threshold has just been hit, with $X_t = \gamma_i^j$. Then the firm will scale up the "baseline" state j debt contract in proportion to the increase in EBIT. That is, the new optimal debt contract will be:

$$\gamma_i^j \Omega_j = (\gamma_i^j c_j, \gamma_i^j \gamma_j^l, \gamma_i^j \gamma_j^h).$$

Of course, it is possible that instead of just reaching the restructuring threshold, through the path-continuous diffusion of the geometric Brownian motion EBIT, there will instead be a jump into restructuring due to a tax rate transition, in which case the debt contract would scale up by $x > \gamma_i^j$, a fact we will account for in the valuations.

Default occurs if EBIT falls below the promised coupon c_i on the current bond outstanding. That is, default results from illiquidity. Liquidity default captures the notion that a financially distressed firm may find it impossible to raise equity externally. An alternative assumption, not adopted here, would be to model default as a stopping problem assuming instead that the firm enjoys frictionless access to external equity injections. Solving the associated stopping time problem would complicate the analysis considerably with little relation to our main argument. In reality, defaults are driven by some combination of illiquidity and insolvency, with the dividing line between the two often difficulty to distinguish.

In the event of default, the firm will be liquidated by the bankruptcy courts, with debtholders receiving unlevered residual value net of liquidation costs. Liquidation costs absorb a fraction α of unlevered asset value.

2.4 Contingent claims

To express the components of levered firm value compactly, we introduce several contingent claims. For a given debt contract $\Omega = (c, \gamma^l, \gamma^h)$, these contingent claims pay off only under specific conditions. In each claim value expression, x denotes the current EBIT value and j denotes the current tax state. We have the following contingent claims.

1. *Contingent Down Claim* with price $d^i(x, j, \Omega)$: This claim pays one if and when default occurs in state i unless it has been knocked out by call or by default in the other tax state j.

2. *Adjusted Contingent Up Claim* with price $m^i(x, j, \Omega)$: This claim pays one if and when call occurs in state i unless it has been knocked out by default or by call in the other tax state j. This claim will be useful in our accounting for cases in which there is a gradual transition into restructuring through the path-continuous diffusion of EBIT.

3. *Contingent Up Claim* with price $u^i(x, j, \Omega)$: This claim pays x/γ^i if and when call occurs in state i unless it has been knocked out by default or by call in the other tax state j. This claim will be useful in our accounting for jumps into restructuring resulting from tax rate transitions.

4. *Contingent Occupation Claim* with price $a^i(x, j, \Omega)$: This claim delivers an instantaneous unit flow of dt whenever the tax state is equal to i unless it has been knocked out by default or call.

In the Appendix we present closed-form expressions for each of these contingent claim prices by solving systems of two simple ODEs.

2.5 Levered firm value

From now on, we fix a pair of debt contracts (Ω_l, Ω_h) and compute levered firm value utilizing the contingent claims defined in the preceding subsection and priced in the Appendix. The levered firm value can be decomposed either into standard real-world financial claims (debt and equity) or into the following four components: (i) unlevered asset value plus (ii) tax benefits less (iii) bankruptcy costs less (iv) issuance costs. Each of the components is defined as the present value of corresponding future cash flows. Initially, we will utilize the latter decomposition of firm value, since equity value can then be readily computed as the residual of total firm value less debt value.

The unlevered asset value has a linear closed-form solution, as shown in equation (2). By definition, this value is independent of the choice of debt contracts (Ω_l, Ω_h). On the other hand, the other three components of firm value depend on (Ω_l, Ω_h) in a complex manner. Conveniently, it is possible to characterize these values analytically by exploiting simple recursive relationships. The next three subsections derive the value of these three components.

2.6 Bankruptcy costs

Let $BC(x, i, \Omega_j)$ denote the present value of bankruptcy costs when EBIT is x, the current tax state is i, and the outstanding debt contract is Ω_j. This valuation considers the costs of any default, not just default on the first bond issued, accounting for the fact that the firm can potentially issue an infinite sequence of bonds in the unlikely event of ever-growing EBIT. At each instant prior to the first default or first restructuring, we have the following analytic expressions for bankruptcy costs:

$$\begin{aligned} BC(x, l, \Omega_i) &= d^l(x, l, \Omega_i)\alpha A(c_i, l) + d^h(x, l, \Omega_i)\alpha A(c_i, h) \\ &\quad + u^l(x, l, \Omega_i)\gamma_i^l BC(1, l, \Omega_l) + u^h(x, l, \Omega_i)\gamma_i^h BC(1, h, \Omega_h); \end{aligned} \tag{3}$$

$$\begin{aligned} BC(x, h, \Omega_i) &= d^l(x, h, \Omega_i)\alpha A(c_i, l) + d^h(x, h, \Omega_i)\alpha A(c, h) \\ &\quad + u^l(x, h, \Omega_i)\gamma_i^l BC(1, l, \Omega_l) + u^h(x, h, \Omega_i)\gamma_i^h BC(1, h, \Omega_h). \end{aligned} \tag{4}$$

In each equation, the first two terms on the right hand side capture the present value of bankruptcy costs under the scenario that default occurs before the first upward levered recapitalization. Because unlevered asset value differs across the two tax states, these cases must be considered separately. The last two terms capture the scaling upward of the present value of bankruptcy costs if the first refinancing occurs prior to a default. The two terms account for the two potential tax states at the time of refinancing.

Conveniently, we can exploit recursive relationships to compute the present value of bankruptcy costs by evaluating the preceding equations at the two possible date-zero configurations of the exogenous state variables. Because the initial EBIT is equal to one and only two potential tax states can occur, bankruptcy costs can be written as

$$\begin{aligned} BC(1, l, \Omega_l) &= d^l(1, l, \Omega_l)\alpha A(c_l, l) + d^h(1, l, \Omega_l)\alpha A(c_l, h) \\ &\quad + u^l(1, l, \Omega_l)\gamma_l^l BC(1, l, \Omega_l) + u^h(1, l, \Omega_l)\gamma_l^h BC(1, h, \Omega_h); \end{aligned} \tag{5}$$

$$\begin{aligned} BC(1, h, \Omega_h) &= d^l(1, h, \Omega_h)\alpha A(c_h, l) + d^h(1, h, \Omega_h)\alpha A(c_h, h) \\ &\quad + u^l(1, h, \Omega_h)\gamma_h^l BC(1, l, \Omega_l) + u^h(1, h, \Omega_h)\gamma_h^h BC(1, h, \Omega_h). \end{aligned} \tag{6}$$

As shown in the Appendix, the two preceding linear equations can be easily solved in order to obtain analytic expressions for the present value of bankruptcy costs evaluated at the initial date. Further, these expressions can then be substituted back into the generic bankruptcy cost equations (3–4) to evaluate bankruptcy costs at each instant prior to the first default or restructuring.

It is clear from the preceding equations that the present value of bankruptcy costs depend on the terms of each tax regime-contingent debt contract (Ω_l, Ω_h). To emphasize this dependence, we denote the date zero bankruptcy costs evaluated at each tax regime by $\{BC_0^l(\Omega_l, \Omega_h), BC_0^h(\Omega_l, \Omega_h)\}$.

2.7 Issuance costs

The present value of future issuance costs is denoted $IC(x, i, \Omega_j)$. This term considers the entire sequence of refinancing possibilities in the future. At each instant prior to the first default or restructuring, we have the following analytic expressions for issuance costs:

$$\begin{aligned} IC(x, l, \Omega_i) \quad = \quad & u^l(x, l, \Omega_i)\gamma_i^l[qD(1, l, \Omega_l) + IC(1, l, \Omega_l)] \\ & + u^h(x, l, \Omega_i)\gamma_i^h[qD(1, h, \Omega_h) + IC(1, h, \Omega_h)]; \end{aligned} \tag{7}$$

$$\begin{aligned} IC(x, h, \Omega_i) \quad = \quad & u^l(x, h, \Omega_i)\gamma_i^l[qD(1, l, \Omega_l) + IC(1, l, \Omega_l)] \\ & + u^h(x, h, \Omega_i)\gamma_i^h[qD(1, h, \Omega_h) + IC(1, h, \Omega_h)]. \end{aligned} \tag{8}$$

In the preceding equation, D denotes the market value of debt, a value function to be derived below. The equation reflects the issuance costs associated with the first potential upward restructuring, as well as the scaling up of the present value of issuance costs in the event of such a restructuring. Because the firm issues different amounts of debt depending on the tax state at the time of refinancing, there are two terms on the right hand sides of each equation. Notice, again we are utilizing the scaling property inherent in the model to evaluate the present value of issuance costs at the moment of refinancing. Finally, it is worth noting that $IC(1, i, \Omega_j)$ excludes the initial issuance costs paid at time zero in connection with the first bond flotation. This cost will be accounted for separately in our date zero firm valuation formulas.

As with bankruptcy costs, we can exploit recursive relationships to compute the present value of issuance costs at date zero by evaluating the two preceding equations at the two possible date-zero configurations of the exogenous state variables, EBIT and tax states. We have:

$$
\begin{aligned}
IC(1,l,\Omega_l) &= u^l(1,l,\Omega_l)\gamma_l^l[qD(1,l,\Omega_l)+IC(1,l,\Omega_l)] \\
&\quad +u^h(1,l,\Omega_l)\gamma_l^h[qD(1,h,\Omega_h)+IC(1,h,\Omega_h)];
\end{aligned}
\tag{9}
$$

$$
\begin{aligned}
IC(1,h,\Omega_h) &= u^l(1,h,\Omega_h)\gamma_h^l[qD(1,l,\Omega_l)+IC(1,l,\Omega_l)] \\
&\quad +u^h(1,h,\Omega_h)\gamma_h^h[qD(1,h,\Omega_h)+IC(1,h,\Omega_h)].
\end{aligned}
\tag{10}
$$

As shown in the Appendix, this pair of linear equations can be solved easily in order to obtain analytic expressions for issuance costs evaluated at the initial date. Further, these expressions can then be substituted back into the generic issuance cost equations (7–8) to compute the present value of issuance costs at each instant prior to the first default or restructuring. Of course, these present values will depend upon the chosen configuration of debt contracts. To emphasize this dependence, we denote the date-zero issuance costs under each tax regime as $\{IC_0^l(\Omega_l,\Omega_h), IC_0^h(\Omega_l,\Omega_h)\}$.

2.8 Tax benefits

The present value of tax benefits is denoted by $TB(x,i,\Omega_j)$. This value accounts for the tax savings generated by the entire sequence of potential debt flotations, not just that associated with the first bond issued. The tax benefits value function satisfies the following system of ODEs:

$$
\begin{aligned}
TB(x,l,\Omega_i) &= \tau_l c_i a^l(x,l,\Omega_i)+\tau_h c_i a^h(x,l,\Omega_i) \\
&\quad +u^l(x,l,\Omega_i)\gamma_i^l TB(1,l,\Omega_l)+u^h(x,l,\Omega_i)\gamma_i^h TB(1,h,\Omega_h);
\end{aligned}
\tag{11}
$$

$$
\begin{aligned}
TB(x,h,\Omega_i) &= \tau_l c_i a^l(x,h,\Omega_i)+\tau_h c_i a^h(x,h,\Omega_i) \\
&\quad +u^l(x,h,\Omega_i)\gamma_i^l TB(1,l,\Omega_l)+u^h(x,h,\Omega_i)\gamma_i^h TB(1,h,\Omega_h).
\end{aligned}
\tag{12}
$$

The first two terms on the right hand side of the preceding equations capture the present value of tax savings generated by the first bond flotation. The two last terms capture the present value of all future tax benefits after a first restructuring. Again, the scaling property inherent in the model is exploited here.

Once again, we can exploit recursive relationships to compute present values at date zero. We must simply evaluate the two preceding equations at the two possible date-zero configurations of the exogenous state variables, EBIT and tax states. We have:

$$\begin{aligned} TB(1,l,\Omega_l) &= \tau_l c_l a^l(1,l,\Omega_l) + \tau_h c_l a^h(1,l,\Omega_l) \\ &\quad + u^l(1,l,\Omega_l)\gamma_l^l TB(1,l,\Omega_l) + u^h(1,l,\Omega_i)\gamma_l^h TB(1,h,\Omega_h); \end{aligned} \tag{13}$$

$$\begin{aligned} TB(1,h,\Omega_h) &= \tau_l c_h a^l(1,h,\Omega_h) + \tau_h c_h a^h(1,h,\Omega_h) \\ &\quad + u^l(1,h,\Omega_h)\gamma_h^l TB(1,l,\Omega_l) + u^h(1,h,\Omega_h)\gamma_h^h TB(1,h,\Omega_h). \end{aligned} \tag{14}$$

As shown in the Appendix, this system of linear equations is easily solved to obtain analytic expressions for tax benefits evaluated at date zero. Further, these expressions can then be substituted back into the generic tax benefit equations (11–12) to evaluate tax benefits at each instant prior to the first default or restructuring. Of course, the present value of tax benefits will hinge upon the chosen configuration of debt contracts. To emphasize this dependence, we denote the date zero tax benefits under each tax regime by $\{TB_0^l(\Omega_l,\Omega_h), TB_0^h(\Omega_l,\Omega_h)\}$.

2.9 Optimal capital structure

At time zero, the firm chooses the optimal capital structure. Once the date zero capital structure is optimized, all future capital structures are also optimized since future debt contracts just scale up the date zero contracts in proportion to the EBIT increase. The optimal capital structure maximizes total firm value and thus solves the following program:

$$\begin{aligned} \Omega_l^* &\in \arg\max_{\Omega_l} \quad TB_0^l(\Omega_l,\Omega_h^*) - BC_0^l(\Omega_l,\Omega_h^*) - IC_0^l(\Omega_l,\Omega_h^*) - qD(1,l,\Omega_l); \\ \Omega_h^* &\in \arg\max_{\Omega_h} \quad TB_0^h(\Omega_l^*,\Omega_h) - BC_0^h(\Omega_l^*,\Omega_h) - IC_0^h(\Omega_l^*,\Omega_h) - qD(1,h,\Omega_h). \end{aligned} \tag{15}$$

It is worth recalling that analytic expressions for tax benefits, bankruptcy costs, and issuance costs were derived in the preceding subsections. Note, the final terms in the preceding equations account for issuance costs incurred on the first bond flotation. At any time after date zero, but prior to the first default or restructuring, total firm value F is computed as

$$F(x,i,\Omega_j) = A(x,i) + TB(x,i,\Omega_j) - BC(x,i,\Omega_j) - IC(x,i,\Omega_j).$$

We next derive debt and equity values. Let $D(x, i, \Omega_j)$ denote the value of a bond that was issued in regime j with the current regime being i. We have:

$$\begin{aligned} D(x, i, \Omega_j) &= [a^l(x, i, \Omega_j) + a^h(x, i, \Omega_j)]c_j + [m^l(x, i, \Omega_j) + m^h(x, i, \Omega_j)]D(1, j, \Omega_j) \\ &\quad +(1-\alpha)[d^l(x, i, \Omega_j)A(c_j, l) + d^h(x, i, \Omega_j)A(c_j, h)]. \end{aligned} \tag{16}$$

The first term in the preceding equation captures the present value of coupon payments. The second term captures the scenario under which the bond is called. The last term captures the liquidation value recovered by lenders if default occurs prior to call.

To pin down the date-zero debt pricing functions, we evaluate the above general debt pricing equation at date zero, so that $j = i$ and $x = 1$. We obtain:

$$D(1, i, \Omega_i) = \frac{[a^l(1, i, \Omega_i) + a^h(1, i, \Omega_i)]c_i + (1-\alpha)[d^l(1, i, \Omega_i)A(c_i, l) + d^h(1, i, \Omega_i)A(c_i, h)]}{1 - m^l(1, i, \Omega_i) - m^h(1, i, \Omega_i)}. \tag{17}$$

The initial debt price can be substituted back into the general debt pricing equation to evaluate debt value at any point in time after issuance. Levered equity value is denoted $E(x, i, \Omega_j)$, and is just equal to the residual of total firm value over and above debt value:

$$E(x, i, \Omega_j) = F(x, i, \Omega_j) - D(x, i, \Omega_j).$$

With all claims priced, we can compute the market leverage ratio L as follows:

$$L(x, i, \Omega_j) \equiv \frac{D(x, i, \Omega_j)}{F(x, i, \Omega_j)}.$$

3 Comparative statics analysis

For public finance economists, the first-order economic question at the heart of our analysis is whether the corporate income tax influences financing decisions. And indeed, in our thought-experiments, which treat the model economy as the true data generating process, the answer to this question is unambiguous. In the posited economy, the tax deductibility of interest expense is *the* cause of all corporate leverage. To see this, note that in the absence of the deductibility of interest expense, each corporation would find it optimal to eschew debt altogether. After all, taking on debt generates positive bankruptcy costs in expectation. Without the tax advantage to debt as a compensating benefit, issuing debt is suboptimal. The key question then is whether empirical tests will confirm what we know to be true in the model economy: taxes matter.

For corporate finance economists, the first-order economic question is whether leverage decisions are explained by the trade-off theory. Here too, in our thought-experiments, which treat the model economy as the true data generating process, the answer to the question is unambiguous. In the model economy, all corporate financing decisions are dictated by trade-off theoretic concerns. Tax advantages to increasing leverage are weighed against bankruptcy costs and issuance costs. The key question then is whether empirical tests will confirm what we know to be true in this economy: trade-off theory explains corporate financing decisions, and taxes matter.

Before going to the data, today's increasingly careful empiricists want to take the underlying theory seriously. And so they will look to the original published versions of models for guidance. Similarly, theorists are today strongly encouraged by editors to furnish empiricists with testable implications that follow from their models. The workhorse methodology for this purpose is comparative statics analysis. Model parameters are varied, with resulting changes in the endogenous variables analyzed. For example, Leland (1994) and Goldstein, Ju and Leland (2001) provide an exhaustive analysis of how optimal leverage changes with the various parameters. The objective of this section is to spell out, within the context of an economy populated by trade-off theoretic firms, the type of testable implications that one would derive by way of numerical comparative statics analysis.

With this in mind, let us now return to the model derived in Section 2. If we treat the tax rate as constant, by letting the transition rates (λ_l, λ_h) go to zero, the model will be nearly identical to that of the trade-off model of Goldstein, Ju, and Leland (2001).[3] This is by now a classical trade-off framework. Our objective is to examine testable implications that would be arrived at by way of comparative statics analysis, with our focus being on the predicted effect of tax rate changes.

Suppose first that we are interested in formulating testable predictions regarding how firms should respond to an increase in the corporate income tax rate from, say, 30% to 40%. Table 1 reports the results of our numerical comparative statics exercise.[4] The table reports the optimal date-zero capital structure of a firm. We focus on three variables: the market leverage ratio L,

[3]One difference is the modeling of default.

[4]Parameter value are discussed in detail below.

debt value D, and interest coverage ratio X_0/c. Of course, given the scaling property inherent in the model, the leverage ratio and coverage ratio shown will also be optimal at each future recapitalization date.

The top row of Table 1 shows the optimal capital structure of a company facing a tax rate equal to 30%. This is the initial capital structure of the firm in the present thought-experiment. Facing this tax rate, the firm optimally chose a leverage ratio of 28.6%, debt value equal to 5.498, and an interest coverage ratio of 3.364. The second row of the table shows the optimal capital structure of a company facing a tax rate of 40%. A firm facing the higher tax rate would adopt a more aggressive financial policy, optimally choosing a higher leverage ratio of 32.7%, a higher debt value of 5.753, and a lower interest coverage ratio of 3.074. Of course, if we were to instead contemplate a tax rate decrease from 40% to 30%, the numerical comparative statics analysis necessarily predicts a symmetric leverage reduction. Starting from an initial leverage ratio of 32.7% at the higher tax rate, optimal leverage falls to 28.6% at the lower tax rate.

In summary, the numerical comparative statics analysis implies a large symmetric tax-sensitivity of leverage. Here the leverage ratio changes by 4.1 percentage points given a 10 percentage point change in the tax rate. The implied mid-point elasticity of leverage is given by:

$$\frac{(.327 - .286)/\frac{1}{2}(.327 + .286)}{(.40 - .30)/\frac{1}{2}(.40 + .30)} = .47.$$

4 Empirical tests using simulated economy data

This section investigates the strengths and weaknesses of state-of-the-art empirical methods for testing whether taxes matter, and whether trade-off theory is a first-order driver of capital structure decisions. Recall, the comparative statics analysis in the preceding section suggested that corporate leverage ratios should be quite responsive to changes in the effective shield value, here the corporate income tax rate, with an implied tax-elasticity of leverage roughly equal to one-half.

We now ask the following question: What would an empiricist, armed with the best empirical methods used for natural experiments, find when the true data generating process is the model itself? To achieve this goal, we first simulate the model to generate panel data on firm leverage ratios and tax rates. Next, we use this data to mimic real-world studies estimating the responsiveness of capital structure to changes in corporate tax rates.

A first-order issue in any real-world empirical study of this type is avoiding the possibility of endogeneity bias. As an example, perhaps the government has decided to lower the corporate tax rate in response to a bad systematic shock? If this is the case, then the observed leverage ratio change is partly attributable to the negative macroeconomic shock, as opposed to tax rate change. Conveniently, the econometrician in our economy enjoys immunity from endogeneity bias since the tax rate process is independent of the EBIT processes. Importantly, this is not to claim that endogeneity bias does not exist or that it is unimportant – it obviously is – but to show that there are other quantitatively significant barriers to correct inference that must be cleared after the first endogeneity hurdle.

As in other similar studies of this kind, the complexity of dynamic effects makes it impossible to derive closed-form expressions for regression coefficients, despite the fact that we have closed-form expressions for all components of firm value. Therefore, we use simulations to generate artificial data from the model, choosing parameter values and simulated experiments to mimic the real-world data as closely as possible.

4.1 Simulation procedure

This subsection describes the simulation procedure. The goal is to generate simulated data that mimics key features of real-world empirical data. Recall, we have already solved from the optimal financial policies of our firms. In particular, we know the firms in our economy will scale up their debt contracts in proportion to the increase in EBIT each time one of the contractually specified call thresholds is crossed, and we know the values of the call thresholds. Thus, simulation simply requires implementing these decision rules for each firm given its respective EBIT path, as well as the path of the corporate income tax rate. There is no need for any optimization at this stage, as optimal policies have already been pinned down.

Of course, in real-world data firms face both idiosyncratic and systematic shocks. We therefore specify an EBIT process for each firm that capture both types of shocks. The EBIT process for an arbitrary firm n evolves as follows:

$$\frac{dX_t^n}{X_t^n} = \mu dt + \sigma_I dZ_t^n + \beta \sigma_S dZ_t^S. \tag{18}$$

In the preceding equation, Z_t^n and Z_t^S are idiosyncratic and systematic shocks, respectively. The parameter β determines the sensitivity of EBIT to systematic shocks. Recall, the sum of two Brownian motions is itself a Brownian motion. We can apply the results from Section 2 by letting

$$\sigma \equiv \sqrt{\sigma_I^2 + \beta^2 \sigma_S^2}. \tag{19}$$

The model determines the optimal capital structure decisions of firms at any point in time, taking into account all the information available up to that point. The optimal decision includes the amount of debt to issue, as well as the timing of refinancing. In particular, if the tax rate changes at any date t, the firm's optimization problem immediately adjusts to take into account the new tax state.

At date zero of the simulation, all firms in the economy are born and choose their optimal capital structure given the tax rate at the time. The initial EBIT of each firm is set to 1. The cross-sectional properties of the economy at date zero are unrealistic because in reality firms refinance at different times and their decisions depend on past realizations of firm-specific shocks. In addition, the capital structure distribution at any point in time is the outcome of past changes in tax rates. Thus, the first step in our simulation is the construction of a stationary cross-sectional distribution of capital structures.

Conveniently, in the model, the leverage ratio can be uniquely derived from EBIT. A firm actively adjusts its capital structure if and only if its EBIT crosses one of the optimally-set tax-specific refinancing thresholds. Therefore, to generate a stationary distribution of leverage ratios, it is sufficient to generate a stationary distribution of EBITs.

To this end, we discretize the continuous-time model and set each period length equal to one quarter. We then simulate quarterly data for 1000 treated firms (and 1000 control firms when performing difference-in-differences estimation). Each simulation consists of two steps. First, to achieve the stationary distribution, we simulate 200 quarters and then drop these initial observations.[5] Next, we continue the simulation until the arrival of a tax rate change that fits the relevant event inclusion criteria, as defined in detail below. We refer to the resulting panel data set as one "simulated economy" or just "economy." It is worth stressing the each economy simply represents one draw of a tax change meeting the stated inclusion criteria. We repeat this procedure 1000 times

[5] Our analysis suggests that dropping the initial 200 quarters is more than sufficient for achieving the stationary distribution. The EBIT distribution is invariant after about 50 quarters.

to study the sampling distribution of test statistics for 1000 stationary economies experiencing the relevant tax rate change.

The simulation procedure runs as follows. In any period, each firm observes the random shock hitting its respective EBIT over the last quarter as well as any changes in the tax rate. To determine whether a tax rate change occurs in the quarter, we draw an independent binomial random variable with transition probability λ_i, where i is the tax state at the beginning of the quarter. If the tax rate switches, the firm's refinancing threshold switches accordingly. And here we recall that the refinancing threshold at each point in time depends on the tax rate at that point in time as well as on the tax rate at the time the outstanding bond was issued.

To simulate the EBIT, we discretize its dynamics as follows:

$$dX^n_{t+\Delta t} = X^n_t \left\{ (\mu + \nu)\Delta t + \sigma_I \Delta Z^n_t + \beta \sigma_S \Delta Z^S_t \right\},$$

and draw $(\{\Delta Z^n_t\}^N_{n=1}, \Delta Z^S_t)$ from independent normal distributions with mean zero and variance Δt. Because the actual stochastic process occurs under the natural distribution, we adjust the instantaneous drift term μ by adding a risk premium ν. If EBIT does not cross any boundary (the default threshold or the state-specific refinancing threshold), the firm takes no action. If EBIT crosses a refinancing threshold, we reset EBIT to 1 at the beginning of next quarter, with the debt contract reset to the state-contingent optimum. Here it is worth noting that we are not interested in the size distribution of firms. And the scalability property inherent in the model ensures that at each refinancing we can simply re-set EBIT to 1 with no contamination of financial ratios. If EBIT reaches the outstanding debt coupon, the firm is liquidated, EBIT is re-set, and the new shareholders re-optimize its capital structure. However, in the analysis of simulated tax reforms, simulated firms that default during the sample window are removed, in order to mimic the real-world data.

An important concern for empiricists is the possibility of closely overlapping events. For example, if a second tax rate change occurs soon after the first change, firm responses to the first reform are difficult to gauge. Empirical studies therefore routinely exclude such contaminated events. We mimic such restrictions in the course of our simulations. In particular, in order for a tax reform to meet our inclusion criteria, there can be no other tax reform within $\pm$ 12quarters. Therefore, we continue a given simulation run until the occurrence of the first tax reform (of the desired sign) meeting these criteria.

4.2 Parameterization

This section describes the choice of parameters. In the interest of transparency, we choose parameter values that are standard in the literature, rather than doing our own moment-matching exercise.

The bankruptcy cost parameter α is set at 0.2. This value is consistent with the findings of Davydenko, Strebulaev and Zhao (2012), who estimate the change in the market value of firm assets in default to be 21.7%. The extant literature on dynamic trade-off models generally assumes α is between 0.05 and 0.30.

The proportional debt issuance cost q is set at 0.01. Kim, Palia, and Saunders (2009), Fischer, Heinkel, and Zechner (1989) and Goldstein, Ju, and Leland (2001) define issuance costs in the same way and utilize a value of 1%. Datta, Iskandar-Datta, and Patel (1997) report total expenses of new debt issuance from 1976 to 1992 equal to 2.96%. Kim, Palia, and Saunders (2003), in a study of underwriting spreads over a 30-year period, arrive at an estimate of 1.15%.

The risk-neutral drift of EBIT, μ, is set at 0.01. The risk premium ν is set at 0.0645. These values are consistent with Strebulaev (2007). Thus, we assume the long-term nominal growth rate of firm payouts and correspondingly returns to be 0.0745. The instantaneous volatility of firm EBIT, σ, is set at 0.2, with the volatility of the idiosyncratic shock, σ_I, set at 0.14. This is consistent with a number of prior studies using volatility values between 0.15 and 0.30. Empirical estimates suggest that the asset volatility distribution can be quite diverse, ranging from 0.05 to more than 0.5 per annum. The constant risk-free rate, r, is assumed to be 0.05.

Two parameters of particular interest are (λ_l, λ_h). Recall, the expected duration of tax regime i is simply λ_i^{-1}. To reduce the number of cases to consider, we assume transition rates are equal across the two states, setting $\lambda_l = \lambda_h = \lambda$. To shed light on the importance of tax regime duration, different values of λ will be considered. For the base case, we will assume $\lambda = .10$, implying an expected regime duration of one decade. By post-war standards this is a conservative assumption given that the effective tax shield value has changed much more frequently due to frequent tax reforms and the effects of inflation.

4.3 Optimal policies at refinancing points

Before analyzing in detail our simulated natural experiments, this subsection discusses the key mechanisms that drive firm financial decisions. Table 2 compares the optimal values of control variables, those written into the date zero debt contract, across alternative environments. In particular, the table contrasts the optimal debt contract in the case of constant tax rates with optimal debt contract terms when tax rates vary stochastically, considering optimal terms for a ten year expected tax regime duration ($\lambda = 0.1$), as well as a one year expected tax regime duration ($\lambda = 1$).

In all cases, one sees that the optimal coupon for debt issued in the high tax state, c_h, is greater than the optimal coupon for debt issued in the low tax state, c_l. Intuitively, the firm is willing to incur greater bankruptcy risk when the flow of debt tax shields is higher. Notice also that the difference between optimal coupons across the low and the high tax states grows larger as the likelihood of a tax rate change falls (lower λ). Intuitively, if the current tax regime is expected to last longer, the firm issues more debt in the high tax regime and less debt in the low tax regime. In fact, in the limit as λ tends to zero, the firm acts as if it will face a constant tax rate forever. Conversely, in the limit, as tax reforms become extremely frequent, the gap between the optimal coupons across the low and high tax rate states vanishes, with the firm acting as if it faces the average of the two tax rates.

To understand firm behavior, it is also critical to understand the optimal refinancing thresholds. To begin, suppose the outstanding debt was issued in the low tax state, and consider the effect of a change in the tax rate from low to high. As shown in Table 2, the optimal refinancing threshold will jump down from γ_l^l to $\gamma_l^h < \gamma_l^l$ implying either an immediate jump into restructuring or a hastening of restructuring. Intuitively, if the tax rate increases, the firm becomes increasingly willing to incur issuance costs in order to capture the higher flow of tax shields. Recall, there is no agency conflict between debt and equity, since lenders and shareholders choose the debt terms to maximize firm value. Rather, the hastening of restructuring after a tax rate increase is due to the negative externality a lower interest coverage ratio imposes on the government. Continuing with the present scenario, comparing across λ values we see that as the expected duration of the tax regime increases, the refinancing threshold for the high rate state (γ_l^h) decreases. The firm becomes more willing to incur new issuance costs if the higher flow of tax savings is more stable. Conversely, as the expected duration of the tax regime increases, the refinancing threshold for the low rate state (γ_l^l) increases. The firm becomes less willing to incur new issuance costs given the lower likelihood of a switch to a higher tax rate, where tax shields are particularly valuable.

Suppose next that the outstanding debt was issued in the high tax state, and consider the effect of a change in the tax rate from high to low. As shown in Table 2, the optimal refinancing threshold will jump up from γ_h^h to $\gamma_h^l > \gamma_h^h$ implying a delay in restructuring. Intuitively, if the tax rate decreases, the firm becomes less willing to incur issuance costs. Since there is no agency conflict between debt and equity, the delay in restructuring must be understood as resulting from the positive externality a lower coupon rate, per unit of EBIT, confers on the government. Continuing with the present scenario, comparing across λ values we see that as the expected duration of the tax regime increases, the refinancing threshold for the high rate state (γ_h^h) decreases. Again, the firm becomes more willing to incur new issuance costs if the high flow of tax savings is more stable. Conversely, as the expected duration of the tax regime increases, the refinancing threshold for the low rate state (γ_h^l) increases. Again, the firm is less willing to incur new issuance costs if there is a lower probability of a switch to a higher tax rate.

In summary, in terms of financial control variables, the optimal interest coverage ratio at refinancing points is lower if the tax rate at that point is high. The wedge between tax rate-contingent coverage ratios widens as the tax rate process becomes more persistent. Refinancing thresholds jump downward (upward) if there is the tax rate jumps from low to high (high to low), implying hastened (delayed) recapitalizations. As tax rates become more persistent, the wedge between tax rate-contingent refinancing thresholds widens further, with high persistence encouraging greatly hastened (greatly delayed) recapitalizations after a jump to high (low) tax rates.

Table 3 reports financial metrics at refinancing points under optimal capital structure policies. The top panel considers financial metrics under constant tax rates, and the bottom panel considers these same financial metrics if the expected duration of the tax rate regime is one decade. Consider first leverage ratios. Under constant tax rates, the optimal leverage ratio is 4.2 percentage points higher at the high tax rate than at the low tax rate, a 14.7% difference. However, with stochastic tax rates, the leverage gap is nearly cut in half, to only 2.2 percentage points, representing a wedge of only 7.4%. And note, this halving of the leverage gap occurs here despite the fact that we have considered a conservative decade-long tax regime duration. Consider next the gap between optimal interest coverage ratios across the two tax rates. Under constant tax rates, the optimal coverage ratio is 9.4% higher at the low tax rate than at the high tax rate. However, with stochastic tax rates, the coverage ratio is only 6.6% higher at the low tax rate.

Of course, as pointed out forcefully by Strebulaev (2007), these findings are not directly informative about empirical tests since, when working with real-world panel data, empiricists do not observe snapshots of firms at their refinancing points. Rather, a subset of firms will be at their refinancing points and the remainder will be in the region of optimal inaction. Our simulated empirical tests will certainly take account of this fact. However, the analysis of this subsection does suggest that even a modest degree of policy transience, e.g. decade-long regimes, may considerably weaken the power of empirical tests, as the leverage ratio wedge narrows under stochastic tax rates. Moreover, the analysis suggests that the power of empirical tests will hinge upon the *direction* of tax rate changes, with leverage ratios, coverage ratios, and refinancing activity predicted to be more responsive to tax rate increases.

4.4 Difference-in-differences

The difference-in-differences method (DD) has recently attracted widespread usage. This is unsurprising given that, in their influential textbook on empirical methods, Angrist and Pischke (2009) treat DD as one of the key tools for achieving sharp identification. Our objective here is not to take exception with the DD method itself, but rather to consider its performance in the specific, and important, context of empirical public finance and corporate finance research on the effect/non-effect of taxation on firm financing policies.

In DD, the behavior of treated agents (e.g. firms) is compared with that of non-treated agents. Of course, a key concern is selection bias, so that in practice great care is taken to verify to the extent possible that the treated and non-treated agents are otherwise comparable, aside from the (hopefully) orthogonal treatment shock. For example, in their study of Belgian tax reforms, Panier, Perez-Gonzalez, and Villanueva (2014) compare the capital structure change of treated Belgian firms with the capital structure change of firms in neighboring countries, such as France and the Netherlands. The firms in those countries were not directly affected by the Belgian reform.

The main assumption underlying the DD analysis is that the treated and the non-treated samples do not differ along other important dimensions that could effect the empirical outcome variable, the change in capital structure. For example, if the firms in France and the Netherlands happened to experience changes in their investment opportunity set, while the firms in Belgium did not, the DD analysis would be undermined.

Our simulated model represents an ideal laboratory for assessing the performance of the DD methodology in the context of capital structure research since, as we describe, we can design the treated and non-treated firms so that they are identical in terms of all key risk factors, aggregate shocks, and deep technological parameters. Specifically, for each simulated economy of 1000 firms, we will generate another parallel economy that we call the non-treated economy. The two economies have exactly the same realized path of systematic shocks. There are only two differences between the treated and non-treated economies. First, as in the real-world data, realized firm-specific idiosyncratic EBIT shocks will differ across all firms. Again, coming back to our example of the study by Panier, Perez-Gonzalez, and Villanueva (2014), while ideally the aggregate shocks affecting France and Belgium would be the same, ensuring common average leverage trends, idiosyncratic shocks affecting individual French and Belgian firms will necessarily differ in reality. Second, in the non-treated economy, there is no tax reform within the simulated event observation window.

It is important to stress that, although the non-treated economy does not experience a tax rate change within the studied event window, the tax rate process in this economy has the exactly the same parameterization as the tax rate process for the treated economy. In other words, the two economies face the same probability and magnitude of tax rate changes, but the realized path of tax rate changes differs. Although we have not seen this assumption discussed in empirical work, commonality of the driving process for the policy shocks is likely to be essential for ensuring common trends, and that is certainly the case in our simulated DD analysis.

After constructing the simulated panels, we run standard DD regressions. For example, in order to study the impact of tax rate changes on leverage ratios, we run the regression

$$\Delta L = \beta_0 + \beta_1 \mathbf{1}(\text{Treated}),$$

where $\mathbf{1}(\text{Treated})$ is a dummy variable that equals one for treated firms, with β_1 being the coefficient of interest.

4.5 Asymmetric responses to tax increases and decreases

In this section, we use the simulated data to further examine our conjecture that there will be an asymmetry in empirical tests regarding tax effects on capital structure. In particular, we conjectured above that firms will respond more aggressively to tax rate increases than to tax rate decreases. Formally, such an effect might well be expected in the simulated model data because tax rate increases (decreases) are associated with a jump downwards (upwards) in optimal refinancing thresholds (Table 2), so that refinancing occurs sooner (later). Intuitively, a tax rate increase (decrease) is associated with an increase (decrease) in the optimal ratio of debt coupons to EBIT, which imposes a negative (positive) externality on the tax collector.

We analyze these conjectures using DD estimation. The analysis in this subsection considers that the difference between the low and high tax rates is 10 percentage points, with $\tau_l = 0.3$ and $\tau_h = 0.4$. The observation window is two years (four quarters before the tax reform and 4 quarters after the tax reform). Such an observation window is standard in empirical studies using annual data.

Figures 1 and 2 report histograms of the distribution of DD estimates from 1000 simulated tax reforms featuring tax rate increases, as well as 1000 simulated tax reforms featuring tax rate decreases. Figure 1 treats the change in the number of refinancing events as the dependent variable. Figure 2 treats the change in the interest coverage ratio as the dependent variable. As both figures show, there is substantially more clustering of DD estimates at zero effect in the case of tax rate decreases. For example, in terms of the change in the number of refinancing events, nearly 40% of the DD estimations analyzing tax rate decreases find zero effect. In contrast, less than 10% of the DD estimations analyzing tax rate increases find zero effect. Similarly, as shown in Figure 2, in terms of the change in interest coverage ratios, more than 35% of the DD estimations analyzing tax rate decreases find zero effect. In contrast, roughly 8% of the DD estimations analyzing tax rate increases find zero effect.

Table 4 reports the average response of the simulated firms to changes in tax rates, focusing on interest coverage ratios and the percentage of firms refinancing. A number of observations stand out. First, tax rate increases are associated with increases in refinancing activity. In contrast, tax rate decreases are associated with a decline in refinancing activity relative to the non-treated control group. After both types of tax rate changes, firms will find themselves some distance from what they would find optimal conditional upon restructuring. However, there is an asymmetry in the gain the firm captures from re-setting the interest coverage ratio upward versus downward. After a tax rate increase (decrease), the firm would like to re-set the coverage ratio upward (downward), imposing a negative (positive) externality on the tax collector, which encourages (discourages) the payment of new issuance costs.

Table 5 considers instead the behavior of market leverage ratios across the same tax rate change. A potential concern for empiricists examining leverage changes is that effects might be mechanical. After all, even if firms did not actively change their outstanding debt, a tax rate increase (decrease) would mechanically cause the market leverage ratio to increase (decrease) since such a rate change would cause total firm value to fall (rise). With this in mind, the final column of Table 5 reports results from a simulated tax rate change in which we deliberately hold fixed debt balances during the event window–a measure of the passive leverage change (ΔL_{pass}). The penultimate column then computes the active leverage ratio change (ΔL_{act}) as the residual of the actual change net of the passive change. Here we see an asymmetry in that the average upward leverage ratio response to a tax rate increase is 29.1% larger in absolute value than the average leverage ratio decrease in response to a tax rate decrease. The final two columns show that most of this difference is due to active changes in financial policies as opposed to mechanical changes in leverage resulting from changes in the market value of the firm. For example, if we look to the active leverage ratio change, the asymmetry is even larger, equal to 38.2%.

5 An examination of statistical power

In this section, we evaluate the power of statistical tests regarding corporate tax effects using simulated tax reform data. To begin, it is useful to consider historical tax reforms in which the tax change was generally accepted as being large. An important tax reform in this regard was the Tax Reform Act of 1986. TRA86 attempted to reduce distortions by way of a combination of lower tax rates and base broadening, moving the code closer to a pure Haig-Simons income tax. The statutory corporate tax rate fell sharply from 46% to 34%, prompting some superficial observers to expect a large corporate leverage response. However, the change in the effective debt tax shield value was substantially lower in magnitude. In fact, although the statutory corporate tax rate fell by 12 percentage points, the effective tax advantage to debt was judged by economists to have increased. For example, Gordon and MacKie-Mason (1990) argue that, due to all the other provisions in TRA86, the effective tax advantage to debt rose by 2.5 cents per dollar of interest expense. The point to note here is that, in reality given the tradeoffs inherent in any tax reform, such as a general desire to maintain rough revenue neutrality, it is unlikely for an empiricist to have access to data associated with massive changes in the effective tax advantage of debt. Therefore, any examination of statistical power must consider realistic changes in tax shield value.

Whether TRA86 led to an increase or decrease in the effective tax shield value is important given the asymmetry results discussed in the previous subsection. After all, if tax reform leads to an increase in the effective tax shield value, we are more likely to detect it empirically. To be on the conservative side, in other words, to increase the probability of finding significant responses to our simulated tax reforms, this section considers an increase in the corporate income tax rate. Further, we'll consider a large increase in tax shield value of 4 percentage points (relative to 2.5 percentage points under TRA86), from $\tau_l = 0.36$ to $\tau_h = 0.4$. We continue to assume tax regimes are expected to last a decade, setting $\lambda = 0.1$ as our baseline. This too may be considered a conservative assumption given that, at the time of TRA86, greater tax rate transiency might well have been expected given the frequency of tax code changes in the run-up to the legislation.

Figure 3 and Table 6 provide detail on the distribution of DD estimates obtained from our 1000 simulated tax reforms, with the change in leverage ratio serving as the dependent variable here. The change in leverage is calculated as leverage one year after the tax reform less leverage one year before the reform for each firm. To set a benchmark, it is useful to first consider the difference in stationary leverage one would observe across economies with permanent tax rates. This difference is equal to 0.0157. With this in mind, consider the results from the simulated DD instead using a ten year ($\lambda = 0.1$) expected regime duration. Here the mean value of β_1 is only 0.0057, which is only 36.4% of the average stationary leverage difference across economies with permanent tax rates.

Returning to Table 6, we see that if attention is turned to statistical significance, the picture is even gloomier. In 45.3% of the simulated tax reforms studied, one fails to find a statistically significant coefficient at the 5% level of confidence. That is, in almost half the studies, we fail to reject the null that taxes matter for leverage decisions despite the fact that taxes are the cause of corporate leverage in the simulated economy. With an expected regime life of two years, one fails to find a significant coefficient in 78% of the simulated tax reforms analyzed.

The next two columns in Table 6 report the results from analogous DD estimations in which the percentage of firms undertaking refinancing activities, the Refinancing Ratio (RR), is used as the dependent variable. Here the results are even more discouraging in terms of statistical power. Even under a decade-long expected tax regime life, one fails to obtain a statistically significant coefficient in 89.4% of the simulated tax reform experiments. The final two columns report results from analogous DD estimations in which the interest coverage ratio is treated as the dependent variable. Here the results are also discouraging in terms of statistical power. Even under a decade-long expected tax regime life, one fails to obtain a statistically significant coefficient in 48.8% of the simulated tax reform experiments.

What explains these troubling findings? First, given that firms face debt issuance costs, one should only expect a minority of firms to react immediately to changes in the tax advantage of debt. The majority of firms will delay debt market activity until there is a sufficient value gain to justify incurring transactions costs. Second, we have seen that transient tax rates cause firms to be less aggressive at restructuring dates. Finally, in reality one works with finite samples, so that the treated and control groups are not identical. Instead they differ in terms of the realization of idiosyncratic shocks. For some sample paths, the non-treated group will experience relatively large increases in EBIT, causing their leverage to increase at a greater rate, despite the lack of a tax increase.

Obviously, the expected duration of the tax regime will influence the magnitude of regression coefficients and statistical power. In particular, as tax regimes become more transient (λ increases) one expects the DD slope coefficient to fall, along with the probability of rejecting the null of zero tax effect. Consistent with this intuition, Figure 4 plots how the results of the present simulation experiment will change as we vary the λ parameter, focusing on the change in market leverage as the dependent variable. We see that soon after λ exceeds .10, so that expected regime duration falls a bit below one decade, one fails to reject the faulty null of zero tax effect in the majority of simulated tax reform experiments.

6 Conclusions

An important question for public finance and corporate finance researchers is whether and how taxation influences financing decisions. Although this question has received considerable attention from empiricists, the ability to interpret this evidence has been severely hampered by the absence of theoretical models speaking to the empirical tests. In particular, empiricists exploit (hopefully) exogenous variation in tax shield value, while null hypotheses are being informed by comparative statics performed on theoretical models in which the tax rate is never expected to change. In this paper, we developed a theoretical model intended to narrow the gap between model and data, developing a dynamic trade-off theoretic model of capital structure in which firms face stochastic tax rates, with leverage adjustments weighing tax benefits of debt against bankruptcy and adjustment costs. Conveniently, all components of firm value are derived in closed-form, which should facilitate future use of the model in considering various empirical thought-experiments.

To this end, we used the model as a basis for generating null hypotheses and assessing statistical power regarding tax effects in simulated laboratory-type controlled experiments. A key prediction of the model is that leverage should be more responsive to increases in debt tax shield value than to decreases. Problematically, we showed that even in ideal settings, the probability of Type II error regarding tax effects is unacceptably high. In particular, in reasonably parameterized economies, with decade-long expected policy regimes, one fails to reject the incorrect null of zero tax effect in roughly one-half of the simulated difference-in-differences studies.

This lack of statistical power for tax effects stems from: a degree of optimal inaction in the face of transactions costs; firms reacting less to current tax rates given that tax rates are stochastic; and noise due to idiosyncratic shocks hitting firms. While some empiricists have listed these factors as caveats, our quantitative analysis shows that these concerns may well be fatal. Moreover, the commonality of false-falsifications in our simulated experiments suggests that these concerns perhaps deserve more attention than inclusion in a list of boilerplate disclaimers after strong conclusions have been reached nevertheless. Our quantitative results suggest that it is still premature to reach any strong consensus regarding tax effects on capital structure.

7 Appendix: Derivation of Contingent Claim Prices

The purpose of this Appendix is to derive closed-form price expressions for contingent claims introduced in Section 2.4: Contingent Down Claim; Contingent Up Claim; Adjusted Contingent Up Claim; and Contingent Occupation Claim.

7.1 Canonical ODE

As a preparation, we solve the following ODE:

$$\rho v = \mu x v' + \frac{1}{2}\sigma^2 x^2 v'' + zx + Z \tag{20}$$

We know the solution is of the form:

$$v = x^{B_1} K_1 + x^{B_2} K_2 + \frac{zx}{\rho - \mu} + \frac{Z}{\rho} \tag{21}$$

where the exponents are the negative and positive roots of

$$\frac{1}{2}\sigma^2 B^2 + \left(\mu - \frac{1}{2}\sigma^2\right) B - \rho = 0. \tag{22}$$

It will be useful to keep in mind this solution as we seek to price the various contingent claims.

7.2 Canonical ODE System

Moreover, the following system of ODEs will also appear frequently in our valuations:

$$\begin{aligned} rv(x,l) &= \mu x v_x(x,l) + \frac{1}{2}\sigma^2 x^2 v_{xx}(x,l) + \lambda_l[v(x,h) - v(x,l)] \\ rv(x,h) &= \mu x v_x(x,h) + \frac{1}{2}\sigma^2 x^2 v_{xx}(x,h) + \lambda_h[v(x,l) - v(x,h)]. \end{aligned} \tag{23}$$

We conjecture solutions of the form:

$$\begin{aligned} v(x,l) &= Lx^\beta \\ v(x,h) &= Hx^\beta \end{aligned} \tag{24}$$

Substituting these derivatives back into the ODEs one obtains:

$$\begin{aligned} (r+\lambda_l)Lx^\beta &= \mu x \beta L x^{\beta-1} + \frac{1}{2}\sigma^2 x^2(\beta^2 - \beta)Lx^{\beta-2} + \lambda_l H x^\beta \\ (r+\lambda_h)Hx^\beta &= \mu x \beta H x^{\beta-1} + \frac{1}{2}\sigma^2 x^2(\beta^2 - \beta)Hx^{\beta-2} + \lambda_h L x^\beta, \end{aligned} \tag{25}$$

which is equivalent to

$$\begin{aligned}\left[(r+\lambda_l)-\left(\mu-\frac{1}{2}\sigma^2\right)\beta-\frac{1}{2}\sigma^2\beta^2\right]L &= \lambda_l H \\ \left[(r+\lambda_h)-\left(\mu-\frac{1}{2}\sigma^2\right)\beta-\frac{1}{2}\sigma^2\beta^2\right]H &= \lambda_h L\end{aligned} \tag{26}$$

Thus we demand:

$$\left[(r+\lambda_l)-\left(\mu-\frac{1}{2}\sigma^2\right)\beta-\frac{1}{2}\sigma^2\beta^2\right]\left[(r+\lambda_h)-\left(\mu-\frac{1}{2}\sigma^2\right)\beta-\frac{1}{2}\sigma^2\beta^2\right]=\lambda_l\lambda_h. \tag{27}$$

Letting

$$\begin{aligned}g_l(\beta) &\equiv (r+\lambda_l)-\left(\mu-\frac{1}{2}\sigma^2\right)\beta-\frac{1}{2}\sigma^2\beta^2 \\ g_h(\beta) &\equiv (r+\lambda_h)-\left(\mu-\frac{1}{2}\sigma^2\right)\beta-\frac{1}{2}\sigma^2\beta^2.\end{aligned} \tag{28}$$

We then demand that any candidate exponent β must satisfy the following characteristic equation:

$$g_l(\beta)g_h(\beta)=\lambda_l\lambda_h. \tag{29}$$

Thus, the general form of the solution is:

$$\begin{aligned}v(x,l) &= L_1x^{\beta_1}+L_2x^{\beta_2}+L_3x^{\beta_3}+L_4x^{\beta_4} \\ v(x,h) &= H_1x^{\beta_1}+H_2x^{\beta_2}+H_3x^{\beta_3}+H_4x^{\beta_4}\end{aligned} \tag{30}$$

where the β_n are the roots of the characteristic equation, with:

$$\beta_1<\beta_2<0<\beta_3<\beta_4. \tag{31}$$

We know the respective constants are linked via:

$$H_n=\left[\frac{g_l(\beta_n)}{\lambda_l}\right]L_n \tag{32}$$

So the Canonical ODE System has solutions of the form:

$$\begin{aligned}v(x,l) &= L_1x^{\beta_1}+L_2x^{\beta_2}+L_3x^{\beta_3}+L_4x^{\beta_4} \\ v(x,h) &= H_1x^{\beta_1}+H_2x^{\beta_2}+H_3x^{\beta_3}+H_4x^{\beta_4} \\ &= L_1\left[\frac{g_l(\beta_1)}{\lambda_l}\right]x^{\beta_1}+L_2\left[\frac{g_l(\beta_2)}{\lambda_l}\right]x^{\beta_2}+L_3\left[\frac{g_l(\beta_3)}{\lambda_l}\right]x^{\beta_3}+L_4\left[\frac{g_l(\beta_4)}{\lambda_l}\right]x^{\beta_4}.\end{aligned} \tag{33}$$

Or for brevity we can write:

$$v(x,l) = \sum_{n=1}^{4} x^{\beta_n} L_n \tag{34}$$

$$v(x,h) = \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] x^{\beta_n} L_n \tag{35}$$

With these solutions in mind, we can now price each of the contingent claims.

7.3 Contingent Down Claim

This claim pays one if and when default occurs in state i unless it has been knocked out by call or default in the other tax state. Let $d^i(x,j,\Omega)$ denote the price of this claim when EBIT is x and the current tax state is j. By the Feynman-Kac formula, this function must satisfy the following system of ODEs:

$$\begin{aligned} rd^i(x,l,\Omega) &= \mu x d^i_x(x,l,\Omega) + \frac{1}{2}\sigma^2 x^2 d^i_{xx}(x,l,\Omega) + \lambda_l[d^i(x,h,\Omega) - d^i(x,l,\Omega)] \\ rd^i(x,h,\Omega) &= \mu x d^i_x(x,h,\Omega) + \frac{1}{2}\sigma^2 x^2 d^i_{xx}(x,h,\Omega) + \lambda_h[d^i(x,l,\Omega) - d^i(x,h,\Omega)] \\ d^i(x,i,\Omega) &= 1 \quad \text{if } x \in (0,c] \\ d^i(x,j,\Omega) &= 0 \quad \text{if } x \in (0,c] \\ d^i(x,l,\Omega) &= 0 \quad \text{if } x \in [\gamma^l,\infty) \\ d^i(x,h,\Omega) &= 0 \quad \text{if } x \in [\gamma^h,\infty). \end{aligned} \tag{36}$$

To solve this system, we need to consider three cases separately depending on the ranking between γ^l and γ^h.

- Case 1: $\gamma^l > \gamma^h$.

  First, we assume that the refinancing threshold is higher in the low tax state. On the region of $[c, \gamma^h]$, equation (36) reduces to a Canonical ODE System with a solution

$$\begin{aligned} d^i(x,l) &= \sum_{n=1}^{4} x^{\beta_n} L_n \\ d^i(x,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] x^{\beta_n} L_n. \end{aligned} \tag{37}$$

On the other hand, when $x \in [\gamma^h, \gamma^l]$, equation (36) reduces to a Canonical ODE:

$$
\begin{aligned}
d^i(x,h) &= 0 \\
(r+\lambda^l)\widetilde{d^i}(x,l) &= \mu x \widetilde{d^i_x}(x,l) + \frac{1}{2}\sigma^2 x^2 \widetilde{d^i_{xx}}(x,l) \\
&\Rightarrow \widetilde{d^i}(x,l) = x^{B_1}K_1 + x^{B_2}K_2
\end{aligned} \tag{38}
$$

The boundary conditions give us six linear equations in the six constants, $(L_1, L_2, L_3, L_4, K_1, K_2)$:

$$
\begin{aligned}
d^i(c,l) &= \sum_{n=1}^{4} c^{\beta_n} L_n = \Phi(i=l) \\
d^i(c,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] c^{\beta_n} L_n = \Phi(i=h) \\
d^i(\gamma^h,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] (\gamma^h)^{\beta_n} L_n = 0 \\
\widetilde{d^i}(\gamma^l,l) &= (\gamma^l)^{B_1}K_1 + (\gamma^l)^{B_2}K_2 = 0 \\
d^i(\gamma^h,l) &= \widetilde{d^i}(\gamma^h,l) \Rightarrow \sum_{n=1}^{4} (\gamma^h)^{\beta_n} L_n = (\gamma^h)^{B_1}K_1 + (\gamma^h)^{B_2}K_2 \\
d^i_x(\gamma^h,l) &= \widetilde{d^i_x}(\gamma^h,l) \Rightarrow \sum_{n=1}^{4} \beta_n (\gamma^h)^{\beta_n - 1} L_n = B_1(\gamma^h)^{B_1-1}K_1 + B_2(\gamma^h)^{B_2-1}K_2
\end{aligned}
$$

Here $\Phi(i=l)$ and $\Phi(i=h)$ are indicator variables. By solving these linear equations, we can pin down the constants and then compute a contingent down claim price.

- Case 2: $\gamma^h > \gamma^l$.

Next, we consider the case where the refinancing threshold is higher in the high tax state. On the region of $[c, \gamma^l]$, equation (36) again reduces to the Canonical ODE System with the solution of form

$$
\begin{aligned}
d^i(x,l) &= \sum_{n=1}^{4} x^{\beta_n} L_n \\
d^i(x,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] x^{\beta_n} L_n.
\end{aligned} \tag{39}
$$

On the other hand, when $x \in [\gamma^l, \gamma^h]$, equation (36) reduces to the Canonical ODE:

$$\begin{aligned} d^i(x,l) &= 0 \\ (r+\lambda^h)\widetilde{d}^i(x,h) &= \mu x \widetilde{d}^i_x(x,h) + \frac{1}{2}\sigma^2 x^2 \widetilde{d}^i_{xx}(x,h) \\ &\Rightarrow \widetilde{d}^i(x,h) = x^{B_1}K_1 + x^{B_2}K_2. \end{aligned} \tag{40}$$

The boundary conditions give us six linear equations in the six constants, $(L_1, L_2, L_3, L_4, K_1, K_2)$:

$$\begin{aligned} d^i(c,l) &= \sum_{n=1}^{4} c^{\beta_n} L_n = \Phi(i=l) \\ d^i(c,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] c^{\beta_n} L_n = \Phi(i=h) \\ d^i(\gamma^l,l) &= \sum_{n=1}^{4} (\gamma^l)^{\beta_n} L_n = 0 \\ \widetilde{d}^i(\gamma^h,h) &= (\gamma^h)^{B_1}K_1 + (\gamma^h)^{B_2}K_2 = 0 \\ d^i(\gamma^l,h) &= \widetilde{d}^i(\gamma^l,h) \Rightarrow \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] (\gamma^l)^{\beta_n} L_n = (\gamma^l)^{B_1}K_1 + (\gamma^l)^{B_2}K_2 \\ d^i_x(\gamma^l,h) &= \widetilde{d}^i_x(\gamma^l,h) \Rightarrow \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] \beta_n (\gamma^l)^{\beta_n - 1} L_n = B_1(\gamma^l)^{B_1-1}K_1 + B_2(\gamma^l)^{B_2-1}K_2 \end{aligned}$$

By solving these linear equations, we can pin down the constants and then compute contingent down claim prices.

- Case 3: $\gamma^h = \gamma^l = \gamma$.

In this case, equation (36) is simply the Canonical ODE System has a solution of the form:

$$\begin{aligned} d^i(x,l) &= \sum_{n=1}^{4} x^{\beta_n} L_n \\ d^i(x,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] x^{\beta_n} L_n. \end{aligned} \tag{41}$$

The boundary conditions give us four linear equations in the four constants, (L_1, L_2, L_3, L_4):

$$\begin{aligned}
d^i(c,l) &= \sum_{n=1}^{4} c^{\beta_n} L_n = \Phi(i=l) \\
d^i(c,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] c^{\beta_n} L_n = \Phi(i=h) \\
d^i(\gamma,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] (\gamma)^{\beta_n} L_n = 0 \\
d^i(\gamma,l) &= \sum_{n=1}^{4} (\gamma)^{\beta_n} L_n = 0.
\end{aligned}$$

By solving these linear equations, we can pin down the constants and then compute contingent down claim prices.

7.4 Contingent Up Claim

This claim pays x/γ^i if and when call occurs under tax regime i unless it has been knocked out by default or call in the other tax state j. Let $u^i(x, j, \Omega)$ denote the price of this claim when EBIT is x and the current tax state is j. By the Feynman-Kac formula, this function must satisfy the following system of ODEs:

$$\begin{aligned}
ru^i(x,l,\Omega) &= \mu x u^i_x(x,l,\Omega) + \frac{1}{2}\sigma^2 x^2 u^i_{xx}(x,l,\Omega) + \lambda_l[u^i(x,h,\Omega) - u^i(x,l,\Omega)] \qquad (42) \\
ru^i(x,h,\Omega) &= \mu x u^i_x(x,h,\Omega) + \frac{1}{2}\sigma^2 x^2 u^i_{xx}(x,h,\Omega) + \lambda_h[u^i(x,l,\Omega) - u^i(x,h,\Omega)] \\
u^i(x,i,\Omega) &= 0 \qquad \text{if } x \in (0, c^i] \\
u^i(x,j,\Omega) &= 0 \qquad \text{if } x \in (0, c^j] \\
u^i(x,i,\Omega) &= \frac{x}{\gamma^i} \qquad \text{if } x \in [\gamma^i, \infty) \\
u^i(x,j,\Omega) &= 0 \qquad \text{if } x \in [\gamma^j, \infty).
\end{aligned}$$

Similarly, we consider two cases separately with different payoff states.

7.4.1 Contingent Up Claim: l-State Payoff

- Case 1: $\gamma^l > \gamma^h$.

On the region of $[c, \gamma^h]$, equation (42) reduces to a Canonical ODE System with a solution

$$\begin{aligned} u^l(x,l) &= \sum_{n=1}^{4} x^{\beta_n} L_n \\ u^l(x,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] x^{\beta_n} L_n. \end{aligned} \tag{43}$$

When $x \in [\gamma^h, \gamma^l]$, the claim is worth 0 in the high tax rate state and so the Canonical ODE System Reduces to a Canonical ODE

$$\begin{aligned} u^l(x,h) &= 0 \\ (r+\lambda_l)\widetilde{u}^l(x,l) &= \mu x \widetilde{u}^l_x(x,l) + \frac{1}{2}\sigma^2 x^2 \widetilde{u}^l_{xx}(x,l) \\ &\Rightarrow \widetilde{u}^l(x,l) = x^{B_1} K_1 + x^{B_2} K_2. \end{aligned}$$

The boundary conditions give us six linear equations in the six constants, $(L_1, L_2, L_3, L_4, K_1, K_2)$:

$$\begin{aligned} u^l(c,l) &= \sum_{n=1}^{4} c^{\beta_n} L_n = 0 \\ u^l(c,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] c^{\beta_n} L_n = 0 \\ u^l(\gamma^h,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] (\gamma^h)^{\beta_n} L_n = 0 \\ \widetilde{u}^l(\gamma^l,l) &= (\gamma^l)^{B_1} K_1 + (\gamma^l)^{B_2} K_2 = 1 \\ u^l(\gamma^h,l) &= \widetilde{u}^l(\gamma^h,l) \Rightarrow \sum_{n=1}^{4} (\gamma^h)^{\beta_n} L_n = (\gamma^h)^{B_1} K_1 + (\gamma^h)^{B_2} K_2 \\ u^l_x(\gamma^h,l) &= \widetilde{u}^l_x(\gamma^h,l) \Rightarrow \sum_{n=1}^{4} \beta_n (\gamma^h)^{\beta_n - 1} L_n = B_1 (\gamma^h)^{B_1 - 1} K_1 + B_2 (\gamma^h)^{B_2 - 1} K_2 \end{aligned}$$

By solving these linear equations, we can pin down the constants and compute the contingent up claim price.

- Case 2: $\gamma^h > \gamma^l$.

On the region of $[c, \gamma^h]$, equation (42) reduces to a Canonical ODE System with a solution of the form

$$u^l(x,l) = \sum_{n=1}^{4} x^{\beta_n} L_n \tag{44}$$

$$u^l(x,h) = \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] x^{\beta_n} L_n.$$

When $x \in [\gamma^l, \gamma^h]$, the claim is worth x/γ^l in the low tax rate state and so the Canonical ODE System Reduces to a Canonical ODE

$$\begin{aligned}
u^l(x,l) &= \frac{x}{\gamma^l} \\
(r+\lambda_h)\widetilde{u}^l(x,h) &= \mu x \widetilde{u}^l_x(x,h) + \frac{1}{2}\sigma^2 x^2 \widetilde{u}^l_{xx}(x,h) + \frac{\lambda_h x}{\gamma^l} \\
&\Rightarrow \widetilde{u}^l(x,h) = x^{B_1}K_1 + x^{B_2}K_2 + \frac{\lambda_h x}{\gamma^l(r+\lambda_h-\mu)}.
\end{aligned}$$

The boundary conditions give us six linear equations in the six constants $(L_1, L_2, L_3, L_4, K_1, K_2)$:

$$\begin{aligned}
u^l(c,l) &= \sum_{n=1}^{4} c^{\beta_n} L_n = 0 \\
u^l(c,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] c^{\beta_n} L_n = 0 \\
u^l(\gamma^l,l) &= \sum_{n=1}^{4} (\gamma^l)^{\beta_n} L_n = 1 \\
\widetilde{u}^l(\gamma^h,h) &= (\gamma^h)^{B_1}K_1 + (\gamma^h)^{B_2}K_2 + \frac{\lambda_h \gamma^h}{\gamma^l(r+\lambda_h-\mu)} = 0 \\
u^l(\gamma^l,h) &= \widetilde{u}^l(\gamma^l,h) \Rightarrow \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] (\gamma^l)^{\beta_n} L_n = (\gamma^l)^{B_1}K_1 + (\gamma^l)^{B_2}K_2 + \frac{\lambda_h}{r+\lambda_h-\mu} \\
u^h_x(\gamma^l,h) &= \widetilde{u}^h_x(\gamma^l,h) \Rightarrow \sum_{n=1}^{4} \beta_n \left[\frac{g_l(\beta_n)}{\lambda_l}\right] (\gamma^l)^{\beta_n-1} L_n = B_1(\gamma^l)^{B_1-1}K_1 + B_2(\gamma^l)^{B_2-1}K_2 + \frac{\lambda_h}{\gamma^l(r+\lambda_h-\mu)}
\end{aligned}$$

By solving these linear equations, we can pin down the constants and compute the contingent up claim price.

- Case 3: $\gamma^h = \gamma^l = \gamma$.

In this case, equation (42) is simply a Canonical ODE System with a solution of the form

$$\begin{aligned} u^l(x,l) &= \sum_{n=1}^{4} x^{\beta_n} L_n \\ u^l(x,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] x^{\beta_n} L_n. \end{aligned} \tag{45}$$

The boundary conditions give us four linear equations in the four constants, (L_1, L_2, L_3, L_4):

$$\begin{aligned} u^l(c,l) &= \sum_{n=1}^{4} c^{\beta_n} L_n = 0 \\ u^l(c,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] c^{\beta_n} L_n = 0 \\ u^l(\gamma,l) &= \sum_{n=1}^{4} (\gamma^l)^{\beta_n} L_n = 1 \\ u^l(\gamma,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] c^{\beta_n} L_n = 0 \end{aligned}$$

By solving these linear equations, we can pin down the constants and compute the contingent up claim price.

7.4.2 Contingent Up Claim: h-State Payoff

- Case 1: $\gamma^l > \gamma^h$.

  On the region of $[c, \gamma^h]$, equation (42) reduces to a Canonical ODE System with a solution of the form

$$\begin{aligned} u^h(x,l) &= \sum_{n=1}^{4} x^{\beta_n} L_n \\ u^h(x,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] x^{\beta_n} L_n. \end{aligned} \tag{46}$$

  When $x \in [\gamma^h, \gamma^l]$, the claim is worth x/γ^h in the high tax rate state and so the Canonical

ODE System Reduces to a Canonical ODE

$$\begin{aligned} u^h(x,h) &= \frac{x}{\gamma^h} \\ (r+\lambda_l)\widetilde{u}^h(x,l) &= \mu x \widetilde{u}^h_x(x,l) + \frac{1}{2}\sigma^2 x^2 \widetilde{u}^h_{xx}(x,l) + \frac{\lambda_l x}{\gamma^h} \\ &\Rightarrow \widetilde{u}^h(x,l) = x^{B_1}K_1 + x^{B_2}K_2 + \frac{\lambda_l x}{\gamma^h(r+\lambda_l-\mu)}. \end{aligned}$$

The boundary conditions give us six linear equations in the six constants, $(L_1, L_2, L_3, L_4, K_1, K_2)$:

$$\begin{aligned} u^h(c,l) &= \sum_{n=1}^{4} c^{\beta_n} L_n = 0 \\ u^h(c,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] c^{\beta_n} L_n = 0 \\ u^h(\gamma^h,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] (\gamma^h)^{\beta_n} L_n = 1 \\ \widetilde{u}^h(\gamma^l,l) &= (\gamma^l)^{B_1}K_1 + (\gamma^l)^{B_2}K_2 + \frac{\lambda_l\gamma^l}{\gamma^h(r+\lambda_l-\mu)} = 0 \\ u^h(\gamma^h,l) &= \widetilde{u}^h(\gamma^h,l) \Rightarrow \sum_{n=1}^{4}(\gamma^h)^{\beta_n} L_n = (\gamma^h)^{B_1}K_1 + (\gamma^h)^{B_2}K_2 + \frac{\lambda_l}{r+\lambda_l-\mu} \\ u^h_x(\gamma^h,l) &= \widetilde{u}^h_x(\gamma^h,l) \Rightarrow \sum_{n=1}^{4}\beta_n(\gamma^h)^{\beta_n-1} L_n = B_1(\gamma^h)^{B_1-1}K_1 + B_2(\gamma^h)^{B_2-1}K_2 + \frac{\lambda_l}{\gamma^h(r+\lambda_l-\mu)} \end{aligned}$$

By solving these linear equations, we can pin down the constants and compute the contingent up claim price.

- Case 2: $\gamma^h > \gamma^l$.

On the region of $[c, \gamma^l]$, equation (42) reduces to a Canonical ODE System with a solution of the form

$$\begin{aligned} u^h(x,l) &= \sum_{n=1}^{4} x^{\beta_n} L_n \qquad (47) \\ u^h(x,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] x^{\beta_n} L_n. \end{aligned}$$

When $x \in [\gamma^l, \gamma^h]$, the claim is worth 0 in the low tax rate state and so the Canonical ODE

System Reduces to a Canonical ODE

$$\begin{aligned} u^h(x,l) &= 0 \\ (r+\lambda_h)\widetilde{u}^h(x,h) &= \mu x \widetilde{u}^h_x(x,h) + \frac{1}{2}\sigma^2 x^2 \widetilde{u}^h_{xx}(x,h) \\ &\Rightarrow \widetilde{u}^h(x,h) = x^{B_1}K_1 + x^{B_2}K_2. \end{aligned}$$

The boundary conditions give us six linear equations in the six constants, $(L_1, L_2, L_3, L_4, K_1, K_2)$:

$$\begin{aligned} u^h(c,l) &= \sum_{n=1}^{4} c^{\beta_n} L_n = 0 \\ u^h(c,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] c^{\beta_n} L_n = 0 \\ u^h(\gamma^l,l) &= \sum_{n=1}^{4} (\gamma^l)^{\beta_n} L_n = 0 \\ \widetilde{u}^h(\gamma^h,h) &= (\gamma^h)^{B_1}K_1 + (\gamma^h)^{B_2}K_2 = 1 \\ u^h(\gamma^l,h) &= \widetilde{u}^h(\gamma^l,h) \Rightarrow \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] (\gamma^l)^{\beta_n} L_n = (\gamma^l)^{B_1}K_1 + (\gamma^l)^{B_2}K_2 \\ u^h_x(\gamma^l,h) &= \widetilde{u}^h_x(\gamma^l,h) \Rightarrow \sum_{n=1}^{4} \beta_n \left[\frac{g_l(\beta_n)}{\lambda_l}\right] (\gamma^l)^{\beta_n-1} L_n = B_1(\gamma^l)^{B_1-1}K_1 + B_2(\gamma^l)^{B_2-1}K_2 \end{aligned}$$

By solving these linear equations, we can pin down the constants and then compute contingent up claim prices.

- Case 3: $\gamma^h = \gamma^l = \gamma$.

In this case, equation (42) is simply a Canonical ODE System with a solution of the form

$$\begin{aligned} u^h(x,l) &= \sum_{n=1}^{4} x^{\beta_n} L_n \qquad (48) \\ u^h(x,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] x^{\beta_n} L_n. \end{aligned}$$

The boundary conditions give us four linear equations in the four constants, (L_1, L_2, L_3, L_4):

$$\begin{aligned}
u^h(c,l) &= \sum_{n=1}^{4} c^{\beta_n} L_n = 0 \\
u^h(c,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] c^{\beta_n} L_n = 0 \\
u^h(\gamma,l) &= \sum_{n=1}^{4} (\gamma^l)^{\beta_n} L_n = 0 \\
u^h(\gamma,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] c^{\beta_n} L_n = 1
\end{aligned}$$

By solving these linear equations, we can pin down the constants and compute the contingent up claim price.

7.5 Adjusted Contingent Up Claim

This claim pays one if and when call occurs under tax regime i unless it has been knocked out by default or upward restructuring in the other tax state j. Let $m^i(x, j, \Omega)$ denote the price of this claim when EBIT is x and the current tax state is j. By the Feynman-Kac formula, this function must satisfy the following system of ODEs:

$$\begin{aligned}
rm^i(x,l,\Omega) &= \mu x m^i_x(x,l,\Omega) + \frac{1}{2}\sigma^2 x^2 m^i_{xx}(x,l,\Omega) + \lambda_l[m^i(x,h,\Omega) - m^i(x,l,\Omega)] \qquad (49) \\
rm^i(x,h,\Omega) &= \mu x m^i_x(x,h,\Omega) + \frac{1}{2}\sigma^2 x^2 m^i_{xx}(x,h,\Omega) + \lambda_h[m^i(x,l,\Omega) - m^i(x,h,\Omega)] \\
m^i(x,i,\Omega) &= 0 \qquad \text{if } x \in (0,c] \\
m^i(x,j,\Omega) &= 0 \qquad \text{if } x \in (0,c] \\
m^i(x,i,\Omega) &= 1 \qquad \text{if } x \in [\gamma^i,\infty) \\
m^i(x,j,\Omega) &= 0 \qquad \text{if } x \in [\gamma^j,\infty).
\end{aligned}$$

In the following, we derive a price expression for each payoff state $(i = l, h)$.

7.5.1 Adjusted Contingent Up Claim: l-State Payoff

- Case 1: $\gamma^l > \gamma^h$.

On the region of $[c, \gamma^h]$: equation (49) reduces to a Canonical ODE System with a solution

$$
\begin{aligned}
m^l(x,l) &= \sum_{n=1}^{4} x^{\beta_n} L_n \qquad (50) \\
m^l(x,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] x^{\beta_n} L_n.
\end{aligned}
$$

When $x \in [\gamma^h, \gamma^l]$, the claim is worth 0 in the high tax rate state and so the Canonical ODE System Reduces to a Canonical ODE

$$
\begin{aligned}
m^l(x,h) &= 0 \\
(r+\lambda_l)\widetilde{m}^l(x,l) &= \mu x \widetilde{m}^l_x(x,l) + \frac{1}{2}\sigma^2 x^2 \widetilde{m}^l_{xx}(x,l) \\
&\Rightarrow \widetilde{m}^l(x,l) = x^{B_1} K_1 + x^{B_2} K_2.
\end{aligned}
$$

The boundary conditions give us six linear equations in the six constants, $(L_1, L_2, L_3, L_4, K_1, K_2)$:

$$
\begin{aligned}
m^l(c,l) &= \sum_{n=1}^{4} c^{\beta_n} L_n = 0 \\
m^l(c,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] c^{\beta_n} L_n = 0 \\
m^l(\gamma^h,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] (\gamma^h)^{\beta_n} L_n = 0 \\
\widetilde{m}^l(\gamma^l,l) &= (\gamma^l)^{B_1} K_1 + (\gamma^l)^{B_2} K_2 = 1 \\
m^l(\gamma^h,l) &= \widetilde{m}^l(\gamma^h,l) \Rightarrow \sum_{n=1}^{4} (\gamma^h)^{\beta_n} L_n = (\gamma^h)^{B_1} K_1 + (\gamma^h)^{B_2} K_2 \\
m^l_x(\gamma^h,l) &= \widetilde{m}^l_x(\gamma^h,l) \Rightarrow \sum_{n=1}^{4} \beta_n (\gamma^h)^{\beta_n - 1} L_n = B_1 (\gamma^h)^{B_1-1} K_1 + B_2 (\gamma^h)^{B_2-1} K_2
\end{aligned}
$$

By solving these linear equations, we can pin down the constants and compute the adjusted contingent up claim price.

- Case 2: $\gamma^h > \gamma^l$.

On the region of $[c, \gamma^l]$, equation (49) reduces to a Canonical ODE System with a solution

$$\begin{aligned} m^l(x,l) &= \sum_{n=1}^{4} x^{\beta_n} L_n \\ m^l(x,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] x^{\beta_n} L_n. \end{aligned} \tag{51}$$

When $x \in [\gamma^l, \gamma^h]$, the claim is worth one in the low tax rate state and so the Canonical ODE System Reduces to a Canonical ODE

$$\begin{aligned} \widetilde{m}^l(x,l) &= 1 \\ (r+\lambda_h)\widetilde{m}^l(x,h) &= \mu x \widetilde{m}^l_x(x,h) + \frac{1}{2}\sigma^2 x^2 \widetilde{m}^l_{xx}(x,h) + \lambda_h \\ &\Rightarrow \widetilde{m}^l(x,h) = x^{B_1}K_1 + x^{B_2}K_2 + \frac{\lambda_h}{r+\lambda_h}. \end{aligned}$$

The boundary conditions give us six linear equations in the six constants, $(L_1, L_2, L_3, L_4, K_1, K_2)$:

$$\begin{aligned} m^l(c,l) &= \sum_{n=1}^{4} c^{\beta_n} L_n = 0 \\ m^l(c,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] c^{\beta_n} L_n = 0 \\ \widetilde{m}^l(\gamma^h,h) &= (\gamma^h)^{B_1}K_1 + (\gamma^h)^{B_2}K_2 + \frac{\lambda_h}{r+\lambda_h} = 0 \\ m^l(\gamma^l,l) &= \sum_{n=1}^{4} (\gamma^l)^{\beta_n} L_n = 1 \\ m^l(\gamma^l,h) &= \widetilde{m}^h(\gamma^l,h) \Rightarrow \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] (\gamma^l)^{\beta_n} L_n = (\gamma^l)^{B_1}K_1 + (\gamma^l)^{B_2}K_2 + \frac{\lambda_h}{r+\lambda_h} \\ m^l_x(\gamma^l,h) &= \widetilde{m}^l_x(\gamma^l,h) \Rightarrow \sum_{n=1}^{4} \beta_n \left[\frac{g_l(\beta_n)}{\lambda_l}\right] (\gamma^l)^{\beta_n-1} L_n = B_1(\gamma^l)^{B_1-1}K_1 + B_2(\gamma^l)^{B_2-1}K_2 \end{aligned}$$

By solving these linear equations, we can pin down the constants and compute the adjusted contingent up claim price.

- Case 3: $\gamma^h = \gamma^l = \gamma$.

In this case, equation (49) is simply a Canonical ODE System with a solution of the form:

$$
\begin{aligned}
m^l(x,l) &= \sum_{n=1}^{4} x^{\beta_n} L_n \qquad (52)\\
m^l(x,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] x^{\beta_n} L_n.
\end{aligned}
$$

The boundary conditions give us four linear equations in the four constants, (L_1, L_2, L_3, L_4):

$$
\begin{aligned}
m^l(c,l) &= \sum_{n=1}^{4} c^{\beta_n} L_n = 0\\
m^l(c,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] c^{\beta_n} L_n = 0\\
m^l(\gamma,l) &= \sum_{n=1}^{4} \gamma^{\beta_n} L_n = 1\\
m^l(\gamma,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] \gamma^{\beta_n} L_n = 0
\end{aligned}
$$

By solving these linear equations, we can pin down the constants and compute the adjusted contingent up claim price.

7.5.2 Adjusted Contingent Up Claim: h-State Payoff

We next consider the pricing of an adjusted contingent up claim with payoff state is h.

- Case 1: $\gamma^l > \gamma^h$.

  On the region of $[c, \gamma^h]$, equation (49) reduces to the Canonical ODE System with the solution form

$$
\begin{aligned}
m^h(x,l) &= \sum_{n=1}^{4} x^{\beta_n} L_n \qquad (53)\\
m^h(x,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] x^{\beta_n} L_n.
\end{aligned}
$$

  When $x \in [\gamma^h, \gamma^l]$, equation (49) instead reduces to the Canonical ODE

$$
\begin{aligned}
m^h(x,h) &= 1\\
(r+\lambda_l)\widetilde{m}^h(x,l) &= \mu x \widetilde{m}^h_x(x,l) + \frac{1}{2}\sigma^2 x^2 \widetilde{m}^h_{xx}(x,l) + \lambda_l\\
&\Rightarrow \widetilde{m}^h(x,l) = x^{B_1}K_1 + x^{B_2}K_2 + \frac{\lambda_l}{r+\lambda_l}.
\end{aligned}
$$

The boundary conditions give us six linear equations in the six constants, $(L_1, L_2, L_3, L_4, K_1, K_2)$:

$$\begin{aligned}
m^h(c,l) &= \sum_{n=1}^{4} c^{\beta_n} L_n = 0 \\
m^h(c,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] c^{\beta_n} L_n = 0 \\
m^h(\gamma^h,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] (\gamma^h)^{\beta_n} L_n = 1 \\
\widetilde{m}^h(\gamma^l,l) &= (\gamma^l)^{B_1} K_1 + (\gamma^l)^{B_2} K_2 + \frac{\lambda_l}{r+\lambda_l} = 0 \\
m^h(\gamma^h,l) &= \widetilde{m}^h(\gamma^h,l) \Rightarrow \sum_{n=1}^{4} (\gamma^h)^{\beta_n} L_n = (\gamma^h)^{B_1} K_1 + (\gamma^h)^{B_2} K_2 + \frac{\lambda_l}{r+\lambda_l} \\
m_x^h(\gamma^h,l) &= \widetilde{m}_x^h(\gamma^h,l) \Rightarrow \sum_{n=1}^{4} \beta_n (\gamma^h)^{\beta_n - 1} L_n = B_1 (\gamma^h)^{B_1 - 1} K_1 + B_2 (\gamma^h)^{B_2 - 1} K_2
\end{aligned}$$

By solving these linear equations, we can pin down the constants and compute the adjusted contingent up claim price.

- Case 2: $\gamma^h > \gamma^l$.

On the region of $[c, \gamma^l]$, equation (49) reduces to a Canonical ODE System with a solution

$$\begin{aligned}
m^h(x,l) &= \sum_{n=1}^{4} x^{\beta_n} L_n \\
m^h(x,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] x^{\beta_n} L_n.
\end{aligned} \tag{54}$$

When $x \in [\gamma^l, \gamma^h]$, the claim is worth 0 in the low tax rate state and so the Canonical ODE System Reduces to a Canonical ODE:

$$\begin{aligned}
\widetilde{m}^h(x,l) &= 0 \\
(r+\lambda_h)\widetilde{m}^h(x,h) &= \mu x \widetilde{m}_x^h(x,h) + \frac{1}{2}\sigma^2 x^2 \widetilde{m}_{xx}^h(x,h) \\
&\Rightarrow \widetilde{m}^h(x,h) = x^{B_1} K_1 + x^{B_2} K_2.
\end{aligned}$$

The boundary conditions give us six linear equations in the six constants, $(L_1, L_2, L_3, L_4, K_1, K_2)$:

$$\begin{aligned}
m^h(c,l) &= \sum_{n=1}^{4} c^{\beta_n} L_n = 0 \\
m^h(c,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] c^{\beta_n} L_n = 0 \\
\widetilde{m}^h(\gamma^h,h) &= (\gamma^h)^{B_1} K_1 + (\gamma^h)^{B_2} K_2 = 1 \\
m^h(\gamma^l,l) &= \sum_{n=1}^{4} (\gamma^l)^{\beta_n} L_n = 0 \\
m^h(\gamma^l,h) &= \widetilde{m}^h(\gamma^l,h) \Rightarrow \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] (\gamma^l)^{\beta_n} L_n = (\gamma^l)^{B_1} K_1 + (\gamma^l)^{B_2} K_2 \\
m_x^h(\gamma^l,h) &= \widetilde{m}_x^h(\gamma^l,h) \Rightarrow \sum_{n=1}^{4} \beta_n \left[\frac{g_l(\beta_n)}{\lambda_l}\right] (\gamma^l)^{\beta_n - 1} L_n = B_1 (\gamma^l)^{B_1 - 1} K_1 + B_2 (\gamma^l)^{B_2 - 1} K_2
\end{aligned}$$

By solving these linear equations, we can pin down the constants and compute the adjusted contingent up claim price.

- Case 3: $\gamma^h = \gamma^l = \gamma$.

  In this case, equation (49) is simply the Canonical ODE System with a solution of form:

$$\begin{aligned}
m^h(x,l) &= \sum_{n=1}^{4} x^{\beta_n} L_n \qquad (55) \\
m^h(x,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] x^{\beta_n} L_n .
\end{aligned}$$

  The boundary conditions give us four linear equations in the four constants, (L_1, L_2, L_3, L_4):

$$\begin{aligned}
m^h(c,l) &= \sum_{n=1}^{4} c^{\beta_n} L_n = 0 \\
m^h(c,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] c^{\beta_n} L_n = 0 \\
m^h(\gamma,l) &= \sum_{n=1}^{4} \gamma^{\beta_n} L_n = 0 \\
m^h(\gamma,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] \gamma^{\beta_n} L_n = 1
\end{aligned}$$

  By solving these linear equations, we can pin down the constants and compute the adjusted contingent up claim price.

7.6 Contingent Occupation Claims

This claim delivers an instantaneous unit flow of dt whenever the tax regime is equal to i. The claim is knocked by default or call. Let $a^i(x, j, \Omega)$ denote the price of this claim when EBIT is x and the current tax state is j. By the Feynman-Kac formula, this function must satisfy the following system of ODEs:

$$\begin{aligned}
ra^i(x,i,\Omega) &= 1 + \mu x a^i_x(x,i,\Omega) + \frac{1}{2}\sigma^2 x^2 a^i_{xx}(x,i,\Omega) + \lambda_i[a^i(x,j,\Omega) - a^i(x,i,\Omega)] \qquad (56)\\
ra^i(x,j,\Omega) &= \mu x a^i_x(x,j,\Omega) + \frac{1}{2}\sigma^2 x^2 a^i_{xx}(x,j,\Omega) + \lambda_j[a^i(x,i,\Omega) - a^i(x,j,\Omega)]\\
a^i(x,i,\Omega) &= 0 \qquad \text{if } x \in (0,c]\\
a^i(x,j,\Omega) &= 0 \qquad \text{if } x \in (0,c]\\
a^i(x,i,\Omega) &= 0 \qquad \text{if } x \in [\gamma^i,\infty)\\
a^i(x,j,\Omega) &= 0 \qquad \text{if } x \in [\gamma^j,\infty).
\end{aligned}$$

In the following, we derive a price expression for each payoff state ($i = l, h$).

7.6.1 Contingent Occupation Claims: l-State Payoff

- Case 1: $\gamma^l > \gamma^h$.

  On the region of $[c, \gamma^h]$, equation (56) reduces to a Canonical ODE System with additional constant terms:

$$\begin{aligned}
ra^l(x,l) &= 1 + \mu x a^l_x(x,l) + \frac{1}{2}\sigma^2 x^2 a^l_{xx}(x,l) + \lambda_l[a^l(x,h) - a^l(x,l)] \qquad (57)\\
ra^l(x,h) &= \mu x a^l_x(x,h) + \frac{1}{2}\sigma^2 x^2 a^l_{xx}(x,h) + \lambda_h[a^l(x,l) - a^l(x,h)].
\end{aligned}$$

  We can also find a closed-form solution:

$$\begin{aligned}
a^l(x,l) &= \sum_{n=1}^{4} x^{\beta_n} L_n + \frac{1}{r} - \frac{\lambda_l}{r(r+\lambda_l+\lambda_h)} \qquad (58)\\
a^l(x,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] x^{\beta_n} L_n + \frac{\lambda_h}{r(r+\lambda_l+\lambda_h)}.
\end{aligned}$$

When $x \in [\gamma^h, \gamma^l]$, equation (56) reduces to a Canonical ODE:

$$\begin{aligned}
a^l(x,h) &= 0 \\
(r+\lambda_l)\widetilde{a}^l(x,l) &= 1 + \mu x \widetilde{a}^l_x(x,l) + \frac{1}{2}\sigma^2 x^2 \widetilde{a}^l_{xx}(x,l) \\
\widetilde{a}^l(x,l) &= x^{B_1}K_1 + x^{B_2}K_2 + \frac{1}{r+\lambda_l}
\end{aligned}$$

The boundary conditions give us six linear equations in the six constants $(L_1, L_2, L_3, L_4, K_1, K_2)$:

$$\begin{aligned}
a^l(c,l) &= \sum_{n=1}^{4} c^{\beta_n} L_n + \frac{1}{r} - \frac{\lambda_l}{r(r+\lambda_l+\lambda_h)} = 0 \qquad (59) \\
a^l(c,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] c^{\beta_n} L_n + \frac{\lambda_h}{r(r+\lambda_l+\lambda_h)} = 0 \\
a^l(\gamma^h,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] (\gamma^h)^{\beta_n} L_n + \frac{\lambda_h}{r(r+\lambda_l+\lambda_h)} = 0 \\
\widetilde{a}^l(\gamma^l,l) &= (\gamma^l)^{B_1}K_1 + (\gamma^l)^{B_2}K_2 + \frac{1}{r+\lambda_l} = 0 \\
a^l(\gamma^h,l) &= \widetilde{a}^l(\gamma^h,l) \Rightarrow \sum_{n=1}^{4} (\gamma^h)^{\beta_n} L_n + \frac{1}{r} - \frac{\lambda_l}{r(r+\lambda_l+\lambda_h)} = (\gamma^h)^{B_1}K_1 + (\gamma^h)^{B_2}K_2 + \frac{1}{r+\lambda_l} \\
a^l_x(\gamma^h,l) &= \widetilde{a}^l_x(\gamma^h,l) \Rightarrow \sum_{n=1}^{4} \beta_n (\gamma^h)^{\beta_n - 1} L_n = B_1(\gamma^h)^{B_1-1}K_1 + B_2(\gamma^h)^{B_2-1}K_2
\end{aligned}$$

By solving these linear equations, we can pin down the constants and then compute contingent occupation claim prices.

- Case 2: $\gamma^h > \gamma^l$

We have the following differential equations that must be satisfied in the interval (c, γ^l):

$$\begin{aligned}
ra^l(x,l) &= 1 + \mu x a^l_x(x,l) + \frac{1}{2}\sigma^2 x^2 a^l_{xx}(x,l) + \lambda_l[a^l(x,h) - a^l(x,l)] \qquad (60) \\
ra^l(x,h) &= \mu x a^l_x(x,h) + \frac{1}{2}\sigma^2 x^2 a^l_{xx}(x,h) + \lambda_h[a^l(x,l) - a^l(x,h)].
\end{aligned}$$

The solution is:

$$\begin{aligned}
a^l(x,l) &= \sum_{n=1}^{4} x^{\beta_n} L_n + \frac{1}{r} - \frac{\lambda_l}{r(r+\lambda_l+\lambda_h)} \qquad (61) \\
a^l(x,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] x^{\beta_n} L_n + \frac{\lambda_h}{r(r+\lambda_l+\lambda_h)}
\end{aligned}$$

And on the interval $[\gamma^l, \gamma^h]$ we have:

$$\begin{aligned} a^l(x,l) &= 0 \\ (r+\lambda_h)\widetilde{a}^l(x,h) &= \mu x \widetilde{a}^l_x(x,h) + \frac{1}{2}\sigma^2 x^2 \widetilde{a}^l_{xx}(x,h) \\ \widetilde{a}^l(x,h) &= x^{B_1}K_1 + x^{B_2}K_2 \end{aligned}$$

The boundary conditions give us six linear equations in the six constants $(L_1, L_2, L_3, L_4, K_1, K_2)$:

$$\begin{aligned} a^l(c,l) &= \sum_{n=1}^{4} c^{\beta_n} L_n + \frac{1}{r} - \frac{\lambda_l}{r(r+\lambda_l+\lambda_h)} = 0 \qquad (62) \\ a^l(c,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] c^{\beta_n} L_n + \frac{\lambda_h}{r(r+\lambda_l+\lambda_h)} = 0 \\ a^l(\gamma^l,l) &= \sum_{n=1}^{4} (\gamma^l)^{\beta_n} L_n + \frac{1}{r} - \frac{\lambda_l}{r(r+\lambda_l+\lambda_h)} = 0 \\ \widetilde{a}^l(\gamma^h,h) &= (\gamma^h)^{B_1}K_1 + (\gamma^h)^{B_2}K_2 = 0 \\ a^l(\gamma^l,h) &= \widetilde{a}^l(\gamma^l,h) \Rightarrow \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] (\gamma^l)^{\beta_n} L_n + \frac{\lambda_h}{r(r+\lambda_l+\lambda_h)} = (\gamma^l)^{B_1}K_1 + (\gamma^l)^{B_2}K_2 \\ a^l_x(\gamma^l,h) &= \widetilde{a}^l_x(\gamma^l,h) \Rightarrow \sum_{n=1}^{4} \beta_n \left[\frac{g_l(\beta_n)}{\lambda_l}\right] (\gamma^l)^{\beta_n-1} L_n = B_1(\gamma^l)^{B_1-1}K_1 + B_2(\gamma^l)^{B_2-1}K_2 \end{aligned}$$

By solving these linear equations, we can pin down the constants and then compute contingent occupation claim prices.

- Case 3: $\gamma^h = \gamma^l = \gamma$

We have the following differential equations that must be satisfied in the interval (c, γ):

$$\begin{aligned} ra^l(x,l) &= 1 + \mu x a^l_x(x,l) + \frac{1}{2}\sigma^2 x^2 a^l_{xx}(x,l) + \lambda_l[a^l(x,h) - a^l(x,l)] \qquad (63) \\ ra^l(x,h) &= \mu x a^l_x(x,h) + \frac{1}{2}\sigma^2 x^2 a^l_{xx}(x,h) + \lambda_h[a^l(x,l) - a^l(x,h)]. \end{aligned}$$

The solution is:

$$\begin{aligned} a^l(x,l) &= \sum_{n=1}^{4} x^{\beta_n} L_n + \frac{1}{r} - \frac{\lambda_l}{r(r+\lambda_l+\lambda_h)} \qquad (64) \\ a^l(x,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] x^{\beta_n} L_n + \frac{\lambda_h}{r(r+\lambda_l+\lambda_h)} \end{aligned}$$

The boundary conditions give us four linear equations in the four constants, (L_1, L_2, L_3, L_4):

$$\begin{aligned} a^l(c,l) &= \sum_{n=1}^{4} c^{\beta_n} L_n + \frac{1}{r} - \frac{\lambda_l}{r(r+\lambda_l+\lambda_h)} = 0 \\ a^l(c,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] c^{\beta_n} L_n + \frac{\lambda_h}{r(r+\lambda_l+\lambda_h)} = 0 \\ a^l(\gamma,l) &= \sum_{n=1}^{4} \gamma^{\beta_n} L_n + \frac{1}{r} - \frac{\lambda_l}{r(r+\lambda_l+\lambda_h)} = 0 \\ a^l(\gamma,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] \gamma^{\beta_n} L_n + \frac{\lambda_h}{r(r+\lambda_l+\lambda_h)} = 0 \end{aligned} \tag{65}$$

By solving these linear equations, we can pin down the constants and then compute contingent occupation claim prices.

7.6.2 Contingent Occupation Claims: h-State Payoff

- Case 1: $\gamma^l > \gamma^h$

  We have the following differential equations that must be satisfied in the interval (c, γ^h):

$$\begin{aligned} ra^h(x,l) &= \mu x a^h_x(x,l) + \frac{1}{2}\sigma^2 x^2 a^h_{xx}(x,l) + \lambda_l[a^h(x,h) - a^h(x,l)] \\ ra^h(x,h) &= 1 + \mu x a^h_x(x,h) + \frac{1}{2}\sigma^2 x^2 a^h_{xx}(x,h) + \lambda_h[a^h(x,l) - a^h(x,h)]. \end{aligned} \tag{66}$$

  The solution is:

$$\begin{aligned} a^h(x,l) &= \sum_{n=1}^{4} x^{\beta_n} L_n + \frac{\lambda_l}{r(r+\lambda_l+\lambda_h)} \\ a^h(x,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] x^{\beta_n} L_n + \frac{1}{r} - \frac{\lambda_h}{r(r+\lambda_l+\lambda_h)}. \end{aligned} \tag{67}$$

  And on the interval $[\gamma^h, \gamma^l]$ we have:

$$\begin{aligned} a^h(x,h) &= 0 \\ (r+\lambda_l)\widetilde{a}^h(x,l) &= \mu x \widetilde{a}^h_x(x,l) + \frac{1}{2}\sigma^2 x^2 \widetilde{a}^h_{xx}(x,l) \\ \widetilde{a}^h(x,l) &= x^{B_1} K_1 + x^{B_2} K_2 \end{aligned}$$

Boundary conditions:

$$
\begin{aligned}
a^h(c,l) &= \sum_{n=1}^{4} c^{\beta_n} L_n + \frac{\lambda_l}{r(r+\lambda_l+\lambda_h)} = 0 && (68)\\
a^h(c,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] c^{\beta_n} L_n + \frac{1}{r} - \frac{\lambda_h}{r(r+\lambda_l+\lambda_h)} = 0\\
a^h(\gamma^h,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] (\gamma^h)^{\beta_n} L_n + \frac{1}{r} - \frac{\lambda_h}{r(r+\lambda_l+\lambda_h)} = 0\\
\widetilde{a}^h(\gamma^l,l) &= (\gamma^l)^{B_1} K_1 + (\gamma^l)^{B_2} K_2 = 0\\
a^h(\gamma^h,l) &= \widetilde{a}^h(\gamma^h,l) \Rightarrow \sum_{n=1}^{4} (\gamma^h)^{\beta_n} L_n + \frac{\lambda_l}{r(r+\lambda_l+\lambda_h)} = (\gamma^h)^{B_1} K_1 + (\gamma^h)^{B_2} K_2\\
a_x^h(\gamma^h,l) &= \widetilde{a}_x^h(\gamma^h,l) \Rightarrow \sum_{n=1}^{4} \beta_n (\gamma^h)^{\beta_n-1} L_n = B_1(\gamma^h)^{B_1-1} K_1 + B_2(\gamma^h)^{B_2-1} K_2
\end{aligned}
$$

- Case 2: $\gamma^h > \gamma^l$

  We have the following differential equations that must be satisfied in the interval (c, γ^l):

$$
\begin{aligned}
ra^h(x,l) &= \mu x a_x^h(x,l) + \frac{1}{2}\sigma^2 x^2 a_{xx}^h(x,l) + \lambda_l[a^h(x,h) - a^h(x,l)] && (69)\\
ra^h(x,h) &= 1 + \mu x a_x^h(x,h) + \frac{1}{2}\sigma^2 x^2 a_{xx}^h(x,h) + \lambda_h[a^h(x,l) - a^h(x,h)].
\end{aligned}
$$

  The solution is:

$$
\begin{aligned}
a^h(x,l) &= \sum_{n=1}^{4} x^{\beta_n} L_n + \frac{\lambda_l}{r(r+\lambda_l+\lambda_h)} && (70)\\
a^h(x,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] x^{\beta_n} L_n + \frac{1}{r} - \frac{\lambda_h}{r(r+\lambda_l+\lambda_h)}.
\end{aligned}
$$

  And on the interval $[\gamma^l, \gamma^h]$ we have:

$$
\begin{aligned}
a^h(x,l) &= 0\\
(r+\lambda_h)\widetilde{a}^h(x,h) &= 1 + \mu x \widetilde{a}_x^h(x,h) + \frac{1}{2}\sigma^2 x^2 \widetilde{a}_{xx}^h(x,h)\\
\widetilde{a}^h(x,h) &= x^{B_1} K_1 + x^{B_2} K_2 + \frac{1}{r+\lambda_h}
\end{aligned}
$$

Boundary conditions:

$$\begin{aligned}
a^h(c,l) &= \sum_{n=1}^{4} c^{\beta_n} L_n + \frac{\lambda_l}{r(r+\lambda_l+\lambda_h)} = 0 \qquad (71)\\
a^h(c,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] c^{\beta_n} L_n + \frac{1}{r} - \frac{\lambda_h}{r(r+\lambda_l+\lambda_h)} = 0\\
a^h(\gamma^l,l) &= \sum_{n=1}^{4} (\gamma^l)^{\beta_n} L_n + \frac{\lambda_l}{r(r+\lambda_l+\lambda_h)} = 0\\
\widetilde{a}^h(\gamma^h,h) &= (\gamma^h)^{B_1} K_1 + (\gamma^h)^{B_2} K_2 + \frac{1}{r+\lambda_h} = 0\\
a^h(\gamma^l,h) &= \widetilde{a}^h(\gamma^l,h) \Rightarrow \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] (\gamma^l)^{\beta_n} L_n + \frac{1}{r} - \frac{\lambda_h}{r(r+\lambda_l+\lambda_h)} = (\gamma^l)^{B_1} K_1 + (\gamma^l)^{B_2} K_2 + \frac{1}{r+\lambda_h}\\
a^h_x(\gamma^l,h) &= \widetilde{a}^h_x(\gamma^l,h) \Rightarrow \sum_{n=1}^{4} \beta_n \left[\frac{g_l(\beta_n)}{\lambda_l}\right] (\gamma^l)^{\beta_n - 1} L_n = B_1(\gamma^l)^{B_1-1} K_1 + B_2(\gamma^l)^{B_2-1} K_2
\end{aligned}$$

- Case 3: $\gamma^h = \gamma^l = \gamma$

  We have the following differential equations that must be satisfied in the interval (c, γ):

$$\begin{aligned}
ra^h(x,l) &= \mu x a^h_x(x,l) + \frac{1}{2}\sigma^2 x^2 a^h_{xx}(x,l) + \lambda_l[a^h(x,h) - a^h(x,l)] \qquad (72)\\
ra^h(x,h) &= 1 + \mu x a^h_x(x,h) + \frac{1}{2}\sigma^2 x^2 a^h_{xx}(x,h) + \lambda_h[a^h(x,l) - a^h(x,h)].
\end{aligned}$$

  The solution is:

$$\begin{aligned}
a^h(x,l) &= \sum_{n=1}^{4} x^{\beta_n} L_n + \frac{\lambda_l}{r(r+\lambda_l+\lambda_h)} \qquad (73)\\
a^h(x,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] x^{\beta_n} L_n + \frac{1}{r} - \frac{\lambda_h}{r(r+\lambda_l+\lambda_h)}.
\end{aligned}$$

  Boundary conditions:

$$\begin{aligned}
a^h(c,l) &= \sum_{n=1}^{4} c^{\beta_n} L_n + \frac{\lambda_l}{r(r+\lambda_l+\lambda_h)} = 0 \qquad (74)\\
a^h(c,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] c^{\beta_n} L_n + \frac{1}{r} - \frac{\lambda_h}{r(r+\lambda_l+\lambda_h)} = 0\\
a^h(\gamma,l) &= \sum_{n=1}^{4} \gamma^{\beta_n} L_n + \frac{\lambda_l}{r(r+\lambda_l+\lambda_h)} = 0\\
a^h(\gamma,h) &= \sum_{n=1}^{4} \left[\frac{g_l(\beta_n)}{\lambda_l}\right] \gamma^{\beta_n} L_n + \frac{1}{r} - \frac{\lambda_h}{r(r+\lambda_l+\lambda_h)} = 0
\end{aligned}$$

7.7 Components of firm value

We here exploit the recursive relationships derived in the body of the paper to pin down the components of firm value. Beginning first with Bankruptcy Costs, we have:

$$BC(1,l,\Omega_l) = \frac{d^l(1,l,\Omega_l)\alpha A(c_l,l) + d^h(1,l,\Omega_l)\alpha A(c_l,h) + u^h(1,l,\Omega_l)\gamma_l^h BC(1,h,\Omega_h)}{1 - u^l(1,l,\Omega_l)\gamma_l^l} \tag{75}$$

$$BC(1,h,\Omega_h) = \frac{d^l(1,h,\Omega_h)\alpha A(c_h,l) + d^h(1,h,\Omega_h)\alpha A(c_h,h) + u^l(1,h,\Omega_h)\gamma_h^l BC(1,l,\Omega_l)}{1 - u^h(1,h,\Omega_h)\gamma_h^h} \tag{76}$$

Consider next Issuance Costs. We have:

$$IC(1,l,\Omega_l) = \frac{u^l(1,l,\Omega_l)\gamma_l^l qD(1,l,\Omega_l) + u^h(1,l,\Omega_l)\gamma_l^h[qD(1,h,\Omega_h) + IC(1,h,\Omega_h)]}{1 - u^l(1,l,\Omega_l)\gamma_l^l} \tag{77}$$

$$IC(1,h,\Omega_h) = \frac{u^h(1,h,\Omega_h)\gamma_h^h qD(1,h,\Omega_h) + u^l(1,h,\Omega_h)\gamma_h^l[qD(1,l,\Omega_l) + IC(1,l,\Omega_l)]}{1 - u^h(1,h,\Omega_h)\gamma_h^h} \tag{78}$$

Consider finally Tax Benefits. We have:

$$TB(1,l,\Omega_l) = \frac{\tau_l c_l a^l(1,l,\Omega_l) + \tau_h c_l a^h(1,l,\Omega_l) + u^h(1,l,\Omega_i)\gamma_l^h TB(1,h,\Omega_h)}{1 - u^l(1,l,\Omega_l)\gamma_l^l} \tag{79}$$

$$TB(1,h,\Omega_h) = \frac{\tau_l c_h a^l(1,h,\Omega_h) + \tau_h c_h a^h(1,h,\Omega_h) + u^l(1,h,\Omega_h)\gamma_h^l TB(1,l,\Omega_l)}{1 - u^h(1,h,\Omega_h)\gamma_h^h} \tag{80}$$

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Table 1: Optimal Capital Structure at Date Zero

	L	D	X_0/c
$\tau = 0.30$	0.286	5.498	3.364
$\tau = 0.40$	0.327	5.753	3.074

This table compares the optimal date zero capital structure for different assumed values of the tax rate parameter in a constant tax rate model. Here L is the market leverage ratio, D is the market value of debt, X is EBIT, and c is the debt coupon.

Table 2: Control Variables

Panel A: $\lambda = 0$

	c_i	γ_i
$i = l$	0.297	1.724
$i = h$	0.325	1.624

Panel B: $\lambda = 0.1$

	c_i	γ_i^l	γ_i^h
$i = l$	0.302	1.724	1.533
$i = h$	0.322	1.837	1.633

Panel C: $\lambda = 1$

	c_i	γ_i^l	γ_i^h
$i = l$	0.311	1.716	1.621
$i = h$	0.315	1.738	1.642

The tables compare the the optimal capital structure policies at date zero. c_i denotes coupon rate when the refinancing state is i, and γ_i^j is the optimal state j refinancing threshold for debt issued in tax state i.

Table 3: Optimal Capital Structure at Date Zero

Panel A: $\lambda = 0$

	L	D	X_0/c
$\tau_L = 30\%$	0.286	5.498	3.364
$\tau_H = 40\%$	0.328	5.753	3.074

Panel B: $\lambda = 0.1$

	L	D	X_0/c
$\tau_L = 30\%$	0.296	5.489	3.313
$\tau_H = 40\%$	0.318	5.802	3.107

This table reports optimal capital structure variables at date zero. Here L is the market leverage ratio, D is the market value of debt, X is EBIT, and c is the debt coupon.

Table 4: Average Response to Tax Rate Changes

	ΔRR	ΔICR
Tax Increase	0.09	-0.19
Tax Decrease	-0.07	0.13

This table reports the average change in the percentage of firms refinancing (RR) and the average change in interest coverage ratio (ICR) for 1000 simulated tax rate changes for 1000 firms with tax rate changes of ten percentage points (30% versus 40%). The observation window is 4 quarters before and 4 quarters after the tax rate change event.

Table 5: Asymmetric Adjustments of Leverage

	ΔL	ΔL_{act}	ΔL_{pass}
Tax Increase (1)	0.0157	0.0113	0.0043
Tax Decrease (2)	-0.0121	-0.0082	-0.0039
-(1)/(2)	129.1%	138.2%	110.1%

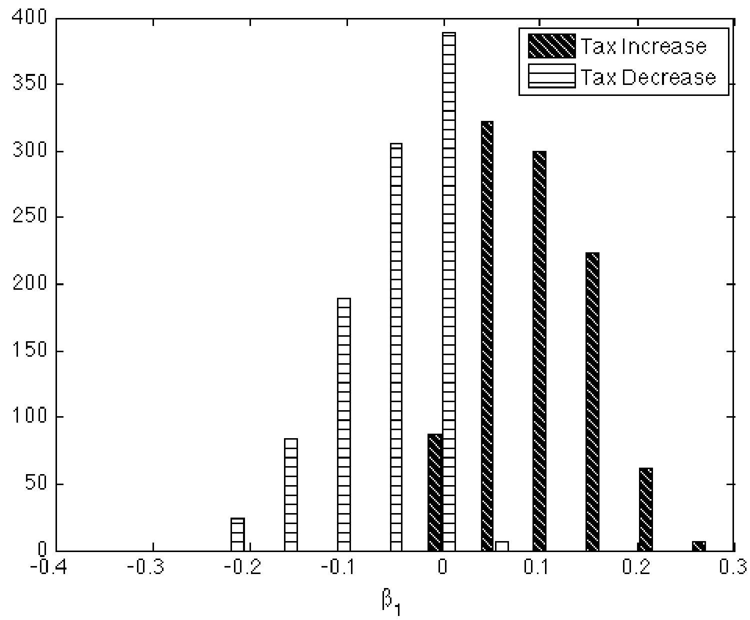
This table reports the average change in the market leverage ratio for 1000 simulated tax rate changes for 1000 firms with tax rate changes of ten percentage points (30% versus 40%). The observation window is 4 quarters before and 4 quarters after the tax rate change event. The final column reports the passive change in the leverage ratio that would occur holding debt fixed but letting the market value of the firm change. The penultimate column reports the active change in the leverage ratio computed by the actual change minus the passive change in the leverage ratio.

Table 6: Properties of Difference in Difference Leverage Regression Coefficient

	Leverage		Refinancing Ratio		Interest Coverage Ratio	
	$\lambda = 0.1$	$\lambda = 0.5$	$\lambda = 0.1$	$\lambda = 0.5$	$\lambda = 0.1$	$\lambda = 0.5$
Mean Beta	0.0057	0.0025	0.0036	0.0021	-0.0595	-0.0310
	[36.4%]	[16.0%]	[82.8%]	[48.7%]	[61.7%]	[32.2%]
Median Beta	0.0057	0.0026	0.0020	0.0010	-0.0539	-0.0281
Standard Deviation of Beta	0.0041	0.0035	0.0112	0.0103	0.0513	0.0391
95% stats significance	54.7%	22.0%	10.6%	8.6%	51.2%	25.3%

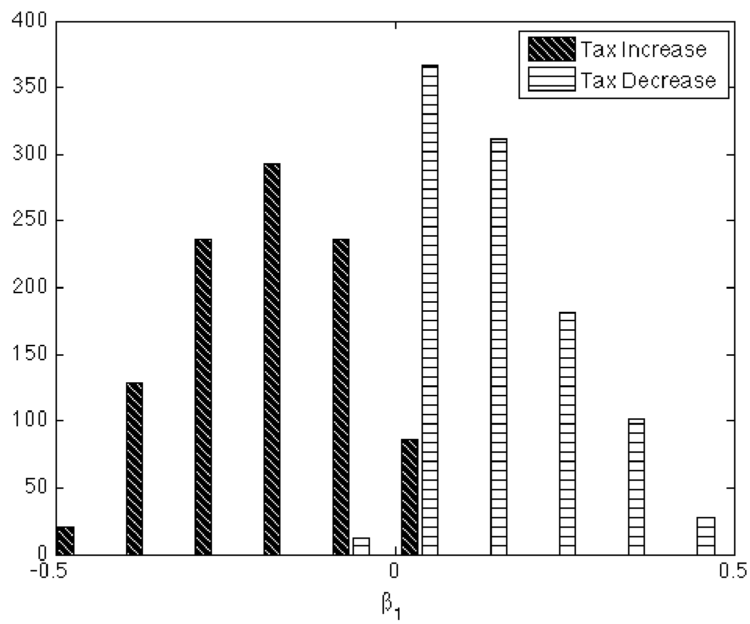
This table reports the properties of the difference in differences slope coefficient in 1000 simulated experiments with tax rate increases from 36% to 40% for 1000 treated and 1000 control firms. The observation window is 4 quarters before and 4 quarters after the tax rate change event. Numbers in brackets show mean beta scaled by differences in the average leverage ratio between the two tax-rate states in the one-state model.

Figure 1: Difference in Differences Coefficient for Number of Refinancing Events



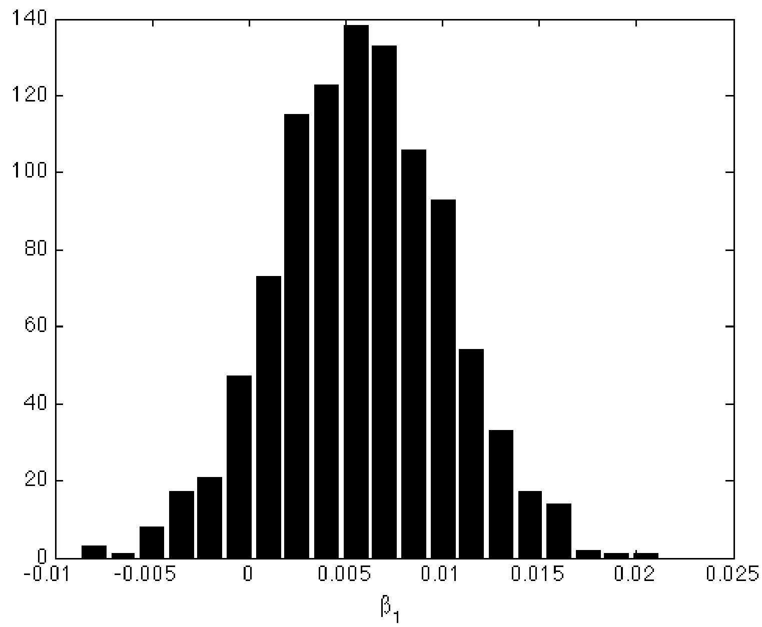
The histogram plots the distribution of slope coefficients from 1000 simulated difference in differences estimations with the change in the number of refinancings regressed on a constant and a dummy variable indicating a tax rate increase or decrease, respectively. The tax rates considered are 30% versus 40%. There are 1000 treated and untreated firms. The observation window is 4 quarters before and 4 quarters after the tax rate change event.

Figure 2: Difference in Differences Coefficient for Interest Coverage Ratios



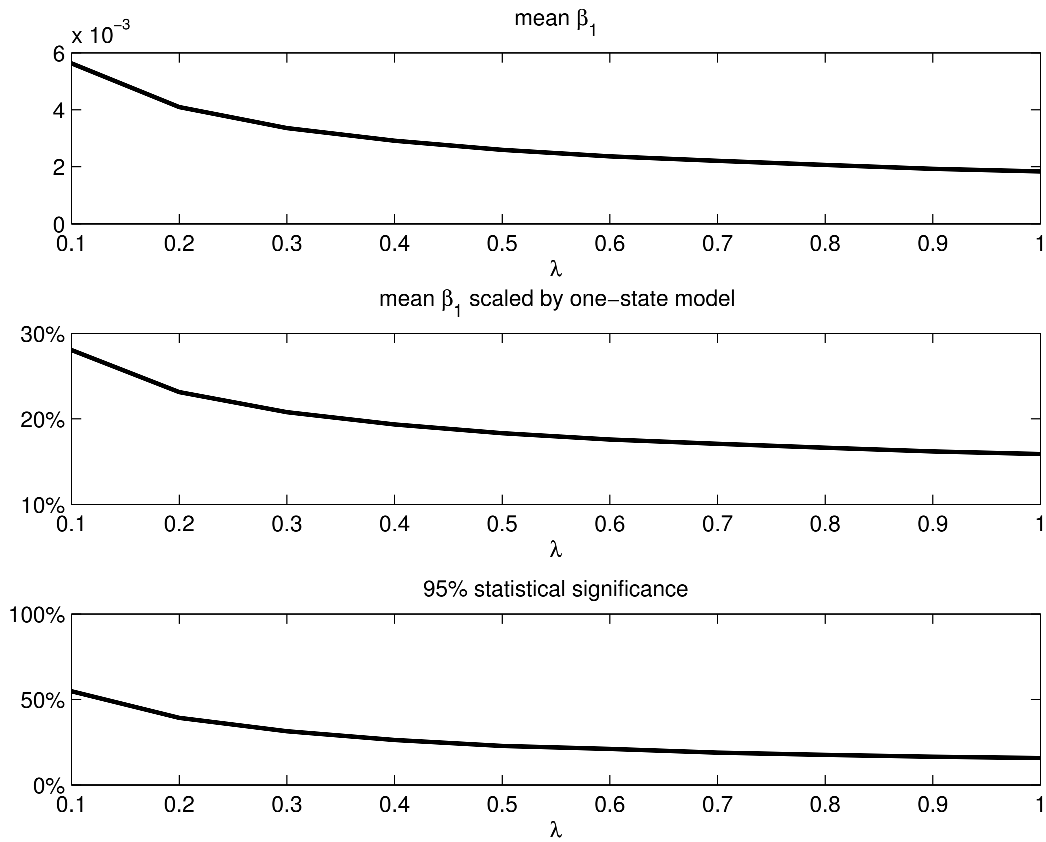
The histogram plots the distribution of slope coefficients from 1000 simulated difference in differences estimations with the change in the interest coverage ratio regressed on a constant and a dummy variable indicating a tax rate increase or decrease, respectively. The tax rates considered are 30% versus 40%. There are 1000 treated and untreated firms. The observation window is 4 quarters before and 4 quarters after the tax rate change event.

Figure 3: Difference in Differences Coefficient for Leverage Ratio



The histogram reports the distribution of the difference in differences slope coefficient in 1000 simulated experiments with tax rate increases from 36% to 40% for 1000 treated and 1000 control firms. The observation window is 4 quarters before and 4 quarters after the tax rate change event.

Figure 4: Leverage Response: DiD Estimates with Different λ after a Tax Increase



This figure plots average DiD estimates of leverage responses to a tax increase with different λ. The second panel shows mean beta scaled by differences in the average leverage ratio between the two tax-rate states in the one-state model.